Revealing the Black-Body Regime of Small-x Deep-Inelastic Scattering through Final-State Signals

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(Received 16 April 2001; published 18 October 2001)

We derive the major characteristics of inclusive and diffractive final states in deep-inelastic scattering off heavy nuclei for the high-energy (small-x) kinematics in which the limit of complete absorption is reached for the dominant hadronic fluctuations in the virtual photon (the black-body limit of the process). Both the longitudinal and transverse distributions of the leading hadrons are found to be strikingly different from the corresponding ones within the leading-twist approximation, and hence provide unambiguous signals for the onset of the black-body limit.

DOI: 10.1103/PhysRevLett.87.192301 PACS numbers: 24.85.+p, 12.38.Aw, 13.60.Hb, 25.30.Dh

The black-body limit (BBL) of deep-inelastic scattering (DIS) from a heavy nucleus was first considered by Gribov in 1969 [1], before the discovery of perturbative QCD (PQCD). He assumed that at ultrahigh energies each hadronic configuration in the photon scatters with the same strength, and is absorbed by the nucleus. If the (virtual) photon is expressed as a linear superposition of partonic components (characterized by the transverse and longitudinal momenta of the individual partons) then the interaction with the target does not mix up these partonic components, i.e., they are eigenstates of the scattering matrix. This implies that the interaction is diagonal for each hadronic fluctuation, and features such as transverse sizes, invariant masses, and longitudinal momentum distributions are preserved by the BBL interaction. Away from the BBL, the diagonality is grossly violated (see, e.g., [2,3]).

The proton structure function, $F_2^p(x, Q^2)$, measured in DIS at DESY ep collider (HERA) exhibits a rapid rise with the energy of the photon-proton subprocess, W, at high energies, or small $x = -q^2/2Pq$, where P is the proton's momentum, and $q, q^2 = -Q^2$ are the photon's momentum and virtuality. The standard leading-twist approximation (LTA) of PQCD describes F_2^p extremely successfully in terms of a convolution of perturbatively calculable hard coefficient functions with quark and gluon parton distributions, which evolve in scale according to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [4] evolution equations. The rise of F_2^p is translated into a rapid rise of the gluon and sea-quark distributions of the proton with $1/x$ at small x. To leading-log accuracy in Q^2, this leads to a numerically large density of partons at small x, which calls into question the applicability of the LTA at even smaller x (for recent reviews, see [5–8]). This concern is best understood in the target rest frame by considering the inelastic interaction cross sections for given partonic configurations of the virtual photon. In particular, the expression for the scattering of a small $q\bar{q}$ dipole of transverse diameter $d_\perp$ at small x is given, in terms of the leading-log gluon distribution of the nuclear target xg^A, by

$$\hat{\sigma}^{\rm inel}_{\rm PQCD}(d_\perp^2, x) = \frac{\pi^2}{3} d_\perp^2 \alpha_s(\bar{Q}^2) x g^A(x, \bar{Q}^2), \tag{1}$$

in which the scale $\bar{Q}^2 = \lambda/d_\perp^2$, with λ a logarithmic function of Q^2. For large gluon densities, this leads to a conflict with unitarity of the S matrix for the interaction of spatially small wave packets in the virtual photon which are almost completely absorbed [9] (especially in the case of heavy nuclei, where the gluon density is enhanced by nucleon number A). This may be conveniently expressed in terms of the geometrical limit for the inelastic "small dipole-nucleus" interaction of

$$\hat{\sigma}^{\rm inel}_{\rm PQCD} \leq \hat{\sigma}_{\rm black} = \pi R^2_{\rm target}, \tag{2}$$

corresponding to the BBL for the total cross section of $2\pi R^2_{\rm target}$. Numerical studies performed using Eqs. (1) and (2) have demonstrated that interactions at the upper range of HERA energies in gluon-induced processes may be fairly close to the BBL in a wide range of impact parameters (see, e.g., [8,10]). This is consistent with an analysis [11] of current fits to HERA diffractive data, which leads to diffractive gluon densities which are significantly larger than the quark ones (see also [12]).

