Quark Models

for baryons

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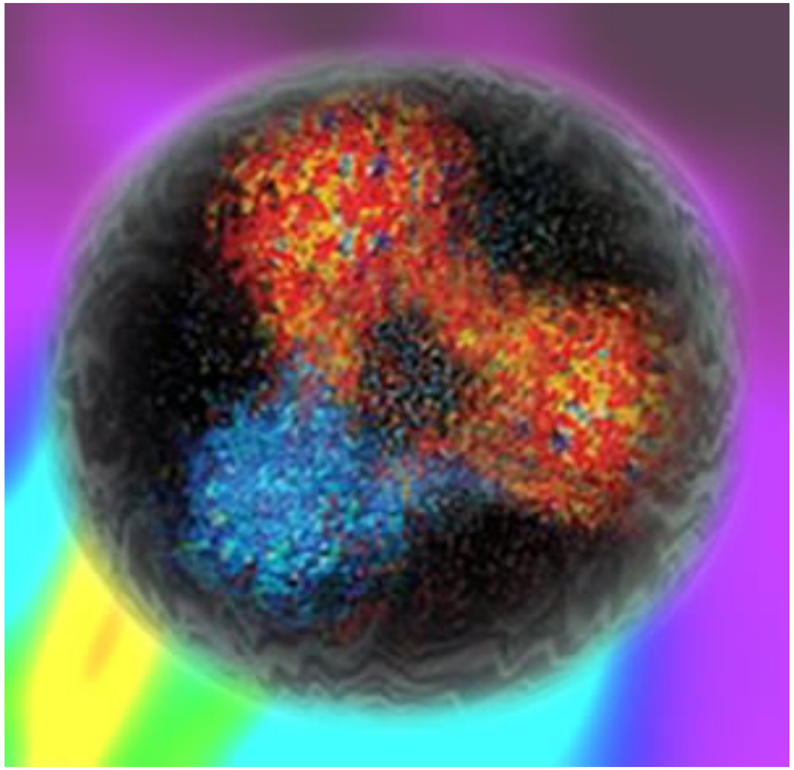
Basic idea of Constituent Quark Models (CQM)



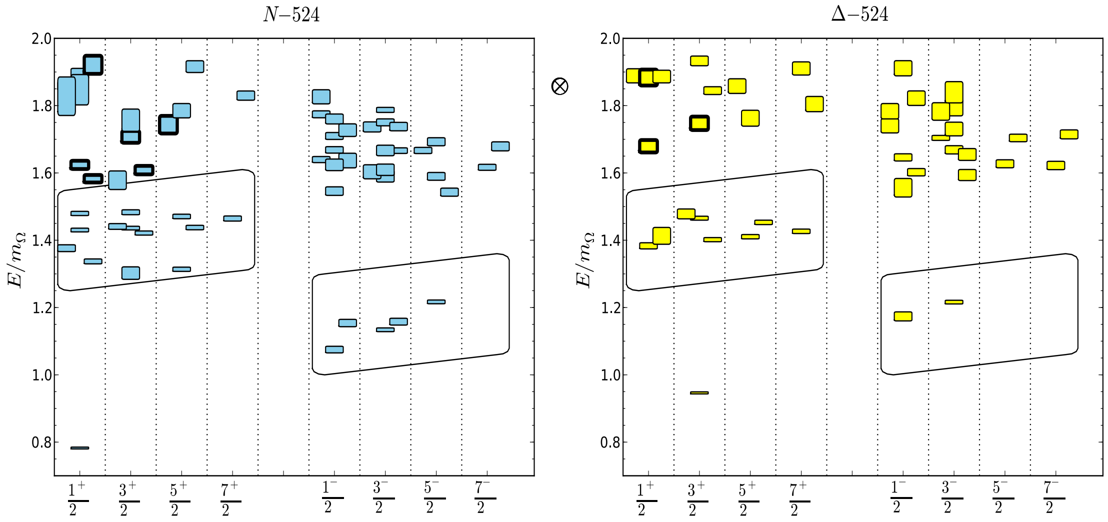
Constituent Quarks

At variance with QCD quarks: CQ acquire mass & size

LQCD results: SU(6)× O(3) QM states up to ≈2.2 GeV

J. J. Dudek, R. G. Edwards

Phys.Rev. D85 (2012) 054016



LQCD (De Rújula, Georgi, Glashow, 1975)

the quark interaction contains

a long range spin-independent confinement

a short range spin dependent term

Spin-independence $\longrightarrow$ SU(6) configurations

SU(3)

3 flavours : $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \equiv (q_\alpha) \quad \alpha = 1,2,3$

* invariance under rotation in u,d,s space
[~ 15 % violation in mass spectrum]

$q \to q' = U q$ U : 3×3 unitary unimodular } SU(3

$\rightsquigarrow e^{i\theta \sum_j n_j \lambda_j}$ $j = 1, \ldots 8 \; [= 3^2 - 1]$

generators of infinitesimal transformations

λ_j : Gell-Mann matrices

$$[\lambda_i, \lambda_j] = 2 i f_{ijk} \lambda_k$$ LIE ALGEBRA

↳ structure constants

- $\lambda_k = \begin{pmatrix} \tau_k & 0 \\ 0 & 0 \end{pmatrix} \quad k = 1,2,3 \qquad SU(2) \subset SU(3)$
- only 2 commute : λ_3, λ_8

$T_z = \frac{1}{2} \lambda_3 \qquad Y = \frac{1}{\sqrt{3}} \lambda_8$

(3^{rd} isospin comp.) (hypercharge)

(u, d) QUARKS AND SU(2)

$\Phi_q = \begin{pmatrix} u \\ d \end{pmatrix}$ in absence of e.m. interactions u and d cannot be distinguished

→ INVARIANCE UNDER ROTATION IN THE CHARGE SPACE

[$p \sim uud$, $n \sim udd$ → nucleon isospin]

$$\phi_q \to \phi'_q = U \phi_q$$

U : 2×2, unitary, $\det(U) = 1$ [unimodular] } element of $SU(2)$

$$U = e^{i \Theta \vec{n} \cdot \frac{1}{2} \vec{\tau}}$$

$\vec{\tau}$ ← generators of infinitesimal transformations

Θ: rotation angle; $\vec{n}$: axis of rotation

$Sp(\vec{\tau}) = 0$

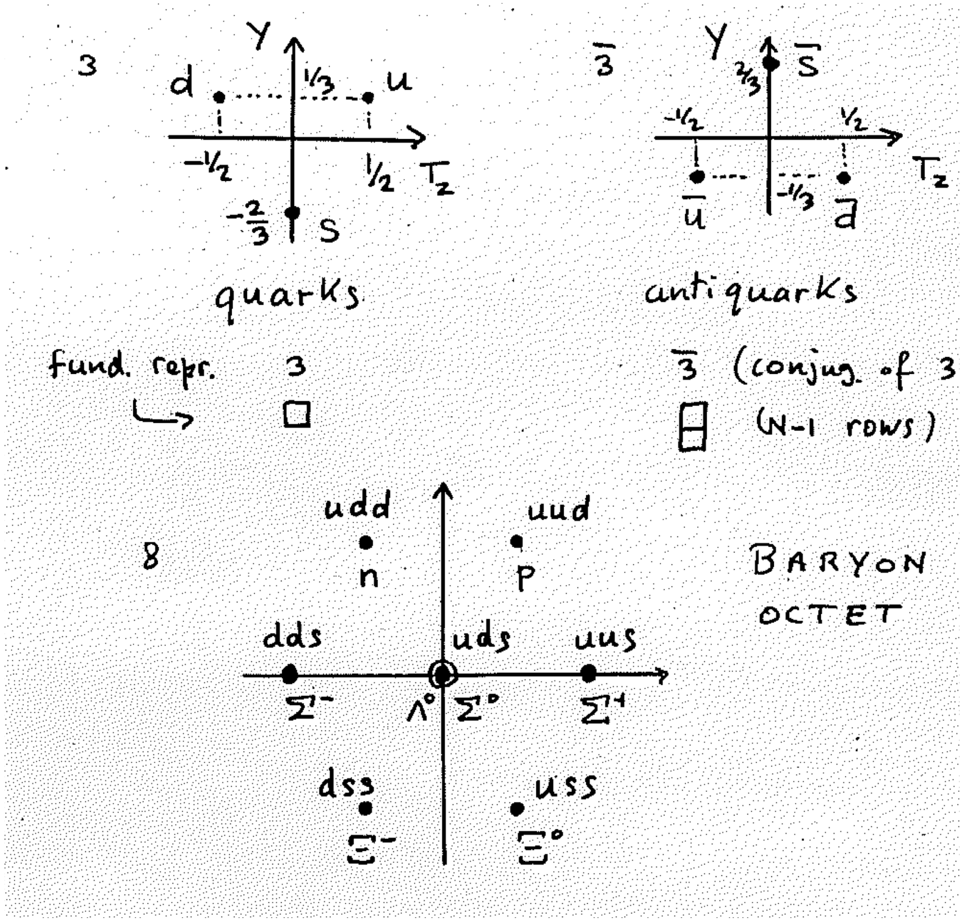
τ_j :

- $j = 1, 2, 3$ (Pauli matrices) $3 = 2^2 - 1$
- satisfy the Lie-algebra

$$[\tau_i, \tau_j] = 2i\, \varepsilon_{ijk} \tau_k \qquad i, j, k = 1, 2, 3$$

- describe the quark isospin
- ε_{ijk} (antisymm. tensor) structure constants

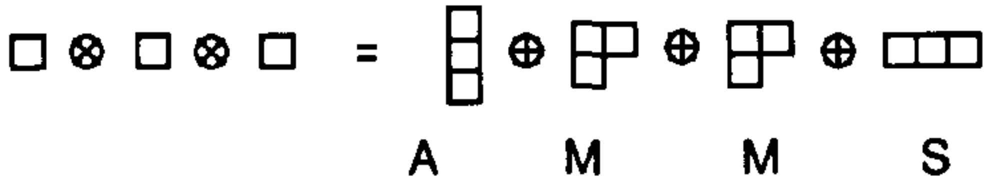
STATES IN A GIVEN IRR. REP. ARE LABELLED BY <u>TWO</u> QUANTUM NUMBERS T_z ; Y



Ex. Construct the baryon decuplet in the Y (Y=B+S) ,T_z plane

Young diagram technique for SU(N)

the fundamental N-dimensional representation is denoted by a box and the irr.rep.of three objects can be obtained as



$$N \otimes N \otimes N = \frac{1}{6} N(N-1)(N-2) + 2 \cdot \frac{1}{3}(N+1)N(N-1) + \frac{1}{6}(N+2)(N+1)N$$

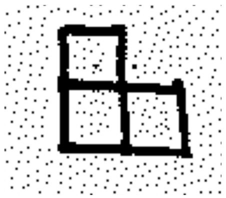
The advantage of this method is two-fold: first the pattern is general, being valid for any SU(N); furthermore each Young tableaux with n boxes defines an irreducible representation of the group Sn containing all the permutations of n objects and therefore it belongs to a definite symmetry type. the labels A,M,S refer to antisymmetry, mixed symmetry and symmetry for the exchange of the 3 quark coordinates. In the case of SU(2) (spin), the antisymmetric 3-quark state does not exist, because only two different states are available for three particles.

Young diagram rules:

1° rule antisymetry for box in columns, symmetry for box in rows

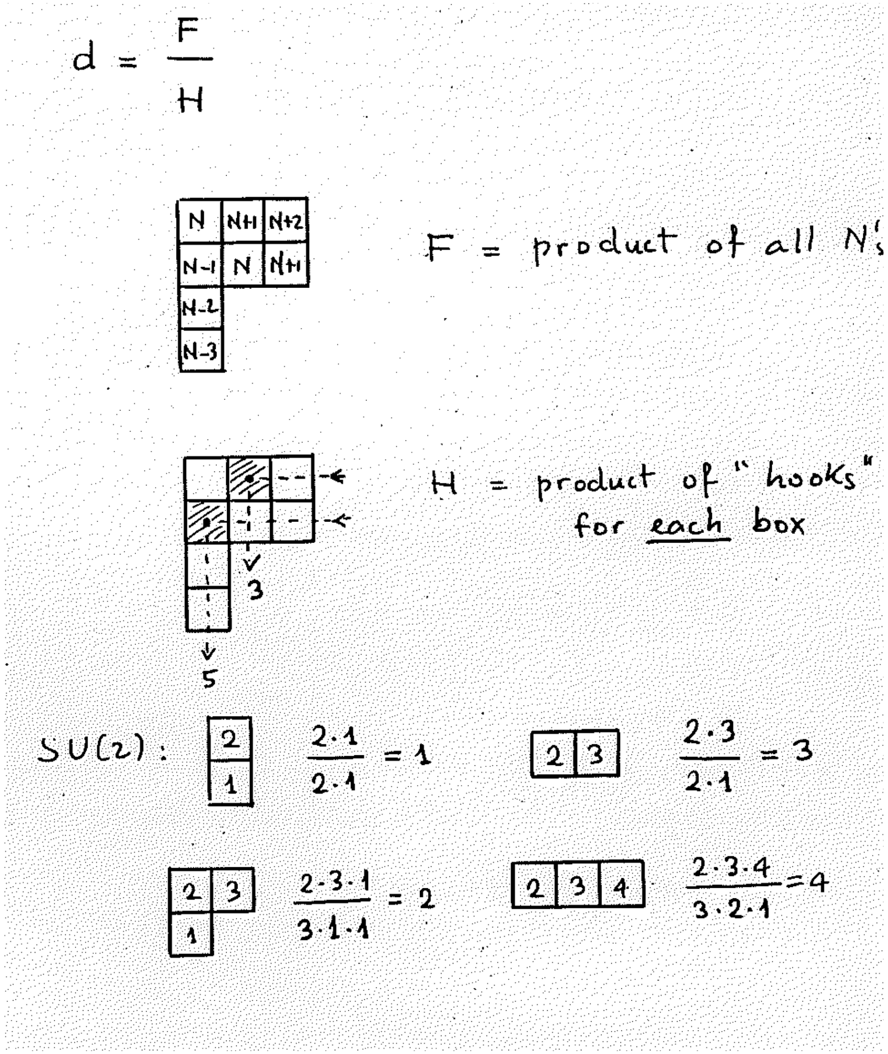
2°rule the number of box in a column cannot exceed the number of states (N) accessible to each particle

3 rule lower rows cannot have more box than the upper one



Not possible

DIMENSIONS for Young diagrams for SU(N). Ex.calculate also for SU(3),SU(6)



Ex.calculate for SU(3) SU(6)

SPIN STATES SU(2)

If we adopt the standard angular momentum notation $|((s1,s2)S\_12,S\_3)S>$,
the explicit form of the 3q spinstates is:

$$\Phi_{MA} = \left|\left(\left(\tfrac{1}{2}\tfrac{1}{2}\right)0,\tfrac{1}{2}\right)\tfrac{1}{2}\right> \equiv \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \\ \cline{1-1}\end{array}$$

$$\Phi_{MS} = \left|\left(\left(\tfrac{1}{2}\tfrac{1}{2}\right)1,\tfrac{1}{2}\right)\tfrac{1}{2}\right> \equiv \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \\ \cline{1-1}\end{array}$$

$$\Phi_{S} = \left|\left(\left(\tfrac{1}{2}\tfrac{1}{2}\right)1,\tfrac{1}{2}\right)\tfrac{3}{2}\right> \equiv \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline\end{array}$$

the suffixes indicate also the symmetry for the exchange of quarks in the pair with total spin S_{12}=0 or 1.

The SU(3) irr.rep. are constructed following to same general scheme, the corresponding dimensions and symmetry types being

$$\text{SU(3)}: \quad 3 \otimes 3 \otimes 3 = \underset{A}{1} \oplus \underset{M}{8} \oplus \underset{M}{8} \oplus \underset{S}{10}$$

In the case of non-strange baryons, the resulting states coincide with the standard isospin ones, detoned, similarly to the spin states by

$$\chi_{MA}, \chi_{MS}, \chi_{S},$$

The strongest component of the quark-quark interaction is spin independent. In this case the flavour and spin states are combined into SU(6) multiplets with the dimensions

$$6 \otimes 6 \otimes 6 = \underset{A}{20} \oplus \underset{M}{70} \oplus \underset{M}{70} \oplus \underset{S}{56}$$

Each SU(6) state can be analyzed with respect to its spin ,SU(2),and flavour SU(3) content. Keeping in mind the symmetry properties of the various states involved, one can easily obtain the following decomposition:

$$\underline{20} = {}^4\underline{1} + {}^2\underline{8}$$

$$\underline{70} = {}^2\underline{1} + {}^2\underline{8} + {}^4\underline{8} + {}^2\underline{10}$$

$$\underline{56} = {}^2\underline{8} + {}^4\underline{10}$$

In the r.h. sides the suffixes denote of course the multiplicity 2S+1 of the 3q spin states, while the underlined numbers are the dimensions of the SU(3) flavour multiplets

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$$\underline{20} = {}^4\underline{1} + {}^2\underline{8}$$

$$\underline{70} = {}^2\underline{1} + {}^2\underline{8} + {}^4\underline{8} + {}^2\underline{10}$$

$$\underline{56} = {}^2\underline{8} + {}^4\underline{10}$$

Multiplication table:

	A	M	S
A	S	M	A
M	M	A,M,S	M
S	A	M	S

⊛ What about colour ?

