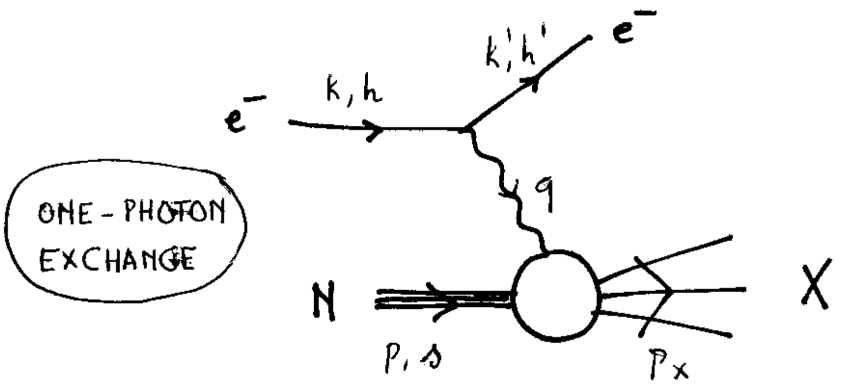
DEEP INELASTIC LEPTON-NUCLEON SCATTERING

PARTON MODEL

⇒ ELECTRON-NUCLEON SCATTERING : KINEMATICS



$h(h')$: INITIAL (FINAL) e^- HELICITIES

$h = \pm\frac{1}{2}$, $h' = \pm\frac{1}{2}$

s : INITIAL NUCLEON SPIN

$s = \pm\frac{1}{2}$

$$\left.\begin{array}{lll} k\ (E_e, \bar{k}) & & E_e = |\bar{k}| \\ k'\ (E_e', \bar{k}') & & E_e' = |\bar{k}'| \end{array}\right\} m_e \approx 0$$

LAB SYSTEM $p\ (M, 0)$

$$q \equiv k - k' = (\nu, \bar{q})$$

$$q^2 = \nu^2 - \bar{q}^2$$

$$= -2k.k' = -4E_eE_e' \sin^2\frac{\theta}{2} < 0$$ SPACELIKE γ

$$-q^2 \equiv +Q^2 > 0$$

⇒ ELECTRON - NUCLEON SCATTERING : CROSS SECTION

IN LAB SYSTEM

SUM OVER FINAL LEPTON HELICITY

$$d\sigma = \frac{1}{v_{rel}} \cdot \frac{1}{(2E_e)(2M)} \sum_{h'} \frac{d^3\bar{k}'}{(2\pi)^3\, 2E'_e} \sum_X (2\pi)^4 \delta^4(q+P-P_X)$$

$$\cdot \left| \bar{u}(k'h')\, \gamma_\nu\, u(k\,h) \cdot \frac{e^2}{Q^2} \cdot \langle X | J^\nu(0) | P\, s \rangle \right|^2$$

$$\downarrow \qquad v_{rel} = \frac{|\bar{k}|}{E_e} + \frac{|\bar{P}|}{E_N} = \frac{|\bar{k}|}{E_e} \approx 1 \qquad \text{(INITIAL FLUX)}$$

$$d\sigma = \frac{(4\pi\alpha)^2}{Q^4} \cdot \frac{1}{4ME_e} \frac{d\Omega'_e \cdot dE_e'\, E_e'}{(2\pi)^3\ 2} \cdot$$

$$\boxed{\alpha \equiv \frac{e^2}{4\pi}} \simeq \frac{1}{137}$$

$$\cdot \sum_{h'} \bar{u}(k\,h)\, \gamma_\mu\, u(k'h')\, \bar{u}(k'h')\, \gamma_\nu\, u(k\,h)$$

$$\cdot \sum_X (2\pi)^4 \delta^4(q+P-P_X) \cdot \langle P s | J^{\mu\dagger}(0) | X \rangle \langle X | J^\nu(0) | P s \rangle$$

$$\downarrow$$

$$\boxed{\left(\frac{d\sigma}{d\Omega'_e\, dE'_e}\right)^{LAB} = \frac{\alpha^2}{Q^4} \frac{1}{2M} \cdot \frac{E'_e}{E_e} \cdot L_{\mu\nu} \cdot W^{\mu\nu}}$$

LEPTON TENSOR $\quad L_{\mu\nu} \equiv \sum_{h'} \bar{u}(k\,h)\, \gamma_\mu\, u(k'h')\, \bar{u}(k'h')\, \gamma_\nu\, u(k\,h)$

HADRON TENSOR $\quad W^{\mu\nu} \equiv \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^4(q+P-P_X)$

$$\cdot \langle P s | J^{\mu\dagger}(0) | X \rangle \langle X | J^\nu(0) | P s \rangle$$

- **LEPTON TENSOR**

$$L_{\mu\nu} \equiv \sum_{h'} \bar{u}(k\,h)\,\gamma_\mu\, u(k'h')\,\bar{u}(k'h')\,\gamma_\nu\, u(k\,h)$$

$$\downarrow \qquad \sum_{h'} u(k'h')\,\bar{u}(k'h') = \not{k}'$$

$$= \mathrm{Tr}\left\{ \gamma_\mu\, \not{k}'\, \gamma_\nu\; u(k\,h)\,\bar{u}(k\,h) \right\}$$

$$\downarrow \qquad u(k\,h)\,\bar{u}(k\,h) = \underset{\substack{\uparrow \\ \text{HELICITY} \\ \text{PROJECTOR}}}{\left(\frac{1+(2h)\gamma_5}{2}\right)} \sum_{h''} u(k\,h'')\,\bar{u}(k\,h'') = \left(\frac{1+2h\,\gamma_5}{2}\right)\not{k}$$

$$L_{\mu\nu} = \mathrm{Tr}\left\{ \gamma_\mu\, \not{k}'\, \gamma_\nu \left(\frac{1+(2h)\gamma_5}{2}\right) \not{k} \right\}$$

$$= \frac{1}{2}\underbrace{\mathrm{Tr}\left\{ \gamma_\mu\, \not{k}'\, \gamma_\nu\, \not{k} \right\}}_{4\left\{ k'_\mu k_\nu + k'_\nu k_\mu - k.k' g_{\mu\nu} \right\}} - \frac{(2h)}{2}\underbrace{\mathrm{Tr}\left\{ \gamma_5\, \gamma_\mu\, \not{k}'\, \gamma_\nu\, \not{k} \right\}}_{\substack{4i\, \varepsilon_{\mu\beta\nu\alpha}\, k'^{\beta}\, k^{\alpha} \\ \parallel \\ -4i\, \varepsilon_{\mu\nu\alpha\beta}\, k^{\alpha}\, q^{\beta} \\ (\varepsilon_{0123} = +1)}}$$

$$\boxed{\begin{array}{ll} L_{\mu\nu} = L^{S}_{\mu\nu} + i\, L^{A}_{\mu\nu} & \\ \text{SYMMETRIC PART:} & L^{S}_{\mu\nu} = 2\left\{ k_\mu k'_\nu + k_\nu k'_\mu - k.k' g_{\mu\nu} \right\} \\ \text{ANTI-SYMMETRIC PART:} & L^{A}_{\mu\nu} = 2(2h)\, \varepsilon_{\mu\nu\alpha\beta}\, k^{\alpha}\, q^{\beta} \end{array}}$$

- HADRON TENSOR

$$W^{\mu\nu} \equiv \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^4(q+p-p_X) \langle p s | J^{\mu +}(0) | X \rangle \langle X | J^{\nu}(0) | p s \rangle$$

$W^{\mu\nu}$ CAN BE PARAMETRIZED IN TERMS OF 4 STRUCTURE FUNCTIONS

$$\boxed{W^{\mu\nu} = W_S^{\mu\nu} + i\, W_A^{\mu\nu}}$$

↑ SYMMETRIC i.e. SPIN INDEPENDENT

↰ ANTI-SYMMETRIC (i.e. SPIN DEPENDENT)

⇒ $W_S^{\mu\nu}$ 2 INDEPENDENT 4 MOMENTA q^μ, p^μ

TENSORS $q^\mu q^\nu,\ p^\mu p^\nu,\ q^\mu p^\nu,\ q^\nu p^\mu,\ g^{\mu\nu}$

\+ GAUGE INVARIANCE $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$

⇓

ONLY 2 TENSORS SURVIVE

$$\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \quad \text{AND} \quad \left(p^\mu - \frac{p.q}{q^2} q^\mu \right)\left(p^\nu - \frac{p.q}{q^2} q^\nu \right)$$

$$\boxed{\begin{aligned} \frac{1}{2M} W_S^{\mu\nu} \equiv & \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) W_1(Q^2, \nu) \\ & + \frac{1}{M^2} \left(p^\mu - \frac{p.q}{q^2} q^\mu \right)\left(p^\nu - \frac{p.q}{q^2} q^\nu \right) W_2(Q^2, \nu) \end{aligned}}$$

W_1 & W_2 -> DEPEND ON Q^2, ν

-> UNPOLARIZED NUCLEON STRUCTURE FUNCTIONS

⇒ $\boxed{W_A^{\mu\nu}}$ DEPENDS LINEARLY ON NUCLEON SPIN

S^μ : 4-VECTOR : NUCLEON POLARIZATION VECTOR

IN NUCLEON REST FRAME $S^\mu = (0, \bar{n})$

$\bar{n}^2 = 1$

$\bar{n}$: AXIS ALONG WHICH NUCLEON SPIN IS QUANTIZED

$$\begin{cases} S_\mu S^\mu = -\bar{n}^2 = -1 \\ S_\mu \cdot p^\mu = 0 . M - \bar{n} . \bar{0} = 0 \end{cases}$$

LORENTZ INV. (HOLD IN ANY FRAME)

$$\underline{\underline{S_\mu S^\mu = -1 \quad , \quad S_\mu p^\mu = 0}}$$

$W_A^{\mu\nu}$ -> DEPENDS LINEARLY ON S

-> SATISFIES $q_\mu W_A^{\mu\nu} = q_\nu W_A^{\mu\nu} = 0$

$W_A^{\mu\nu}$ INVOLVES -> $\varepsilon^{\mu\nu\alpha\beta} q_\alpha S_\beta$

-> $\varepsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta . (S . q)$

OR INSTEAD OF

$$\varepsilon^{\mu\nu\alpha\beta}\, q_\alpha\, P_\beta\, (S.q)$$

WE CAN USE

$$\varepsilon^{\mu\nu\alpha\beta}\, q_\alpha\, \underbrace{\left(S_\beta - \frac{S.q}{P.q}\, P_\beta \right)}_{\text{'TRANSVERSE' SPIN VECTOR}}$$

$$\equiv (S_\perp)_\beta \qquad (q_\beta \cdot S_\perp^\beta = 0)$$

$$\boxed{\begin{aligned} \frac{1}{2M} W_A^{\mu\nu} \equiv \varepsilon^{\mu\nu\alpha\beta}\, q_\alpha \Big\{ & M\, S_\beta\ G_1(Q^2, \nu) \\ & + \frac{P.q}{M} \left[S_\beta - \frac{S.q}{P.q}\, P_\beta \right] G_2(Q^2, \nu) \Big\} \end{aligned}}$$

G_1, G_2 : SPIN - DEPENDENT NUCLEON STRUCTURE FUNCTIONS

- 'SCALING' FUNCTIONS

INSTEAD OF ν : USE $\boxed{x_B \equiv \frac{Q^2}{2M\nu}}$ BJORKEN SCALING VARIABLE

$$\boxed{\begin{aligned} M W_1 &\equiv F_1(Q^2, x_B) \\ \nu W_2 &\equiv F_2(Q^2, x_B) \\ M^2 \nu G_1 &\equiv g_1(Q^2, x_B) \\ M \nu^2 G_2 &\equiv g_2(Q^2, x_B) \end{aligned}}$$