For heavy nuclei and central impact parameters, the perturbative x, Q^2 region, in which the interaction should be close to the BBL in both quark and gluon channels, is much broader than for nucleons, where scattering amplitudes at peripheral impact parameters are important and are far from the BBL. For example, for DIS on Pb at $x \sim 10^{-3}$, at small impact parameters the BBL may be reached for $Q^2 \leq 7(15)$ GeV^2 for the quark (gluon) channel, that is, for the region where $\ln Q^2/\Lambda^2_{\rm QCD} \geq \ln x_0/x$ ($x_0 \sim 0.05-0.1$ is the starting scale of the evolution at small

x). This indicates that DGLAP becomes inapplicable in an x, Q^2 range, where the expansion of a small dipole in the transverse direction (diffusion) due to gluon bremsstrahlung is still small and hence the coupling constant is small. This indicates an exciting possibility of the existence of the BBL in QCD in the perturbative domain for the kinematics which will be probed at Cern Large Hadron Collider (LHC), and which could also be probed at Relativistic Heavy-Ion Collider (RHIC) (at BNL) in pA collisions and at HERA/THERA in eA mode.

The experimental and theoretical challenge is to find unambiguous signals for the BBL. One problem is that predictions for the inclusive structure functions do not contain "smoking gun" signatures since they must account for leading twist shadowing, uncertainties in the input gluon densities, etc. This is especially acute in the case of the nucleon scattering where, even in the BBL, the structure function $F_2^p(x, Q^2)$ is expected to increase rather rapidly, $F_2^p \propto \ln^3 1/x$, of which $\ln^2 1/x$ is due to a Froissart-like increase of the radius of interaction $r_{\text{int}} \propto \ln 1/x$ ($\alpha'_{\text{eff}} \propto \ln 1/x$) and the remaining $\ln 1/x$ reflects the infinite renormalization of the electric charge [1]. As discussed in [10], the latter results from the fact that the photon couples in a pointlike manner to quarks. This also implies that for any energy there will always be small configurations in the photon wave function which have not yet reached the BBL. However, we expect that the BBL formulas will give a reasonable description of structure functions and hard diffractive processes for heavy nuclear targets, where the blackness of the interaction is enhanced by the number of nucleons. The growth in the radius is essentially screened by the nuclear environment so that, for large nuclei, $F_2^A \propto \ln 1/x$.

To deduce predictions for the properties of the final states it is useful to work within the $q\bar{q}$ dipole approximation for the photon wave function. In the BBL the elastic and inelastic contributions are equal so that the total cross section is given by twice the inelastic contribution (summed over all dipole configurations):

$$\sigma_{\gamma^* A} = 2 \int \frac{dz\, d^2 p_\perp}{2(2\pi)^3} |\psi_{\gamma^*}^{q\bar{q}}(z, p_\perp, Q^2)|^2 \pi R_A^2 . \qquad (3)$$

Here z is the fraction of the photon plus momentum, $q_+ = q_0 + q_3$, carried by the quark, $z = p_+/q_+$, and $p_\perp$ is the modulus of its transverse momentum in the plane perpendicular to $\vec{q}$ [the antiquark has $(1 - z)$ and $-p_\perp$). In Eq. (3) we implicitly use the fact that the interaction is diagonal in z and $p_\perp$ and black for each configuration. It is convenient to define

$$M^2 = (p_\perp^2 + m_q^2)/[z(1 - z)] \qquad (4)$$

for the mass squared of the $q\bar{q}$ system and angle θ between the direction of the momentum of the quark in the center of mass frame and the photon direction (the transverse plane is defined to be perpendicular to this). Neglecting m_q^2 compared to M^2, Q^2 gives

$$\sin\theta = 2p_\perp/\sqrt{M^2}, \qquad z = (1 + \cos\theta)/2 . \qquad (5)$$

Hence for the nuclear structure functions we get

$$F_T^A(x, Q^2) = \int_0^{M_{\max}^2} dM^2 \frac{2\pi R_A^2}{12\pi^3} \frac{Q^2 M^2 \rho(M^2)}{(M^2 + Q^2)^2} \times \int_{-1}^{1} d\cos\theta \, \frac{3}{8} (1 + \cos^2\theta) , \qquad (6)$$

$$F_L^A(x, Q^2) = \int_0^{M_{\max}^2} dM^2 \frac{2\pi R_A^2}{12\pi^3} \frac{Q^4 \rho(M^2)}{(M^2 + Q^2)^2} \times \int_{-1}^{1} d\cos\theta \, \frac{3}{4} \sin^2\theta , \qquad (7)$$

$$\rho(M^2) = \sigma^{e^+e^- \to \text{hadrons}}/\sigma^{e^+e^- \to \mu^+\mu^-} . \qquad (8)$$