3q-states : $q^{a}_{\alpha}(1)\; q^{b}_{\beta}(2)\; q^{c}_{\gamma}(3)$

← colour → $SU(3)_{colour}$
← flavour → $SU(3)_{flavour}$
} indices

- Y.D. above o.k.
- there is a $SU(3)_{colour}$ -singlet 

baryons are colourless

-  is completely antisymmetric

$$\frac{1}{\sqrt{6}}\, \varepsilon^{abc}\, q^{a}(1)\, q^{b}(2)\, q^{c}(3)$$

Casimir Operators

- DEFINED IN TERMS OF THE GENERATORS
- COMMUTE WITH ALL GENERATORS

$\longrightarrow$ they have a definite value for __each__ irreducible representation

THEY LABEL THE I.R.

SU(N) : N-1 Casimir op.

SU(2) : $t_i = \frac{1}{2}\tau_i$

$$C = \sum_{i=1}^{3} t_i^2 \quad \rightarrow \quad T(T+1)$$

SU(3) : $F_i = \frac{1}{2}\lambda_i \quad i = 1, \ldots 8$

$$C_1 = \sum_{i=1}^{8} F_i^2$$

unitary spin F^2

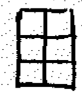
$$C_2 \rightarrow O(F_i^3)$$

dim. irr. repr.	1	3	$\bar{3}$	8	6	10
F^2	0	4/3	4/3	3	10/3	6

PROBLEMS

1. – Calculate the dimensions of

 ,  ,  in SU(3) and in SU(6)

2. – Show that  is a SU(3)-singlet

3. – Show that in SU(3) one has $3 \otimes \bar{3} = 1 \oplus 8$

4. – Prove the SU(6) decomposition scheme

THREE-QUARK WAVE FUNCTION

$$\Psi_{3q} = \theta_{colour} \times \chi_{spin} \times \phi_{iso} \times \psi_{space}$$

SU(3)$_c$ **SU(2)** **SU(3)$_f$** **O(3)**

SU(6) limit: (spin-independent interaction)

SU(3)$_c$ **SU(6)** **O(3)**

Permutation symmetry: $\boldsymbol{\Psi_{3q}}$ must be antisymmetric

θ_{colour} is a colour singlet => *A*

the rest must be symmetric

SU(6) & O(3) wf have the same symmetry (A, MS, MA, S)

SU(6) configurations for three quark states

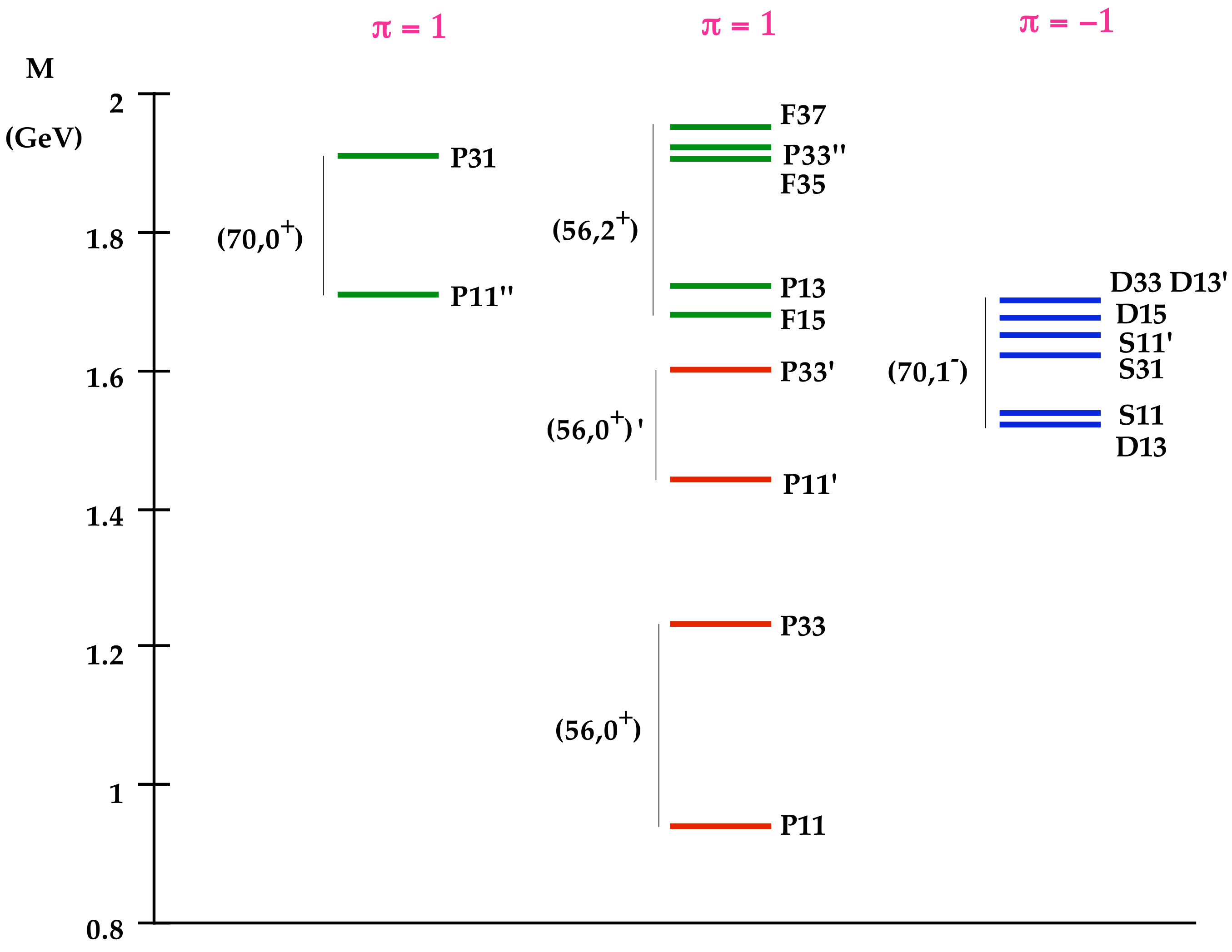
$$6 \text{ X } 6 \text{ X } 6 = \underset{A}{20} + \underset{M}{70} + \underset{M}{70} + \underset{S}{56}$$

Notation

$$(d, L^{\pi})$$

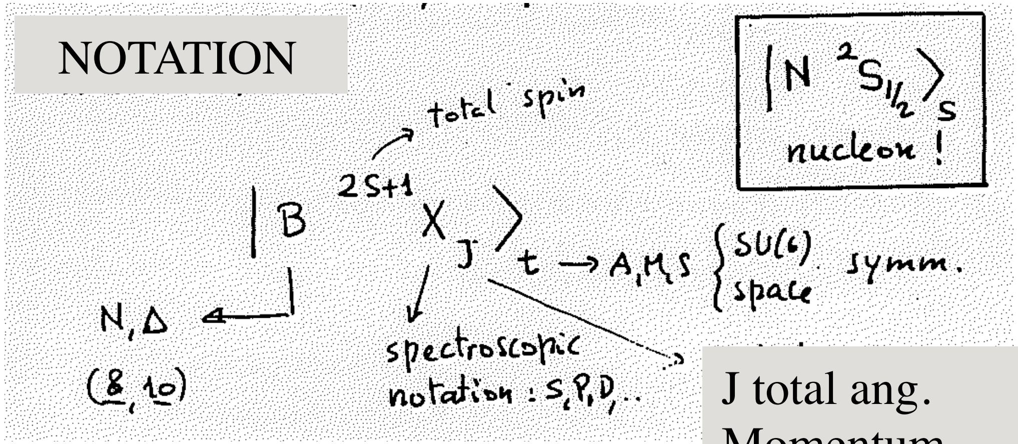
d = dim of SU(6) irrep
L = total orbital angular momentum
π = parity

PDG 4* & 3*



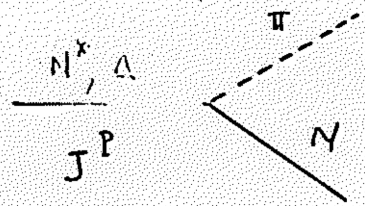
π = 1
π = 1
π = −1
M
(GeV)
2
1.8
1.6
1.4
1.2
1
0.8
P31
(70,0+)
P11''
F37
P33''
F35
(56,2+)
P13
F15
P33'
(56,0+)'
P11'
P33
(56,0+)
P11
D33 D13'
D15
S11'
S31
(70,1-)
S11
D13

NOTATION



J total ang. Momentum.

now New notation on PDG



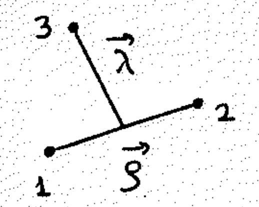
$X_{2I\,2J}$

$X = S, P, D, \ldots$

referred to the wave of the outgoing pion

$I = \frac{3}{2}$	$J^P = \frac{3}{2}^+$	P_{33} (Δ)
$I = \frac{1}{2}$	$J^P = \frac{5}{2}^+$	F_{15}
$I = \frac{1}{2}$	$J^P = \frac{3}{2}^-$	D_{13}
$I = \frac{1}{2}$	$J^P = \frac{1}{2}^+$	P_{11}

Harmonic Oscillator States



$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)$$

$$\vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

$$H = 3m + \frac{p_\rho^2 + p_\lambda^2}{2m} + \underbrace{\frac{1}{2}\sum_{i<j} \kappa\, r_{ij}^2}_{\frac{3}{2}\kappa(\rho^2+\lambda^2) \,=\, \frac{3}{2}\kappa\, x^2}$$

$x = (\rho^2 + \lambda^2)^{1/2}$ hyperradius

$\xi \quad E = (3 + N_{h.o.})\,\hbar\omega \qquad \omega = \frac{\alpha^2}{m}$

$$\psi_{3q} \propto e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)} \longleftrightarrow \begin{cases} e^{-\frac{\alpha^2}{2}(r_1^2+r_2^2+r_3^2)} \\ \text{without c.m. motion} \end{cases}$$

$\rightsquigarrow$ **Quark Shell Model**

$$\alpha^2 = (3\kappa m)^{1/2}$$

$$N_{h.o.} = 2(\nu + n) + \ell_\rho + \ell_\lambda$$

$(\nu \swarrow x, \quad n \downarrow \xi)$

$$E = (3 + N_{h.o.})\,\hbar\omega \qquad \omega = \frac{\alpha^2}{m}$$

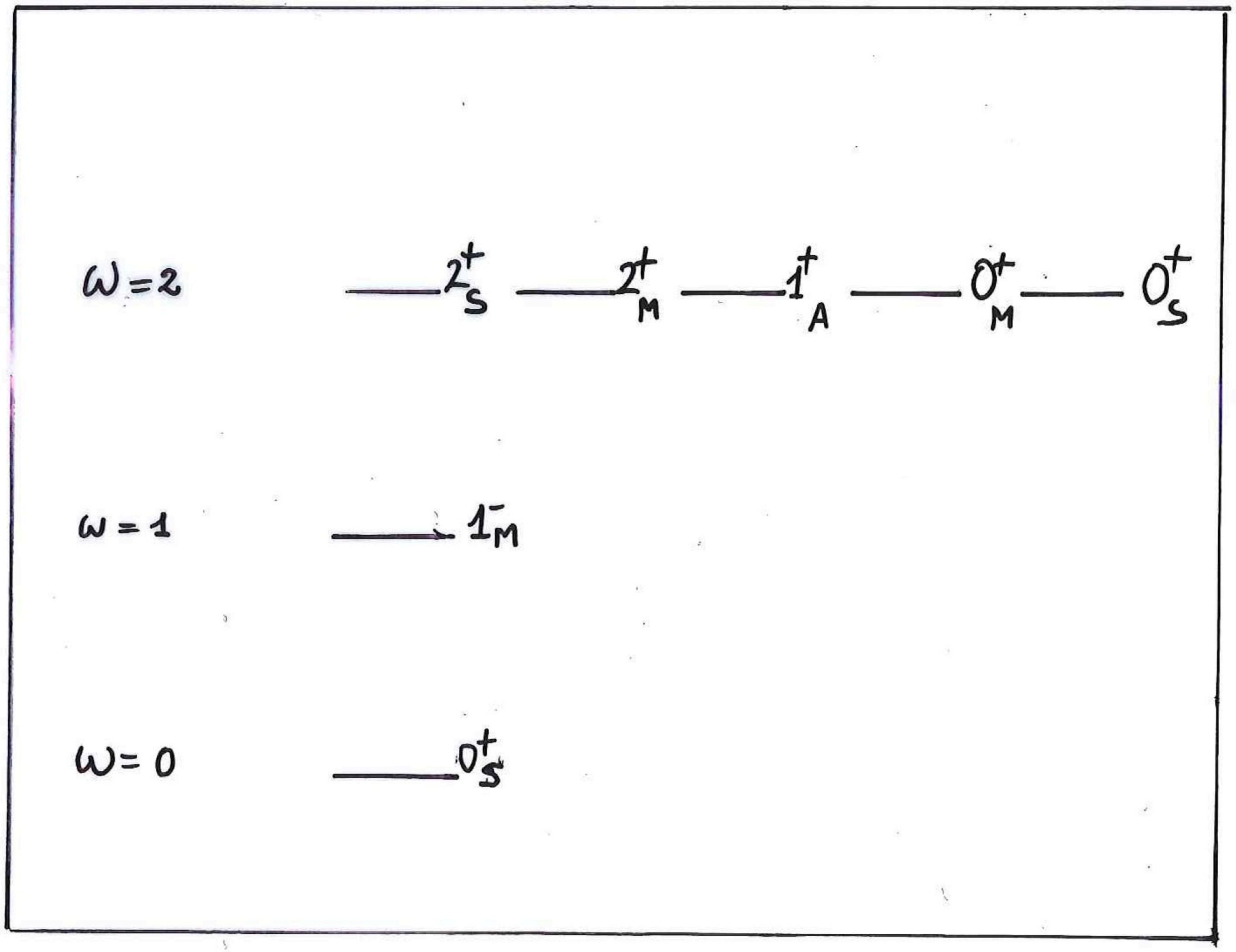
$$N_{h.o.} = 2(\nu + n) + \ell_\rho + \ell_\lambda \qquad \alpha^2 = (3Km)^{1/2}$$

$\nu \to x$, $n \to \xi$

General structure of the h.o. wave functions

$$\Psi_{NLt} = \mathcal{N}\; P_N(\rho,\lambda)\; e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)}\; Y_{\ell_\rho}(\Omega_\rho)\, Y_{\ell_\lambda}(\Omega_\lambda)$$

- t: symm. type
- $\mathcal{N}$: norm. factor
- P_N: polyn. degree N
- $Y_{\ell_\rho}(\Omega_\rho)\, Y_{\ell_\lambda}(\Omega_\lambda)$: combined to total ang. mom. L


ω=2
2^+_S
2^+_M
1^+_A
0^+_M
0^+_S
ω=1
1^-_M
ω=0
0^+_S

The general structure of the h.o. wave functions is

$$\Psi_{NLt} = \mathcal{N} P_N(\rho,\lambda)\, e^{-\frac{\alpha^2}{2}(\rho^2+\lambda^2)}\, Y_{l_\rho}(\Omega_\rho)\, Y_{l_\lambda}(\Omega_\lambda)$$

TABLE 2 - The harmonic oscillator wave functions for the 3-quark system [5,6] according to eq. (8). The presence of two items in the same line means that the correct symmetry property is obtained with a linear combination of the two wave functions. The quantity J_2 in the normalization factor is given by $4\alpha^3/\sqrt{\pi}$ and t(=A,M,S) denotes the type of permutation symmetry. The parity Π is $(-)^N$.

