ELECTROMAGNETIC INTERACTIONS OF HADRONS

1) ELECTROMAGNETIC FORM FACTORS
   - E.M INTERACTIONS IN STANDARD MODEL
   - E.M. MATRIX ELEMENT FOR DIRAC PARTICLE
   - E.M. MATRIX ELEMENT FOR COMPOSITE PARTICLE (HADRON)

2) PHYSICS INTERPRETATION OF E.M. FFs
   - 3 DIM FOURIER TF. IN BREIT FRAME FOR NON RELATIVISTIC BOUND STATE
   - 2 DIM FOURIER TF IN LIGHT-FRONT FRAME FOR RELATIVISTIC BOUND STATES
   - DISPERSION THEORY / VECTOR MESON DOMINANCE
   - PERTURBATIVE QCD

3) ELASTIC ELECTRON - NUCLEON SCATTERING

4) DEEP - INELASTIC ELECTRON SCATTERING

⇒ 1) ELECTROMAGNETIC FORM FACTORS (FF)

- E.M. INTERACTIONS IN STANDARD MODEL

↳ MATTER IN S.M (LEPTONS, QUARKS)

e.g. QUARKS $\mathcal{L}_{QUARKS} = \bar{q}\,(i\gamma^\mu \partial_\mu - m)\, q$

REQUIRE THAT $\mathcal{L}$ SATISFIES INVARIANCE UNDER LOCAL PHASE TF. $U(1)$

$$q(x) \xrightarrow{U(1)} q(x)\, e^{i e_q \chi(x)}$$

FEYNMAN:

$$\partial_\mu q \xrightarrow{U(1)} e^{i e_q \chi(x)} \left[\partial_\mu q + i e_q\, q\, (\partial_\mu \chi) \right]$$

VECTOR FIELD A^μ REQUIRES TO COMPENSATE FOR DIFFERENT PHASE CHOICES IN DIFFERENT SPACE-TIME POINTS

↳ COVARIANT DERIVATIVE

$$\partial_\mu q \Rightarrow D_\mu q \equiv (\partial_\mu + i e_q A_\mu)\, q$$

AND CHOOSE TRANSFORMATION OF A^μ UNDER $U(1)$ SUCH THAT IT COMPENSATES TF OF $\partial^\mu q$

$$A^\mu \xrightarrow{U(1)} A^\mu - \partial^\mu \chi \qquad \text{GAUGE TF.}$$

$$\therefore \; D_\mu q \xrightarrow{U(1)} e^{ie_q \chi(x)} \left[\partial_\mu q + \cancel{ie_q q (\partial_\mu \chi)} + ie_q A_\mu q - \cancel{ie_q (\partial_\mu \chi) q} \right]$$

$$D_\mu q \xrightarrow{U(1)} e^{ie_q \chi(x)} D_\mu q$$

$$\bar{q} \gamma^\mu D_\mu q \xrightarrow{U(1)} \bar{q} \gamma^\mu D_\mu q$$

$$\underline{\underline{\mathcal{L}_{QUARKS} + \mathcal{L}_{INT} = \bar{q} (i \gamma^\mu D_\mu - m) q.}}$$

↳ IN INVARIANT UNDER $U(1)$

↳ EM INTERACTION

$$\mathcal{L}_{INT} = - e_q \, \bar{q} \gamma^\mu q \, A_\mu.$$

e_q : CHARGE OF MATTER PARTICLE

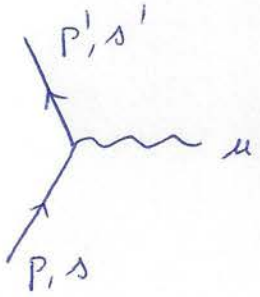
$$e_u = + \frac{2}{3} e \; , \quad e_d = e_s = - \frac{1}{3} e$$

$$\frac{e^2}{4\pi} \simeq \frac{1}{137} \qquad \text{STRENGTH OF EM INTERACTION}$$

EM INTERACTION SATISFIES $\mathcal{P}$ & $\mathcal{T}$ SYMMETRIES

PARTICLES & ANTI-PARTICLES $\quad e_{\bar{q}} = - e_q$

FEYNMAN RULE : S-MATRIX ELEMENT $\quad (i \mathcal{L}_{INT})$



$$\underline{\underline{- i e_q \gamma^\mu}}$$

- E.M. MATRIX ELEMENT FOR DIRAC PARTICLE

↳ E.M. CURRENT

IN GENERAL $\quad \mathcal{L}_{INT} = - e \, J^\mu_{EM}(x) \, A_\mu(x)$

E.M. CURRENT OPERATOR

↳ DIRAC PARTICLE (SPIN 1/2 POINT PARTICLE)

$$\boxed{\langle p', s' \,|\, J^\mu_{EM}(0) \,|\, p, s \rangle = \bar{u}(p', s') \, \gamma^\mu \, u(p, s)}$$

p : 4-MOMENTUM

s : SPIN PROJECTION / HELICITY

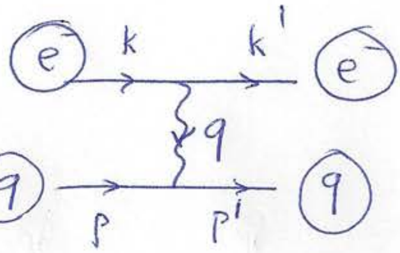
SPINOR $\quad u(p, s) = N \begin{pmatrix} \chi_s \\ \frac{\bar{\sigma} \cdot \bar{p}}{E_p + m} \chi_s \end{pmatrix}$

