Let's create the simplest model we can imagine

$$f_\ell(t) = \frac{1}{2}\int_{-1}^{1} A(z,t)\, P_\ell(z)\, dz$$

$$f(\ell,t) = \frac{1}{2\pi}\int_{z_0}^{\infty}\int_{-1}^{1} D_s A(z',t)\, \frac{P_\ell(z)}{z'-z}\, \hat{dz}\, dz' + \frac{1}{2\pi}\int_{-\infty}^{-z_0}\int_{-1}^{1} D_u A(z',t)\, \frac{P_\ell(z')}{z'-z}\, \hat{dz}\, dz'$$

$$Q_\ell(-z) = (-)^{\ell+1}\, Q_\ell(z) \qquad Q_\ell(z) = \frac{1}{2}\int_{-1}^{1} \frac{P_\ell(z_t)}{z-z_t}\, dz_t$$

$$f^{\pm}(\ell,t) = \frac{1}{\pi}\int_{z_0}^{\infty} \Big[D_s A(z',t) \ \pm\ D_u A(z',t) \Big]\, Q_\ell(z')\, dz'$$

$$e^{-i\pi\ell}$$

$f^{+}(\ell,t)$ matches $f_\ell(t)$ ℓ even

$f^{-}(\ell,t)$ " " " odd

$\pi^- p \to \eta n$: simplest model

Step 1 (signature $\tau_{a_2} = +1$):

$$H_f = \beta_f \frac{\tau_{a_2} + e^{-i\pi\alpha_{a_2}(t)}}{2\sin\pi\alpha_{a_2}(t)} s^{\alpha_{a_2}(t)}$$

$$H_{nf} = \beta_{nf} \frac{\tau_{a_2} + e^{-i\pi\alpha_{a_2}(t)}}{2\sin\pi\alpha_{a_2}(t)} s^{\alpha_{a_2}(t)}$$

i.e. no t-dependence in residues. Phenomenology: we should get $\beta_f > \beta_{nf}$.

Remove the ghost poles with $m^2 < 0$

$\pi^- p \to \eta n$: removing ghosts

Step 2:

$$\beta_f \to \beta_f(t) = \alpha_{a_2}(\alpha_{a_2}+2)(\alpha_{a_2}+4)\cdots$$

idem for $\beta_{nf} \to \beta_{nf}(t)$.

$$A_{H_t}(s,t) = \sum_{J=\max\{|\lambda|,|\lambda'|\}}^{\infty} (2J+1) A^J_{H_t}(t) d^J_{\lambda\lambda'}(z_t)$$

We are lacking t-dependence (especially at small $|t|$)

Angular momentum conservation: t-singularities in the s-channel helicity amplitudes can only stem from

$$
\begin{aligned}
d^J_{\mu\mu'}(z_s) \sim \xi_{\mu\mu'}(z_s) &\equiv \left(\frac{1-z_s}{2}\right)^{\frac{1}{2}|\mu-\mu'|} \left(\frac{1+z_s}{2}\right)^{\frac{1}{2}|\mu+\mu'|} \\
&\sim (-t')^{\frac{1}{2}|\mu-\mu'|}
\end{aligned}
$$

since

$$
\begin{aligned}
z_s &= \cos\theta_s \\
&= \frac{s^2 + s(2t-\Sigma) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{4s q_{s12} q_{s34}} \\
&= 1 + \frac{2st'}{\mathcal{S}_{12}(s)\mathcal{S}_{34}(s)}
\end{aligned}
$$

where $t' = t - t_{\text{min}}$ and $t_{\text{min}} = t(z_s = +1)$.
Hence, angular momentum conservation requires at least

$$
\beta_{\mu_1\mu_2\mu_3\mu_4} \sim \sqrt{-t'}^{|\mu-\mu'|} \qquad (t = t' \text{ for } s \to \infty)
$$

Reggeon: definite parity. In the t-channel we have

$$P\,|J,\lambda,\lambda_1,\lambda_3\rangle = P_1 P_3 (-1)^{J-\sigma_1-\sigma_3}\,|J,\lambda,-\lambda_1,-\lambda_3\rangle$$

$$|J,\lambda,\lambda_1,\lambda_3,\eta\rangle \equiv \frac{1}{\sqrt{2}}\left[|J,\lambda,\lambda_1,\lambda_3\rangle + \eta P_1 P_3 (-1)^{\sigma_1+\sigma_3}\,|J,\lambda,-\lambda_1,-\lambda_3\rangle\right]$$