↳ F_1, F_2, g_1, g_2 ARE DIMENSIONLESS

$$\boxed{\begin{aligned} W_S^{\mu\nu} = 2 \Big\{ & \Big(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big) F_1 \\ & + \frac{1}{p\cdot q} \Big(p^\mu - \frac{p\cdot q}{q^2} q^\mu\Big)\Big(p^\nu - \frac{p\cdot q}{q^2} q^\nu\Big) F_2 \Big\} \end{aligned}}$$

$$\boxed{\begin{aligned} W_A^{\mu\nu} = 2\, \varepsilon^{\mu\nu\alpha\beta} q_\alpha \frac{1}{\nu} \Big\{ & S_\beta\, g_1 \\ & + \Big(S_\beta - \frac{S\cdot q}{p\cdot q} p_\beta\Big) g_2 \Big\} \end{aligned}}$$

- UNPOLARIZED CROSS SECTION

$$\left(\frac{d\sigma}{d\Omega'_e\, dE'_e}\right)^{LAB} = \frac{\alpha^2}{Q^4}\,\frac{E'_e}{E_e}\cdot\frac{1}{2M}\cdot L^S_{\mu\nu}\cdot W_S^{\mu\nu}$$

$$\Rightarrow \quad L^S_{\mu\nu}\cdot H^{\mu\nu}_S = 2\left\{ k_\mu k'_\nu + k_\nu k'_\mu - k.k'\, g_{\mu\nu}\right\}$$

$$\cdot\, 2\left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 + \frac{1}{P.q}\left(P^\mu - \frac{P.q}{q^2} q^\mu\right)\left(P^\nu - \frac{P.q}{q^2} q^\nu\right) F_2 \right\}$$

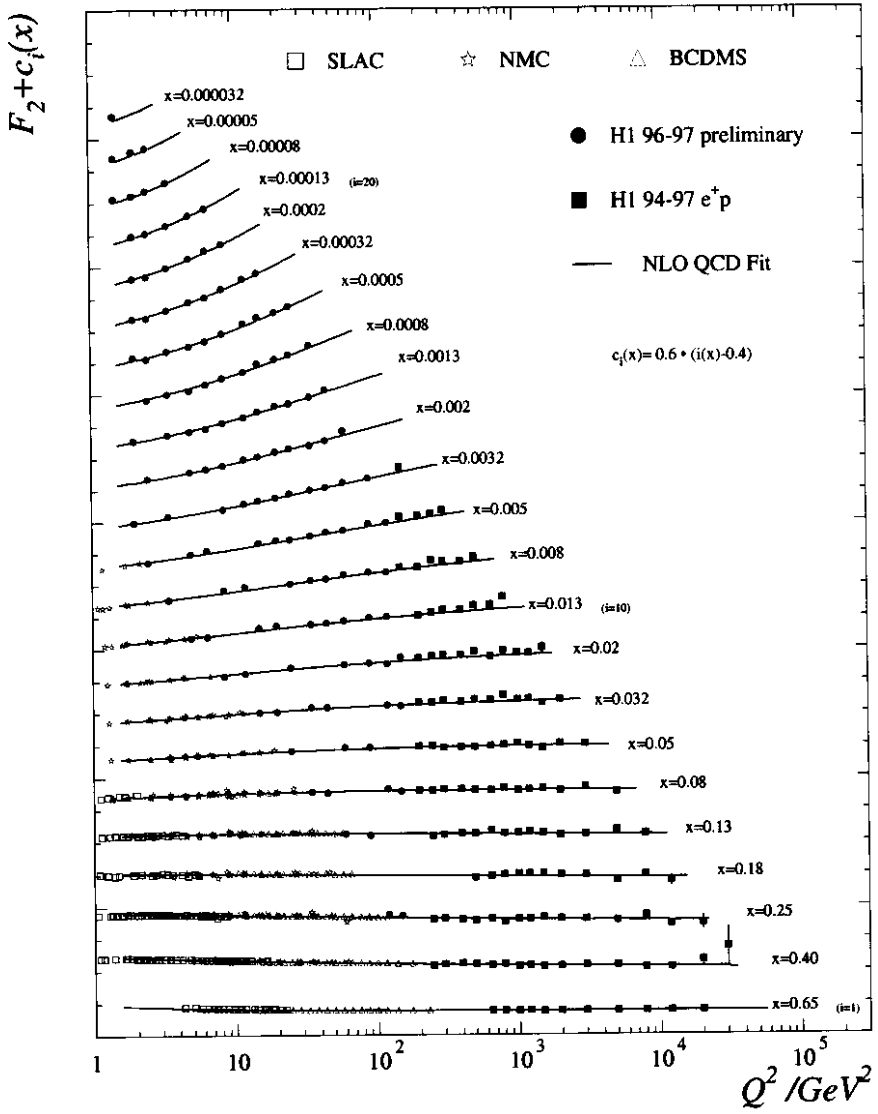
$$= 4\left\{ 2\, k.k'\, F_1 + \frac{1}{P.q}\Big(2\,(P.k)(P.k') - k.k'\, M^2\Big) F_2 \right\}$$

$$= 4\left\{ Q^2 F_1 + \frac{M^2}{P.q}\, E_e E'_e\, (1+\cos\theta)\, F_2 \right\}$$

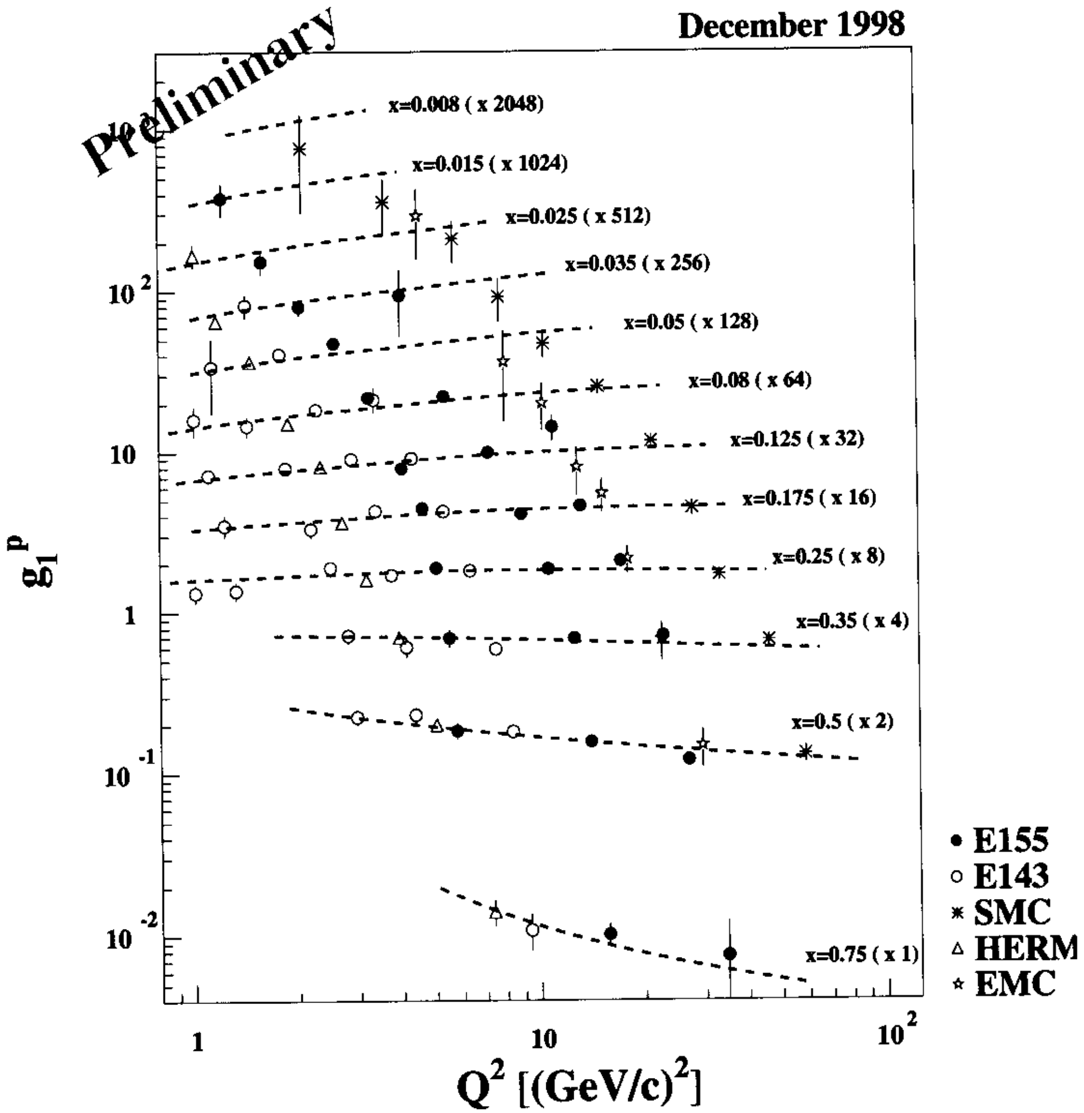
$$= 8\, E_e E'_e \left\{ 2\sin^2\theta/2\; F_1 + \frac{M}{\nu}\cos^2\theta/2\; F_2 \right\}$$

$$\boxed{\left(\frac{d\sigma}{d\Omega'_e\, dE'_e}\right)^{LAB}_{UNPOL} = \frac{\alpha^2}{Q^4}\,\frac{4E'^{\,2}_e}{M}\left\{ 2\sin^2\theta/2\; F_1 + \frac{M}{\nu}\cos^2\theta/2\; F_2 \right\}}$$

World data on F_1^p



World data on g_1^p



- BJORKEN SCALING

IN LIMIT $Q^2 \gg$

$\nu \gg$

$$x_B = \frac{Q^2}{2M\nu} = \text{CONSTANT}$$

$$\left.\begin{array}{l} F_1(x_B, Q^2) \longrightarrow F_1(x_B) \\ F_2(x_B, Q^2) \longrightarrow F_2(x_B) \end{array} \quad \text{AND} \right\} \; F_2 = 2 x_B F_1$$

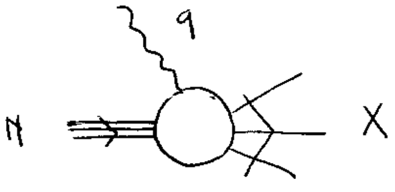
(CALLAN - GROSS RELATION)

$$g_1(x_B, Q^2) \longrightarrow g_1(x_B)$$

$$g_2(x_B, Q^2) \longrightarrow g_2(x_B)$$

⇒ HADRONIC TENSOR IN PARTON MODEL

- $$W^{\mu\nu} = \frac{1}{2\pi} \sum_X (2\pi)^4 \, \delta^4(q+p-P_X) \, \langle N | J^{\mu\dagger}(0) | X \rangle \langle X | J^\nu(0) | N \rangle$$



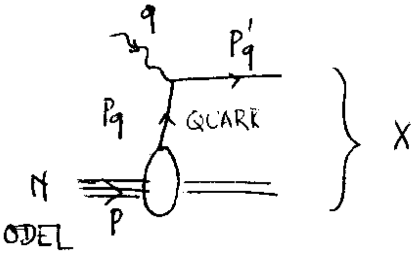
$\approx$

IN PARTON MODEL

$Q^2 \gg$

$\nu \gg$

$x_B = \frac{Q^2}{2M\nu}$ = CONSTANT



EVALUATE TENSOR IN FRAME WHERE PROTON MOVES WITH LARGE MOMENTUM (INFINITE MOMENTUM FRAME)

↳ THE QUARKS MOVE NEARLY COLLINEAR WITH PROTON (i.e. NEGLECT THEIR SMALL TRANSVERSE MOMENTUM $\overline{p}_{q\perp}$)