Ψ_{nLt}	N	ν	n	l_ρ	l_λ	L	Π	$\mathcal{N}/J_2$	P_N
Ψ_{00S}	0	0	0	0	0	0	+	1	1
Ψ^{ρ}_{11M}	1	0	0	1	0	1	-	$\alpha\sqrt{2/3}$	ρ
Ψ^{λ}_{11M}	1	0	0	0	1	1	-	$\alpha\sqrt{2/3}$	λ
Ψ_{20S}	2	1	0	0	0	0	+	$1/\sqrt{3}$	$\alpha^2(\rho^2+\lambda^2)-3$
Ψ_{20M}	2	0	1	0	0	0	+	$\alpha^2/\sqrt{3}$	$\rho^2-\lambda^2$
$\Psi_{22S/22M}$	2	0	0	2	0	2	+	$2\alpha^2/\sqrt{15}$	ρ^2
$\Psi_{22S/22M}$	2	0	0	0	2	2	+	$2\alpha^2/\sqrt{15}$	λ^2
Ψ'_{20M}	2	0	0	1	1	0	+	$2\alpha^2/3$	$\rho\lambda$
Ψ_{21A}	2	0	0	1	1	1	+	$2\alpha^2/3$	$\rho\lambda$
Ψ'_{22M}	2	0	0	1	1	2	+	$2\alpha^2/3$	$\rho\lambda$

Table 6. Three-quark states with positive parity. For simplicity of notation, we have omitted the coupling to the total angular momentum L of the second column

Resonance	$L^P_{S_3}$	S	T	SU(6) configurations
$P11$	0^+_S	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{00}Y_{[0]00}\Omega_S$
	0^+_S	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{10}Y_{[0]00}\Omega_S$
	0^+_S	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{20}Y_{[0]00}\Omega_S$
	0^+_M	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[Y_{[2]00}\Omega_{MS}+Y_{[2]11}\Omega_{MA}]$
	2^+_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(Y_{[2]20}-Y_{[2]02})\phi_{MS}+Y_{[2]11}\phi_{MA}]\chi_S$
$P13$	2^+_M	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(Y_{[2]20}-Y_{[2]02})\Omega_{MS}+Y_{[2]11}\Omega_{MA}]$
	2^+_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(Y_{[2]20}-Y_{[2]02})\phi_{MS}+Y_{[2]11}\phi_{MA}]\chi_S$
	0^+_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[Y_{[2]00}\phi_{MS}+Y_{[2]11}\phi_{MA}]\chi_S$
	2^+_S	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[Y_{[2]20}+Y_{[2]02}]\Omega_S$
$F15$	2^+_M	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(Y_{[2]20}-Y_{[2]02})\Omega_{MS}+Y_{[2]11}\Omega_{MA}]$
	2^+_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(Y_{[2]20}-Y_{[2]02})\phi_{MS}+Y_{[2]11}\phi_{MA}]\chi_S$
	2^+_S	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[Y_{[2]20}+Y_{[2]02}]\Omega_S$
$F17$	2^+_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(Y_{[2]20}-Y_{[2]02})\phi_{MS}+Y_{[2]11}\phi_{MA}]\chi_S$
$P31$	2^+_S	$\frac{3}{2}$	$\frac{3}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[(Y_{[2]20}+Y_{[2]02}]\chi_S\phi_S$
	0^+_M	$\frac{1}{2}$	$\frac{3}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[Y_{[2]00}\chi_{MS}+Y_{[2]11}\chi_{MA}]\phi_S$
$P33$	0^+_S	$\frac{3}{2}$	$\frac{3}{2}$	$\psi_{00}Y_{[0]00}\chi_S\phi_S$
	0^+_S	$\frac{3}{2}$	$\frac{3}{2}$	$\psi_{10}Y_{[0]00}\chi_S\phi_S$
	0^+_S	$\frac{3}{2}$	$\frac{3}{2}$	$\psi_{20}Y_{[0]00}\chi_S\phi_S$
	2^+_S	$\frac{3}{2}$	$\frac{3}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[Y_{[2]20}+Y_{[2]02}]\chi_S\phi_S$
	2^+_M	$\frac{1}{2}$	$\frac{3}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(Y_{[2]20}-Y_{[2]02})\chi_{MS}+Y_{[2]11}\chi_{MA}]\phi_S$
$F35$	2^+_M	$\frac{1}{2}$	$\frac{3}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(Y_{[2]20}-Y_{[2]02})\chi_{MS}+Y_{[2]11}\chi_{MA}]\phi_S$
	2^+_S	$\frac{3}{2}$	$\frac{3}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[Y_{[2]20}+Y_{[2]02}]\chi_S\phi_S$
$F37$	2^+_S	$\frac{3}{2}$	$\frac{3}{2}$	$\psi_{22}\frac{1}{\sqrt{2}}[Y_{[2]20}+Y_{[2]02}]\chi_S\phi_S$

$$\chi_{MS} = |((\frac{1}{2},\frac{1}{2})1,\frac{1}{2})\frac{1}{2}>,$$

$$\chi_{MA} = |((\frac{1}{2},\frac{1}{2})0,\frac{1}{2})\frac{1}{2}>,$$

$$\chi_S = |((\frac{1}{2},\frac{1}{2})1,\frac{1}{2})\frac{3}{2}>,$$

$$\Omega_S = \frac{1}{\sqrt{2}}[\chi_{MA}\phi_{MA}+\chi_{MS}\phi_{MS}],$$

$$\Omega_{MS} = \frac{1}{\sqrt{2}}[\chi_{MA}\phi_{MA}-\chi_{MS}\phi_{MS}],$$

$$\Omega_{MA} = \frac{1}{\sqrt{2}}[\chi_{MA}\phi_{MS}+\chi_{MS}\phi_{MA}],$$

$$\Omega_A = \frac{1}{\sqrt{2}}[\chi_{MA}\phi_{MS}-\chi_{MS}\phi_{MA}],$$

Table 7. Three quark states with negative parity

Resonances	$L^P_{S_3}$	S	T	States
$S11$	1^-_M	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{11}\frac{1}{\sqrt{2}}[Y_{[1]10}\Omega_{MA} + Y_{[1]01}\Omega_{MS}]$
	1^-_M	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{21}\frac{1}{\sqrt{2}}[Y_{[1]10}\Omega_{MA} + Y_{[1]01}]\Omega_{MS}$
	1^-_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{11}\frac{1}{\sqrt{2}}[Y_{[1]10}\phi_{MA} + Y_{[1]01}\phi_{MS}]\chi_S$
	1^-_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{21}\frac{1}{\sqrt{2}}[Y_{[1]10}\phi_{MA} + Y_{[1]01}\phi_{MS}]\chi_S$
$D13$	1^-_M	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{11}\frac{1}{\sqrt{2}}[Y_{[1]10}\Omega_{MA} + Y_{[1]01}\Omega_{MS}]$
	1^-_M	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_{21}\frac{1}{\sqrt{2}}[Y_{[1]10}\Omega_{MA} + Y_{[1]01}]\Omega_{MS}$
	1^-_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{11}\frac{1}{\sqrt{2}}[Y_{[1]10}\phi_{MA} + Y_{[1]01}\phi_{MS}]\chi_S$
	1^-_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{21}\frac{1}{\sqrt{2}}[Y_{[1]10}\phi_{MA} + Y_{[1]01}\phi_{MS}]\chi_S$
$D15$	1^-_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{11}\frac{1}{\sqrt{2}}[Y_{[1]10}\phi_{MA} + Y_{[1]01}\phi_{MS}]\chi_S$
	1^-_M	$\frac{3}{2}$	$\frac{1}{2}$	$\psi_{21}\frac{1}{\sqrt{2}}[Y_{[1]10}\phi_{MA} + Y_{[1]01}\phi_{MS}]\chi_S$
$S31$	1^-_M	$\frac{1}{2}$	$\frac{3}{2}$	$\psi_{11}\frac{1}{\sqrt{2}}[Y_{[1]10}\chi_{MA} + Y_{[1]01}\chi_{MS}]\phi_S$
	1^-_M	$\frac{1}{2}$	$\frac{3}{2}$	$\psi_{21}\frac{1}{\sqrt{2}}[Y_{[1]10}\chi_{MA} + Y_{[1]01}\chi_{MS}]\phi_S$
$S33$	1^-_M	$\frac{1}{2}$	$\frac{3}{2}$	$\psi_{11}\frac{1}{\sqrt{2}}[Y_{[1]10}\chi_{MA} + Y_{[1]01}\chi_{MS}]\phi_S$
	1^-_M	$\frac{1}{2}$	$\frac{3}{2}$	$\psi_{21}\frac{1}{\sqrt{2}}[Y_{[1]10}\chi_{MA} + Y_{[1]01}\chi_{MS}]\phi_S$

Flavor wave functions

the flavor wave functions $|(p,q),I,M_I,Y\rangle$

(i) The octet baryons $(p,q)=(1,1)$:

$$
\begin{aligned}
|(1,1),1/2,1/2,1\rangle \;&:\; \phi_\rho(p) \;=\; [|udu\rangle - |duu\rangle]/\sqrt{2}\ , \\
&:\; \phi_\lambda(p) \;=\; [2|uud\rangle - |udu\rangle - |duu\rangle]/\sqrt{6}\ , \\
|(1,1),1,1,0\rangle \;&:\; \phi_\rho(\Sigma^+) \;=\; [|suu\rangle - |usu\rangle]/\sqrt{2}\ , \\
&:\; \phi_\lambda(\Sigma^+) \;=\; [|suu\rangle + |usu\rangle - 2|uus\rangle]/\sqrt{6}\ , \\
|(1,1),0,0,0\rangle \;&:\; \phi_\rho(\Lambda) \;=\; [2|uds\rangle - 2|dus\rangle - |dsu\rangle + |sdu\rangle - |sud\rangle + |usd\rangle]/\sqrt{12}\ , \\
&:\; \phi_\lambda(\Lambda) \;=\; [-|dsu\rangle - |sdu\rangle + |sud\rangle + |usd\rangle]/2\ ,
\end{aligned}
$$

$$
\begin{aligned}
|(1,1),1/2,1/2,-1\rangle \;&:\; \phi_\rho(\Xi^0) \;=\; [|sus\rangle - |uss\rangle]/\sqrt{2}\ , \\
&:\; \phi_\lambda(\Xi^0) \;=\; [2|ssu\rangle - |sus\rangle - |uss\rangle]/\sqrt{6}\ .
\end{aligned}
$$

(ii) The decuplet baryons $(p,q)=(3,0)$:

$$
\begin{aligned}
|(3,0),3/2,3/2,1\rangle \;&:\; \phi_S(\Delta^{++}) \;=\; |uuu\rangle\ , \\
|(3,0),1,1,0\rangle \;&:\; \phi_S(\Sigma^+) \;=\; [|suu\rangle + |usu\rangle + |uus\rangle]/\sqrt{3}\ , \\
|(3,0),1/2,1/2,-1\rangle \;&:\; \phi_S(\Xi^0) \;=\; [|ssu\rangle + |sus\rangle + |uss\rangle]/\sqrt{3}\ , \\
|(3,0),0,0,-2\rangle \;&:\; \phi_S(\Omega^-) \;=\; |sss\rangle\ .
\end{aligned}
$$

(iii) The singlet baryons $(p,q)=(0,0)$:

$$
|(0,0),0,0,0\rangle \;:\; \phi_A(\Lambda) \;=\; [|uds\rangle - |dus\rangle + |dsu\rangle - |sdu\rangle + |sud\rangle - |usd\rangle]/\sqrt{6}
$$

The S_3 invariant space-spin-flavor ($\Psi = \psi\chi\phi$) baryon wave functions are given by

$$
\begin{aligned}
{}^2 8[56, L^P] &\quad : \quad \psi_S(\chi_\rho\phi_\rho + \chi_\lambda\phi_\lambda)/\sqrt{2} \ , \\
{}^2 8[70, L^P] &\quad : \quad [\psi_\rho(\chi_\rho\phi_\lambda + \chi_\lambda\phi_\rho) + \psi_\lambda(\chi_\rho\phi_\rho - \chi_\lambda\phi_\lambda)]/2 \ , \\
{}^4 8[70, L^P] &\quad : \quad (\psi_\rho\phi_\rho + \psi_\lambda\phi_\lambda)\chi_S/\sqrt{2} \ , \\
{}^2 8[20, L^P] &\quad : \quad \psi_A(\chi_\rho\phi_\lambda - \chi_\lambda\phi_\rho)/\sqrt{2} \ , \\
{}^4 10[56, L^P] &\quad : \quad \psi_S\chi_S\phi_S \ , \\
{}^2 10[70, L^P] &\quad : \quad (\psi_\rho\chi_\rho + \psi_\lambda\chi_\lambda)\phi_S/\sqrt{2} \ , \\
{}^2 1[70, L^P] &\quad : \quad (\psi_\rho\chi_\lambda - \psi_\lambda\chi_\rho)\phi_A/\sqrt{2} \ , \\
{}^4 1[20, L^P] &\quad : \quad \psi_A\chi_S\phi_A \ .
\end{aligned}
$$

Magnetic moments of Baryons

Single quark magnetic moment operator

$$\vec{\mu}_j = \frac{e_j}{2m_j}\vec{\sigma}_j$$

μ_B is given by the matrix element

$$< B\frac{1}{2}, \frac{1}{2})| \sum_{j=1,2,3} \frac{e_j}{2m_j}\vec{\sigma}_j |B(\frac{1}{2}, \frac{1}{2}) >$$

proton

the u pair is in a symmetric (triplet) spin state $\chi(1,m)$

(the antisimmetry is ensured by the color wf)

$\phi(1/2,s)$ spin state of the third quark

proton state

$$\psi(\frac{1}{2},\frac{1}{2}) = \sqrt{\frac{2}{3}}\chi(1,1)\phi(\frac{1}{2},-\frac{1}{2}) - \sqrt{\frac{1}{3}}\chi(1,0)\phi(\frac{1}{2},\frac{1}{2})$$

$$\mu_p = \frac{2}{3}(2\mu_u - \mu_d) + \frac{1}{3}\mu_d = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$

neutron (u and interchanged)

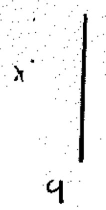
$$\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u \qquad \frac{\mu_n}{\mu_p} = -\frac{2}{3} \qquad (exp - 0.685)$$

Baryon magnetic moments in nuclear magnetons (n.m.), normalised to proton and lambda moments

Baryon	Magnetic moment in quark model	Predicted, n.m.	Observed, n.m.
p	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	+2.79	+2.793
n	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	−1.86	−1.913
Λ	μ_s	−0.61	-0.614 ± 0.005
Σ^+	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	+2.68	$+2.46 \pm 0.01$
Σ^-	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_s$	−1.04	-1.16 ± 0.03
Ξ^0	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	−1.44	-1.25 ± 0.014
Ξ^-	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	−0.51	-0.65 ± 0.01
Ω^-	$3\mu_s$	−1.84	-2.02 ± 0.05

The baryon current

QUARKS ARE THE FUNDAMENTAL CARRIERS OF THE BARYON CHARGE



$$j_\mu^{(B)} = \sum_{i=1}^{3} j_\mu^{(q)}(i)$$

$j_\mu^{(q)} \uparrow \bar{q}\gamma_\mu q$ quark current (pointlike quarks)

non relativistic reduction:

$$\rho(\vec{q}) = \sum_i e_i \, e^{i\vec{q}\cdot\vec{r}_i}$$

$$\vec{j}(\vec{q}) = \frac{1}{2m}\sum_i e_i \left[\vec{p}_i^{\,\prime} + \vec{p}_i + i\,\vec{\sigma}_q(i)\times\vec{q}\right] e^{i\vec{q}\cdot\vec{r}_i}$$

$\vec{\sigma}_q(i)$ ⟶ quark spin

quark charge:

$$e_i = \frac{1}{2}\left[\frac{1}{3} + \tau_0^q(i)\right]$$

$\tau_0^q(i)$ ⟶ quark isospin

(u, d only)

$\rho_B = \langle B | \rho | B \rangle$ $\qquad\qquad$ $\vec{J}_B = \langle B | \vec{J} | B \rangle$

- $\langle B | \to$ 3q state
- $\rho \to \sum_i e_i e^{i\vec{q}\cdot\vec{r}_i}$
- $\langle B' |$ for e.m. excitation

example : magnetic moments $\begin{cases} N \\ H_{hyp} = \end{cases}$

$$\mu = \langle \underbrace{N, J_z = +\tfrac{1}{2}}_{3q} | \sum_i \frac{e_i \hbar}{2mc} \sigma_z^q(i) | \underbrace{N, J_z = +\tfrac{1}{2}}_{3q} \rangle$$

$m \to \frac{M_N}{3}$

$$= 3 \cdot \frac{e\hbar}{2M_N c} 3 \underbrace{\langle e_3 \sigma_z^q(3) \rangle}_{\frac{1}{2}\langle \phi\chi | e_3 \sigma_z^{(q)}(3) | \phi\chi \rangle}$$

(ψ_{3q} symm.) (ψ_{space} norm.)

nucleon isospin

$$= \frac{e\hbar}{2M_N c} \cdot \frac{1 + 5\tau_0}{2} = \begin{cases} 3\,\mu_N & \text{proton} \\ -2\,\mu_N & \text{neutron} \end{cases}$$

Note: $\langle \vec{\sigma}^q \rangle = \frac{1}{3} \vec{\sigma}_N$

$\langle \vec{\sigma}^q \tau_0^q \rangle = \frac{5}{9} \vec{\sigma}_N \tau_0$

Ex. $\langle r^2 \rangle_p = 1/\alpha^2 = 0.89\ \text{fm}^2$, $\quad \alpha = 1.23\ \text{fm}^{-1}$

The Isgur and Karl model
(how to correct the defect of to the H.O. model)

$$H = 3m + \frac{p_\rho^2 + p_\lambda^2}{2m} + L(\vec{\rho}, \vec{\lambda}) + H_{hyp}(\vec{\rho}, \vec{\lambda}, \vec{\sigma}_i)$$

$$L = \sum_{i<j} \left(\frac{1}{2} K r_{ij}^2 + U(r_{ij})\right) \equiv V_{conf}$$

The term $L(\vec{\rho}, \vec{\lambda})$ provides confinement again through a h.o. potential, to which however an anharmonic term U is added

$$L = \sum_{i<j} \left(\frac{1}{2} K r_{ij}^2 + U(r_{ij})\right) \equiv V_{conf}$$