$$A^{\eta}_{H_s} = A_{\mu_4\mu_3,\mu_2\mu_1} + \eta P_2 P_4 (-1)^{\sigma_4+\sigma_2}(-1)^{\mu_2-\mu_4} A_{-\mu_4\mu_3,-\mu_2\mu_1}$$
$$A^{-\eta}_{H_s} = A_{\mu_4\mu_3,\mu_2\mu_1} - \eta P_2 P_4 (-1)^{\sigma_4+\sigma_2}(-1)^{\mu_2-\mu_4} A_{-\mu_4\mu_3,-\mu_2\mu_1}$$

$$A^{\eta}_{H_s} = A_{\mu_4\mu_3,\mu_2\mu_1} + \eta P_2 P_4 (-1)^{\sigma_2+\sigma_4}(-1)^{\mu_4-\mu_2} A_{-\mu_4\mu_3,-\mu_2\mu_1}$$
$$A^{-\eta}_{H_s} = A_{\mu_4\mu_3,\mu_2\mu_1} - \eta P_2 P_4 (-1)^{\sigma_2+\sigma_4}(-1)^{\mu_4-\mu_2} A_{-\mu_4\mu_3,-\mu_2\mu_1}$$

Hence, a definite parity (η) Reggeon pole requires for $t \to 0$

$$A_{\mu_4\mu_3,\mu_2\mu_1} \sim \sqrt{-t}^{|\mu-\mu'|} \qquad A_{-\mu_4\mu_3,-\mu_2\mu_1} \sim \sqrt{-t}^{|\mu+\mu'|}$$

$$A^{\eta}_{H_s} \to A^{-\eta}_{H_s} = 0$$

$$\sqrt{-t}^{|(\mu_1-\mu_2)-(\mu_3-\mu_4)|}\beta_{\mu_4\mu_3\mu_2\mu_1} = \pm\sqrt{-t}^{|(\mu_1+\mu_2)-(\mu_3+\mu_4)|}\beta_{-\mu_4\mu_3-\mu_2\mu_1}$$
$$\sqrt{-t}^{|\omega-\omega'|}\beta_{\mu_4\mu_3\mu_2\mu_1} = \pm\sqrt{-t}^{|\omega+\omega'|}\beta_{-\mu_4\mu_3-\mu_2\mu_1}$$

$$\beta_{\mu_4\mu_3\mu_2\mu_1}(t) = (-t)^{\frac{1}{2}(\max\{|\omega+\omega'|,|\omega-\omega'|\}-|\omega-\omega'|)}\, g_{\mu_4\mu_3\mu_2\mu_1}(t)$$

Reggeon: factorization. $\beta_{\mu_4\mu_3\mu_2\mu_1} = \beta_{\mu_4\mu_2}\beta_{\mu_3\mu_1}$

$$\beta_{H_s}(t) = \sqrt{-t}^{|\omega|+|\omega'|}\gamma_{H_s}(t)$$

$\pi^- p \to \eta n$: parity and factorization

Step 3:

$$\beta_{\mu_1\mu_2\mu_3\mu_4}(\pi^- p \underset{a_2}{\to} \eta n) = \beta^{a_2\pi^-\eta}_{\mu_1\mu_3}\beta^{a_2 pn}_{\mu_2\mu_4}$$

Together with parity

$$\beta_{\mu_i\mu_f}(t) \sim \sqrt{-t}^{|\mu_f-\mu_i|}$$

In words: $\sqrt{-t}$ for each unit of helicity flip at a vertex. Hence (NOTE: $\sqrt{-t}^{|\mu-\mu'|} = \sqrt{-t}^{|\omega|+|\omega'|}$)