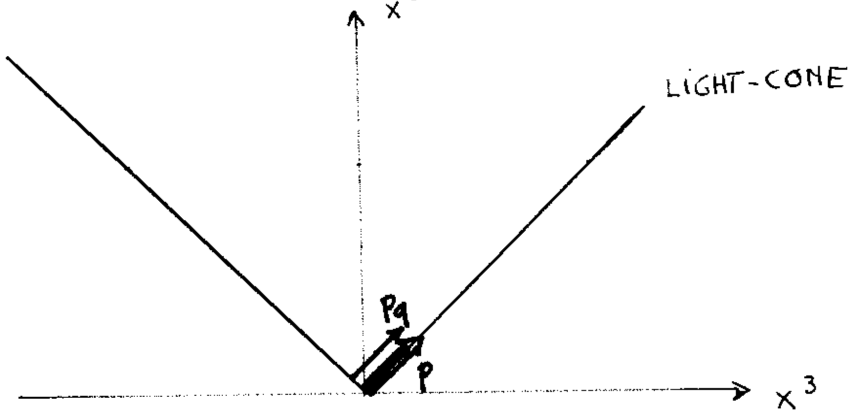
$$\langle \overline{p}_{q\perp}^{\,2} \rangle \approx (0.3 \text{ GeV})^2 \ll Q^2$$

$$\boxed{p_q \approx x\, p}$$

$p_q^2 \approx 0$

(QUARKS ARE NEARLY MASSLESS)

SPACE-TIME PICTURE



IN PARTON MODEL (INCOHERENT SCATTERING OFF INDIVIDUAL QUARKS)

$$\boxed{W^{\mu\nu}_{\substack{\text{PARTON}\\ \text{MODEL}}} = \sum_q \sum_s \int_0^1 \frac{dx}{x} \; \underbrace{n_q(x,s)} \cdot \underbrace{e_q^2 \; w^{\mu\nu}}}$$

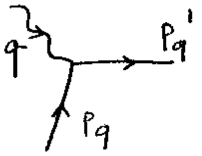
SUM OVER QUARK SPECIES (FLAVORS)

SUM OVER INITIAL QUARK SPIN PROJECTIONS

INITIAL FLUX $E_q = x\, E_N$

NUMBER DENSITY OF QUARKS WITH MOMENTUM FRACTION x & SPIN PROJECTION $s = \pm\frac{1}{2}$

SCATTERING OFF A SINGLE QUARK



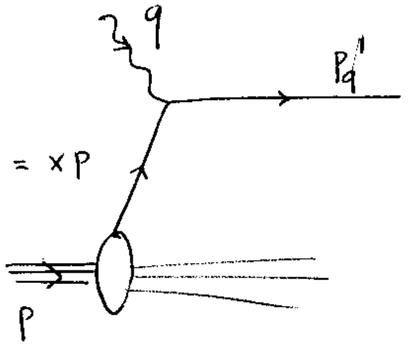
- SCATTERING OFF AN INDIVIDUAL QUARK : $w^{\mu\nu}$

$$w^{\mu\nu} = \frac{1}{2\pi} \underbrace{\sum_{s'} \int \frac{d^3\bar{p}_q'}{(2\pi)^3\, 2E_q'}}_{\sum_X} (2\pi)^4 \delta^4(q + p_q - p_q') \cdot \bar{u}(p_q\, s)\, \gamma^\mu\, u(p_q'\, s')\, \bar{u}(p_q'\, s')\, \gamma^\nu\, u(p_q\, s)$$

$$= \underbrace{\frac{1}{2E_q'}\, \delta(p_q'^0 - E_q')}_{\substack{|||\\ \delta(p_q'^2 - \underset{\substack{\parallel\\ 0}}{m_q^2})}} \; (p_q'^0 > 0) \quad \cdot \quad \text{Tr}\left\{ \left(\frac{1 + (2s)\gamma_5}{2}\right) \not{p}_q\, \gamma^\mu\, \not{p}_q'\, \gamma^\nu \right\}$$

$$\downarrow \qquad \delta(p_q'^2) = \delta\Big((q + \overset{\substack{p_q = xp\\ \downarrow}}{xp})^2 \Big) = \delta(x\, 2p\cdot q - Q^2)$$

$$= \frac{1}{2p\cdot q}\, \delta\left(x - \frac{Q^2}{2p\cdot q} \right)$$



$$\delta\left(p_q'^2\right) = \frac{1}{Q^2}\left(\frac{Q^2}{2P\cdot q}\right)\delta\left(x - \frac{Q^2}{2P\cdot q}\right)$$

$\Downarrow$ USING $x_B \equiv \dfrac{Q^2}{2P\cdot q}$ $\left(= \dfrac{Q^2}{2M\nu}\right.$ IN LAB SYSTEM$\left.\right)$

$$\delta\left(p_q'^2\right) = \frac{x_B}{Q^2}\,\delta\left(x - x_B\right)$$

$\Downarrow$

PHOTON SCATTERS OFF QUARK WHICH HAS MOMENTUM FRACTION x_B OF PROTON MOMENTUM

$$\therefore \quad w^{\mu\nu} = \frac{x_B}{Q^2}\,\delta\left(x - x_B\right) \cdot \frac{1}{2}\left\{ \mathrm{Tr}\left\{ \not{p}_q \gamma^\mu \not{p}_q' \gamma^\nu \right\} + (2s)\,\mathrm{Tr}\left\{ \gamma_5 \not{p}_q \gamma^\mu \not{p}_q' \gamma^\nu \right\} \right\}$$

$$w^{\mu\nu} = \frac{x_B}{Q^2}\,\delta\left(x - x_B\right) \cdot 2\left\{ p_q^\mu p_q'^\nu + p_q^\nu p_q'^\mu - \left(p_q \cdot p_q'\right) g^{\mu\nu} + (2s)\, i\, \varepsilon^{\mu\nu\alpha\beta}\, p'_{q\alpha}\, p_{q\beta} \right\}$$

$$\downarrow \qquad \begin{aligned} P_q &= x P \\ P_q' &= q + x P \end{aligned}$$

$$w^{\mu\nu} = \frac{x_B}{Q^2} \, \delta(x - x_B)$$

$$\cdot 2 \cdot \Big\{ x_B^2 \Big[P^\mu \Big(P^\nu + \frac{1}{x_B} q^\nu \Big) + P^\nu \Big(P^\mu + \frac{1}{x_B} q^\mu \Big) \Big]$$

$$- \frac{Q^2}{2} g^{\mu\nu} + (2s) \, i \, \varepsilon^{\mu\nu\alpha\beta} \, x_B \, q_\alpha P_\beta \Big\}$$

$$\downarrow$$

$$w^{\mu\nu} = \frac{x_B}{Q^2} \, \delta(x - x_B)$$

$$\cdot 2 \cdot \Big\{ 2 x_B^2 \Big(P^\mu - \frac{P \cdot q}{q^2} q^\mu \Big) \Big(P^\nu - \frac{P \cdot q}{q^2} q^\nu \Big)$$

$$+ \frac{Q^2}{2} \Big(- g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \Big)$$

$$+ x_B \, (2s) \, i \, \varepsilon^{\mu\nu\alpha\beta} \, q_\alpha P_\beta \Big\}$$

↳
$$\boxed{\begin{aligned} W^{\mu\nu}_{\substack{\text{PARTON}\\ \text{MODEL}}} &= \sum_q e_q^2 \sum_s n_q(x_B, s) \cdot \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \\ &+ \sum_q e_q^2 \sum_s n_q(x_B, s) \cdot \frac{4x_B^2}{Q^2}\left(p^\mu - \frac{P.q}{q^2} q^\mu\right)\left(P^\nu - \frac{P.q}{q^2} q^\nu\right) \\ &+ \sum_q e_q^2 \sum_s (2s)\, n_q(x_B, s) \cdot \frac{2x_B}{Q^2}\, i\, \varepsilon^{\mu\nu\alpha\beta} q_\alpha P_\beta \end{aligned}}$$

COMPARE THIS WITH GENERAL EXPRESSION

$$\begin{aligned} W^{\mu\nu} = 2 \Big\{ &\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 \\ &+ \frac{1}{P.q}\left(p^\mu - \frac{P.q}{q^2} q^\mu\right)\left(P^\nu - \frac{P.q}{q^2} q^\nu\right) F_2 \\ &+ i\, \varepsilon^{\mu\nu\alpha\beta} q_\alpha \frac{M}{P.q}\left[S_\beta\, g_1 + S_{\perp\beta}\, g_2\right] \Big\} \end{aligned}$$

- UNPOLARIZED STRUCTURE FUNCTIONS

  IN PARTON MODEL

$$F_1(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 \sum_s n_q(x_B, s)$$ FUNCTION OF x_B ONLY!

$$F_2(x_B, Q^2) = x_B \sum_q e_q^2 \sum_s n_q(x_B, s)$$ FUNCTION OF x_B ONLY!

$\Downarrow$

BJORKEN
SCALING

$$\sum_{s} n_q(x_B, s) = n_q\left(x_B, +\tfrac{1}{2}\right) + n_q\left(x_B, -\tfrac{1}{2}\right)$$

$$\equiv \mathbf{q(x_B)}$$ UNPOLARIZED QUARK DISTRIBUTION IN NUCLEON

FOR EACH FLAVOR

$$u(x_B),\ d(x_B),\ s(x_B),\ \dots$$ QUARK NUMBER DENSITIES IN NUCLEON

IN PARTON MODEL

$$\boxed{F_1(x_B, \cancel{Q^2}) = \frac{1}{2} \sum_q e_q^2 \left\{ q(x_B) + \bar{q}(x_B) \right\}}$$

↳ ANTI-QUARK CONTRIBUTION

$$= \frac{1}{2} \left\{ \frac{4}{9} u(x_B) + \frac{1}{9} d(x_B) + \frac{1}{9} s(x_B) + \cdots \right.$$
$$\left. + \frac{4}{9} \bar{u}(x_B) + \frac{1}{9} \bar{d}(x_B) + \frac{1}{9} \bar{s}(x_B) + \cdots \right\}$$

$$\boxed{F_2(x_B, \cancel{Q^2}) = \sum_q e_q^2 \, \underbrace{x_B \left\{ q(x_B) + \bar{q}(x_B) \right\}}}$$

↳ QUARK MOMENTUM DENSITIES

$$F_2(x_B, \cancel{Q^2}) \underset{\substack{\uparrow \\ \text{IN PARTON} \\ \text{MODEL}}}{=} 2 x_B \, F_1(x_B, \cancel{Q^2})$$

: CALLAN - GROSS RELATION

IS A CONSEQUENCE OF SPIN 1/2 NATURE OF PARTONS

↓

QUARKS ARE DIRAC PARTICLES

- SPIN DEPENDENT STRUCTURE FUNCTIONS

NUCLEON MOVING WITH HIGH VELOCITY

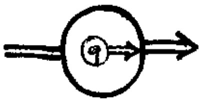
↳ SPIN ALIGNED ALONG MOMENTUM (POSITIVE HELICITY)

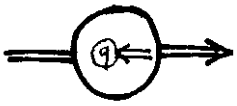
$$S^\beta \approx \frac{P^\beta}{M} \qquad S \cdot p = M \approx 0 \quad (E_N \gg M)$$

$$S_\perp^\beta \approx 0 \Rightarrow \text{IN PARTON MODEL}$$

$g_2(x_B, Q^2)$ CANNOT BE ACCESSED

$$g_1(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 \sum_s (2s)\, n_q(x_B, s)$$





$$\sum_s (2s)\, n_q(x_B, s) = n_q(x_B, +\tfrac{1}{2}) - n_q(x_B, -\tfrac{1}{2})$$

$$\equiv \mathbf{\Delta q\,(x_B)}$$ QUARK HELICITY DISTRIBUTION IN NUCLEON

$$\boxed{g_1(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 \left\{ \Delta q(x_B) + \Delta \bar{q}(x_B) \right\}}$$ BJORKEN SCALING

$$= \frac{1}{2} \left\{ \frac{4}{9} \Delta u(x_B) + \frac{1}{9} \Delta d(x_B) + \frac{1}{9} \Delta s(x_B) + \cdots \right.$$

$$\left. + \frac{4}{9} \Delta \bar{u}(x_B) + \frac{1}{9} \Delta \bar{d}(x_B) + \frac{1}{9} \Delta \bar{s}(x_B) + \cdots \right\}$$