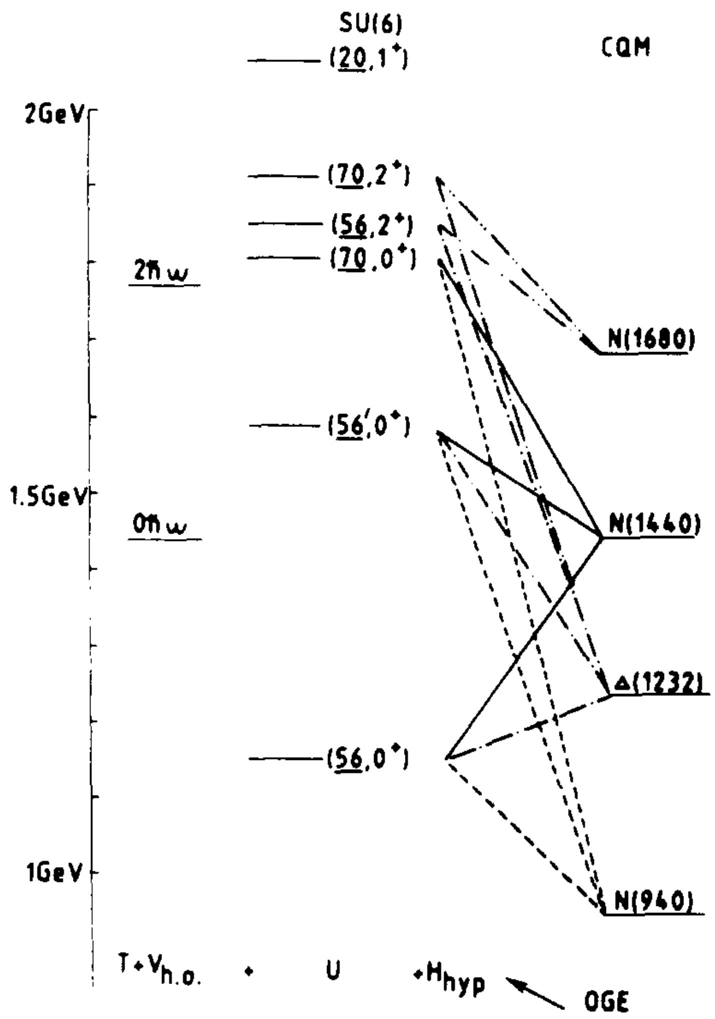
$$H_{hyp} = \frac{2\alpha_s}{3m^2} \sum_{i<j} \left\{ \frac{8\pi}{3} \vec{S}_i \, \vec{S}_j \, \delta(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} \left[\frac{3 (\vec{S}_i \cdot \vec{r}_{ij}) (\vec{S}_j \cdot \vec{r}_{ij})}{r_{ij}^2} - \vec{S}_i \cdot \vec{S}_j \right] \right\}$$

$$a_m = 3 (\alpha/\pi^{\frac{1}{2}})^3 \int d\rho \, (\alpha\rho)^m \, U(\sqrt{2}\,\rho) \exp(-\alpha^2\rho^2)$$

through the quantities

$$E_0 = \varepsilon_0 + a_0 \; ; \qquad \Omega = \hbar\omega - a_0/2 + a_2/3 \; ; \qquad \Delta = -5/4\, a_0 + 5/3\, a_2 - 1/3\, a_4$$

$E(\underline{56}, 0^+) = E_0$	$E(\underline{70}, 1^-) = E_0 + \Omega$
$E(\underline{56}', 0^+) = E_0 + 2\Omega - \Delta$	$E(\underline{70}, 0^+) = E_0 + 2\Omega - \Delta/2$
$E(\underline{56}, 2^+) = E_0 + 2\Omega - 2\Delta/5$	$E(\underline{70}, 2^+) = E_0 + 2\Omega - \Delta/5$



SU(6)
CQM
2GeV
1.5GeV
1GeV
2ħω
0ħω
(20,1+)
(70,2+)
(56,2+)
(70,0+)
(56',0+)
(56,0+)
N(1680)
N(1440)
Δ(1232)
N(940)
T+Vh.o. + U +Hhyp
OGE

$\rho_B = \langle B | \rho | B \rangle$ $\qquad$ $\vec{J}_B = \langle B | \vec{J} | B \rangle$

$\langle B' |$ for e.m. excitation

$\rho \to \sum_i e_i e^{i\vec{q}\cdot\vec{r}_i}$

$|B\rangle \to$ 3q state

<u>example</u> : magnetic moments $\left\{ \begin{array}{l} N \\ H_{hyp} = \end{array} \right.$

$$\mu = \langle \underbrace{N, J_z = +\tfrac{1}{2}}_{3q} | \sum_i \frac{e_i \hbar}{2mc} \sigma_z^q(i) | \underbrace{N, J_z = +\tfrac{1}{2}}_{3q} \rangle$$

$\frac{e_i \hbar}{2mc} \to \frac{M_N}{3}$

$$= 3 \cdot \frac{e\hbar}{2M_N c} 3 \underbrace{\langle e_3 \sigma_z^q(3) \rangle}_{\frac{1}{2} \langle \phi\chi | e_3 \sigma_z^{(q)}(3) | \phi\chi \rangle}$$

(ψ_{3q} symm.) (ψ_{space} norm.)

nucleon isospin (τ_0)

$$= \frac{e\hbar}{2M_N c} \cdot \frac{1 + 5\tau_0}{2} = \begin{cases} 3\,\mu_N & \text{proton} \\ -2\,\mu_N & \text{neutron} \end{cases}$$

Note: $\langle \vec{\sigma}^q \rangle = \frac{1}{3} \vec{\sigma}_N$

$\langle \vec{\sigma}^q \tau_0^q \rangle = \frac{5}{9} \vec{\sigma}_N \tau_0$

Ex. $\langle r^2 \rangle_p = 1/\alpha^2$

Algebraic solution of the Coulomb problem

A) **three dimensions** $\frac{1}{r}$

$\therefore$ **angular momentum** $\rightarrow$ **O(3) symmetry**

$\therefore$ **Runge-Lenz vector** $\rightarrow$ **O(4) symmetry**

$$H = -\frac{1}{2\,[C_2(O(4)) + 1]} \rightarrow E = -\frac{1}{2\,n^2}$$

[$C_2(O(4))$ Casimir for O(4)]

since

$$C_2(O(4)) = \omega\,(\omega + 2) \qquad n = \omega + 1$$

In general the eigenvalues of C_2(O(N))=ω (ω+N-2)

algebraic solution to the hyperCoulomb problem

B) six dimensions $\frac{1}{x}$

∴ hyperangular invariance → O(6) symmetry

∴ "Runge-Lenz vector" → O(7) symmetry

$$H = -\frac{1}{2\,[C_2(O(7)) + (5/2)^2]} \rightarrow E = -\frac{1}{2\,n^2}$$

$\nu + 1$ $((\nu-1)/2)^2$

$$C_2(O(7)) = \omega\,(\omega + 5) \qquad n = \omega + 5/2$$

$\omega\,(\omega + \nu - 1)$ $\omega + (\nu-1)/2$

(iachello, Giannini, Santopinto)

$$H = \sum_{i=1}^{6} \frac{p_i^2}{2m} - \frac{\tau}{x}$$

$$p_i = (\vec{p}_\rho, \vec{p}_\lambda)$$

$$q_i = (\vec{\rho}, \vec{\lambda})$$

$$x = \left(\sum q_i^2\right)^{1/2}$$

$$\underbrace{L_{ij} = q_i p_j - q_j p_i}_{O(6)}$$

15 generators

$$[H, L_{ij}] = 0$$

"Runge-Lenz" vector :

$$M_i = \frac{1}{2m}\left(p_j L_{ij} + L_{ij} p_j\right) - \frac{\tau}{x} q_i$$

$$[H, M_i] = 0$$

generalization of classical R.L. vector

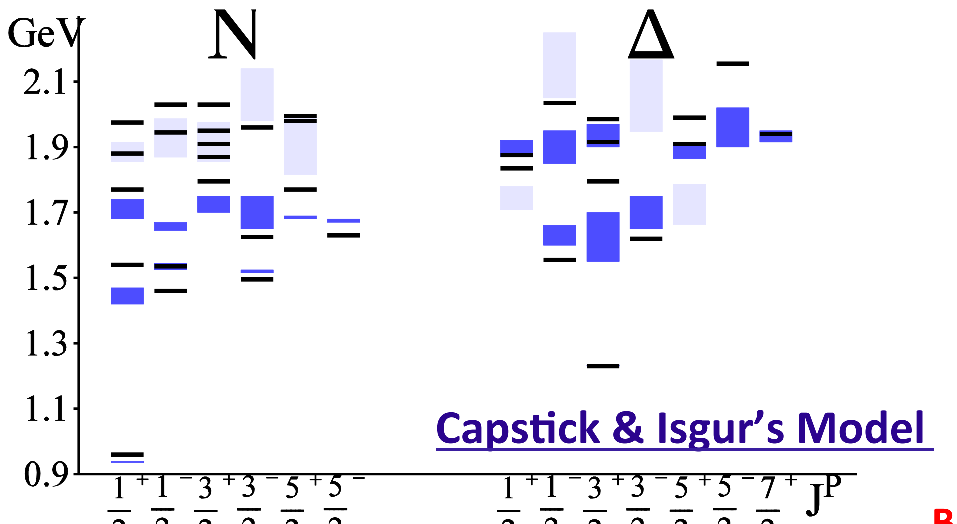
The Models
(CQM)

some other **C**onstituent **Q**uark **M**odel

different CQMs for bayons

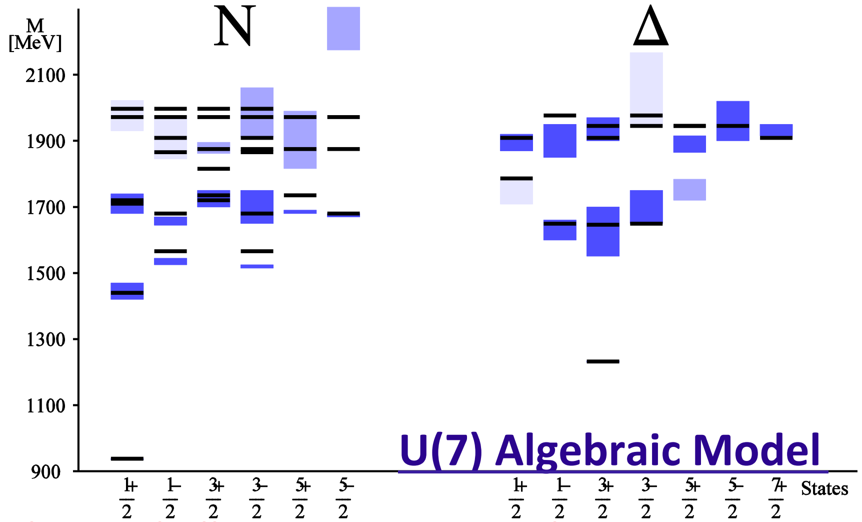
	Kin. Energy	SU(6) inv	SU(6) viol	date
Isgur-Karl	non rel	h.o. + shift	OGE	1978-9
Capstick-Isgur	rel	string + coul-like	OGE	1986
U(7) B.I.L.	rel M^2	vibr+L	Guersey-R	1994
Hyp. O(6)	non rel/rel	hyp.coul+linear	OGE	1995
Glozman Riska	non rel/rel**Plessas**	h.o./linear	GBE	1996
Bonn	rel	linear 3-body	instanton	2001

Non strange spectrum



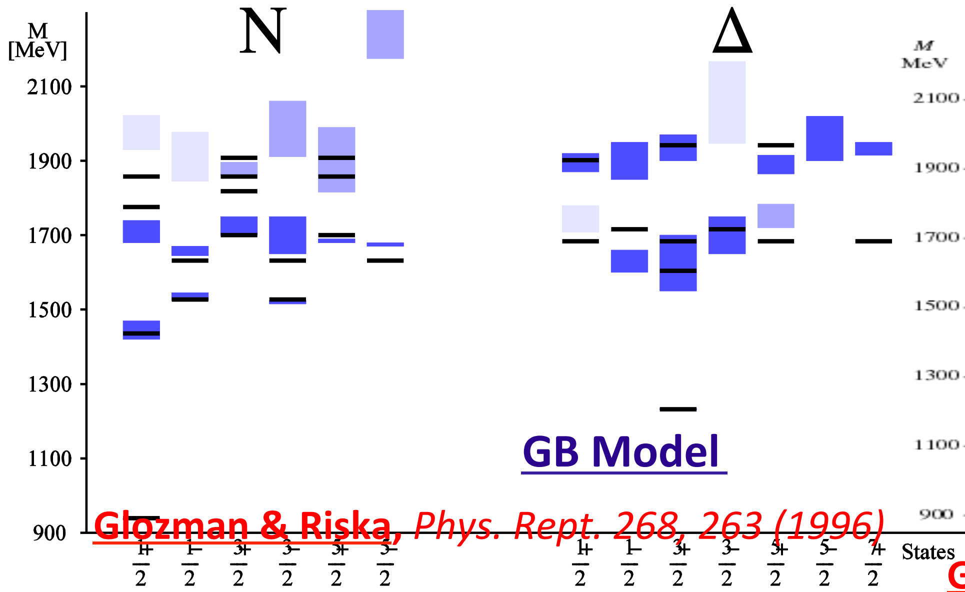
Capstick & Isgur's Model

Capstick and Isgur, *Phys. Rev. D34, 2809.*



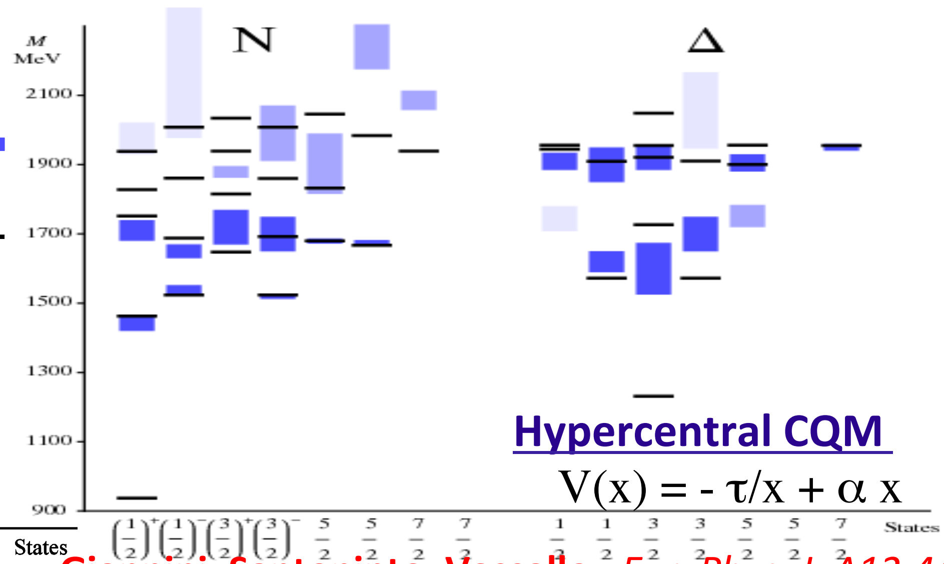
U(7) Algebraic Model

Bijker, Iachello, Leviatan, Ann. Phys. 236, 69 (1994)



GB Model

Glozman & Riska, *Phys. Rept. 268, 263 (1996)*



Hypercentral CQM

$V(x) = -\tau/x + \alpha x$

Giannini, Santopinto, Vassallo, *Eur. Phys. J .A12:447*

Hypercentral Constituent Quark Model
hCQM

free parameters fixed from the spectrum

Comment
The description of the spectrum is the first task of a model builder

Predictions for:

- photocouplings
- transition form factors
- elastic from factors
- ....

describe data (if possible)
understand what is missing

Hypercentral Constituent Quark Model
hCQM

free parameters fixed from the spectrum

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The description of the spectrum is the first task of a model builder

Predictions for:
- photocouplings
- transition form factors
- elastic from factors
- ....

describe data (if possible)
understand what is missing

LQCD (De Rújula, Georgi, Glashow, 1975)

the quark interaction contains

a long range spin-independent confinement

a short range spin dependent term

Spin-independence $\longrightarrow$ SU(6) configurations

SU(6) configurations for three quark states

$$6 \times 6 \times 6 = 20 + 70 + 70 + 56$$

A M M S

Notation

$$(d, L^{\pi})$$

d = dim of SU(6) irrep

L = total orbital angular momentum

π = parity

SU(6) configurations for three quark states

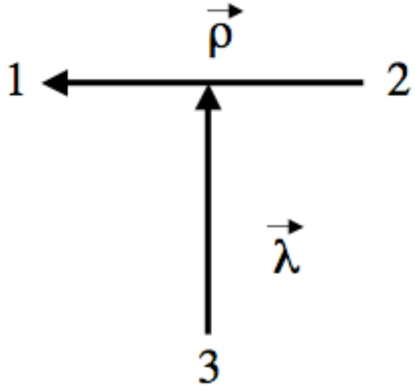
$$6 \text{ X } 6 \text{ X } 6 = \underset{A}{20} + \underset{M}{70} + \underset{M}{70} + \underset{S}{56}$$

Notation

(d, L^{π})

d = dim of SU(6) irrep
L = total orbital angular momentum
π = parity

Jacobi coordinates



Hyperspherical Coordinates

$$(\rho, \Omega_\rho, \lambda, \Omega_\lambda) \Rightarrow (x, t, \Omega_\rho, \Omega_\lambda)$$

$$x = \sqrt{\rho^2 + \lambda^2}$$ hyperradius

$$t = arctg\frac{\rho}{\lambda}$$ hyperangle

$$L^2(\Omega) Y_{[\gamma]}(\Omega) = -\gamma(\gamma+4) Y_{[\gamma]}(\Omega)^{\gamma = 2n + l_\rho + l_\lambda}$$

$$L^2(\Omega) \Leftrightarrow C_2(O(6))$$

γ grand angular quantum number

$Y_{[\gamma]}(\Omega)$ Hyperspherical harmonics

$$\sum_{i<j} V(\mathbf{r}_{ij}) \approx V(x) + \ldots$$

$\gamma = 2n + l_\rho + l_\lambda$

Hasenfratz et al. 1980:
$\Sigma\, V(r_i,r_j)$ is approximately hypercentral

Hypercentral Hypothesis

$$V = V(x)$$

Factorization

$$\psi(x, t, \Omega_\rho, \Omega_\lambda) = \quad \psi_{\nu\gamma}(x) \quad Y_{[\gamma, l_\rho, l_\lambda]}$$