$$H_f \sim \sqrt{-t}$$
$$H_{nf} \sim 1$$

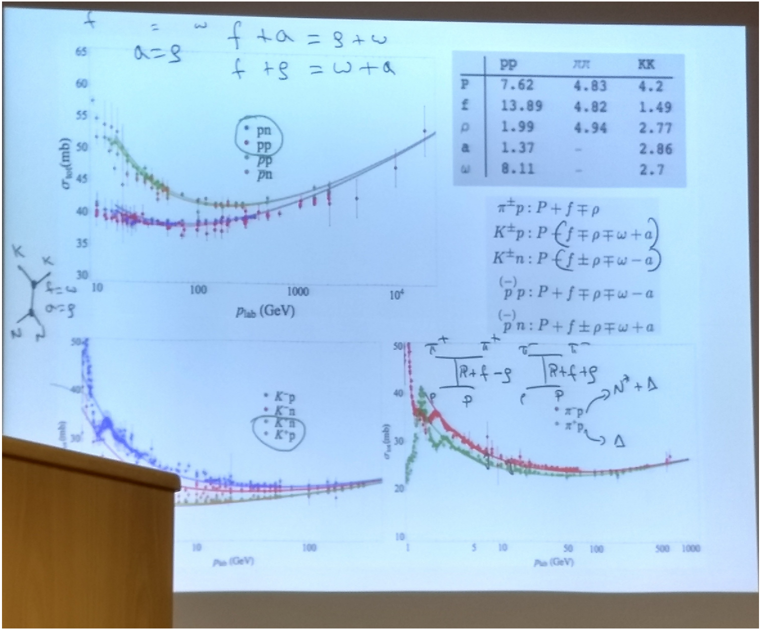
In principle

$$\beta_{\mu_i\mu_f}(t) = \sqrt{-t}^{|\mu_f-\mu_i|}P(t)$$

where $P(t)$ is a polynomial in t.

Other reactions

- πN CEX
- KN CEX


f + a = ρ + ω
f + ρ = ω + a
a = ρ
pn
pp
p̄p
p̄n
σ_tot(mb)
p_lab (GeV)
	pp	ππ	KK
P	7.62	4.83	4.2
f	13.89	4.82	1.49
ρ	1.99	4.94	2.77
a	1.37	-	2.86
ω	8.11	-	2.7
π±p : P + f ∓ ρ
K±p : P + f ∓ ρ ∓ ω + a
K±n : P + f ± ρ ∓ ω − a
(−)p p : P + f ∓ ρ ∓ ω − a
(−)p n : P + f ± ρ ∓ ω + a
K
K
f = ω
a = ρ
N
N
K⁻p
K⁻n
K⁺n
K⁺p
π⁺
π⁺
π⁻
π⁻
P + f − ρ
P + f + ρ
p
p
π⁻p
π⁺p
N* + Δ
Δ
p_lab (GeV)

SUMMARY

To remember

1. Signature factor
2. Remove ghost poles ($m^2 < 0$)
3. Small $|t|$: $\beta_{\mu_i \mu_f} \sim \sqrt{-t}^{|\Delta\mu|}$
4. EXD, definite parity and factorization have non-trivial concequences

Deviations: non-factorizable contributions

1. At small $|t|$, we found $\beta_{\mu_i \mu_f} \sim \sqrt{-t}^{|\Delta\mu|}$. *Charged pion* exchange shows deviations (see photoproduction). Can be related to the low energy amplitude using FESR!
2. Dip in πN data doesn't go to 0 like the model. The dip is filled up.

Additional ingredients in the literature:

1. Absorption [Perl]
2. Cuts [Irving and Worden]
3. Daughters [Collins]
4. Conspirators [Cohen-Tannoudji, Nuovo Cim. A55 (1968)]
5. *s*-channel electric terms [GLV, Nucl. Phys. A627 (1997)]

For an overview of all: [Irving and Worden Phys. Rept. 34 (1977)]

Cuts

Why do we know Regge poles should provide us a good first-order data description?

- We get the right s-dependence (s-dependence of cuts are modified, so one cut cannot reproduce correct s-dependence).
- Factorization often not too bad.
- Reggeons have definite S, B and I, cuts don't necessarily (unless RxP). Need to fine tune cuts for each reaction individually.
- Faith (or at least hope).

How to make a model in general?

Building a model with 'helicity' amplitudes

1. Derive the possible quantum numbers
2. Determine independent amplitudes (parity, time reversal)
3. Derive definite parity amplitudes in s or t-channel

$$\hat{A}^{\eta}_{H_t} = \hat{A}_{H_t} + \eta P_1 P_3 (-1)^{\lambda' + M + \sigma_1 + \sigma_3} \hat{A}_{\overline{H}_t}$$

4. Fit the residues, imposing factorization (start with EXD)
5. Extrapolate residues to the pole and compare to literature

First order approximation using single-particle exchange model

1. Feynman diagrams
2. 'Covariant' reggeization

‘Reggeization’ procedure

How do we connect Feynman diagrams to Reggeon diagrams?
[Nucl. Phys. A627 (1997) 645]