Integration over θ gives the transverse and longitudinal structure functions in the BBL as an integral over produced masses. If one takes into account only the suppression induced by the square of the nuclear form factor in the rescattering amplitude, then $M_{\max}^2 \le W^2/(m_N R_A)$ and Eqs. (6) and (7) coincide with the original result of [1]. However, in QCD, due to color transparency, $M_{\max}^2$ is determined by the unitarity constraint and is substantially smaller than $W^2/(m_N R_A)$ (for a recent discussion see [8]). The contribution of larger M^2 is suppressed by a smaller interaction cross section ($\propto 1/M^2$, in the LTA with small enough gluon densities). In order to visualize manifestations of the BBL, in the following discussion we neglect this contribution, which has a stronger A dependence than the BBL term. The inclusion of $\rho(M^2)$ corrects the dipole formula, for which Eq. (3) holds, for higher order corrections (in α_s) contributing at a given M^2. A relation between $\rho(M^2)$ and nuclear structure functions was also suggested in [13]. We also obtain, for the ratio of the deeply virtual Compton scattering and elastic amplitudes,

$$\frac{M(\gamma_T^* A \to \gamma A)}{M(\gamma_T^* A \to \gamma_T^* A)} = \frac{\int_0^{M_{\max}^2} \frac{dM^2\, \rho(M^2)}{(M^2+Q^2)}}{\int_0^{M_{\max}^2} \frac{dM^2\, \rho(M^2) M^2}{(M^2+Q^2)^2}} . \qquad (9)$$

The ability to neglect nondiagonal transitions in the BBL [1] justifies the removal of the integral over the masses in Eqs. (6) and (7) and, since diffraction is 50% of the total cross section, we immediately find for the spectrum of diffractive masses:

$$\frac{dF_T^{D(3)}(x, Q^2, M^2)}{d\cos\theta\, dM^2} + \frac{\epsilon\, dF_L^{D(3)}(x, Q^2, M^2)}{d\cos\theta\, dM^2} = \frac{\pi R_A^2}{12\pi^3} \frac{Q^2 \rho(M^2)}{(M^2 + Q^2)^2} \left(\frac{3}{8} M^2 (1 + \cos^2\theta) + \frac{3}{4} \epsilon Q^2 \sin^2\theta \right) , \qquad (10)$$

where ϵ is the photon polarization. Thus, the spectrum of hadrons in the center of mass of the diffractively produced system should be the same as in e^+e^- annihilation. Hence, the dominant diffractively produced final state will have two jets (with fractal substructure) with a distribution in the center of mass emission angle proportional to $1 + \cos^2\theta$ for the transverse case and $\sin^2\theta$ for the longitudinal case. The diffractive cross section, integrated over θ, is obtained from Eq. (10) by removing the $\frac{3}{8}(1 + \cos^2\theta)$, $\frac{3}{4}\sin^2\theta$ factors. It follows from Eq. (10) that in the BBL diffractive production of high $p_\perp$ jets is $\propto M^2$ (while in the LTA it is $\propto \ln Q^2$) and hence is enhanced: $\langle p_\perp^2(\text{jet})\rangle_T = 3M^2/20$, $\langle p_\perp^2(\text{jet})\rangle_L = M^2/5$.

The relative rate and distribution of jet variables for three jet events (originating from $q\bar{q}g$ configurations) will also be the same as in e^+e^- annihilation and hence is given by the standard expressions for the process $e^+e^- \to q\bar{q}g$. In addition, in the BBL, the production of jets is flavor democratic (weighted by quark charges but unrelated to the quark content of the target).

An important advantage of the diffractive BBL signal is that these features of the diffractive final state should hold for $M^2 \leq Q^2_{\rm BBL}$ even for $Q^2 \geq Q^2_{\rm BBL} \gg \Lambda^2_{\rm QCD}$ because configurations with transverse momenta $\leq Q_{\rm BBL}/2$ are perturbative but still interact in the black regime (and correspond to transverse size fluctuations for which the interaction is already black).

