

Summer Workshop on the Reaction Theory Exercise sheet 4

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To be discussed on Thursday of Week-I.

Classwork

4.1 Dalitz plot representations

A representation of the relativistic kinematics of three hadrons which originate from the decay of one mother particle were suggested by Dalitz. Let us consider decay $0 \rightarrow 123$, e.g., $K^+ \rightarrow \pi^+\pi^+\pi^-$. Here and below we denote four momenta of the daughter particles as p_i , $i = 0, 1, 2, 3$; m_i , E_i are corresponding masses and energies in the rest frame of the mother particle.

Two dimensional representation of the decay kinematics is called Dalitz plot. There are two commonly exploited representations of the plot

- “Triangle” representation (Mandelstam representation). Kinetic energies were used to parametrize kinematics. $T_i = E_i - m_i$, $i = 1, 2, 3$.
- Cartesian representation. The dynamics is shown in Mandelstam variables $s_{i_1} = (p_{i_2} + p_{i_1})^2$, where i_1, i_2, i_3 are numbers 1, 2, 3 or possible circular permutations.

In the examples which we consider, a density function drawn in the Dalitz plot is double differential partial width distribution $d^2\Gamma/(dT_1dT_3)$ for the “triangle” representation and $d^2\Gamma/(ds_1ds_3)$ for the Cartesian representation.

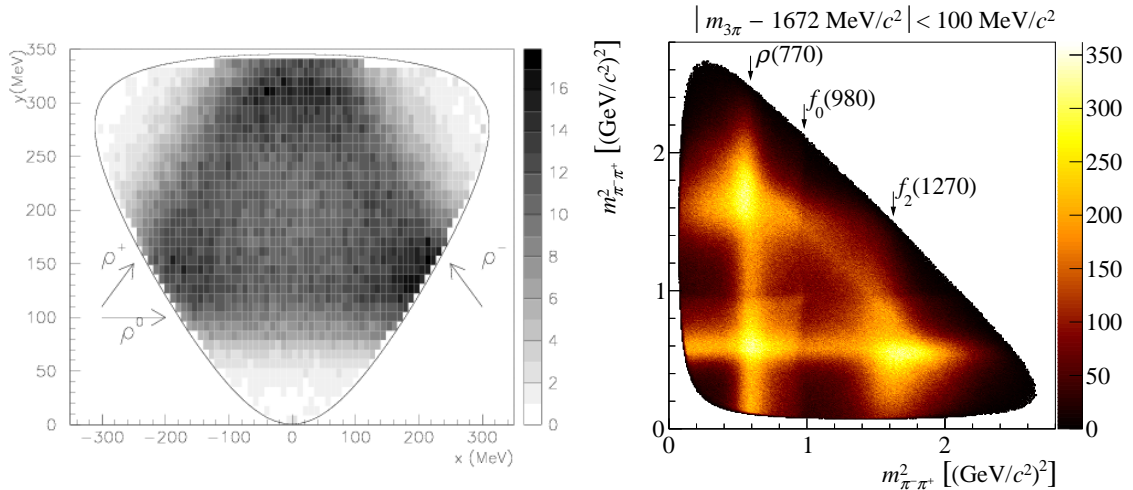


Figure 1: Two examples of Dalitz plot in different representations. The plot on the left is the Dalitz plot in the triangle representation for decay $\phi \rightarrow \pi^+\pi^-\pi^0$ from KLOE experiment [PLB 561 (2003) 55-60], the plot on right is the Dalitz plot in the Cartesian representation for the decay $\pi_2^- \rightarrow \pi^-\pi^-\pi^+$ from COMPASS experiment [PRD 95 (2017) 032004]

The key feature of the Dalitz plot is that in absent of dynamics the distribution is constant. Any deviation from the flat distribution suggests a presence of non-trivial physics in the decay.

- (a) Counting degrees of freedom and conservation laws for the decay process, show that the amplitude of the process depends on only two variables.

The three-body phase space integral has nine integrals and four delta-functions therefore five variables remain.

$$d\Phi_3 = \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3} (2\pi)^4 \delta^4(p_0 - p_1 - p_2 - p_3).$$

How does it much to two degrees of freedom which we found above?