⇒ VALENCE & SEA-QUARK DISTRIBUTIONS

$q(x_B)$: QUARK DISTRIBUTION IN PROTON

$\bar{q}(x_B)$: ANTI-QUARK DISTRIBUTION IN PROTON

- "VALENCE" DISTRIBUTION

$$q_V(x_B) \equiv q(x_B) - \bar{q}(x_B)$$

- "SEA" DISTRIBUTION

$$q_S(x_B) \equiv \bar{q}(x_B)$$

$$\rightsquigarrow \boxed{q(x_B) = q_V(x_B) + q_S(x_B)}$$

⇒ UNPOLARIZED SUM RULES

- PROTON : $|p\rangle = c_1 |uud\rangle + c_2 |uud\, u\bar{u}\rangle$
  $+ c_3 |uud\, d\bar{d}\rangle$
  $+ \dots$

$$\int_0^1 dx \quad u_V(x) = 2$$

$$\int_0^1 dx \quad d_V(x) = 1$$

$$\int_0^1 dx \left[s(x) - \bar{s}(x) \right] = 0$$

NET STRANGENESS OF PROTON $= 0$

- **NEUTRON**

q^p : QUARK DISTR IN PROTON

q^n : QUARK DISTR IN NEUTRON

$$|n\rangle = c_1\,|ddu\rangle + c_2\,|ddu\,d\bar{d}\rangle + c_3\,|ddu\,u\bar{u}\rangle + \cdots$$

↳ OBTAINED FROM $|p\rangle$ BY $u \leftrightarrow d$

$$u^n(x) = d^p(x) \equiv d(x)$$

$$d^n(x) = u^p(x) \equiv u(x)$$

SU(2) SYMMETRY

$$\int_0^1 dx\;\; d_V^n(x) = \int_0^1 dx\;\; u_V(x) = 2$$

$$\int_0^1 dx\;\; u_V^n(x) = \int_0^1 dx\;\; d_V(x) = 1$$

$$F_1^p = \frac{1}{2}\left\{\frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s})\right\}$$

$$F_1^n = \frac{1}{2}\left\{\frac{1}{9}(d+\bar{d}) + \frac{4}{9}(u+\bar{u}) + \frac{1}{9}(s+\bar{s})\right\}$$

∴ BY DOING EXPERIMENTS ON <u>BOTH</u> PROTON & NEUTRON

⟹ u, d QUARK FLAVOR SEPARATION

⇒ SEPARATION OF QUARK & ANTI-QUARK DISTRIBUTIONS

HOW CAN WE SEPARATE q & $\bar{q}$?

↳ EM PROBE COUPLES IN SAME WAY TO q & $\bar{q}$

$$F_1 = \frac{1}{2} e_q^2 \left(q + \bar{q} \right)$$

↳ NEED PROBE WHICH COUPLES DIFFERENTLY TO q & $\bar{q}$

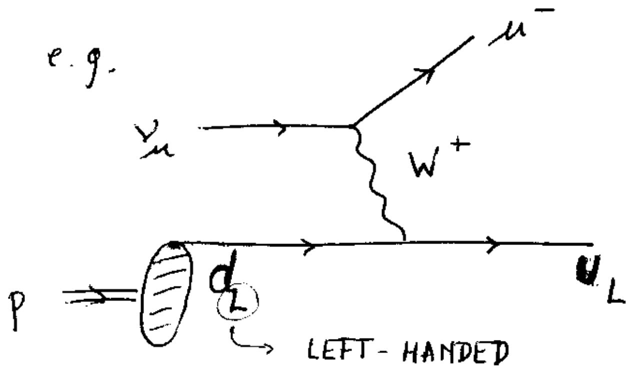
MASSLESS q : LEFT-HANDED

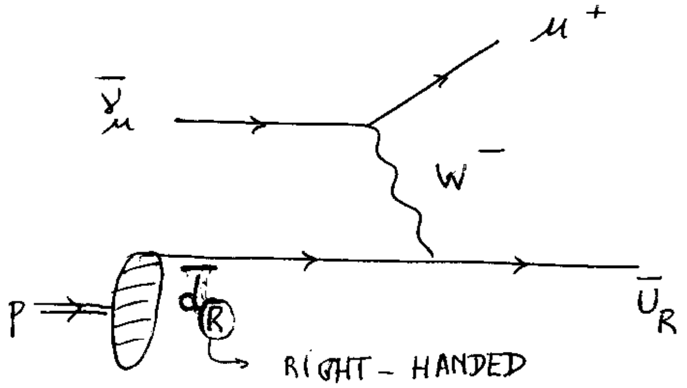
MASSLESS $\bar{q}$: RIGHT-HANDED

⇓

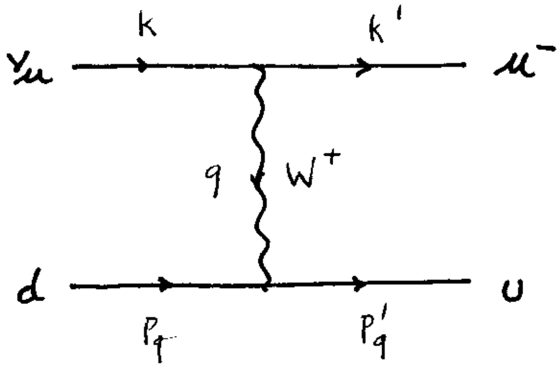
WEAK INTERACTION COUPLES DIFFERENTLY TO q & $\bar{q}$

e.g.





⇒ NEUTRINO – QUARK SCATTERING



$$(-i)\,\frac{g_W}{2\sqrt{2}}\,\gamma_\alpha(1-\gamma_5)$$

$$-\,\frac{i\,g^{\alpha\beta}}{q^2-m_W^2}$$

$$(-i)\,\frac{g_W}{2\sqrt{2}}\,\gamma_\beta(1-\gamma_5)$$ (NEGLECT $d \leftrightarrow s$ MIXING

WITH $\boxed{\dfrac{g_W^2}{8m_W^2} \equiv \dfrac{G_F}{\sqrt{2}}}$ → FERMI WEAK COUPLING CONSTANT

FOR $-q^2 \ll m_W^2$ ⇝ NEGLECT q^2 IN W PROPAGATOR

- $$\mathcal{M} = -i\left(\frac{G_F}{\sqrt{2}}\right)\cdot \bar{u}_{\mu^-}\gamma_\nu(1-\gamma_5)u_{\nu_\mu}\cdot \bar{u}_u\gamma^\nu(1-\gamma_5)u_d$$

- CROSS SECTION

$$d\sigma = \frac{1}{2E_k\,2E_{p_q}\,v_{rel}}\cdot\frac{d^3\bar{k}'}{(2\pi)^3\,2E_k'}\,\frac{d^3\bar{p}_q'}{(2\pi)^3\,2E_{p_q}'}\cdot(2\pi)^4\,\delta^4(k+p_q-k'-p_q')\cdot|\mathcal{M}|^2$$

↓ NEGLECT MASSES OF ALL PARTICLES

+ CONSIDER c.m. SYSTEM $|\bar{k}| = |\bar{p}_q| = |\bar{k}'| = |\bar{p}_q'| = \frac{\sqrt{s}}{2}$

$$\left.\begin{aligned}\hat{s} &= (k+p_q)^2\\ \hat{u} &= (k'-p_q)^2\\ \hat{t} &= (k-k')^2 = q^2 = -Q^2\end{aligned}\right\}\quad \hat{s}+\hat{u} = Q^2$$

↳ $$2E_k \; 2E_{p_q} v_{rel} = 4E_k E_{p_q}\left(\frac{|\bar{k}|}{E_k} + \frac{|\mathbf{k}|}{E_{p_q}}\right)$$

$$= 4|\bar{k}|\sqrt{\hat{s}} = 2\hat{s}$$

↳ $$\hat{t} = -2k.k' = -2|\bar{k}|^2(1-\cos\Theta_{cm})$$

$$d\hat{t} = +2|\bar{k}|^2 \, d\cos\Theta_{cm}$$

↳ $\int d\phi \rightsquigarrow 2\pi$ ↳ $\delta(\sqrt{\hat{s}} - 2|\bar{k}'|) = \frac{1}{2}\delta\left(|\bar{k}'| - \frac{\sqrt{\hat{s}}}{2}\right)$

$$\frac{d\sigma}{d\hat{t}} = \underbrace{\frac{1}{2\hat{s}}}_{\frac{1}{4E_k E_{p_q} v_{rel}}} \cdot \underbrace{\frac{1}{2\pi}}_{\text{FROM } \int d\phi} \cdot \underbrace{\frac{1}{2\hat{s}}}_{\frac{1}{2 \,\cdot\, 4E_{k'}E'_{p_q}}} \quad \underbrace{\frac{1}{2}}_{\text{FROM } \delta} \cdot |\mathcal{M}|^2$$

$$\boxed{\frac{d\sigma}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2} \cdot \left(\frac{G_F}{\sqrt{2}}\right)^2 \cdot L_{\mu\nu} H^{\mu\nu}}$$

WITH

→ NO POLARIZATION AVERAGE FOR ν_μ (LEFT-HANDED)

$$L_{\mu\nu} = \text{Tr}\left\{\gamma_\mu(1-\gamma_5)\not{k}'\gamma_\nu(1-\gamma_5)\not{k}\right\}$$

$$H^{\mu\nu} = \underbrace{\frac{1}{2}}_{\uparrow} \text{Tr}\left\{\gamma^\mu(1-\gamma_5)\not{p}'_q\gamma^\nu(1-\gamma_5)\not{p}_q\right\}$$

AVERAGE OVER
POLARIZATION
OF INITIAL
QUARK

- $$L_{\mu\nu} = 2\,\mathrm{Tr}\left\{\gamma_\mu \not{k}' \gamma_\nu \not{k}\right\} + 2\,\mathrm{Tr}\left\{\gamma_5 \gamma_\mu \not{k}' \gamma_\nu \not{k}\right\}$$

$$= 8\left\{k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} + i\,\varepsilon_{\mu\nu\alpha\beta}\, k^\alpha k'^\beta\right\}$$

↳ SAME AS LEPTON TENSOR FOR e^- SCATTERING
WHEN PUTTING $(2h) = -1$ (ν ARE LEFT-HANDED)

- $$H^{\mu\nu} = \frac{8}{2}\left\{p_q^\mu p_q'^\nu + p_q^\nu p_q'^\mu - p_q\cdot p_q'\, g^{\mu\nu} + i\,\varepsilon^{\mu\nu\gamma\delta}(p_q)_\gamma (p_q')_\delta\right\}$$

- $$L_{\mu\nu} H^{\mu\nu} = 32\left\{k_\mu k'_\nu + k_\nu k'_\mu - \frac{Q^2}{2} g_{\mu\nu}\right\}\left\{2 p_q^\mu p_q^\nu - \frac{Q^2}{2} g^{\mu\nu}\right\}$$

$$- 32 \underbrace{\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{\mu\nu\gamma\delta}}_{-2\left(g_\alpha^\gamma g_\beta^\delta - g_\alpha^\delta g_\beta^\gamma\right)} k^\alpha k'^\beta (p_q)_\gamma (p_q')_\delta$$

$$= 32\left\{4(p_q\cdot k)(p_q\cdot k') + \frac{1}{2}(Q^2)^2 + 2(p_q\cdot k)(p_q'\cdot k') - 2(p_q\cdot k')(p_q'\cdot k)\right\}$$