("dynamics") ("geometry")

Only one differential equation in x (hyperradial equation)

Hypercentral Model

Genoa group, 1995

$V(x) = -\tau/x + \alpha\, x$

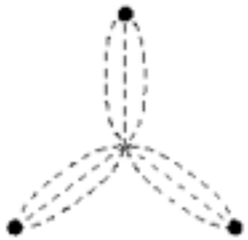
Hypercentral approximation of $V = - b/r + c\, r$

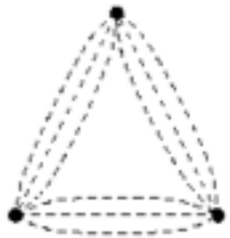
- QCD fundamental mechanism

3-body forces

Carlson et al, 1983
Capstick-Isgur 1986
hCQM 1995

- Flux tube model



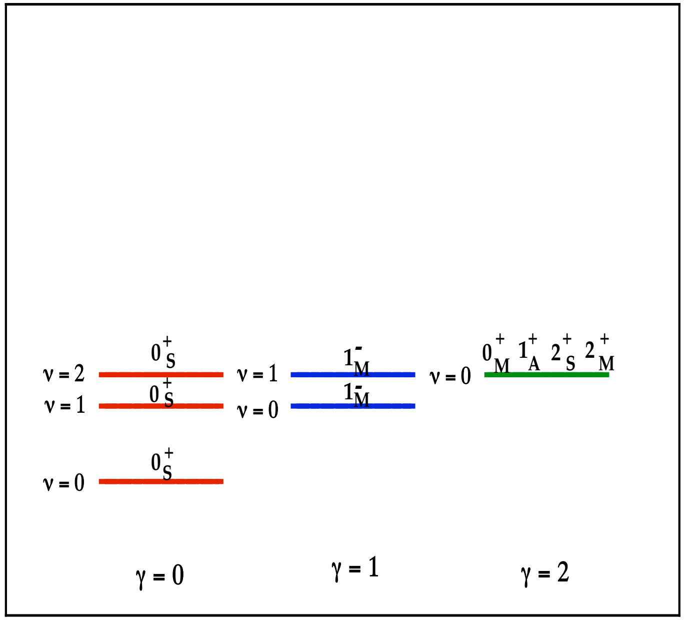


Two analytical solutions

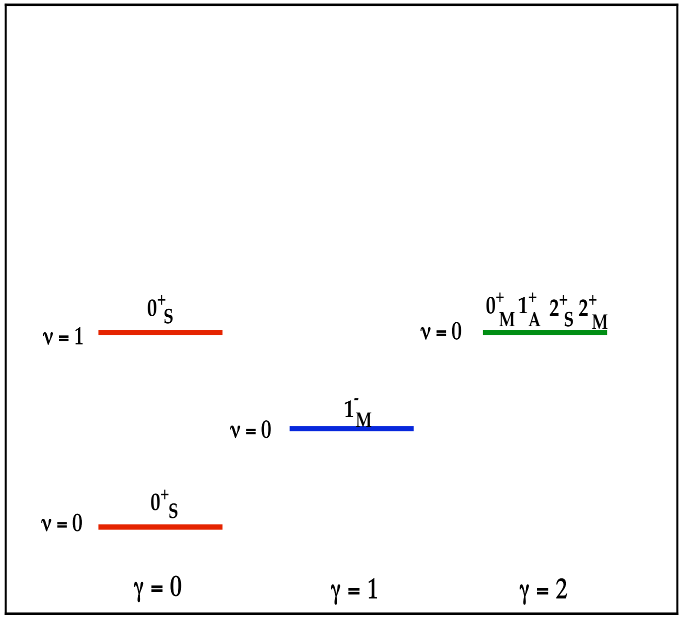
hyperCoulomb $\qquad - \tau/x$

h. o. $\qquad \Sigma_{i<j} 1/2\ k\ (r_i - r_j)^2 = 3/2\ \ k\ x^2$

a) HYPERCOULOMB



b) H. O.



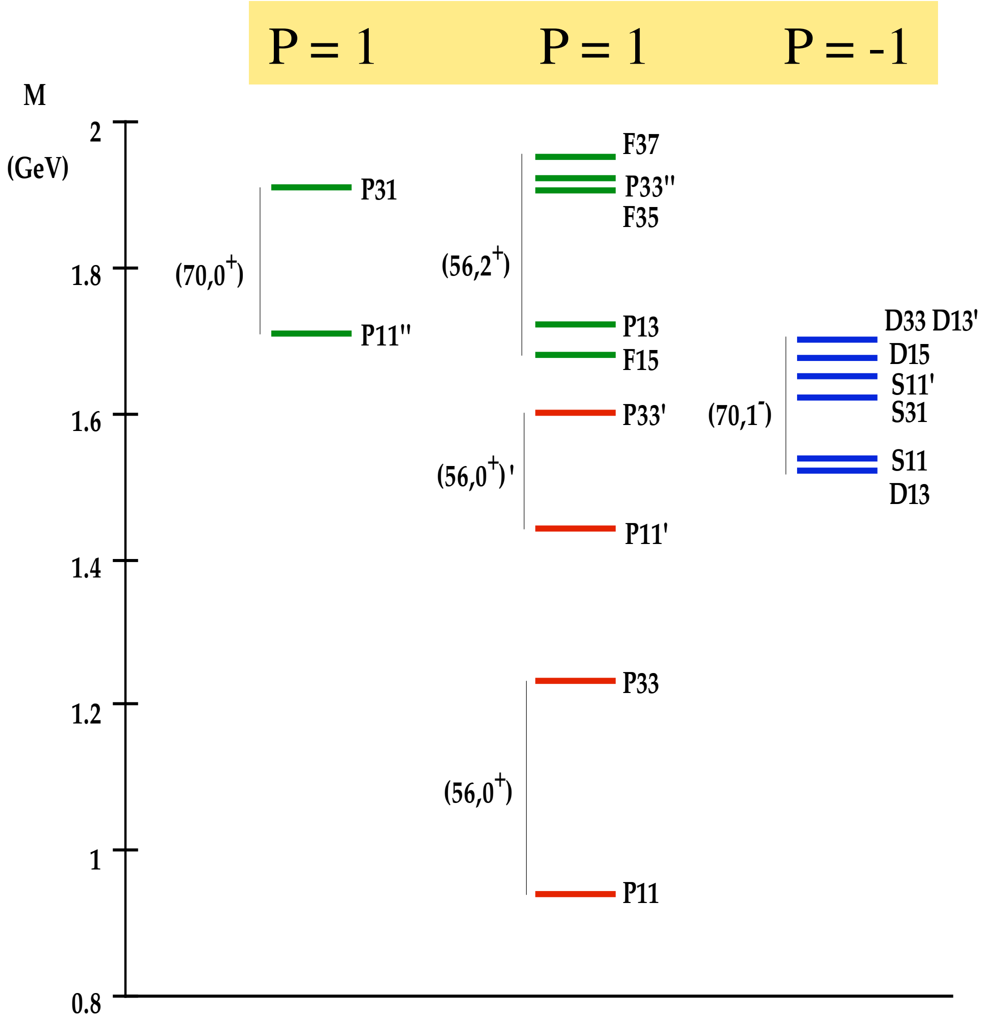
- H.O.
- W.fs $e^{-\alpha 2 r 2}$
- F.F. $e^{-\alpha 2 r 2/6}$
- Transition form factor :
- Polynomial $\times e^{-\alpha 2 r 2/6}$

- Hyp.
- W.fs Polinomial e^{-br}
- F.F. $:1/(1+Q^2{}_{/b\wedge 2})^{7/2}$
- Transition f.f.:

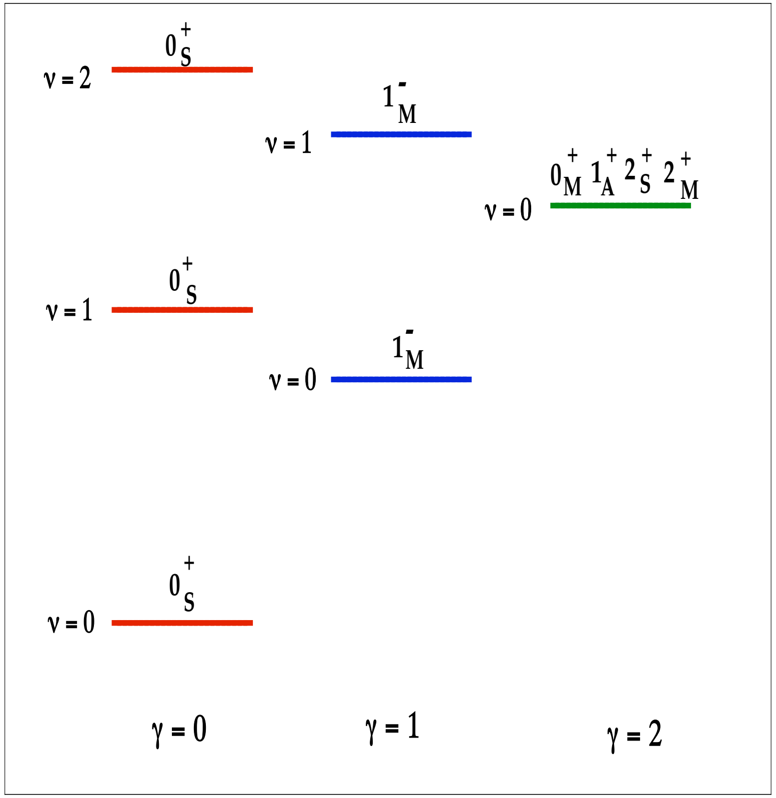
Polynomial×

$1/(1+Q^2{}_{/b\wedge 2})^{(7+n)/2}$

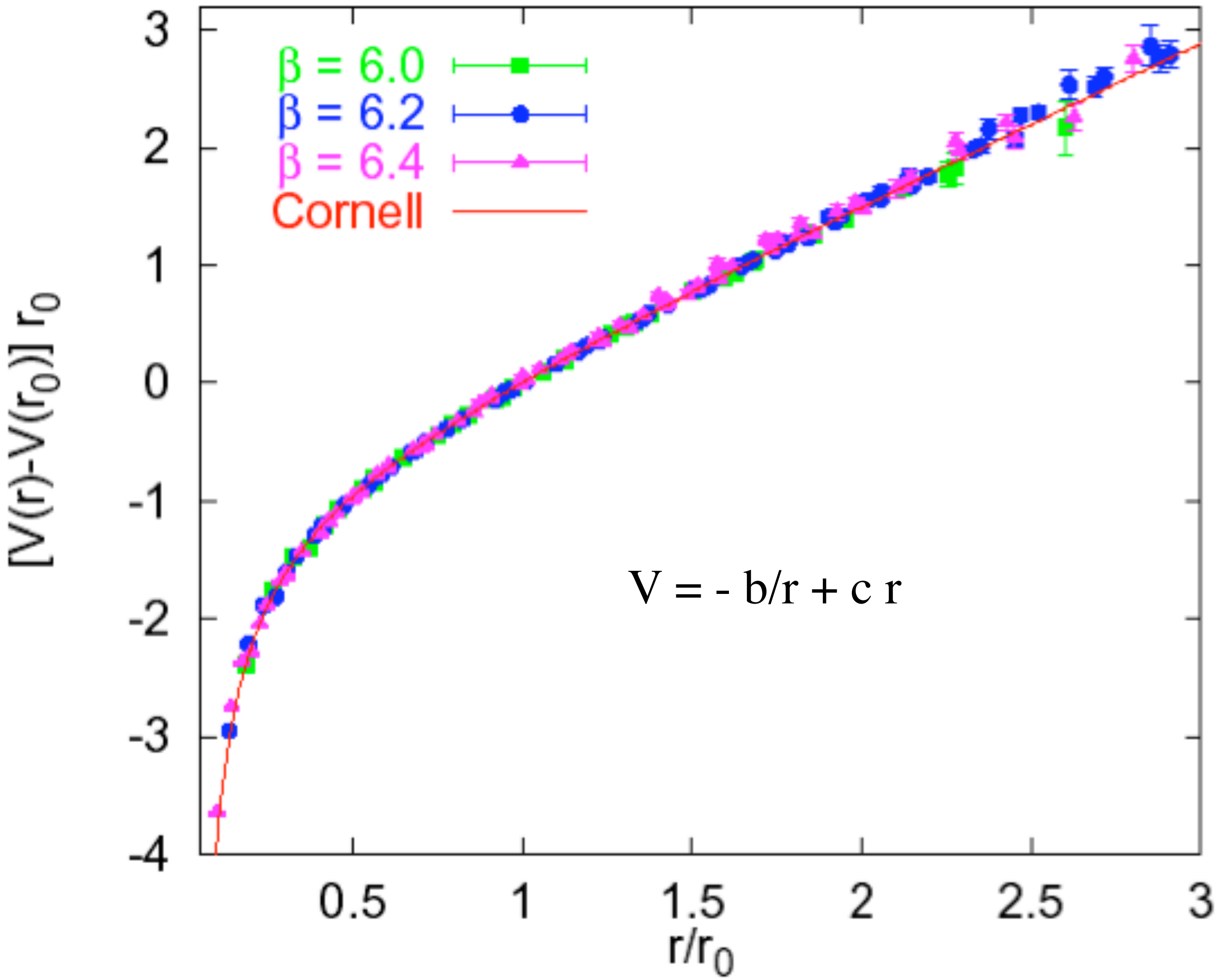
PDG 4* & 3*



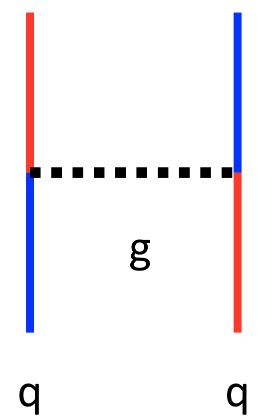
$$V(x) = -\tau/x + \alpha x$$



Quark-antiquark lattice potential G.S. Bali Phys. Rep. 343, 1 (2001)



Introducing SU(6) violation



One Gluon Exchange

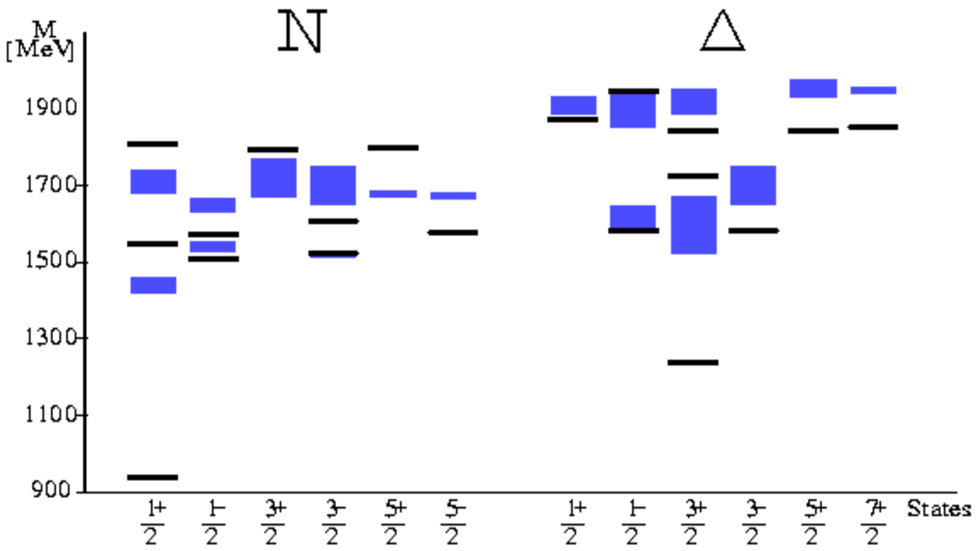
V_{OGE} = -a/r + Hyperfine interaction

Hypercentral Model (1)

$$H_{3q} = 3m + \sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m} + V(x) + H_{hyp}$$

M. Ferraris, M. M. Giannini, M. Pizzo, E. Santopinto, L. Tiator, Phys. Lett. B364 (1995), 231

- $V(x) = -\frac{\tau}{x} + \alpha x;$ $\quad H_{hyp} = A \left[\sum_{i<j} V^S(\mathbf{r}_i, \mathbf{r}_j) \ \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \text{tensor} \right]$
- 3 parameters τ α A $\leftarrow$ fixed to the spectrum, $m = \frac{M}{3}$



$\tau = 4.59$

$\alpha = 1.61 \text{ fm}^{-1}$

$A \Leftarrow (N - \Delta)$

$$x = \sqrt{\rho^2 + \lambda^2}$$

hyperradius

Results (predictions) with the Hypercentral Constituent Quark Model

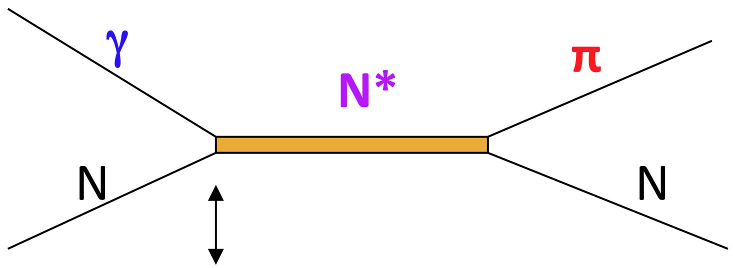
for

- Helicity amplitudes
- Elastic nucleon form factors

The helicity amplitudes

HELICITY AMPLITUDES

Extracted from electroproduction of mesons



$A_{1/2}$ $A_{3/2}$ $S_{1/2}$

Definition

$$A_{1/2} = \langle N^* \; J_z = 1/2 \mid H^T_{em} \mid N \; J_z = -1/2 \rangle \qquad \S$$
$$A_{3/2} = \langle N^* \; J_z = 3/2 \mid H^T_{em} \mid N \; J_z = 1/2 \rangle \qquad \S$$
$$S_{1/2} = \langle N^* \; J_z = 1/2 \mid H^L_{em} \mid N \; J_z = 1/2 \rangle$$