COVARIANT NORMALIZATION : $\quad \bar{u}(p, s) \, u(p, s) = 2m$

$$N = \sqrt{E_p + m}$$

↳ PHOTON VIRTUALITY

e.g. $e^- q$ SCATTERING



MOMENTUM TRANSFER $q = k - k' = p' - p$

$$p' = p + q \;\curvearrowright\; \underbrace{p'^2}_{m^2} = \underbrace{p^2}_{m^2} + q^2 + 2p \cdot q$$

ON-SHELL

$$q^2 = -2p \cdot q \underset{\uparrow}{=} -2m\, q^0_{LAB}$$

IN LAB $p^\mu = (m, 0)$

AS $q^0_{LAB} = k^0_{LAB} - k'^0_{LAB} > 0$

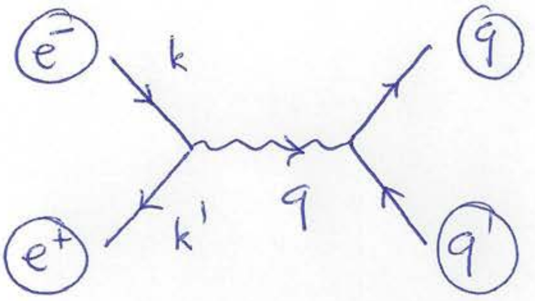
$$q^2 = -2m\, q^0_{LAB} < 0$$

$q^2 < 0$: SPACE-LIKE PHOTON

$$\underline{\underline{Q^2 \equiv -q^2 = 2p \cdot q}}$$

ELASTIC SCATTERING CONDITION

e.g. **e^-e^+ ANNIHILATION**



$$q = k + k' \qquad k' : \text{MOM OF INCOMING } e^+$$

$$q^2 = (k^2 + k')^2 = s \qquad \text{IN } e^-e^+ \text{ CM}$$

$$(k + k')^\mu = (\sqrt{s}, 0)$$

$q^2 > 0$: TIME-LIKE PHOTON

↳ GORDON IDENTITY

$$q = p' - p$$

$$\bar{u}(p', s')\, \gamma^\mu\, u(p, s)$$

$$= \bar{u}(p', s') \left\{ \frac{(p + p')^\mu}{2m} + i\sigma^{\mu\nu} \frac{q_\nu}{2m} \right\} u(p, s)$$

$\Uparrow$ CONVECTION CURRENT

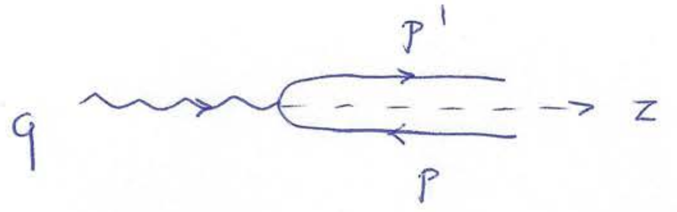
$\Uparrow$ MAGNETIZATION CURRENT

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

↳ CONSERVATION

$$q_\mu \langle p' s' | J^\mu_{EM}(0) | p\, s \rangle = 0$$

↳ BREIT - FRAME INTERPRETATION



$$q^\mu\,(0,0,0,Q)$$
$$p^\mu\,\left(E_p,0,0,-\frac{Q}{2}\right)$$
$$p'^\mu\,\left(E_p,0,0,\frac{Q}{2}\right)$$

WITH $E_p = \sqrt{\frac{Q^2}{4}+m^2} \equiv m\sqrt{1+\tau}$

$$\tau \equiv \frac{Q^2}{4m^2}$$

- CHARGE J^0_{EM}

$$\underline{\underline{\langle p' s' \,|\, J^0_{EM}(0)\,|\, p\, s\rangle_{BREIT}}}$$

$$= u^\dagger(p' s')\, u(p\, s)$$

$$= \left(\sqrt{E_p+m}\right)^2 \left(\chi^\dagger_{s'},\ \chi^\dagger_{s'}\frac{\sigma_3\, Q/2}{(E_p+m)}\right)\begin{pmatrix}\chi_s \\ \dfrac{\sigma_3}{E_p+m}\left(-\dfrac{Q}{2}\right)\chi_s\end{pmatrix}$$

$$= (E_p+m)\,\chi^\dagger_{s'}\chi_s\left(1-\frac{Q^2}{4}\frac{1}{(E_p+m)^2}\right)$$

$$\downarrow \quad \frac{Q^2}{4} = E_p^2 - m^2$$

$$= \chi^\dagger_{s'}\chi_s\,(E_p+m)\left(1-\frac{E_p-m}{E_p+m}\right)$$

$$= 2m\,\chi^\dagger_{s'}\chi_s = \underline{\underline{2m\,\delta_{s's}}} \qquad \text{(NO SPIN FLIP)}$$

• MAGNETIZATION

TERM $\sim (p+p')^i$ IS ZERO IN BRETT FRAME

$$\underline{\underline{\langle p', s' \,|\, J^i_{EM}(0) \,|\, p, s \rangle_{BRETT}}}$$

$$= \bar{U}(p', s') \underbrace{\left\{ - i\sigma^{i3} \frac{Q}{2m} \right\}}_{i\frac{Q}{2m}\varepsilon^{ik3}\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}} U(p, s)$$