- (b) Obtain the expression for the differential width.

$$\frac{d\Gamma}{ds_1 ds_3} = \frac{|\mathcal{M}|^2}{256\pi^3 M^3}$$

Hints:

- Use definition of the width $d\Gamma$ to relate it the matrix element and the phase space (see PDG (47.11)).
- Use recursive relation to rewrite the three body phase space to the convolution of the two invariant two-body phase spaces via e.g. the invariant mass of the system (12).

$$d\Phi_3(p_0; p_1, p_2, p_3) = \int \frac{ds_3}{2\pi} d\Phi_2(p_0; p_{(12)}, p_3) d\Phi_2(p_{(12)}; p_1 p_2).$$

- The first two-body phase space $0 \rightarrow (12) 3$ can be calculated at the mother particle reference frame. The second phase space $(12) \rightarrow 12$ can be calculated in the (12) -reference frame. It azimuthal angle θ_3 in the (12) -frame is chosen between the momenta \vec{p}_1 and $-\vec{p}_3$, it can be related to s_1 .

$$s_1 = m_2^2 + m_3^2 + \frac{(s_3 + m_2^2 - m_1^2)(M^2 - s_3 - m_3^2)}{2s_3} - \frac{\lambda^{1/2}(s_3, m_2^2, m_1^2)\lambda^{1/2}(s_3, M^2, m_3^2)}{2s_3} \cos \theta_3 \quad (1)$$

- (c) Note that the energies of particles in the rest frame of particle 0 can also be used as kinematics representation quantities $E_1 + E_2 + E_3 = M$. There is a linear relation between E_i and s_i , $i = 1, 2, 3$. It makes Dalitz plot shown in s_i -variables look the same as one drawn in E_i quantities. Find the relation:

$$E_i = \frac{M^2 + m_i^2 - s_i}{2M}, \quad i = 1, 2, 3. \quad (2)$$

As well as E_i , kinematic energies $T_i = E_i - m_i$ are can be used as representation variables.

- (d) Figure out the expression for the differential cross section $d\Gamma/(dT_1 dT_3)$

4.2 Reproduce Dalitz plot

Work in groups. The Dalitz plots for the problem can be at the end of the documents. It is given that the decaying particle is D^+ for CLEO experiment and D_s for BaBar experiment. The particles labels are hidden by purpose.

The exercise is to reproduce the Dalitz plot approximately using the isobar approximation presented in the lecture. To proceed with the exercise you can follow the hints as follows.

- Estimate masses of the daughter particles at the decay. Find out the mass of the mother particle.
- Read off structures, identify decay chains.
- Estimate spin of the resonances, consult the PDG booklet.
- Write down matrix element for every decay chain simplifying down the general equation.

$$\mathfrak{M}_{\text{isobar}} = \frac{c_i}{m_i^2 - s_i - im_i\Gamma_i} h_L(|\vec{p}_i|) h_S(|\vec{p}_{i_2}^*|) A_{LS}^{JM}(\Omega_i, \Omega_{i_2i_3}),$$

$$A_{LS}^{JM}(\Omega_i, \Omega_{i_2i_3}) = \left(\frac{2L+1}{2J+1}\right)^{1/2} \sum_{\lambda} \langle L0S\lambda | J\lambda \rangle \left(\frac{2J+1}{4\pi}\right)^{1/2} D_{M\lambda}^{J*}(\Omega_{i_1}) \left(\frac{2S+1}{4\pi}\right)^{1/2} D_{M\lambda}^{S*}(\Omega_{i_2i_3}),$$

where m_i, Γ_i are parameters of the subchannel resonance (isobar). Ω_i are angles of the isobar direction in the mother particle frame, $\Omega_{i_2i_3}$ are angles of the particle i_2 in the isobar rest frame. The indexes i_1, i_2, i_3 are 1, 2, 3 with possible circular permutations. The functions $h_L(|\vec{p}|) = (R^2|\vec{p}|^2/(1+R^2|\vec{p}|^2))^{L/2}$.

- Plot the differential cross-section trying to reproduce the Dalitz plot at the exercise sheet as close as possible.

To make the plots it is offered to use jupyter-notebook with the julia kernel. Find the technical instructions below.

4.3 Technical aspects

Jupyter-notebook with julia kernel in browser

If there is no jupyter-julia at your machine you can run it in a browser. To do this please follow instructions below.

- (a) Go to the juliabox webpage <https://juliabox.com/>,
- (b) Sign in using your Google/GitHub/LinkedIn account,
- (c) Check Sync-tab,
- (d) Clone git repository from the GitHub inserting the link <https://github.com/mmikhasenko/SWRT2017.git> to the Git Clone URL field and pressing plus sign. New repository is supposed to appear to the list of The Git Repositories.
- (e) Check the tab Jupyter. Select the folder SWRT2017/notebooks/
- (f) Click to Day4.Slot2.DalitzPlot.ipynb. The notebooks will be opened, the kernel will start automatically.
- (g) Minimal set of short cuts:
 - **Enter** to edit cell content (edit mode), **Esc** to get to command mode.
 - **Shift-Enter** to evaluate a cell.
 - **b** to create a new cell below, **a** to create a new cell above.
 - **m** in the command mode will change cell type to Markdown, **y** will change the cell type to Run cell.

More instructions at <http://jupyter-notebook.readthedocs.io/en/latest/notebook.html>, list of shortcuts is at <https://www.cheatography.com/weidadeyue/cheat-sheets/jupyter-notebook/>.