N, N* nucleon and resonance as 3q states

H^T_{em} H^l_{em} model transition operator

§ results for the negative parity resonances:
M. Aiello, M.G., E. Santopinto J. Phys. G24, 753 (1998)

Systematic predictions for transverse and longitudinal amplitudes
E. Santopinto, M.G., submitted to PR C

Definition

$$A_{1/2} = < N^* \; J_z = 1/2 \mid H^T_{em} \mid N \; J_z = -1/2 > \qquad \S$$
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N, N* nucleon and resonance as 3q states

H^T_{em} H^l_{em} model transition operator

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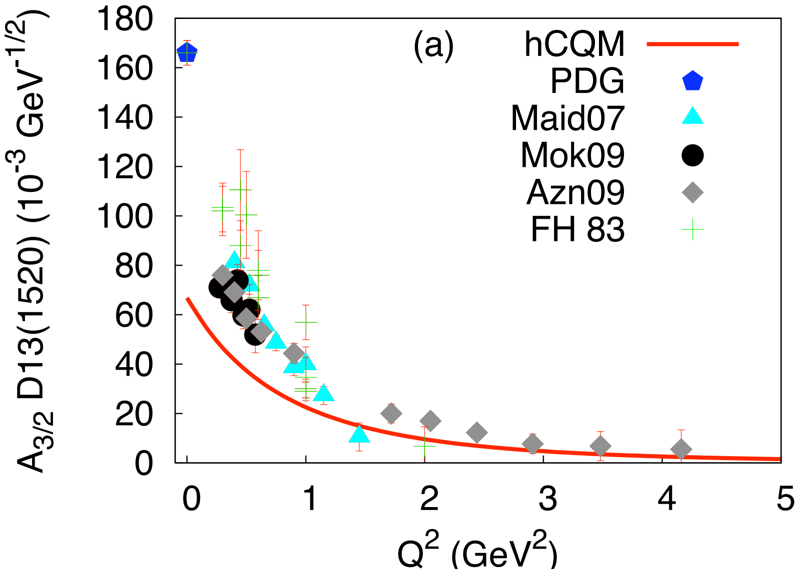
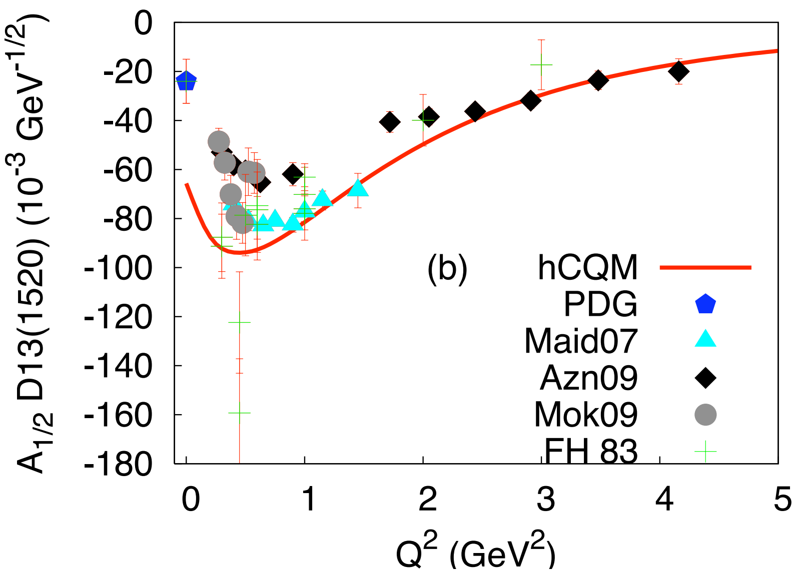
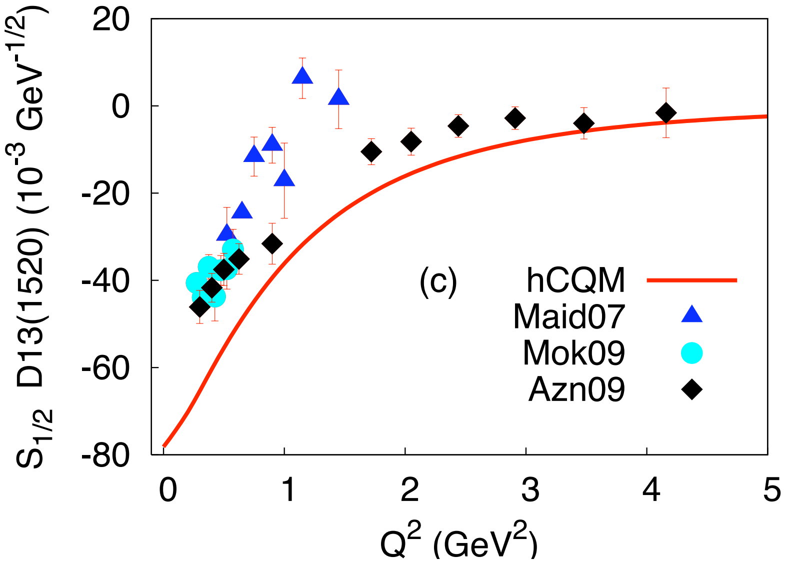
Systematic predictions for transverse and longitudinal amplitudes
E. Santopinto et al. , Phys. Rev. C86, 065202 (2012)

Proton and neutron electro-excitation to 14 resonances

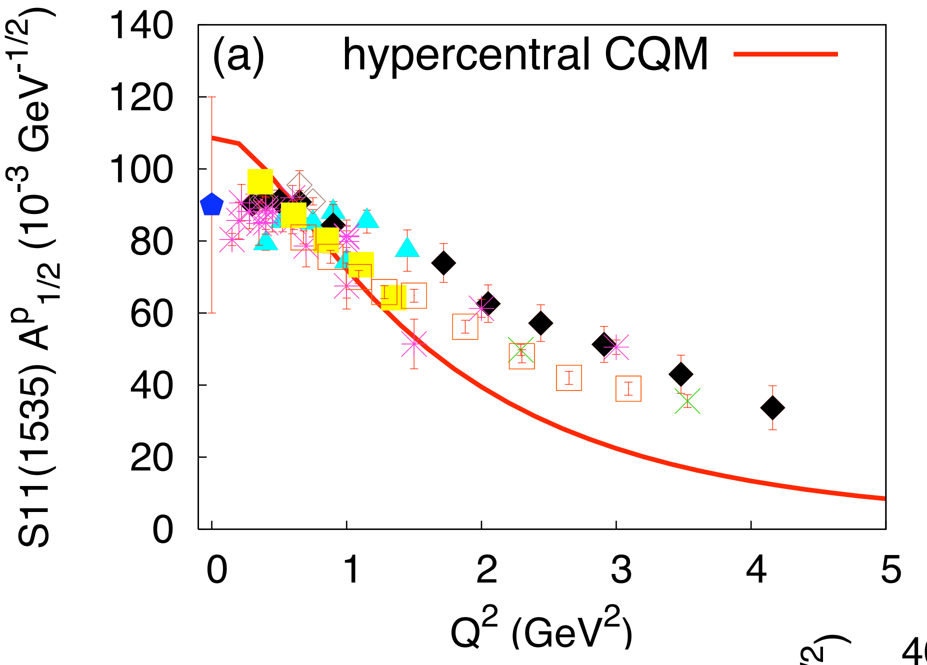
N(1520) 3/2- transition amplitudes



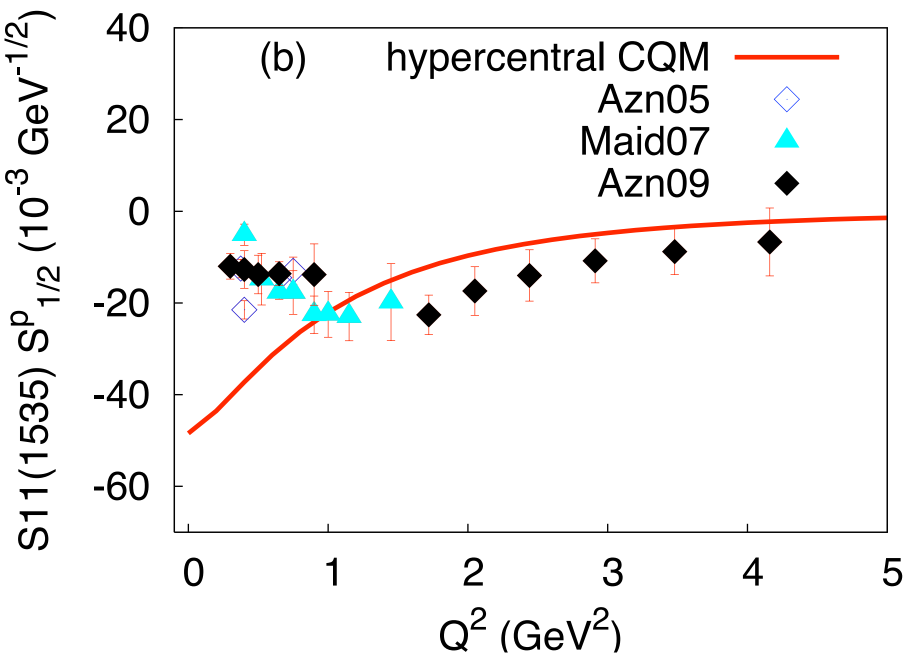




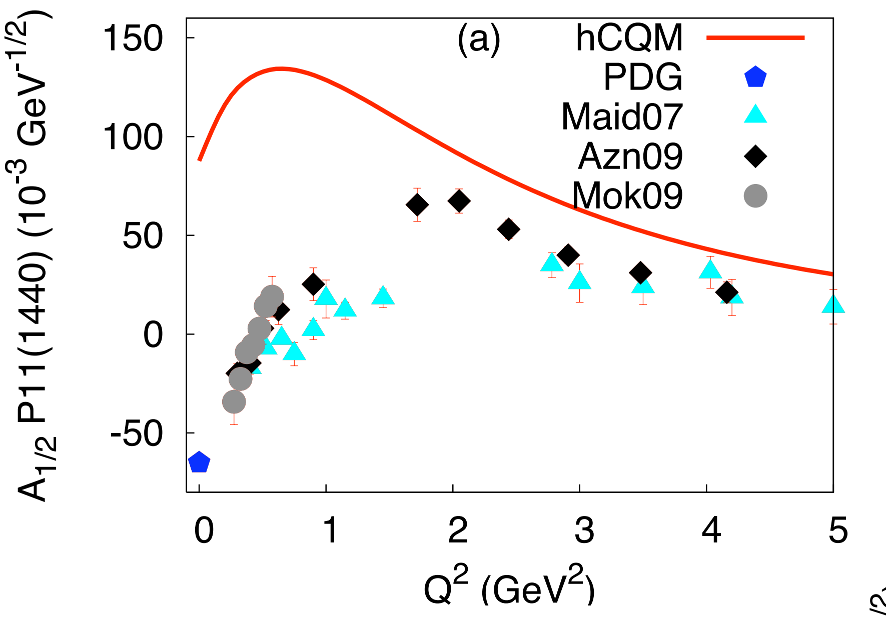
E. Santopinto, M. Giannini. Phys. Rev. C86, 065202 (2012)



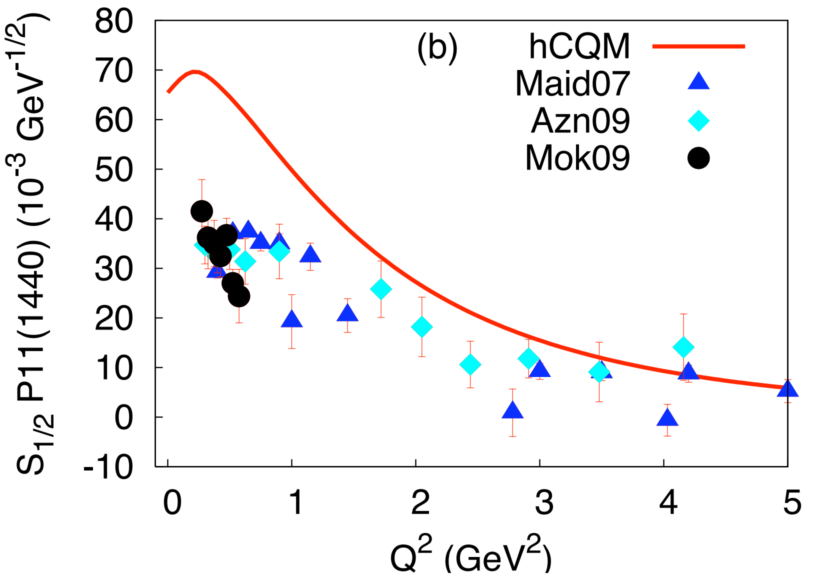
N(1535) ½ ⁻ transition amplitudes



E. Santopinto, M.G.
Phys. Rev. C86,
065202 (2012)

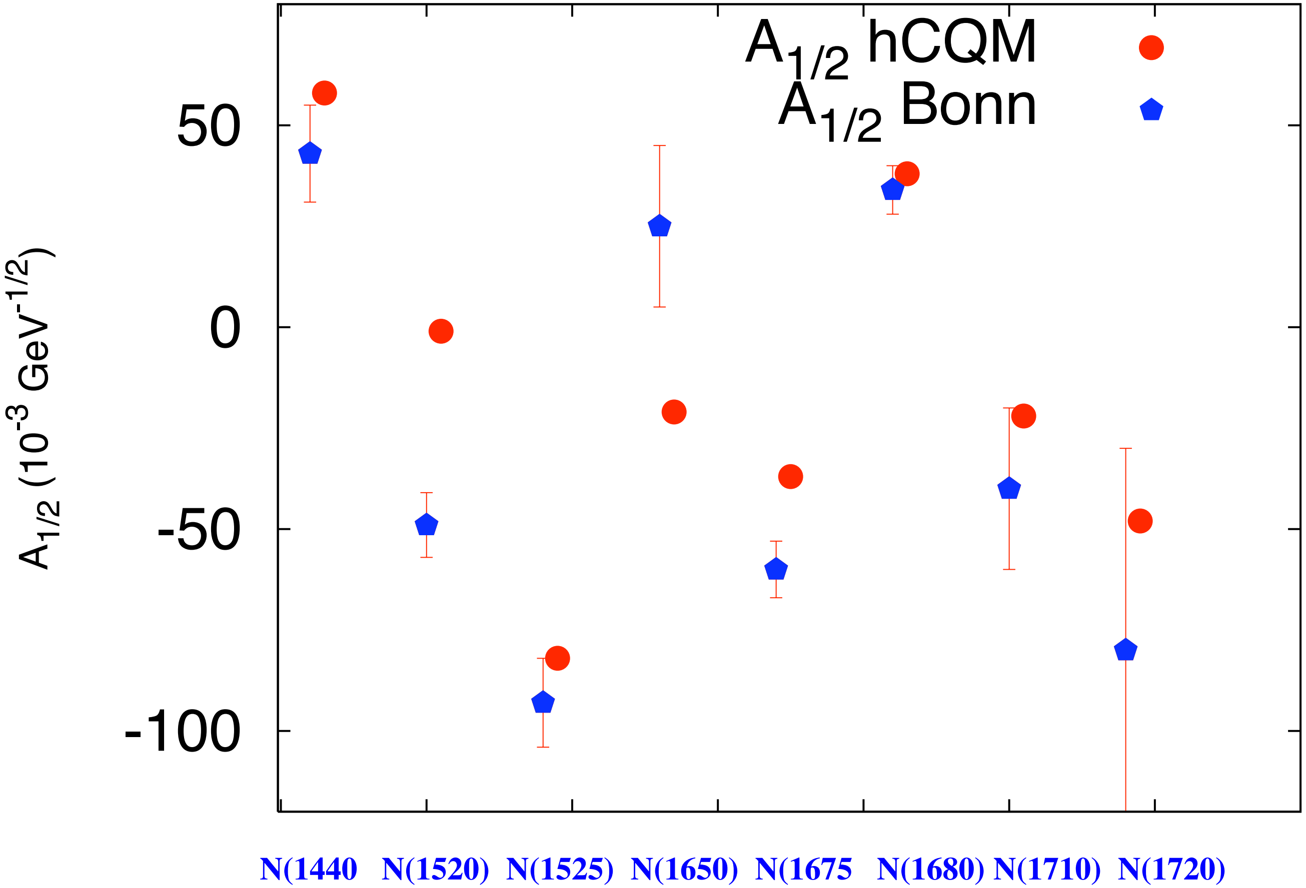


N1440) ½ +
(Roper)
transition amplitudes



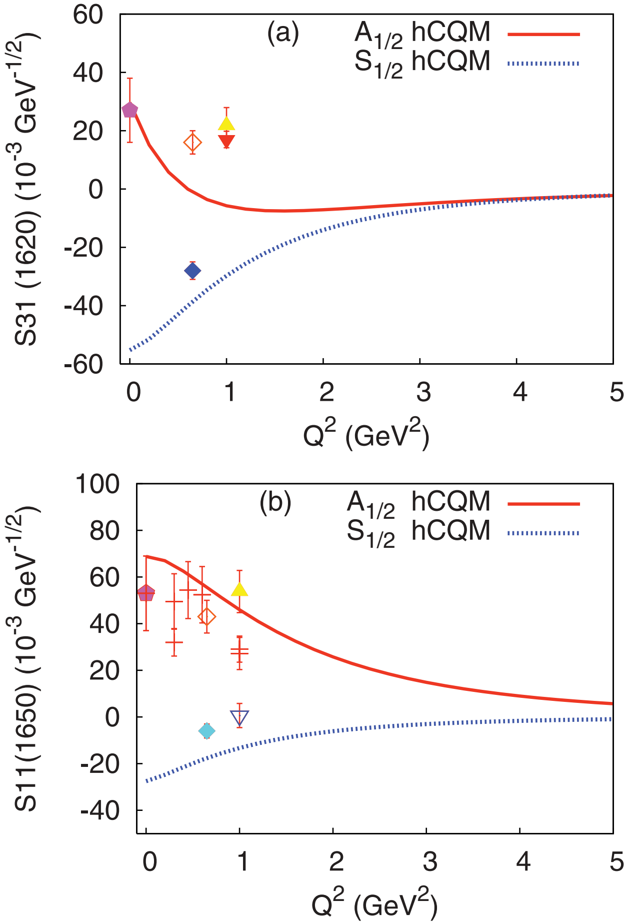
E. Santopinto, M.Giannini,Phys. Rev. C86, 065202 (2012)