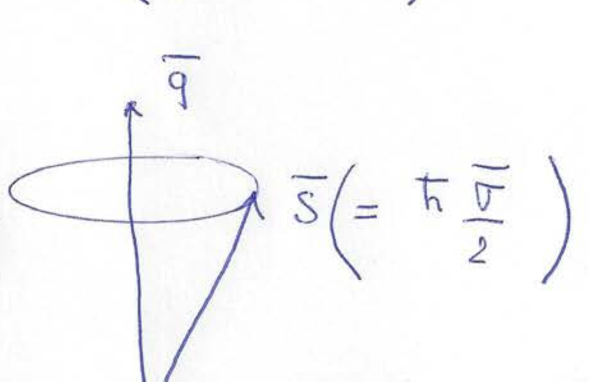
$$= i\varepsilon^{ik3}\frac{Q}{2m}(E_p + m)\left[\chi^+_{s'} \quad -\chi^+_{s'}\frac{\sigma_3 Q/2}{E_p + m} \right] \begin{bmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{bmatrix} \begin{bmatrix} \chi_s \\ -\frac{\sigma_3 Q/2}{E_p + m}\chi_s \end{bmatrix}$$

$$= i\varepsilon^{ik3}\frac{Q}{2m}(E_p + m)\underbrace{\left(1 - \frac{Q^2/4}{(E_p + m)^2}\right)}_{\frac{2m}{E_p + m}} \chi^+_{s'}\,\sigma^k\,\chi_s$$

$$= Q\, i\varepsilon^{ik3}\,\chi^+_{s'}\,\sigma^k\,\chi_s$$

$$= \underline{\underline{\chi^+_{s'}\, i(\bar{\sigma} \times \bar{q})^i\, \chi_s}} \qquad (\text{SPIN FLIP})$$

$\Downarrow$



DIRAC MAGNETIC MOMENT

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}$$

$$\mu_z = g \cdot \left(\frac{e}{2m}\right)\frac{\hbar}{2} \qquad \text{WITH} \quad \boxed{g = 2}$$

• EM MATRIX ELEMENT FOR COMPOSITE PARTICLE

(HADRON)

↳ SPIN 1/2 , e.g. NUCLEON

$$\langle N(p',s') \,|\, J^\mu(0) \,|\, N(p,s) \rangle$$

$$= \bar{N}(p',s') \; \Gamma^\mu \; N(p,s)$$

Γ^μ RESPECTING T, P INVARIANCE

LORENTZ VECTOR

$$\gamma^\mu, \quad i\sigma^{\mu\nu} q_\nu, \quad q^\mu$$

+ CURRENT CONSERVATION

$$q_\mu J^\mu(0) = 0$$

$\Downarrow$

STRUCTURE $\sim q^\mu$ HAS TO VANISH

$$\therefore \quad \boxed{\Gamma^\mu = F_1(q^2)\,\gamma^\mu + F_2(q^2)\, i\sigma^{\mu\nu}\frac{q_\nu}{2M}}$$

F_1, F_2 : FORM FACTORS

FUNCTIONS OF 1 SCALAR q^2

AS $p^2 = p'^2 = M^2 \qquad p' = p + q$

↳ F_1 DIRAC FF $\quad F_1^{p}(0) = 1$ (PROTON), $\quad F_1^{n}(0) = 0$ (NEUTRON) $\Big\}$ CHARGE

↳ F_2 PAULI FF : VALUE AT $q^2 = 0$ GIVES ANOMALOUS MAGNETIC MOMENT

$$F_1^{p}(0) \simeq 1.79 = K^{p}$$

FIRST MEASUREMENT
OTTO STERN 1933
(NOBEL PRIZE PHYSICS 1943)

FIRST INDICATION THAT
PROTON IS COMPOSITE!
(NOT DIRAC PARTICLE)

$$F_1^{n}(0) \simeq -1.91 = K^{n}$$

TOTAL MAGNETIC MOMENT

$$\mu^{p} = 1 + K^{p} \simeq 2.79$$

$$\mu^{n} = 0 + K^{n} \simeq -1.91$$

↑
DIRAC

↳ ISOSPIN DEPENDENCE OF E.M. INTERACTION

- $$J^{\mu}_{EM}(0) = \frac{2}{3}\bar{u}\gamma^{\mu}u - \frac{1}{3}\bar{d}\gamma^{\mu}d - \frac{1}{3}\bar{s}\gamma^{\mu}s$$

IN ABSENCE OF s-QUARKS

QCD HAS $SU(2)_V$ ISOSPIN SYMM.

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$J^{\mu}_{EM}(0) = \bar{q}\left\{\frac{1}{6}\mathbb{1} + \frac{\tau_3}{2}\right\}q$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\equiv \underset{\substack{\uparrow \\ \text{ISOSCALAR}}}{J^{\mu}_{S}(0)} + \underset{\substack{\uparrow \\ \text{ISOVECTOR}}}{J^{\mu}_{V}(0)}$$

$$\langle N(p',s') | J^{\mu}_{S,V}(0) | N(p,s)\rangle = \bar{N}(p',s')\,\Gamma^{\mu}_{S,V}\,N(p,s)$$

$$\Gamma^{\mu}_{S,V} = F_1^{S,V}(q^2)\,\gamma^{\mu} + F_2^{S,V}(q^2)\,i\sigma^{\mu\nu}\frac{q_\nu}{2M}$$

- ISOSPIN STRUCTURE IN HADRONIC BASIS

$$\eta \equiv \begin{pmatrix} p \\ n \end{pmatrix} \begin{matrix} \leftarrow \text{PROTON} \\ \leftarrow \text{NEUTRON} \end{matrix}$$

FUND. REPR. SU(2)

$$\langle \eta(p',s') \,|\, J^{\mu}_{EM}(0) \,|\, \eta(p,s) \rangle$$

$$= \bar{\eta} \left\{ \frac{1}{2} \Gamma^{\mu}_{S} + \frac{1}{2} \tau_3 \Gamma^{\mu}_{V} \right\} \eta$$

$$\begin{cases} F^{p}_{1,2}(q^2) = \frac{1}{2} F^{S}_{1,2} + \frac{1}{2} F^{V}_{1,2} \\ F^{n}_{1,2}(q^2) = \frac{1}{2} F^{S}_{1,2} - \frac{1}{2} F^{V}_{1,2} \end{cases}$$

$$F^{S}_{1}(0) = 1 \qquad F^{V}_{1}(0) = 1.$$

$$F^{S}_{2}(0) = K^{p} + K^{n} \simeq -0.12 \qquad F^{V}_{2}(0) = K^{p} - K^{n} \simeq 3.70$$