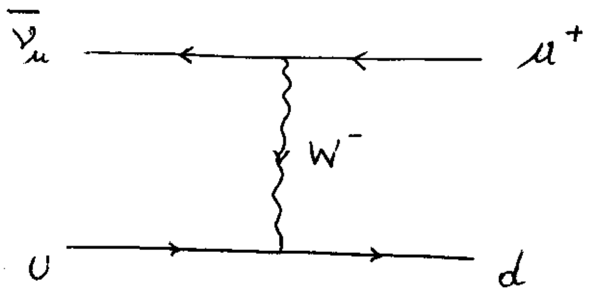
$$= 32\left\{-\hat{s}\hat{u} + \frac{1}{2}(\hat{s}+\hat{u})^2 + \frac{1}{2}\hat{s}^2 - \frac{1}{2}\hat{u}^2\right\}$$

$$= \underline{32\,\hat{s}^2}$$

$$\hookrightarrow \boxed{\frac{d\sigma}{d\hat{t}}\left(\nu_\mu d \to \mu^- u\right) = \frac{G_F^2}{32\pi\hat{s}^2}\cdot 32\hat{s}^2 = \frac{G_F^2}{\pi}}$$

$$= \frac{d\sigma}{d\hat{t}}\left(\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}\right)$$

↳ ANALOGOUSLY $\bar{\nu}_\mu u \to \mu^+ d$



ANTI-ν IS RIGHT-HANDED

W^- COUPLES ONLY TO LEFT-HANDED QUARKS

$$L_{\mu\nu} = \mathrm{Tr}\left\{\gamma_\nu(1-\gamma_5)\not{k}'\gamma_\mu(1-\gamma_5)\not{k}\right\}$$

↳ $\mu \leftrightarrow \nu$ COMPARED TO $L_{\mu\nu}$ FOR $\nu_\mu d \to \mu^- u$

$H^{\mu\nu}$ SAME AS FOR $\nu_\mu d \to \mu^- u$

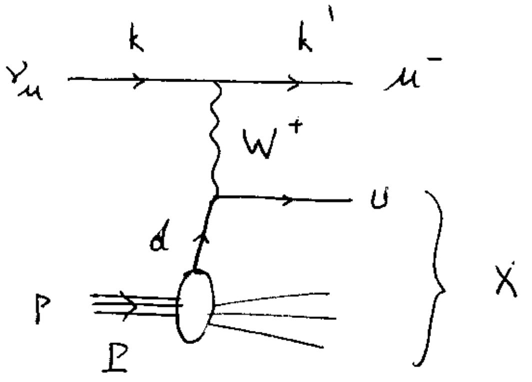
$$\therefore \; L_{\mu\nu}H^{\mu\nu} = 32\left\{-\hat{s}\hat{u} + \frac{1}{2}(\hat{s}+\hat{u})^2 \underset{\uparrow}{\textcircled{$-$}}\left[\frac{1}{2}\hat{s}^2 - \frac{1}{2}\hat{u}^2\right]\right\} = 32\,\hat{u}^2$$

DIFFERENT RELATIVE SIGN

$$\boxed{\frac{d\sigma}{d\hat{t}}\left(\bar{\nu}_\mu u \to \mu^+ d\right) = \frac{G_F^2}{\pi}\cdot\frac{\hat{u}^2}{\hat{s}^2}}$$

$$= \frac{d\sigma}{d\hat{t}}\left(\nu_\mu \bar{u} \to \mu^- \bar{d}\right)$$

⇒ NEUTRINO - PROTON SCATTERING (NEGLECT ALL MASSES)



$t = k - k'^{2}$

$s = (k + P)^{2}$

$$\frac{d\sigma}{dt}(\nu p \to \mu^- X) = \sum_q \int_0^1 dx \; q(x) \cdot \underbrace{\frac{d\sigma}{dt}(\nu q \to \mu^- q')}_{\text{WITH } \underline{\underline{\hat{s} = x s}}}$$

PARTONIC

$$\boxed{\frac{d\sigma}{dt\, dx_B}(\nu p \to \mu^- X) = \sum_q q(x_B) \cdot \frac{d\sigma}{dt}(\nu q \to \mu^- q')\Big|_{\hat{s} = x_B s}}$$

↓ INTRODUCE DIMENSIONLESS VARIABLE

$$y \equiv \frac{q \cdot P}{k \cdot P} = \frac{2\, q \cdot P}{s} = \frac{Q^2}{s\, x_B}$$

$$\underline{\underline{Q^2 = s\, x_B\, y}} \qquad dQ^2 = s\, x_B\, dy$$

$$y = \frac{q \cdot (x P)}{k \cdot (x P)} = \frac{(k - k') \cdot P_q}{k \cdot P_q} = \frac{\hat{s} + \hat{u}}{\underset{\uparrow}{\hat{s}}}$$

PARTONIC
VARIABLES

$$\frac{\hat{u}}{\hat{s}} = -(1-y)$$

$$\Downarrow$$

$$\frac{d\sigma}{d\hat{t}}\left(\bar{\nu}\, u \to \mu^+ d\right) = \frac{G_F^2}{\pi}(1-y)^2$$

$$\frac{d\sigma}{dx_B\, dy}\left(\nu p \to \mu^- X\right)$$

$$= s\, x_B \cdot \frac{d\sigma}{dt\, dx_B}\left(\nu p \to \mu^- X\right)$$

$$= s\, x_B \cdot \frac{G_F^2}{\pi}\left\{ d(x_B) + \bar{u}(x_B) \cdot (1-y)^2 \right\}$$

$$\Longrightarrow \boxed{\frac{d\sigma}{dx_B\, dy}\left(\nu p \to \mu^- X\right) = \frac{G_F^2 s}{\pi}\left\{ x_B\, d(x_B) + x_B\, \bar{u}(x_B)\,(1-y)^2 \right\}}$$

$$\Longrightarrow \boxed{\frac{d\sigma}{dx_B\, dy}\left(\bar{\nu} p \to \mu^+ X\right) = \frac{G_F^2 s}{\pi}\left\{ x_B\, u(x_B)\,(1-y)^2 + x_B\, \bar{d}(x_B) \right\}}$$

QUALITATIVE DIFFERENCE ν CROSS SECTION $\sim d(x_B)$

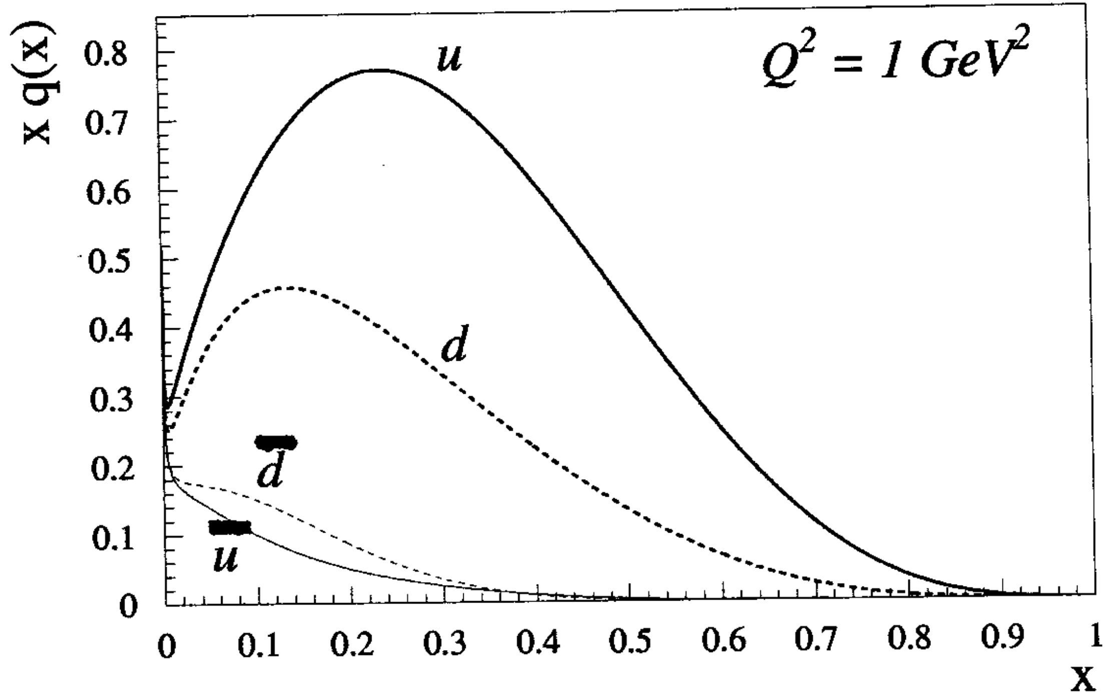
$\bar{\nu}$ CROSS SECTION $\sim \underline{\underline{u(x_B)\,(1-y)^2}}$

& VICE VERSA FOR $\bar{q}$

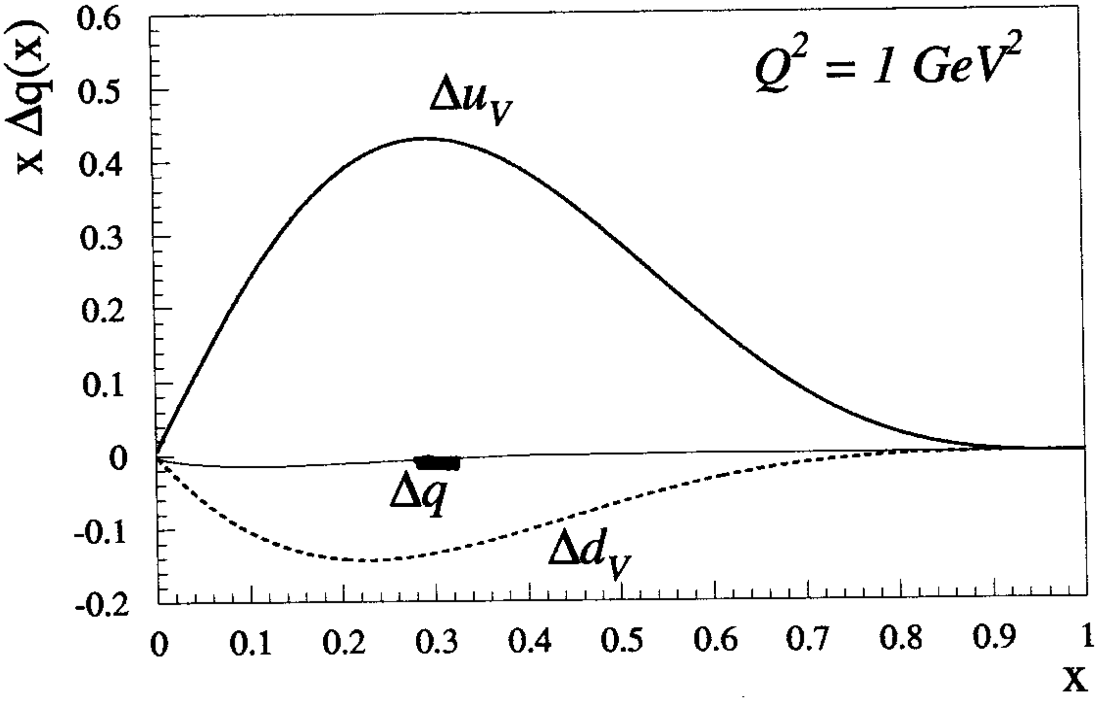
QUARK DISTRIBUTIONS in the PROTON

$q(x) = q_v(x) + q_s(x)$ $q_s(x) = \bar{q}(x)$

⇒ MRST 98 NLO QUARK DISTR. $q(x)$



⇒ LEADER, SIDOROV, STAMENOV 98 NLO $\Delta q(x)$



UNPOLARIZED SUM RULES

$$q(x) = q_V(x) + q_S(x)$$
$$\bar{q}(x) = q_S(x)$$

⇒ **SUM RULES FOR 1 FLAVOR** (PROTON)

$$\int_0^1 dx \; u_V(x) = 2$$

$$\int_0^1 dx \; d_V(x) = 1$$

$$\int_0^1 dx \left[s(x) - \bar{s}(x) \right] = 0$$

$$|p\rangle = c_1 |uud\rangle + c_2 |uud\,u\bar{u}\rangle + c_3 |uud\,d\bar{d}\rangle + \cdots$$