Neutron photocouplings



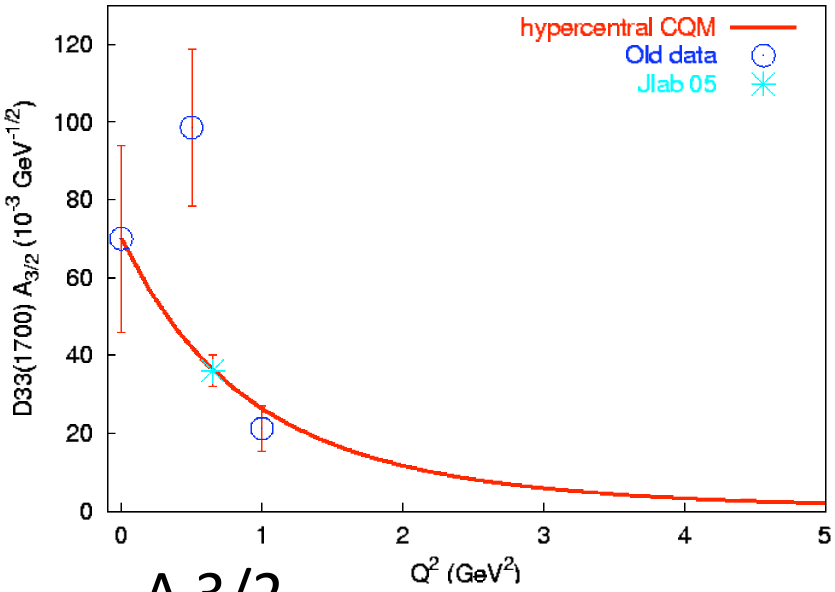
hCQM: E. Santopinto, M.G. Phys. Rev. C86, 065202 (2012)

Bonn: A.V. Anisovich et al., EPJ A49, 67 (2013)



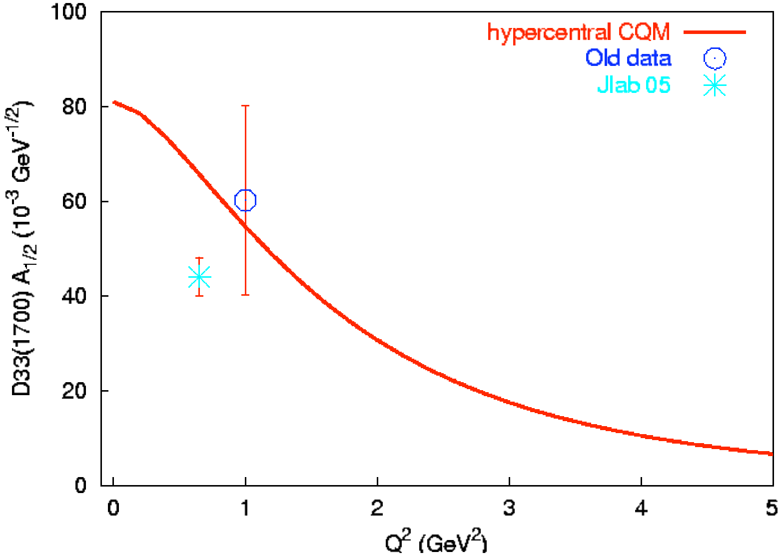
E. Santopinto, M.Giannini,Phys. Rev. C86, 065202 (2012)

D33(1700)



A 3/2

A 1/2



E. Santopinto, M.Giannini,Phys. Rev. C86, 065202 (2012)

- The hCQM seems to provide realistic three-quark wave functions
- The main reason is the presence of the **hypercoulomb** term

Solvable model

$V(x) = -\tau/x + \alpha\, x$ linear term treated as a perturbation

wf mainly concentrated in the low x region

- energy levels expressed analytically
- unperturbed wf given by the 1/x term
- major contribution to the helicity amplitudes

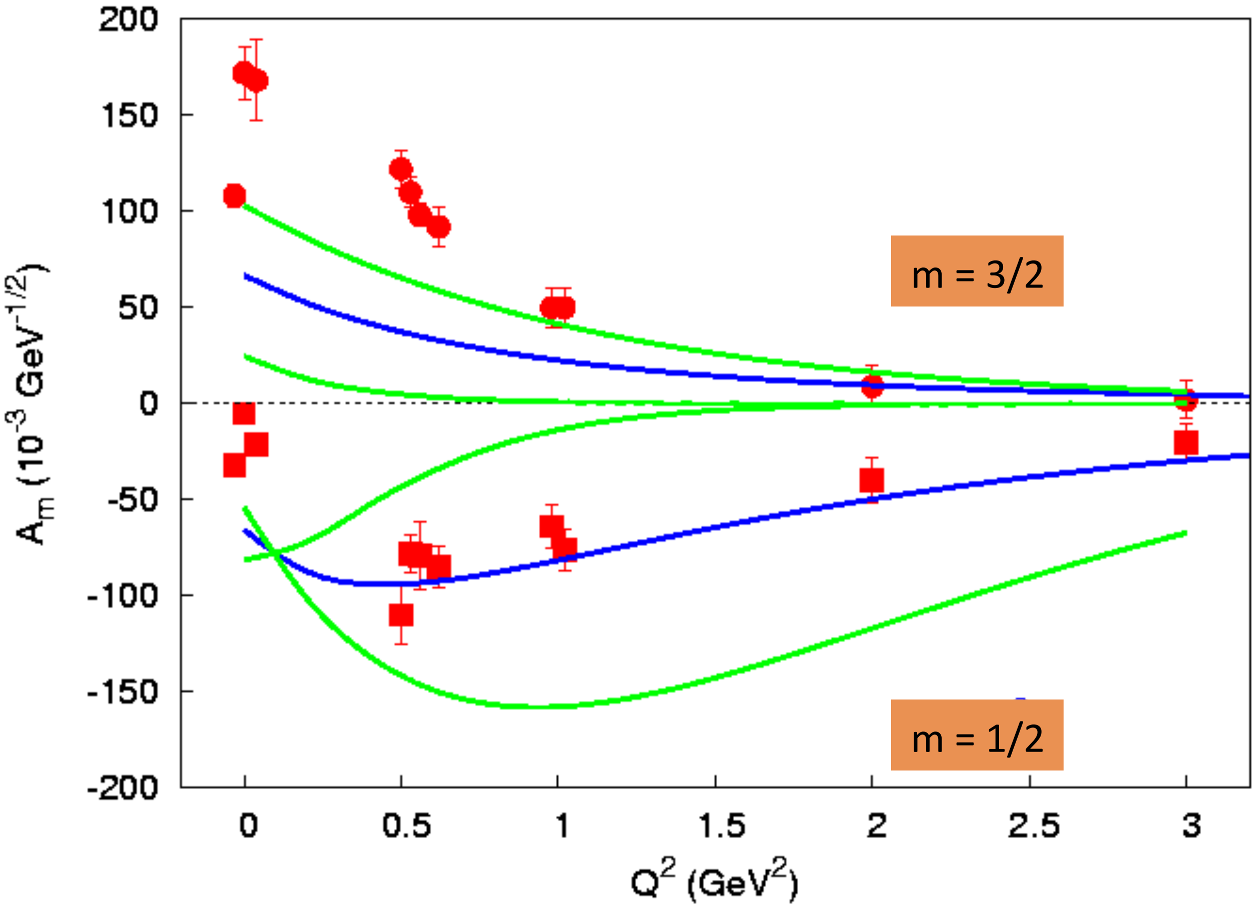
Good results due to semplicity

E. Santopinto, F. Iachello, M.Giannini, Eur. Phys. J. A **1**, 307 (1998)

$A^p_m \; N(1520)D13$

r_p 0.5 fm

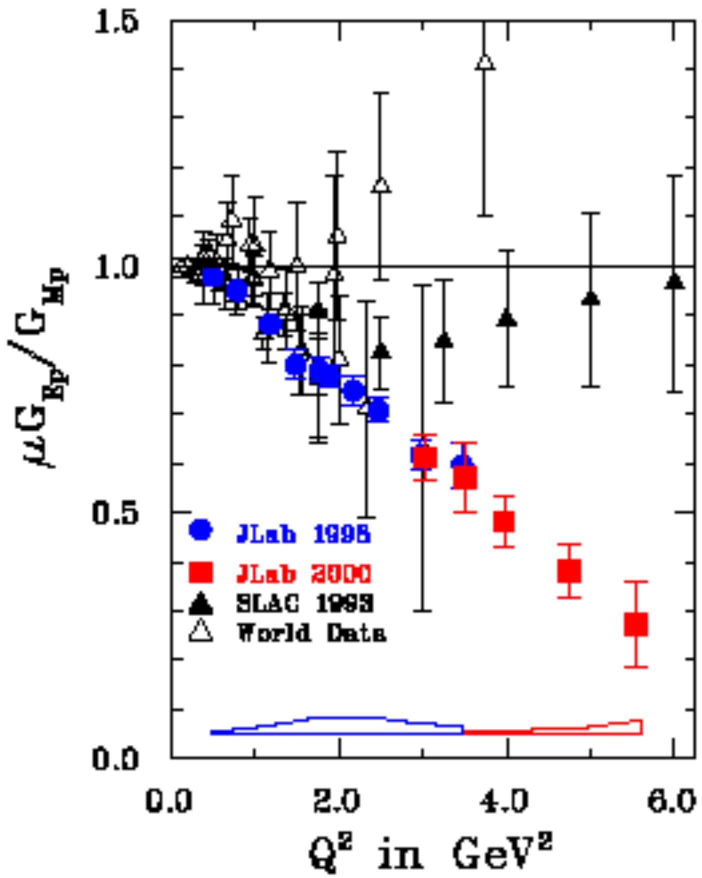
r_p 0.86 fm



Green curves H.O.

Blue curves hCQM

The nucleon elastic form factors



- elastic scattering of polarized electrons on polarized protons

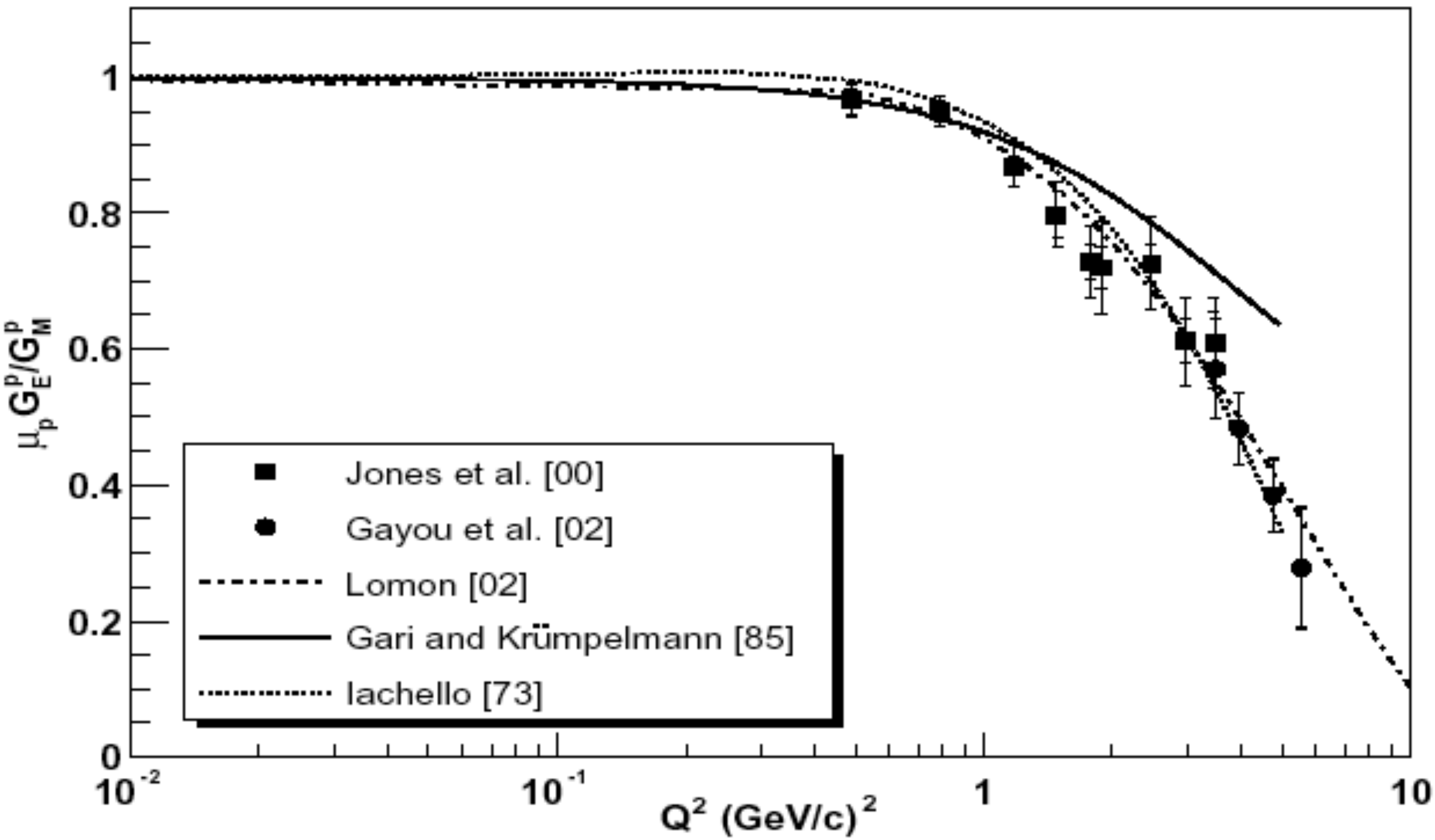
- measurement of polarizations asymmetry gives directly the ratio G^p_E/G^p_M

- discrepancy with Rosenbluth data (?)

- linear and strong decrease

- pointing towards a zero (!)

- new data (jan 2010) seem to confirm the behaviour



1
0.8
0.6
0.4
0.2
0
$\mu_p G_E^p / G_M^p$
10^{-2}
10^{-1}
1
10
Q^2 (GeV/c)^2
Jones et al. [00]
Gayou et al. [02]
Lomon [02]
Gari and Krümpelmann [85]
Iachello [73]

With a calculated radius of about 0.5 fm
the e.m. form factors predicted by the hCQM
are not good!