⟹ 2) PHYSICS INTERPRETATION OF E.M. FFs

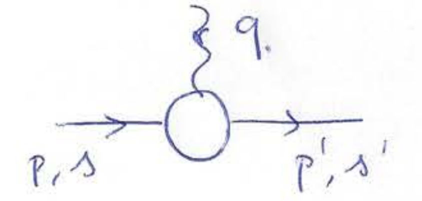
- 3 DIM FOURIER TF. IN BREIT FRAME FOR NONRELATIVISTIC BOUND STATE

e.g. ATOMIC NUCLEUS
NONRELATIVISTIC BOUND STATE
CONSTITUENTS (p, n) FIXED (NO PAIR PRODUCTION)
BINDING ENERGY $\sim$ 8 MeV / NUCLEON

$$E_B \simeq 8 \text{ MeV} \ll M_p \simeq 939 \text{ MeV}$$

$$(\sim 1\%)$$

IN BREIT FRAME (SPIN 1/2) (EXERCISE)



$(q^2 < 0)$

$\hookrightarrow \langle N(p', s') \,|\, J^0_{EM}(0) \,|\, N(p, s) \rangle_{\text{BREIT}} = 2M \, \delta_{ss'} \, G_E(q^2)$

$\hookrightarrow \langle N(p', s') \,|\, J^i_{EM}(0) \,|\, N(p, s) \rangle_{\text{BREIT}} = \chi^+_{s'} \, i(\vec{\sigma} \times \vec{q})^i \chi_s \cdot G_M(q^2)$

$\hookrightarrow$ E.M. SACHS FFS

ELECTRIC $G_E(q^2) \equiv F_1 - \tau F_2$ $\qquad \tau \equiv \dfrac{Q^2}{4M^2}$

MAGNETIC $G_M(q^2) \equiv F_1 + F_2$

↳ CHARGE / MAGNETIZATION DENSITIES

$$\rho_{ch}(r) \qquad \int_0^\infty dr\, r^2 \rho_{ch}(r) = \begin{cases} 1, & p \\ 0, & n \end{cases}$$

$\rho_m(r)$: DEFINED THROUGH

$$\bar{\mu}(r) = \mu\, \rho_m(r)\, \bar{\sigma}$$

$$\mu^p = 2.79$$

$$\mu^n = -1.91$$

$$\int_0^\infty dr\, r^2 \rho_m(r) = 1.$$

↳ FOURIER TF.

$$\tilde{\rho}_{ch}(|\bar{q}|) = \frac{1}{4\pi} \int d^3\bar{r}\, e^{i\bar{q}\cdot\bar{r}} \rho_{ch}(r)$$

↓ SPHERICAL SYMM.

$$= \frac{1}{4\pi} \sum_{\ell=0}^\infty (2\ell+1)\, i^\ell \int_0^\infty dr\, r^2 \rho_{ch}(r)\, j_\ell(qr)$$

$$\cdot \int d\Omega\, P_\ell(\underset{\hat{q}\cdot\hat{r}}{\underset{\parallel}{\cos\theta}})$$

$$= \int_0^\infty dr\, r^2 j_0(qr)\, \rho_{ch}(r)$$

NOTE $\tilde{\rho}_{ch}(0) = \begin{cases} 1 & , p \\ 0 & , n \end{cases}$

$$\tilde{\rho}_{ch}(|\bar{q}|) = \int_0^\infty dr \; r^2 \; j_0(qr) \; \rho_{ch}(r)$$

FOURIER - BESSEL TF.

ANALOGOUSLY FOR $\tilde{\rho}_m(|\bar{q}|)$

↳ IN NON-REL. LIMIT (BREIT FRAME)

$$\tilde{\rho}_{ch}(|\bar{q}|) = G_E(q^2) \qquad q^2 = -Q^2$$
$$|\bar{q}_B| = Q = \sqrt{Q^2}$$

↳ REL. CORRECTIONS

$$|\bar{q}|^2 = \frac{Q^2}{1+\tau} \qquad \text{KINEMATIC}$$

DYNAMIC CORRECTIONS : NEED TO BOOST W.F.
SOLVE NON-PERT. PROBLEM !
(MODELS)

↳ AT LOW Q^2 : TAYLOR EXPANSION

$$G_E^p(q^2) = 1 - \frac{1}{6} \langle r_E^2 \rangle Q^2 + O(Q^4)$$

CHARGE RADIUS

MAGNETIC RADIUS

$$G_M^p(q^2) / \mu_p = 1 - \frac{1}{6} \langle r_M^2 \rangle Q^2 + O(Q^4)$$

- 2 DIM FOURIER TF IN LIGHT-FRONT FRAME FOR RELATIVISTIC BOUND STATE

LIGHT HADRONS $(p, n, \rho, \dots)$

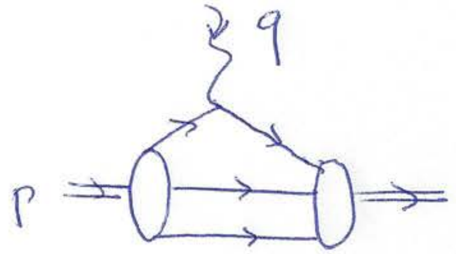
COMPOSED OF u, d, s QUARKS

$$m_{u,d} \simeq 2 - 5 \text{ MeV}$$

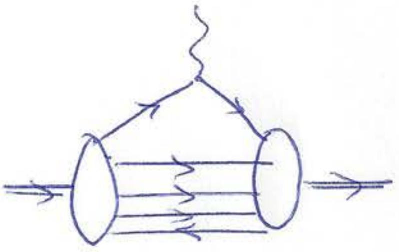
$$m_s \simeq 100 \text{ MeV}$$

$$m_{u,d} \ll m_s \ll M_p \sim 1 \text{ GeV}$$

↳ PAIR CREATION ABUNDANT !