Another interesting feature of the BBL is the spectrum of leading hadrons in the photon fragmentation region. It is essentially given by Eq. (10). Since the distributions in z (or equivalently θ) do not depend on M, the jet distribution in z is given by

$$\frac{d(\sigma_T + \epsilon\sigma_L)}{dz} \propto \frac{M^2}{Q^2}\frac{1 + (2z - 1)^2}{8} + \epsilon(z - z^2). \tag{11}$$

Exclusive vector meson production in the BBL corresponds, in a sense, to a resurrection of the original vector meson dominance model [14] without off-diagonal transitions. The amplitude for the vector meson-nucleus interaction is proportional to $2\pi R_A^2$ (since each configuration in the virtual photon interacts with the same BBL cross section). This is markedly different from the requirements [3] for matching the generalized vector dominance model (see, e.g., [2]) with QCD in the scaling limit, where the off-diagonal matrix elements are large and lead to strong cancellations. We can factorize out the universal black interaction cross section for the dipole interaction from the overlap integral between wave functions of virtual photon and vector mesons to find, for the dominant electroproduction of vector mesons,

$$\frac{d\sigma^{\gamma_T^* + A \to V + A}}{dt} = \frac{M_V^2}{Q^2}\frac{d\sigma^{\gamma_L^* + A \to V + A}}{dt} = \frac{(2\pi R_A^2)^2}{16\pi}\frac{3\Gamma_V M_V^3}{\alpha(M_V^2 + Q^2)^2}\frac{4|J_1(\sqrt{-t}\,R_A)|^2}{-tR_A^2}, \tag{12}$$

where Γ_V is the electronic decay width $V \to e^+e^-$, α is the fine-structure constant. Thus the parameter-free prediction is that, in the BBL (complete absorption) at large Q^2, vector meson production cross sections have a $1/Q^2$ behavior. This is in stark contrast to the asymptotic behavior of $1/Q^6$ predicted in PQCD [15] since a factor $1/Q^4$, due to the square of the cross section of interaction of a small dipole with the target (color transparency), disappears in the BBL.

In the LTA, the factorization theorem is valid and leads to a universal (i.e., target-independent) spectrum of leading particles for scattering off partons of the same flavor. Fundamentally, this can be explained by the fact that, in the Breit frame, the fast parton which is hit by the photon carries practically all of its light-cone momentum ($z \to 1$). As a result of QCD evolution, this parton acquires virtuality, $\sim Q^2$, and a rather large transverse momentum, k_t (which is still $\ll Q^2$). So, in PQCD, quarks and gluons emitted in the process of QCD evolution and in the fragmentation of heavily virtual partons together still carry all the photon momentum. In contrast, in the BBL, the leading particles originate from coherent diffraction (peripheral collisions) and central highly inelastic collisions. These contributions come from the fragmentation of a highly virtual $q\bar{q}$ pair with similar light-cone fractions of longitudinal momenta and large relative transverse momenta [see, e.g., Eqs. (10) and (11)].

The inclusive spectrum of leading hadrons can be assumed, neglecting energy losses, as being due to the independent fragmentation of quark and antiquark of virtualities $\geq Q^2$, with z and $p_\perp$ distributions given by Eqs. (10) and (11) (cf. diffractive production of jets discussed above). Note that energy losses of partons calculated in the limit of small nuclear parton densities do not lead to a change of the z fraction carried by a parton, and hence do not violate the LTA (see, e.g., [16]). In the BBL, energy losses may be larger, further suppressing the spectrum as compared to Eq. (13).