⇒ **GROSS - LLEWELLYN - SMITH SUM RULE** (# VALENCE QUARK

$$S_{GLS} \equiv \frac{1}{2} \int_0^1 dx \left[F_3^{\nu p}(x,Q^2) + F_3^{\bar{\nu} p}(x,Q^2) \right]$$

$$= \int_0^1 dx \; [u(x,Q^2) - \bar{u}(x,Q^2) + d(x,Q^2) - \bar{d}(x,Q$$

$$= 3 \left[1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2) \right]$$

(CCFR) EXP. $S_{GLS}(Q^2 = 3\,\text{GeV}^2) = 2.50 \pm 0.018\,(\text{stat}) \pm 0.078\,($

THEORY $S_{GLS}(Q^2 = 3\,\text{GeV}^2) = 2.66 \pm 0.04$ ✓

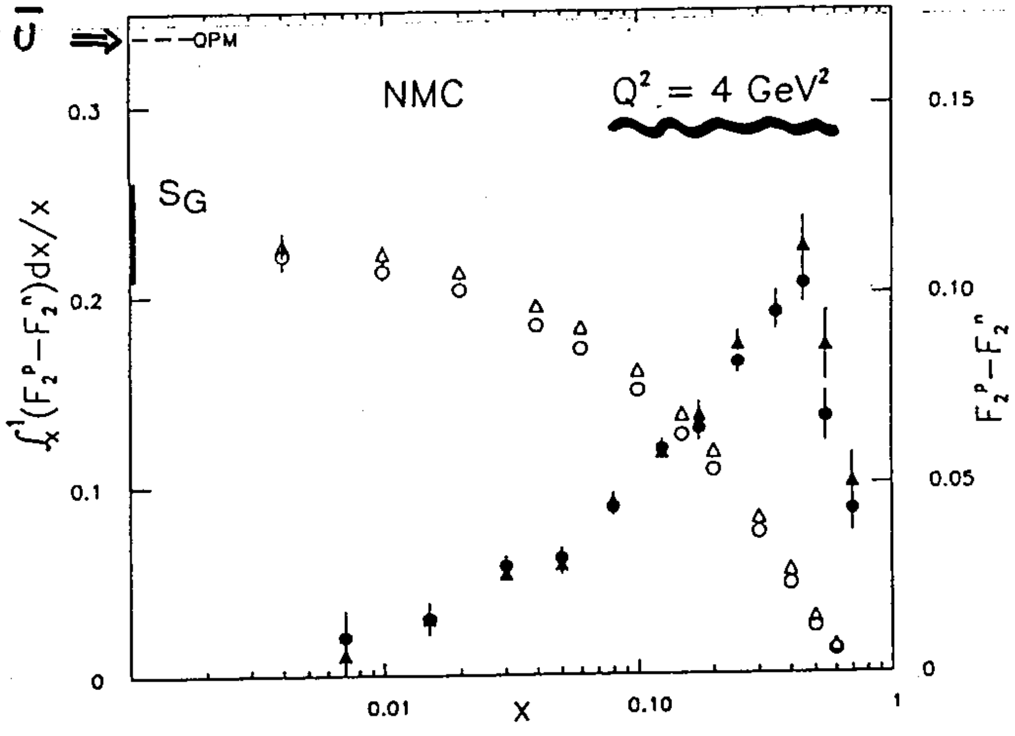
GOTTFRIED SUM RULE

$$\Rightarrow \quad \underline{F_2^p - F_2^n} = \frac{x}{3}\left[u(x) + \bar{u}(x) - d(x) - \bar{d}(x)\right]$$

$$= \frac{x}{3}\left[u_v(x) - d_v(x)\right] + \frac{2x}{3}\left[\bar{u}(x) - \bar{d}(x\right.$$

$$S_G \equiv \boxed{\int_0^1 dx\, \frac{1}{x}\left(F_2^p - F_2^n\right) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\left[\bar{d}(x) - \bar{u}(x\right.}$$

iF $\bar{d} = \bar{u} \Rightarrow$



(NMC 97) $\underline{\underline{S_G = 0.2281 \pm 0.0065 \ (stat)}}$

$$\Downarrow \qquad \bar{d} > \bar{u}$$

$$\int_0^1 dx\left(\bar{d} - \bar{u}\right) \simeq 0.15$$

MOMENTUM SUM RULE

$$\boxed{M_2^q(Q^2) \equiv \int_0^1 dx \;\; x \left[q(x,Q^2) + \bar{q}(x,Q^2) \right]}$$

↳ MOMENTUM FRACTION OF PROTON CARRIED BY QUARK OF FLAVOR q

⇒ M_2^q AT LOW SCALE : $Q^2 = 1$ GeV^2

⇒ MRST 98 NLO QUARK DISTR.

q	$M_2^q\,(Q^2 = 1\,\text{GeV}^2)$
u	0.40
d	0.22
s	0.03
SUM	0.65

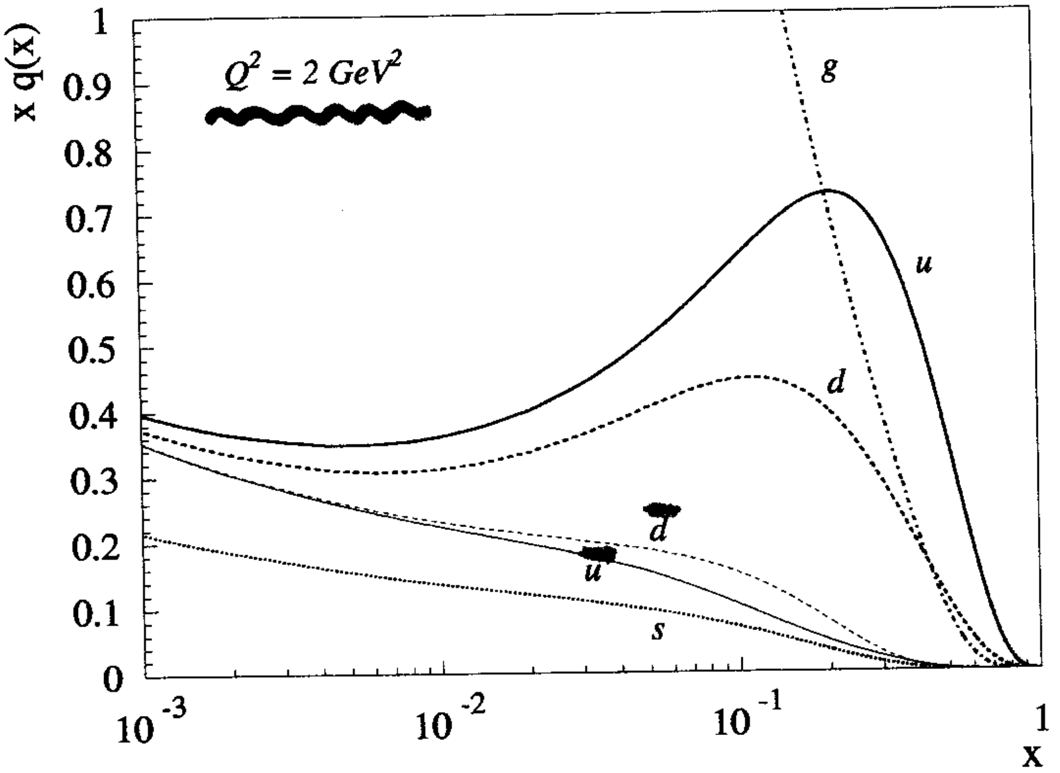
⇒ M_2^q IN LIMIT $Q^2 \to \infty$

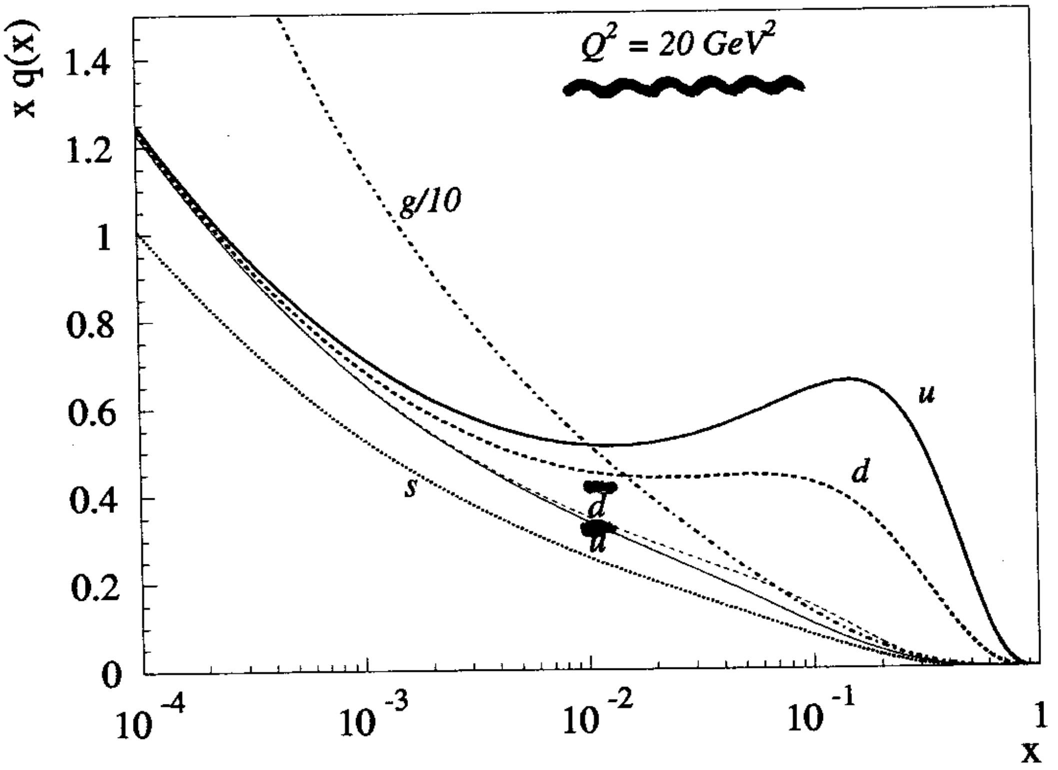
$$M_2^q\,(Q^2 \to \infty) = \frac{3N_f}{16 + 3N_f} \underset{\boxed{N_f = 3}}{=} \underset{\sim\sim}{0.36}$$

(FIXED POINT SOLUTION OF RENORMALIZATION GROUP EQ.)