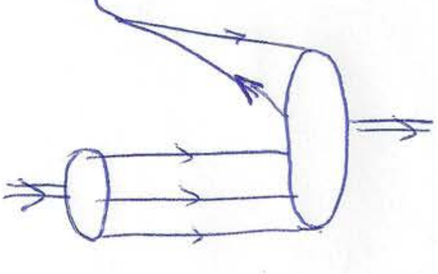
3 FOCK COMPONENT



5 FOCK COMPONENT

INFINITE TOWER

BUT ALSO !



NON-DIAGONAL OVERLAPS OF 3 & 5 QUARK STATES

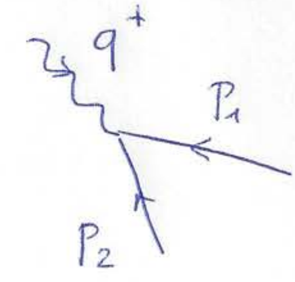
↳ NO DENSITY INTERPRETATION

TO SUPPRESS NON-DIAGONAL OVERLAPS

GO TO SPECIAL FRAME $q^+ = 0$ $\quad (q^+ = q^0 + q^3)$

(POSSIBLE FOR SPACE LIKE PHOTON)

LIGHT-FRONT FRAME (INF. MOM. FRAME)



$$q^+ = P_1^+ + P_2^+$$

$$P_1^+ , P_2^+ > 0$$

(ON-SHELL QUARKS
ANTI-QUARKS)

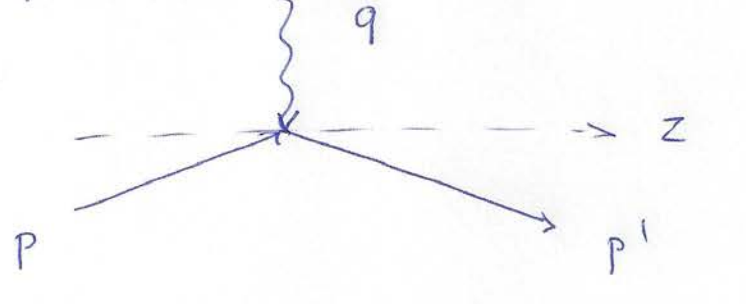
IN $q^+ = 0$ FRAME

PAIR PRODUCTION FORBIDDEN

$\Downarrow$

ALLOWS DENSITY INTERPRETATION

↳ SYMMETRIC LIGHT-FRONT FRAME



$a \, (a^+, a^-, \vec{a}_\perp)$ $\qquad a^\pm \equiv a^0 \pm a^3$

$q \, (0, 0, \vec{q}_\perp)$ $\qquad |\vec{q}_\perp| = Q = \sqrt{Q^2}$

$p \left(P^+, \frac{M^2}{P^+} \tau, -\frac{\vec{q}_\perp}{2} \right)$ $\qquad P^+$: MOM. OF FAST MOVING PROTON

$p' \left(P^+, \frac{M^2}{P^+} \tau, \frac{\vec{q}_\perp}{2} \right)$ $\qquad P^+ \equiv \frac{1}{2}(p + p')^+$

↳ LIGHT-FRONT HELICITY MATRIX ELEMENTS

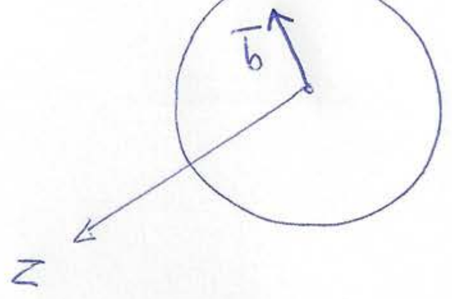
$$\langle \pm \,|\, J^{+}_{EM}(0) \,|\, \pm \rangle = (2P^{+})\, F_1(q^2)$$

$$\langle \mp \,|\, J^{+}_{EM}(0) \,|\, \pm \rangle = (2P^{+})\,(\pm)\left(\frac{q^1 \pm i q^2}{2M}\right) F_2(q^2)$$

↑ HELICITY

↳ 2 DIM FOURIER TF

$\bar{q}_\perp \longleftrightarrow$ CONJUG. POSITION $\bar{b}$ ⊥ TO DIRECTION OF FAST MOVING HADRON (Z-AXIS)



$$\boxed{\rho_0(b) \equiv \int \frac{d^2\bar{q}_\perp}{(2\pi)^2}\, e^{-i\bar{b}\cdot\bar{q}_\perp}\, F_1(-\bar{q}_\perp^{\,2})}$$

↓ FOR AXIAL SYMM

$$-\bar{q}_\perp^{\,2} = -Q^2 < 0$$

$$Q = |\bar{q}_\perp|$$

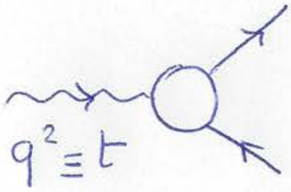
$$\rho_0(b) = \int_0^\infty \frac{dQ}{2\pi}\, Q\, J_0(bQ)\, F_1(-Q^2)$$