The independence of fragmentation is justified because large transverse momenta of quarks dominate in the photon wave function [cf. Eqs. (4)–(7)] and because of the weakness of the final-state interaction between q and $\bar{q}$, since the α_s is small and the rapidity interval is of the order of 1. Obviously, this leads to a gross depletion of the leading hadron spectrum as compared to the LTA situation in which leading hadrons are produced in the fragmentation region of the parton which carries essentially all momentum of the virtual photon. If we neglect gluon emissions in the photon wave function, we find, for instance, for the differential multiplicity of leading hadrons, $dN^{\gamma_T^*/h}/dz$, produced by transverse virtual photons, in the BBL,

$$\frac{dN^{\gamma_T^*/h}}{dz} = 2\int_z^1 D^{q/h}\left(\frac{z}{y}\right)\frac{3}{4}[1 + (2y - 1)^2]\,dy. \tag{13}$$

Here $D^{q/h}(z/y, Q^2)$ is the fragmentation function of a quark, with any flavor q, into hadrons. To simplify

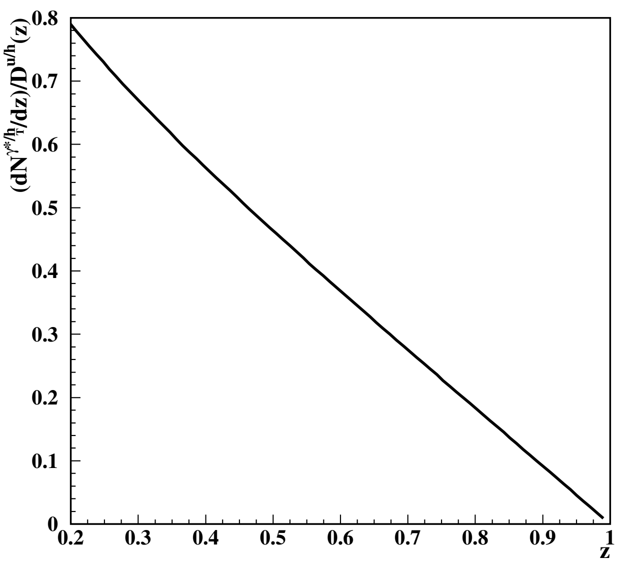

FIG. 1. The ratio $(dN^{\gamma_T^*/h}/dz)/D^{u/h}(z, Q^2)$ as a function of z at $Q^2 = 2\ \text{GeV}^2$. $D^{u/h}(z, Q^2)$ is from Ref. [17].

Eq. (13) we used $D^{u/h} = D^{d/h}$ and neglected a rather small difference between $D^{u/h}$ and $D^{s,c/h}$.

Calculations using Eq. (13) show a strong suppression of the leading hadron spectrum as compared to the LTA predictions (see Fig. 1). Moreover we expect a further softening as Q^2 increases, resulting from increased parton emission in the virtual photon wave function: progressively more configurations contain extra hard gluons, each fragmenting independently in the BBL, further amplifying deviations from the standard LTA predictions.

Another important signature of the BBL is the hardening of $p_\perp$ distributions with decreasing x (at fixed Q^2). Hence, an efficient experimental strategy would be to select leading jets in the current fragmentation region and examine their z and $p_\perp$ dependence as a function of x. Qualitatively, the effect of broadening of $p_\perp$ distributions is similar to the increase of the $p_\perp$ distribution in [18], although final states in DIS were not discussed in this model. However, due to the distribution of the available $p_\perp$ between the produced hadrons, this effect would be more difficult to observe than the z depletion one, and would require detailed modeling.

An important advantage of nondiffractive scattering is the ability to select scattering at central impact parameters (for example, via studies of inelastic interactions as a function of the number of nucleons produced in the nucleus fragmentation region). Such a selection allows the effective thickness of the nucleus to be increased, as compared to the inclusive situation, by a factor of $\sim$1.5, and hence allows the BBL to be reached at significantly larger x. An important signature will be a strong depletion of the leading hadron spectrum with centrality (in the BBL) and a lack of correlations with centrality (in the LTA).

To summarize, we have demonstrated that the study of final states in DIS off nuclei would provide several stringent signals for the onset of the BBL. In contrast to inclusive observables they will not depend on details of the input parton distributions, and hence will be far more reliable. In the nucleon target case, despite the fact that many photon configurations are far away from the BBL, an examination of these final-state observables appears to be a promising way to search for precursors of the BBL. In particular, looking for a strong depletion of hadrons in the current fragmentation region, in combination with the detection of particles in the proton fragmentation region, looks promising.

We thank A. H. Mueller for a useful discussion and GIF, ARC, PPARC, and DOE for support. L. F. and M. S. thank INT for hospitality during the time this work was completed.

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