∴ **GLUONS** CARRY AN IMPORTANT FRACTION OF PROTON MOMENTUM

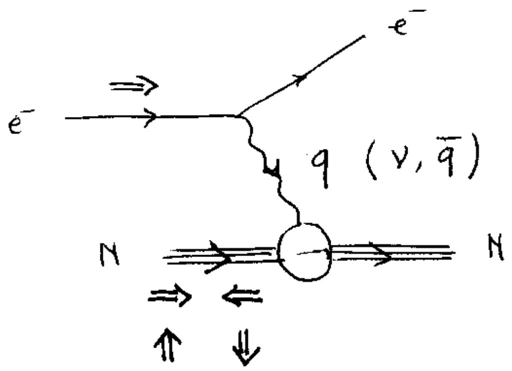
MRST98 UNPOLARIZED parton distributions





POLARIZED DIS
SPIN OF THE NUCLEON

⇒ ASYMMETRIES

- 

$$\underline{\underline{W^{\mu\nu} = W_S^{\mu\nu} + i\, W_A^{\mu\nu}}}$$

$$\boxed{W_A^{\mu\nu} = 2\, \varepsilon^{\mu\nu\alpha\beta}\, q_\alpha\, \frac{1}{\nu} \left\{ S_\beta\, g_1 + \left(S_\beta - \frac{S.q}{p.q}\, p_\beta \right) g_2 \right\}}$$

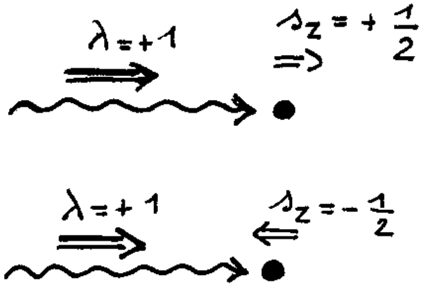
$$W_S^{\mu\nu} = 2 \left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1 + \frac{1}{p.q} \left(p^\mu - \frac{p.q}{q^2} q^\mu \right) \left(p^\nu - \frac{p.q}{q^2} q^\nu \right) F_2 \right\}$$

POLARIZATION OF e^- (HELICITY)
IS TRANSFERED TO VIRTUAL PHOTON
↓
3 POLARIZATION STATES

$$\left.\begin{array}{ll} \varepsilon^\mu(\lambda = +1) = -\frac{1}{\sqrt{2}}\,(0, 1, i, 0) & \text{RIGHT-HANDED} \\ \varepsilon^\mu(\lambda = -1) = \frac{1}{\sqrt{2}}\,(0, 1, -i, 0) & \text{LEFT-HANDED} \end{array}\right\} \text{CIRCULAR POL.}$$

$$\varepsilon^\mu(\lambda = 0) = \frac{1}{Q}\,(|\vec{q}|, 0, 0, \nu) \qquad \text{LONGITUDINAL}$$

• TRANSVERSE γ^* POLARIZATION



$\sigma_{3/2}$ TOTAL HELICITY : 3/2

$\sigma_{1/2}$ TOTAL HELICITY . 1/2

$$\sigma_{3/2} \sim \varepsilon^*_\mu(\lambda=+1)\,\varepsilon_\nu\,(\lambda=+1)\,W^{\mu\nu}(s_z=+\tfrac{1}{2})$$

$$\downarrow$$

$S^\alpha\ (0,0,0,1)$

PROTON SPIN POL. ALONG
z - AXIS IN ITS REST FRAME

$$\sigma_{1/2} \sim \varepsilon_\mu(\lambda=+1)\,\varepsilon_\nu\,(\lambda=+1)\,W^{\mu\nu}(s_z=-\tfrac{1}{2})$$

$$\downarrow$$

$S^\alpha\ (0,0,0,-1)$

$$\hookrightarrow\quad \varepsilon^*_\mu(\lambda=1)\,\varepsilon_\nu\,(\lambda=1)\,W_S^{\mu\nu}(s_z=+\tfrac{1}{2})$$

$$= -2\,(\varepsilon.\varepsilon^*)\,F_1 = 2\,F_1$$

$$\hookrightarrow\quad \varepsilon^*_\mu(\lambda=1)\,\varepsilon_\nu\,(\lambda=1)\,W_A^{\mu\nu}\,(s_z=+\tfrac{1}{2})$$

$$= \varepsilon^{*\mu}.\,\varepsilon^{\nu}.\left[2\,\varepsilon_{\mu\nu\alpha 3}\,q^\alpha\,\frac{1}{\nu}\left\{g_1+g_2\right\} + 2\,\varepsilon_{\mu\nu\alpha 0}\,q^\alpha\,\frac{1}{\nu^2}\,|\bar{q}|\,g_2\right]$$

$$= \varepsilon^{*\mu}\,\varepsilon^{\nu}\left[2\,\varepsilon_{0\mu\nu 3}\,(g_1+g_2) - 2\,\varepsilon_{0\mu\nu 3}\,\frac{|\bar{q}|^2}{\nu^2}\,g_2\right]$$

$$= \left[\frac{1}{2}\,(1)(+i)\,\varepsilon_{0123} + \frac{1}{2}\,(-i)(1)\,\varepsilon_{0213}\right]\cdot\left\{2(g_1+g_2) - 2\,\frac{|\bar{q}|^2}{\nu^2}g_2\right\}$$

$$= +i\;2\left\{g_1 - \frac{Q^2}{\nu^2}\,g_2\right\}$$

$$\therefore \quad \varepsilon_\mu(\lambda=+1)\,\varepsilon^*_\nu(\lambda=+1)\,\underbrace{W^{\mu\nu}(s_z=+\tfrac{1}{2})}_{\substack{\| \\ W_S^{\mu\nu}+i\,W_A^{\mu\nu}}}$$

$$= 2\left\{F_1 - \left(g_1 - \frac{Q^2}{\nu^2}g_2\right)\right\}$$

$$\boxed{\begin{aligned} \sigma_{3/2} &\sim 2\left\{F_1 - \left(g_1 - \frac{Q^2}{\nu^2}g_2\right)\right\} \\ \sigma_{1/2} &\sim 2\left\{F_1 + \left(g_1 - \frac{Q^2}{\nu^2}g_2\right)\right\} \end{aligned}}$$

$$\Downarrow$$

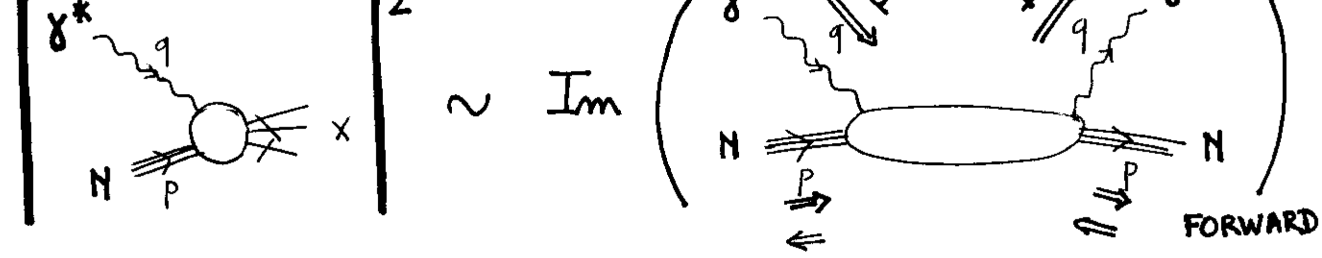
ASYMMETRY

$$\boxed{\frac{\sigma_{1/2}-\sigma_{3/2}}{\sigma_{1/2}+\sigma_{3/2}} = \frac{g_1 - \frac{Q^2}{\nu^2}g_2}{F_1}} \qquad \text{WITH } \frac{Q^2}{\nu^2} = \frac{(2Mx)^2}{Q^2}$$

$$\xrightarrow[Q^2 \gg]{} \frac{g_1}{F_1}$$

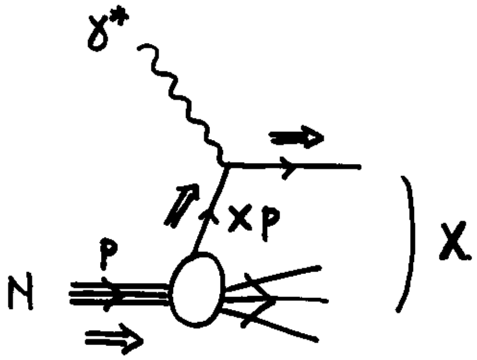
(cf. RESULT IN QUARK PARTON MODEL)

OPTICAL THEOREM

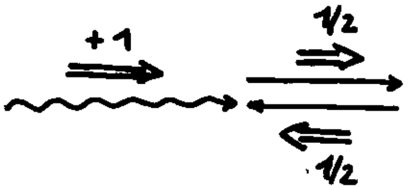


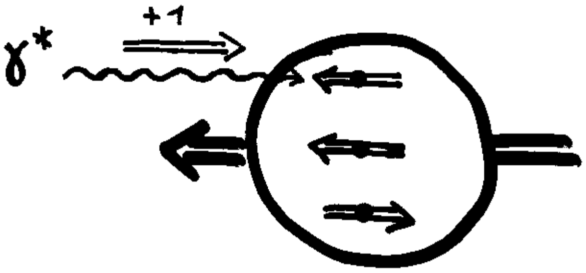
POLARIZED DIS IN QUARK PARTON MODEL

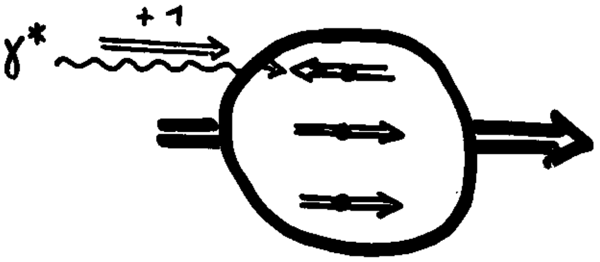
- $\boxed{g_1(x)}$



IN LIMIT $m_q \to 0$: PHOTON & QUARK HELICITIES ARE OPPOSITE







$\sigma_{1/2}$ $\sigma_{3/2}$

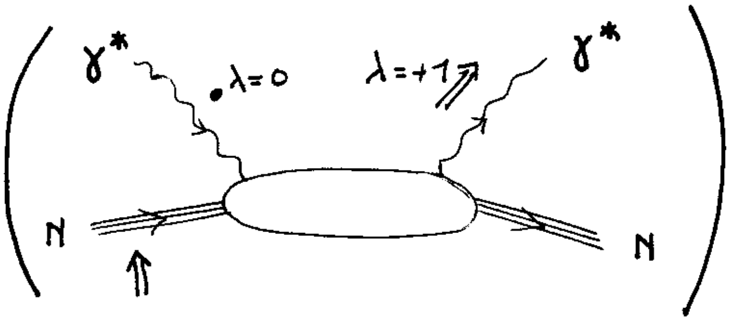
$q_\uparrow(x)$ $q_\downarrow(x)$

$$\sigma_{1/2} - \sigma_{3/2} \sim q_\uparrow(x) - q_\downarrow(x) = \underset{\Uparrow}{\Delta q(x)}$$

$$\boxed{g_1(x) = \frac{1}{2} \sum_q e_q^2 \, \Delta q(x)}$$

QUARK HELICIT
DISTRIBUTION

• LONGITUDINAL γ^* POLARIZATION

Im


TRANSVERSE SPIN

$S^\alpha(0,1,0,0)$

$$\sigma_{LT} \sim \varepsilon^{\mu *}(\lambda=+1)\,\varepsilon^{\nu}(\lambda=0)\,W_{\mu\nu}$$

$$= i\,\varepsilon^{\mu *}(\lambda=+1)\,\varepsilon^{\nu}(\lambda=0)\,W^{A}_{\mu\nu}$$

$$= 2i\,\varepsilon^{\mu *}(\lambda=+1)\frac{1}{\nu Q}\left\{|\bar{q}|^2\,\varepsilon_{\mu 0 3 \beta}\left[S^\beta g_1 + S^\beta g_2\right] + \gamma^2\,\varepsilon_{\mu 3 0 \beta}\left[S^\beta g_1 + S^\beta g_2\right]\right\}$$

$$\downarrow \quad \beta = 1 \Rightarrow \mu = 2$$

$$= 2i\left(\frac{i}{\sqrt{2}}\right)\frac{1}{\nu Q}\left\{|\bar{q}|^2\,\underbrace{\varepsilon_{2031}}_{-1} + \nu^2\,\underbrace{\varepsilon_{2301}}_{+1}\right\}(g_1+g_2)$$

$$= \sqrt{2}\,\frac{Q}{\nu}\,(g_1+g_2)$$

$$\sigma_{LT} \sim \sqrt{2}\,\frac{2Mx}{Q}\,(g_1+g_2)$$

ASYMMETRY

$$\boxed{\frac{2\sigma_{LT}}{\sigma_{1/2}+\sigma_{3/2}} = \sqrt{2}\left(\frac{2Mx}{Q}\right)\cdot\frac{g_1+g_2}{2F_1}}$$