↳ HAS PROPER DENSITY INTERPRETATION

• DISPERSION THEORY / VECTOR MESON DOMINANCE

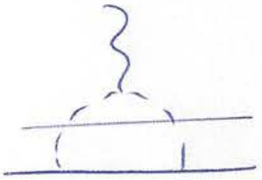
↳ ANALYTIC STRUCTURE OF FF (e.g. F_1)

$F_1(q^2)$



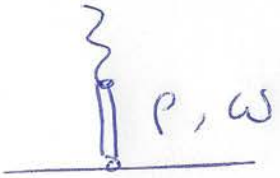
PHOTON SEES PROTON THROUGH ALL HADRONIC STATES (WITH VECTOR QUANTUM NUMBERS)

IN EFT



2π INTERMEDIATE STATE (NON-RESONANT)

+ RESONANCES (VECTOR MESONS)



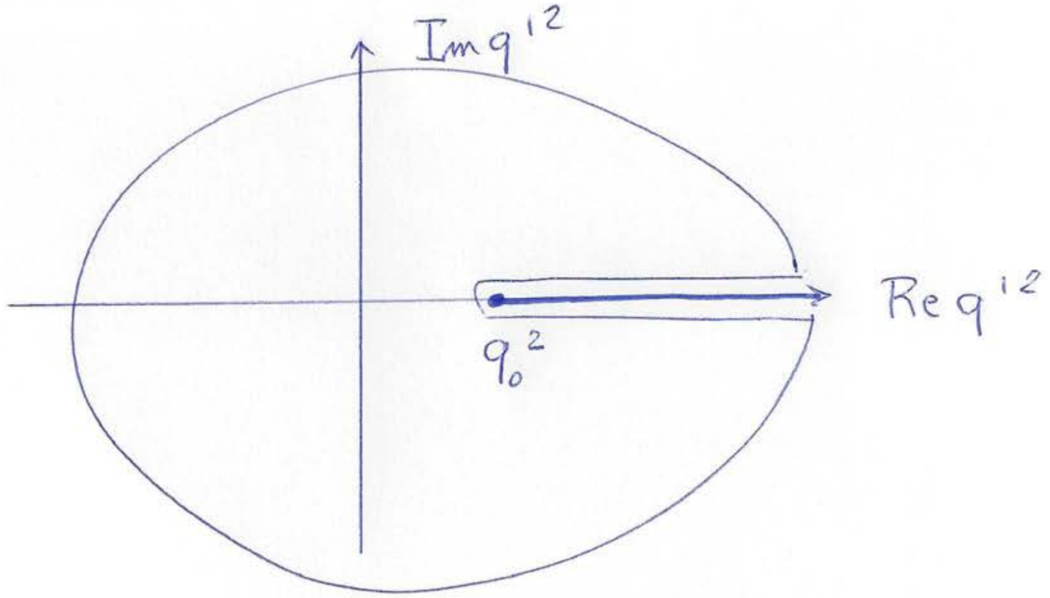
ρ : ISOVECTOR
ω : ISOSCALAR

CONSIDER $F_1(q^2)$ AS ANALYTIC FUNCTION IN q^2

BRANCH CUT FOR $q^2 > q_0^2$

↗ ISOVECTOR $q_0^2 = (2m_\pi)^2$
↘ ISOSCALAR $q_0^2 = (3m_\pi)^2$

↳ SUBTRACTED DISPERSION RELATION

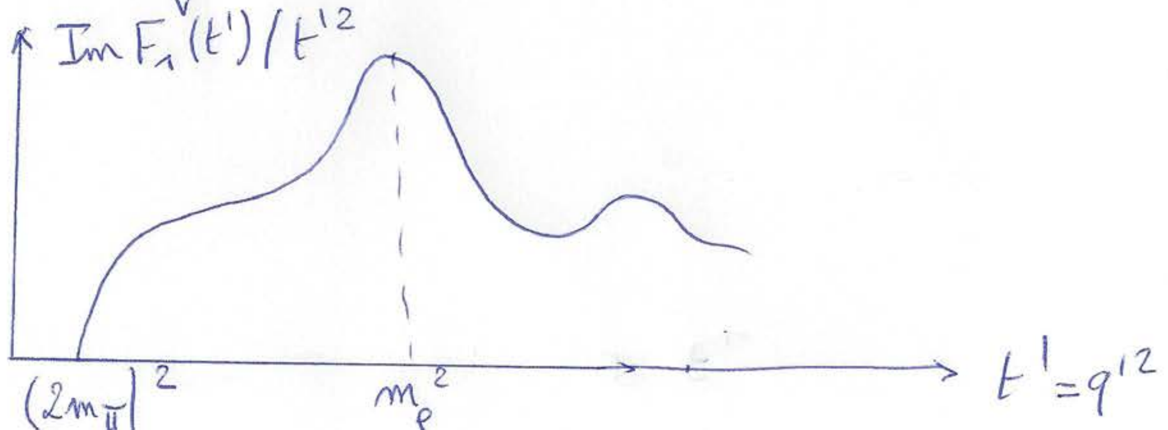


$$F_1(q^2) = \underset{\substack{\| \\ 1}}{F_1(0)} + \frac{q^2}{\pi} \int_{q_0^2}^{\infty} dq'^2 \, \frac{\mathrm{Im}\, F_1(q'^2)}{q'^2(q'^2 - q^2)}$$

e.g. RADIUS (ISOVECTOR)

$$F_1^V(q^2) = 1 - \frac{1}{6} \langle r_{1,V}^2 \rangle \, Q^2 + \dots$$

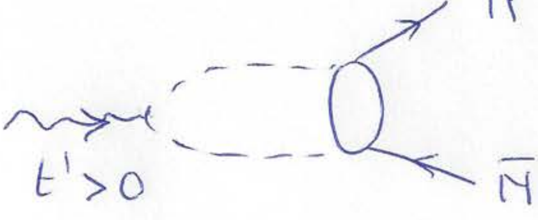
$$\langle r_{1,V}^2 \rangle = \frac{6}{\pi} \int_{(2m_\pi)^2}^{\infty} dt' \, \frac{\mathrm{Im}\, F_1^V(t')}{t'^2}$$



LEFT SIDE OF DR : SPACE-LIKE FF (REAL FUNCTION)

RIGHT SIDE OF DR : TIME-LIKE FF (HAS IMAG PART)

e.g.