Feynman diagrams (Gribov Section 8.4)

$$A_{H_s}(s,t) = \Gamma_{\{\kappa\}}(p_1,p_3)\frac{\sum_{m=1}^{2J_e+1} e^m_{\{\kappa\}}(q) e^m_{\{\nu\}}(q)}{m_e^2 - t}\Gamma_{\{\nu\}}(p_2,p_4)$$

$$\xrightarrow[s\to\infty]{} \beta_{H_s,J_e}(t)\frac{s^{J_e}}{m_e^2 - t}$$

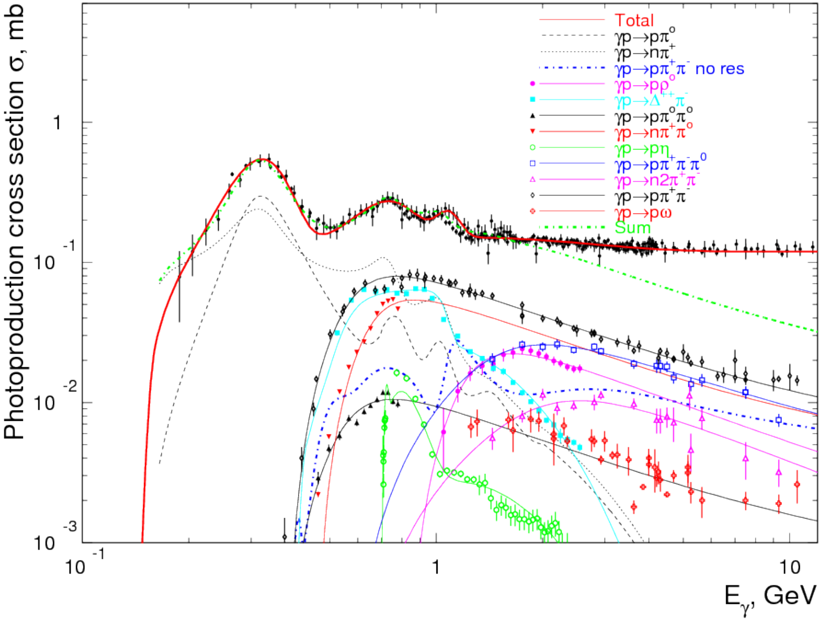
Reggeon diagrams

$$A_{H_s}(s,t) = \beta_{H_s}(t)\frac{\tau + e^{-i\pi\alpha}}{2\sin\pi\alpha}s^{\alpha}$$

“Covariant” reggeization

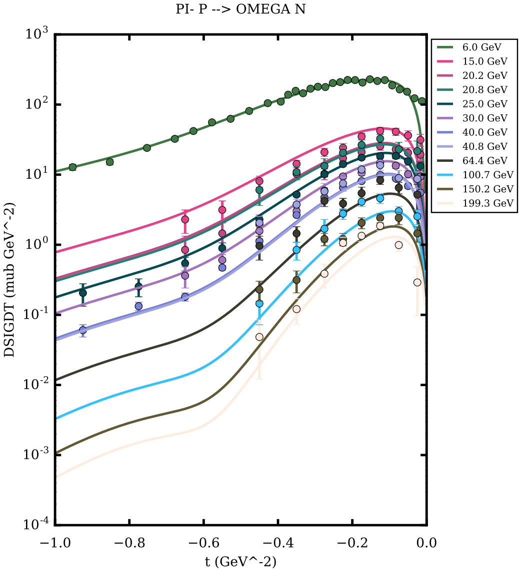
$$\frac{1}{m_e^2 - t} \rightarrow \frac{1}{\Gamma(\alpha + 1 - l_e)}\frac{\pi\alpha'}{2}\frac{\tau + e^{-i\pi\alpha}}{\sin\pi\alpha}s^{\alpha - J_e} \qquad (1)$$

BE AWARE: we assume $\beta(t)$ to be that of the lightest meson on the trajectory!



Photoproduction cross section σ, mb
E_γ, GeV
1
10^{-1}
10^{-2}
10^{-3}
10^{-1}
1
10
Total
$\gamma p \to p\pi^o$
$\gamma p \to n\pi^+$
$\gamma p \to p\pi^+\pi^-$ no res
$\gamma p \to p\rho^o$
$\gamma p \to \Delta^{++}\pi^-$
$\gamma p \to p\pi^o\pi^o$
$\gamma p \to n\pi^+\pi^o$
$\gamma p \to p\eta$
$\gamma p \to p\pi^+\pi^-\pi^0$
$\gamma p \to n2\pi^+\pi^-$
$\gamma p \to p\pi^+\pi^-$
$\gamma p \to p\omega$
Sum

A tougher cookie



Beam asymmetry: $\overrightarrow{\gamma} p \to \pi^- \Delta^{++}$