⇒ NUCLEON SPIN STRUCTURE IN NON-RELATIVISTIC CONSTITUENT QUARK MODEL

- SU(6) SPIN-FLAVOR WAVEFUNCTION

$$|P_\uparrow\rangle = \frac{1}{\sqrt{6}}\left\{2\,|u_\uparrow u_\uparrow d_\downarrow\rangle - |u_\uparrow u_\downarrow d_\uparrow\rangle - |u_\downarrow u_\uparrow d_\uparrow\rangle\right\}$$

$$+ \ SU(6) \ \text{PERMUTATIONS}$$

- $\langle u_\uparrow\rangle = \langle P_\uparrow|\,\frac{1}{2}(1+\hat{\sigma}_3)\,\frac{1}{2}(1+\hat{\tau}_3)\,|P_\uparrow\rangle$

$$\boxed{\begin{aligned}\hat{\sigma}_3 &\equiv \sum_{i=1}^{3}\hat{\sigma}_3^{(i)} \quad \text{↳ FOR EACH OF 3 QUARKS}\\ \hat{\tau}_3 &\equiv \sum_{i=1}^{3}\hat{\tau}_3^{(i)}\end{aligned}}$$

$$\begin{aligned}\Delta u &= \langle u_\uparrow\rangle - \langle u_\downarrow\rangle\\ &= \frac{1}{2}\langle P_\uparrow|\,\hat{\sigma}_3\,(1+\hat{\tau}_3)\,|P_\uparrow\rangle\\ &= \frac{1}{2}\left\{\underbrace{\langle\hat{\sigma}_3\rangle}_{\frac{1}{6}(4+4-2)} + \underbrace{\langle\hat{\sigma}_3\hat{\tau}_3\rangle}_{\frac{1}{6}(4+4+2)}\right\}\end{aligned}$$

$$\frac{1}{6}\left(\underset{\text{QUARK 1}}{\overset{}{4}} + \underset{\text{QUARK 2}}{4} - \underset{\text{QUARK 3}}{2}\right)$$

$$\Delta u = \frac{1}{2}\left\{1+\frac{5}{3}\right\} \Rightarrow \boxed{\Delta u = \frac{4}{3}}$$

TOTAL NUCLEON SPIN CONTRIBUTION CARRIED BY U-QUARK

- $\langle\Delta d\rangle = \frac{1}{2}\left\{\langle\hat{\sigma}_3\rangle - \langle\hat{\sigma}_3\hat{\tau}_3\rangle\right\}$

$$= \frac{1}{2}\left\{1-\frac{5}{3}\right\} \Rightarrow \boxed{\Delta d = -\frac{1}{3}}$$

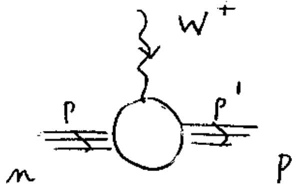
- IN NON-REL. QUARK MODEL $\underline{\underline{\Delta\Sigma \equiv \Delta u + \Delta d = 1}}$

AXIAL CURRENT

- $$\langle p | \frac{1}{2} \left(\bar{u} \gamma^\mu \gamma_5 u - \bar{d} \gamma^\mu \gamma_5 d \right) | p \rangle$$

$$\equiv \bar{N} \quad G_A(t) \; \gamma^\mu \gamma_5 \; N \quad + \quad \text{TERM WHICH VANISHES FOR } p' = p$$

AXIAL FORM FACTOR



$$t \equiv (p' - p)^2$$

$$G_A(t=0) \equiv g_A \quad \text{AXIAL COUPLING CONSTANT}$$

EXPERIMENTALLY : $g_A \approx 1.267$ FROM n - BETA DECAY

$$\hookrightarrow \quad \langle p_\uparrow | \frac{1}{2} \left(\bar{u} \gamma^i \gamma_5 u - \bar{d} \gamma^i \gamma_5 d \right) | p_\uparrow \rangle_{t=0}$$

$$= g_A \quad \bar{N}(p, \uparrow) \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} N(p, \uparrow)$$

$$= g_A \quad N^+(p, \uparrow) \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} N(p, \uparrow)$$

$\downarrow$ cfr. SPIN VECTOR OPERATOR

$$\Downarrow$$

$$g_A = \langle \hat{\sigma}_3 \hat{\tau}_3 \rangle$$

$$\boxed{g_A = \frac{5}{3} = \Delta u - \Delta d} \quad > 1.267 \; !$$

- RELATIVISTIC QUARK MODEL

$\Delta u \approx 1.0$, $\Delta d \approx -0.25$ $\Rightarrow$ $\Delta\Sigma \approx 0.75$

FIRST MOMENT OF g_1

- $$g_1^p(x) = \frac{1}{2}\Big\{ \frac{4}{9}(\Delta u + \Delta\bar{u}) + \frac{1}{9}(\Delta d + \Delta\bar{d}) + \frac{1}{9}(\Delta s + \Delta\bar{s}$$

  $$g_1^n(x) = \frac{1}{2}\Big\{ \frac{4}{9}(\Delta d + \Delta\bar{d}) + \frac{1}{9}(\Delta u + \Delta\bar{u}) + \frac{1}{9}(\Delta s + \Delta\bar{s})$$

  $$\boxed{\Gamma_1^{p,n} \equiv \int_0^1 dx \; g_1^{p,n}(x)}$$

- <u>$SU(3)_f$ AXIAL CHARGES</u>

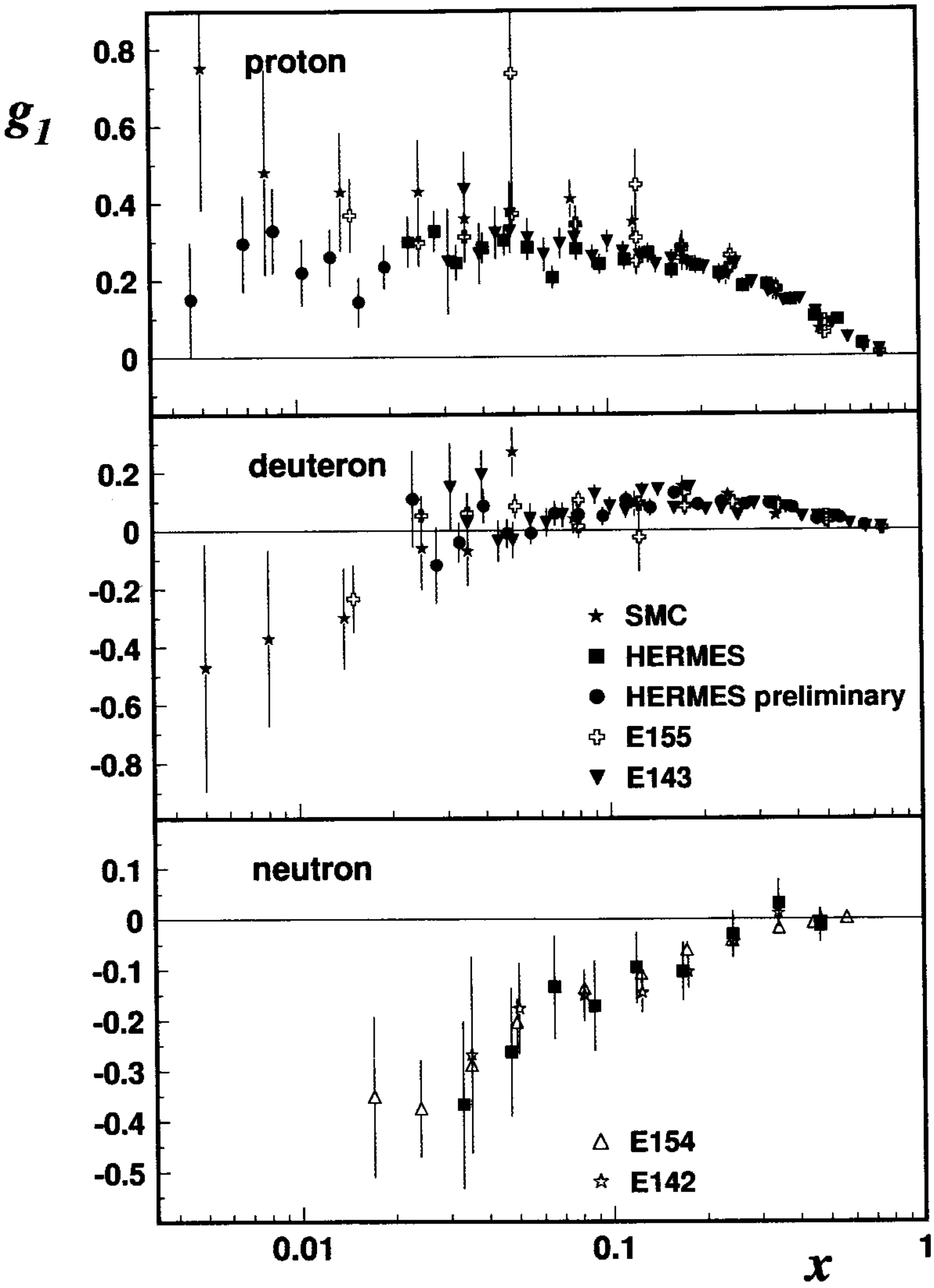
  $$\Delta q \equiv \int_0^1 dx \; \Delta q(x)$$

  $$\begin{aligned} a_0 &= (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) + (\Delta s + \Delta\bar{s}) \equiv \Delta\Sigma \\ a_3 &= (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d}) \\ a_8 &= (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) - 2(\Delta s + \Delta\bar{s}) \end{aligned}$$

  n β-DECAY $\Rightarrow$ $a_3 = g_A = 1.267$

  HYPERON β-DECAY $\Rightarrow$ $a_8 \simeq 0.58 \pm 0.03$

g_1 STRUCTURE FUNCTIONS of p, d, and n



BJORKEN SUM RULE

- $$\boxed{\begin{aligned} \Gamma_1^{P} - \Gamma_1 &= \frac{1}{6}\int_0^1 dx \left\{ (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d}) \right\} \\ &= \frac{1}{6}\, a_3 \\ &= \underbrace{\frac{1}{6}\, g_A}_{\simeq 0.21} \left[1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2) \right] \end{aligned}}$$

  BASED ON ISOSPIN SYMMETRY & CURRENT ALGEBRA

- EXPERIMENT

(E155 '00) $\Gamma_1^{P} - \Gamma_1^{n}(Q^2 = 5\,\text{GeV}^2) = 0.176 \pm 0.003 \pm 0.007$

THEORY : $\Gamma_1^{P} - \Gamma_1^{n}(Q^2 = 5\,\text{GeV}^2) = 0.181 \pm 0.005$ ✓

ELLIS - JAFFE SUM RULES

- $$\boxed{\Gamma_1^{p,n} = \frac{1}{12}\left\{\frac{4}{3}a_0 \pm a_3 + \frac{1}{3}a_8\right\}}$$

- ELLIS - JAFFE : ASSUME $\Delta s = \Delta \bar{s} = 0$

$$\Downarrow$$

$$a_0 = a_8 \iff \Delta\Sigma \simeq 0.6$$

$$\Gamma_{1,EJ}^{p,n} = \frac{1}{12}\left\{\frac{5}{3}a_8 \pm a_3\right\}$$

$$Q^2 = 5\,GeV^2 \quad \nearrow \Gamma_{1,EJ}^{p} = 0.163 \pm 0.004$$

$$\searrow \Gamma_{1,EJ}^{n} = -0.019 \pm 0.004$$

- EXPERIMENT (SLAC, SMC, HERMES)

(E155)

$$\Gamma_1^{p}(Q^2 = 5\,GeV^2) = 0.118 \pm 0.004 \pm 0.007$$

$$\Gamma_1^{n}(Q^2 = 5\,GeV^2) = -0.058 \pm 0.005 \pm 0.008$$

ELLIS- JAFFE SUM RULES **NOT** VERIFIED

EXPERIMENT $\Rightarrow$ $\boxed{\Delta\Sigma \simeq 0.3}$

NOTE HOWEVER
SCHEME DEPENDE
AT NLO