↳ VECTOR MESON DOMINANCE (VMD)

PROMINENT CONTRIBUTIONS TO $F_1^V(t')$

FROM VECTOR MESONS $\rho, \rho', \ldots$

CONSIDER UNSUBTRACTED DR

$$F_1^V(q^2) = \frac{1}{\pi} \int_{q_0^2}^{\infty} dt' \, \frac{\mathrm{Im}\, F_1^V(t')}{(t' - q^2)}$$

AND ASSUME 2 NARROW RESONANCES ρ, ρ'

i.e. $\frac{1}{\pi} \mathrm{Im}\, F_1^V(t') \simeq a_\rho \, \delta(t' - m_\rho^2) + a_{\rho'} \, \delta(t' - m_{\rho'}^2)$

$\Downarrow$

$$F_1^V(q^2) \simeq \frac{a_\rho}{m_\rho^2 - q^2} + \frac{a_{\rho'}}{m_{\rho'}^2 - q^2}$$

\+ IMPOSE $F_1^V(0) = 1$

$\Downarrow$

$$\left(\frac{a_\rho}{m_\rho^2} + \frac{a_{\rho'}}{m_{\rho'}^2} \right) = 1.$$

IF $a_{\rho'} \simeq - a_\rho \simeq - \dfrac{m_\rho^2 \, m_{\rho'}^2}{(m_{\rho'}^2 - m_\rho^2)}$

$$F_1^V(q^2) \simeq \frac{a_\rho (m_{\rho'}^2 - m_\rho^2)}{(m_\rho^2 - q^2)(m_{\rho'}^2 - q^2)}$$

$$\simeq \frac{1}{\left(1 - \frac{q^2}{m_\rho^2}\right)\left(1 - \frac{q^2}{m_{\rho'}^2}\right)}$$

$F_1^V(q^2)$ HAS APPROXIMATE DIPOLE BEHAVIOR

EXP. SCALE $\Lambda^2 = (0.843 \text{ GeV})^2$

$$m_\rho^2 = (0.77 \text{ GeV})^2$$

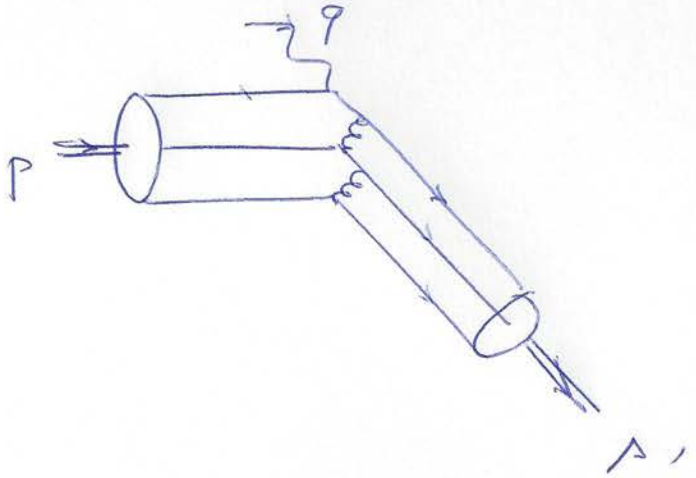
$$m_{\rho'}^2 = (1.45 \text{ GeV})^2$$

APPROXIMATE DIPOLE BEHAVIOR
ARISES FROM RESIDUES OF NEARBY
POLES (RESONANCES) WITH
NEAR EQUAL MAGNITUDE & OPPOSITE SIGN

• PERTURBATIVE QCD

IN LIMIT $-q^2 = Q^2 \to \infty$

CONFIGURATION WITH MINIMUM # QUARKS WILL DOMINATE



2 HARD GLUONS $\quad F_1(q^2) \underset{-q^2 \to \infty}{\sim} \dfrac{1}{q^4}$

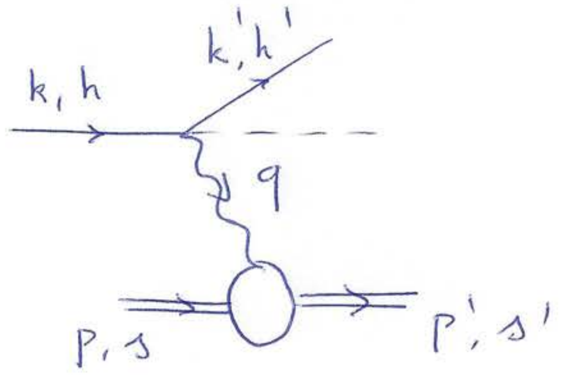
WHEN DOES pQCD BEHAVIOR SET IN ?

NON-PERTURBATIVE CALCULATION NEEDED

EXPERIMENT INDICATES $Q^2 > 10 \text{ GeV}^2$

3) ELASTIC ELECTRON - NUCLEON SCATTERING

- DETERMINATION OF ELASTIC FF
  THROUGH ELASTIC e^- - HADRON SCATTERING
  IN 1γ - EXCHANGE APPROXIMATION
  ↳ POSSIBLE DUE TO SMALLNESS OF $\alpha_{EM} = \frac{e^2}{4\pi} \simeq \frac{1}{137}$



$2 \to 2$ PROCESS : 2 KINEMATICAL VARIABLES
s AND t

$$s = (k+p)^2 = M^2 + \cancel{m_e^2} + 2 M E_e$$

$m_e \ll M$ $\qquad$ e^- LAB ENERGY

$$t = q^2 = -Q^2 < 0$$

$$Q^2 = 2 M q^0_{LAB}$$ (ELASTIC SCATTERING CONDITION)

- CROSS SECTION (UNPOLARIZED)

$$\hookrightarrow \left(\frac{d\sigma}{d\Omega_{e'}}\right)_{LAB} \sim L_{\mu\nu} H^{\mu\nu}_{el}$$

$H^{\mu\nu}_{el}$ ELASTIC $\hookrightarrow$ HADRON TENSOR

$L_{\mu\nu}$ $\hookrightarrow$ LEPTON TENSOR

$$L_{\mu\nu} = \frac{1}{2} \mathrm{Tr}\left\{ \not{k}' \gamma_\mu \not{k} \gamma_\nu \right\}$$

$$= 2 \left\{ k'_\mu k_\nu + k'_\nu k_\mu - (k.k') g_{\mu\nu} \right\}$$

$$H^{\mu\nu}_{el} = \frac{1}{2} \mathrm{Tr}\left\{ (\not{p}' + M) \Gamma^\mu (\not{p} + M) \Gamma^\nu \right\}$$

NOTE $\quad q^\mu L_{\mu\nu} = 0 \qquad q^\nu L_{\mu\nu} = 0$

$\quad q_\mu H^{\mu\nu}_{el} = 0 \qquad q_\nu H^{\mu\nu}_{el} = 0$

$$\hookrightarrow \boxed{\left(\frac{d\sigma}{d\Omega_{e'}}\right)_{LAB} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)^{MOTT} \cdot \left(\frac{E'_e}{E_e}\right) \cdot \frac{1}{(1+\tau)} \cdot \frac{1}{\varepsilon} \left\{ \tau G_M^2 + \varepsilon G_E^2 \right\}}$$

ROSENBLUTH FORMULA (EXERCISE)

$$\left(\frac{d\sigma}{d\Omega_{e'}}\right)^{MOTT} = \frac{\alpha^2 \cos^2\theta_e/2}{4E_e^{\,2} \sin^4\theta_e/2} \qquad \text{(ALL IN LAB)}$$

↳ CROSS SECTION FOR SCATTERING FROM SPIN-0 RELATIVISTIC POINT PARTICLE

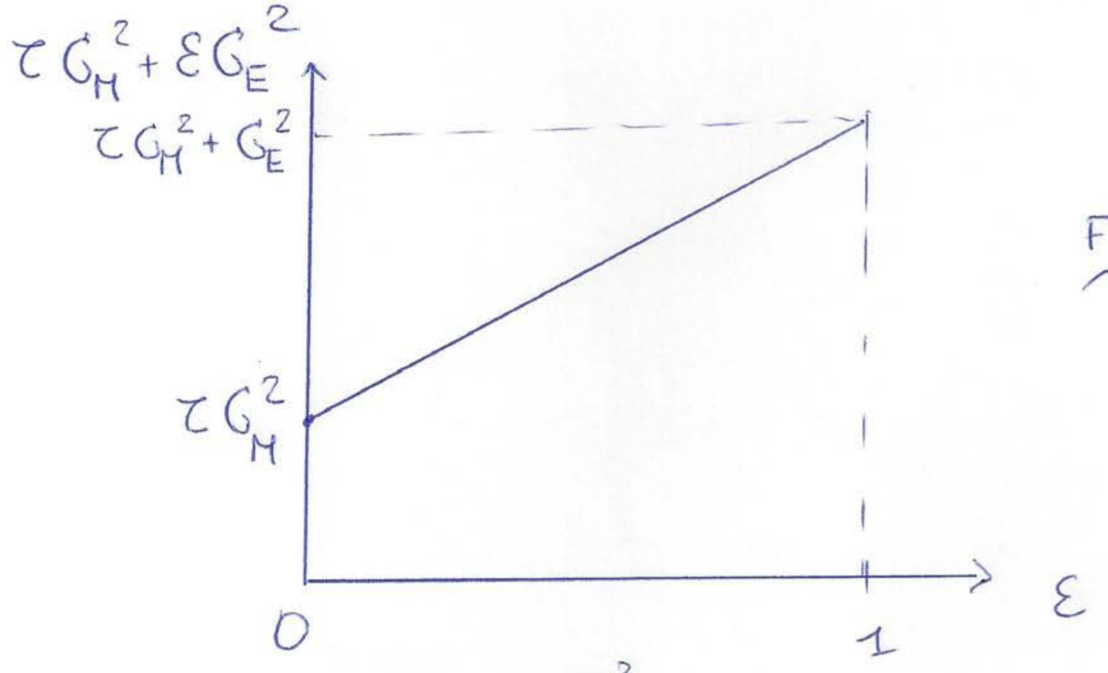
$$\varepsilon \equiv \left[1 + \frac{2|\vec{q}|^2}{Q^2}\tan^2\theta_e/2\right]^{-1}$$

WITH $|\vec{q}|^2 = Q^2 + q_0^{\,2} = Q^2(1+\tau)$.

VIRTUAL PHOTON POLARIZATION

$\varepsilon = 1 \iff \theta_e = 0$ MAX. CONTRIBUTION OF LONGITUDINAL PHOTON POL.

$\varepsilon = 0 \iff \theta_e = 180°$ ONLY TRANSVERSE PHOTON POL.



FIXED Q^2

INTERCEPT $\Rightarrow G_M^2$

SLOPE $\Rightarrow G_E^2$