BUT

relativity is needed

RELATIVITY

Various levels

- relativistic kinetic energy
- Lorentz boosts
- Relativistic dynamics
- quark-antiquark pair effects (meson cloud)
- relativistic equations (BS, DS)

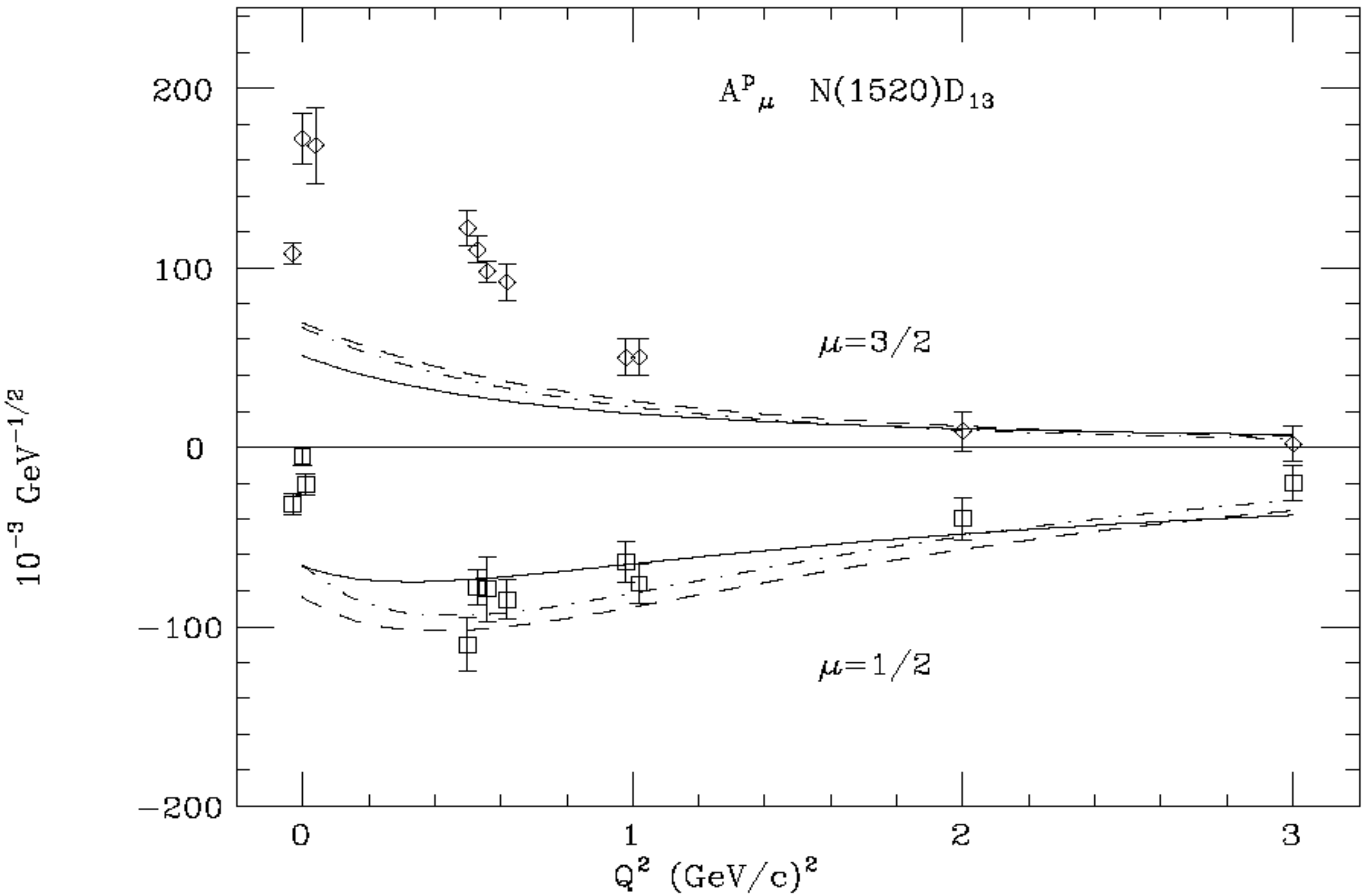
Relativistic corrections to form factors

- Breit frame
- Lorentz boosts applied to the initial and final state
- Expansion of current matrix elements up to first order in quark momentum
- Results

$$A_{rel}(Q^2) = F\ A_{n.rel}(Q^2_{eff})$$

F = kin factor $\qquad Q^2_{eff} = Q^2\,(M_N/E_N)^2$

De Sanctis et al. EPJ 1998

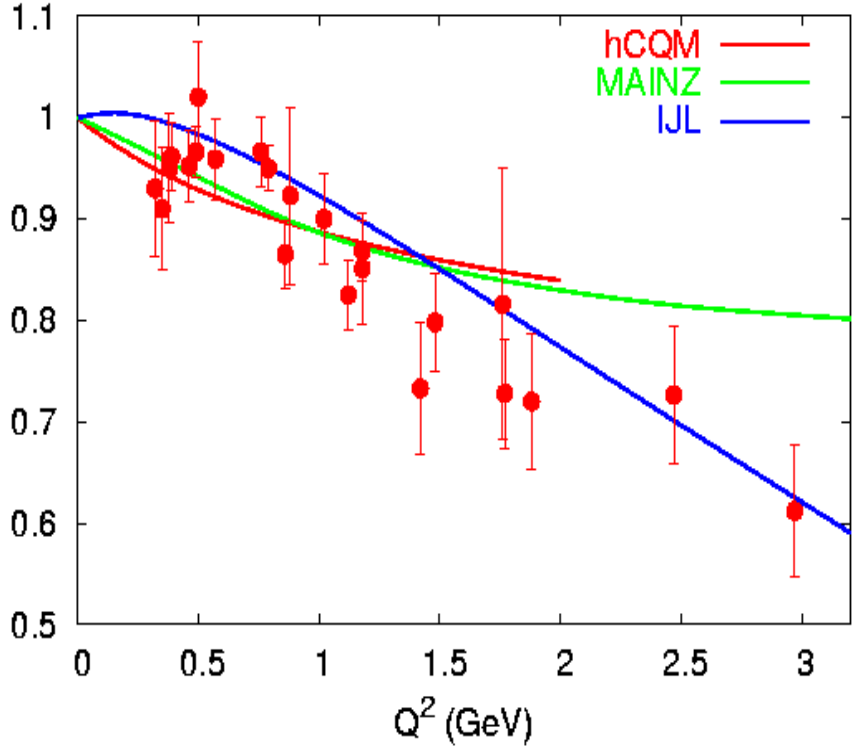


Full curves: hCQM with relativistic corrections
Dashed curves: hCQM in different frames

Elastic Form Factors in the hCQM

M. De Sanctis, M.M. Giannini, L. Repetto, E. Santopinto, Phys. Rev. C62, (2000) 025208.

$\mu_p\, G_E^p(Q^2)/G_M^p(Q^2)$ calculated



NO QUARK FORM FACTORS

T_{NR}, BOOSTS, CURRENT EXPANSION

Construction of a fully relativistic theory
Relativistic Dynamics

Three forms (Dirac):
Light (LF), Instant (IF), Point (PF)

Point form:

Composition of angular momentum states as in the non relativistic case

Moving three-quark states are obtained through (interaction free) Lorentz boosts (velocity states)

Construction of a fully relativistic theory
Relativistic Dynamics

Relativistic Hamiltonian Dynamics
for a fixed number of particles (Dirac)

Construction of a representation of the Poincaré generators
P_μ (tetramomentum), J_k (angular momenta), K_i (boosts)
obeying the Poincaré group commutation relations
in particular

$$[P_k , K_i] = i \, \delta_{kj} \, H$$

Moving three-quark states are obtained through
(interaction free) Lorentz boosts (velocity states)

Three forms:
Light (LF), Instant (IF), Point (PF)
Differ in the number and type of (interaction) free generators

Point form: P_μ interaction dependent

J_k and K_i free

Composition of angular momentum states as in the non relativistic case

Mass operator $M = M_0 + M_I$

$$M_0 = \Sigma_i \sqrt{\vec{\mathbf{p}}_i^{\,2} + m^2}$$

$$\Sigma_i \vec{\mathbf{p}}_i = 0$$

$\vec{\mathbf{P}}_i$ undergo the same Wigner rotation -> M_0 is invariant

Similar reasoning for the hyperradius

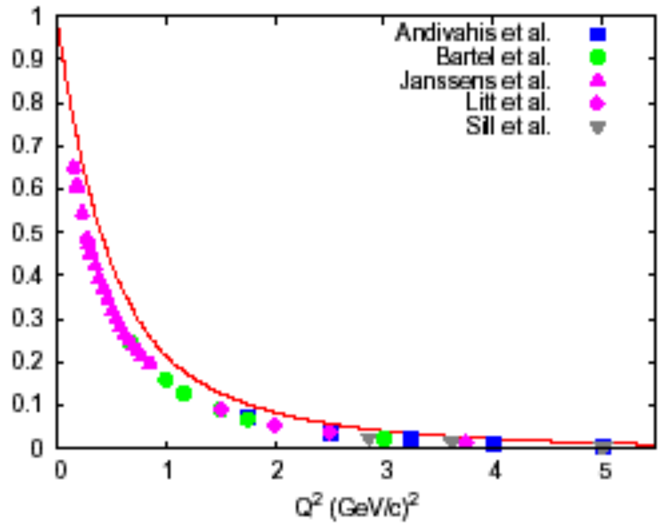
The eigenstates of the relativistic hCQM are interpreted as eigenstates of the mass operator M

Moving three-quark states are obtained through (interaction free) Lorentz boosts (velocity states)

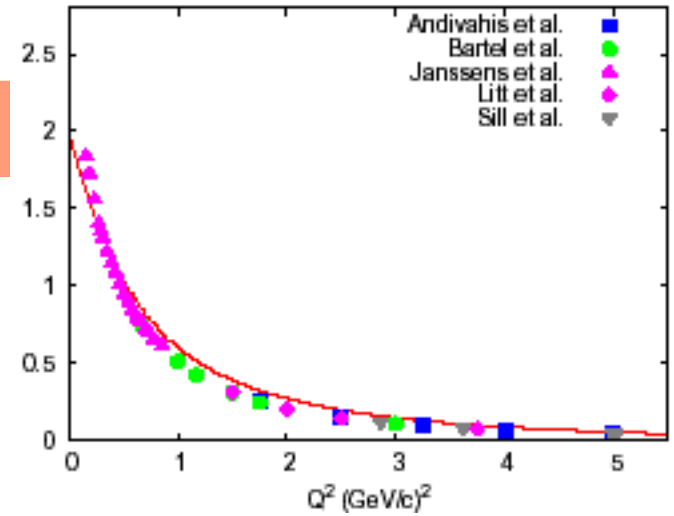
Calculated values!

- Boosts to initial and final states
- Expansion of current to any order
- Conserved current

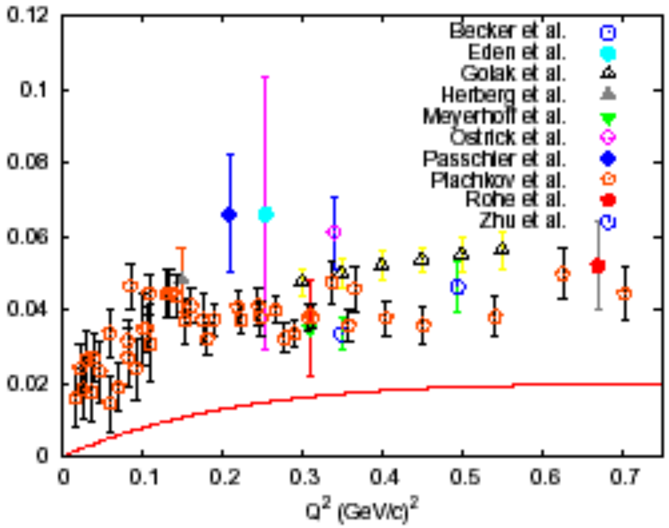
G_E^p



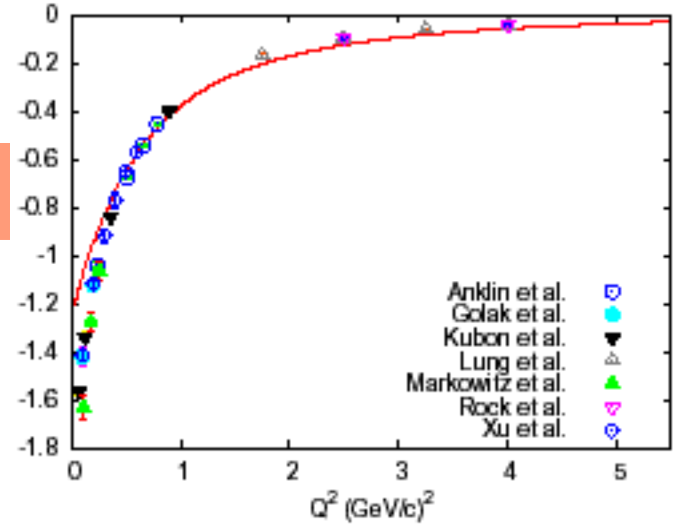
G_M^p

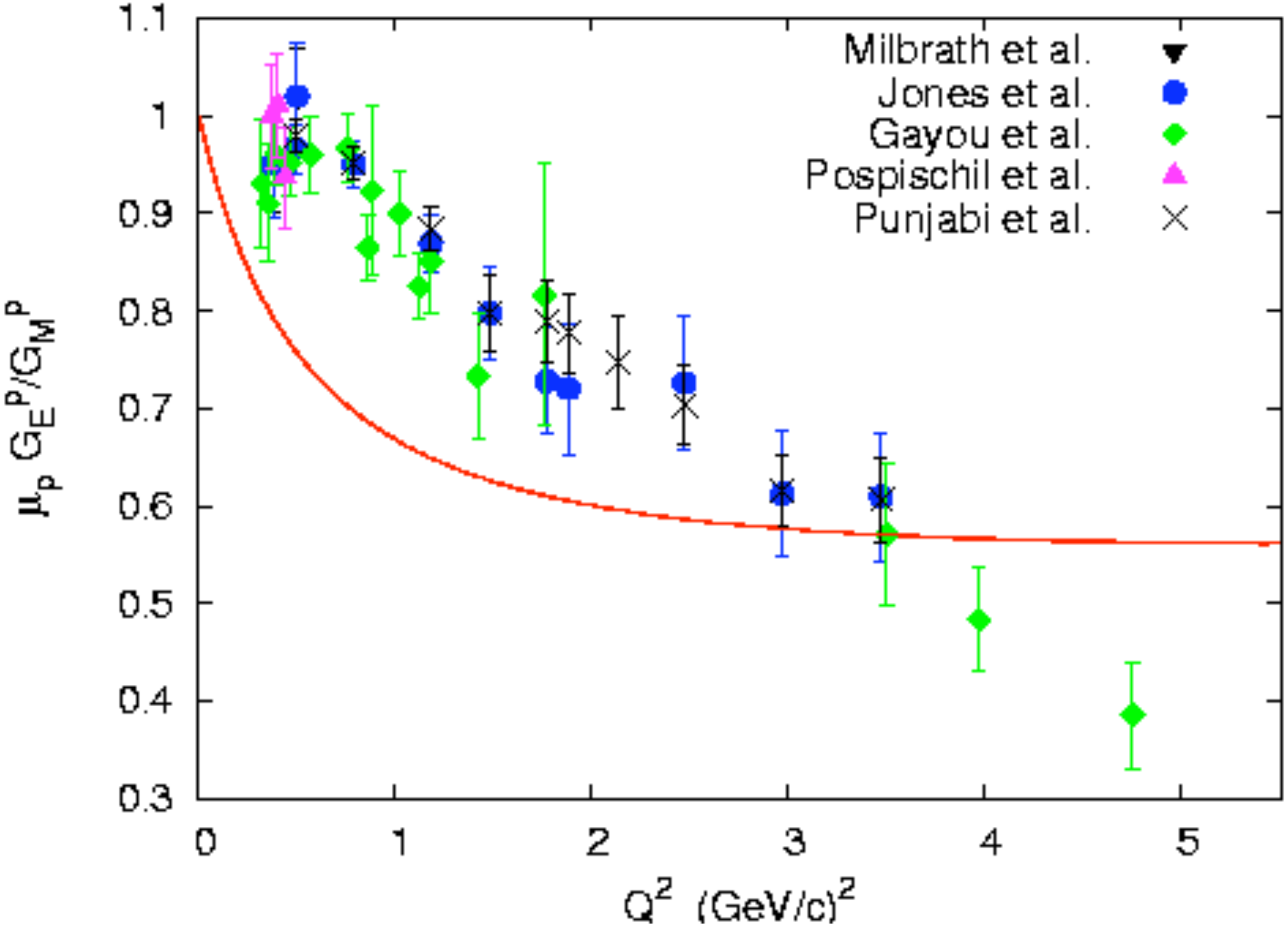


G_E^n



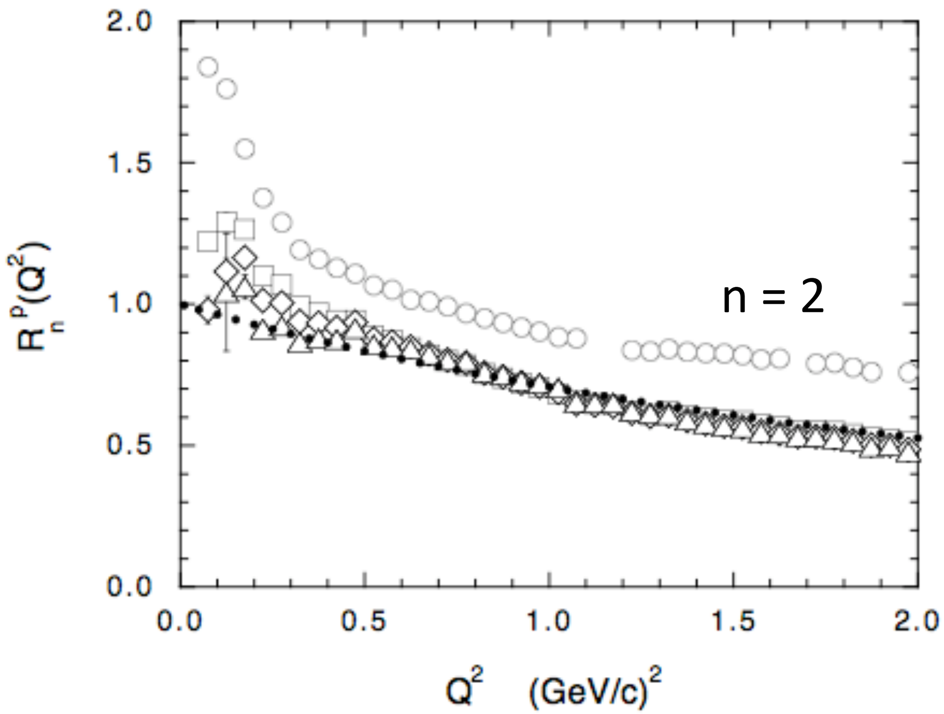
G_M^n





Milbrath et al.
Jones et al.
Gayou et al.
Pospischil et al.
Punjabi et al.
$\mu_p G_E^p / G_M^p$
Q^2 (GeV/c)^2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
1
2
3
4
5

Further support 2



Ratio between
proton Nachtmann moments &
CQ distribution

Bloom-Gilman duality

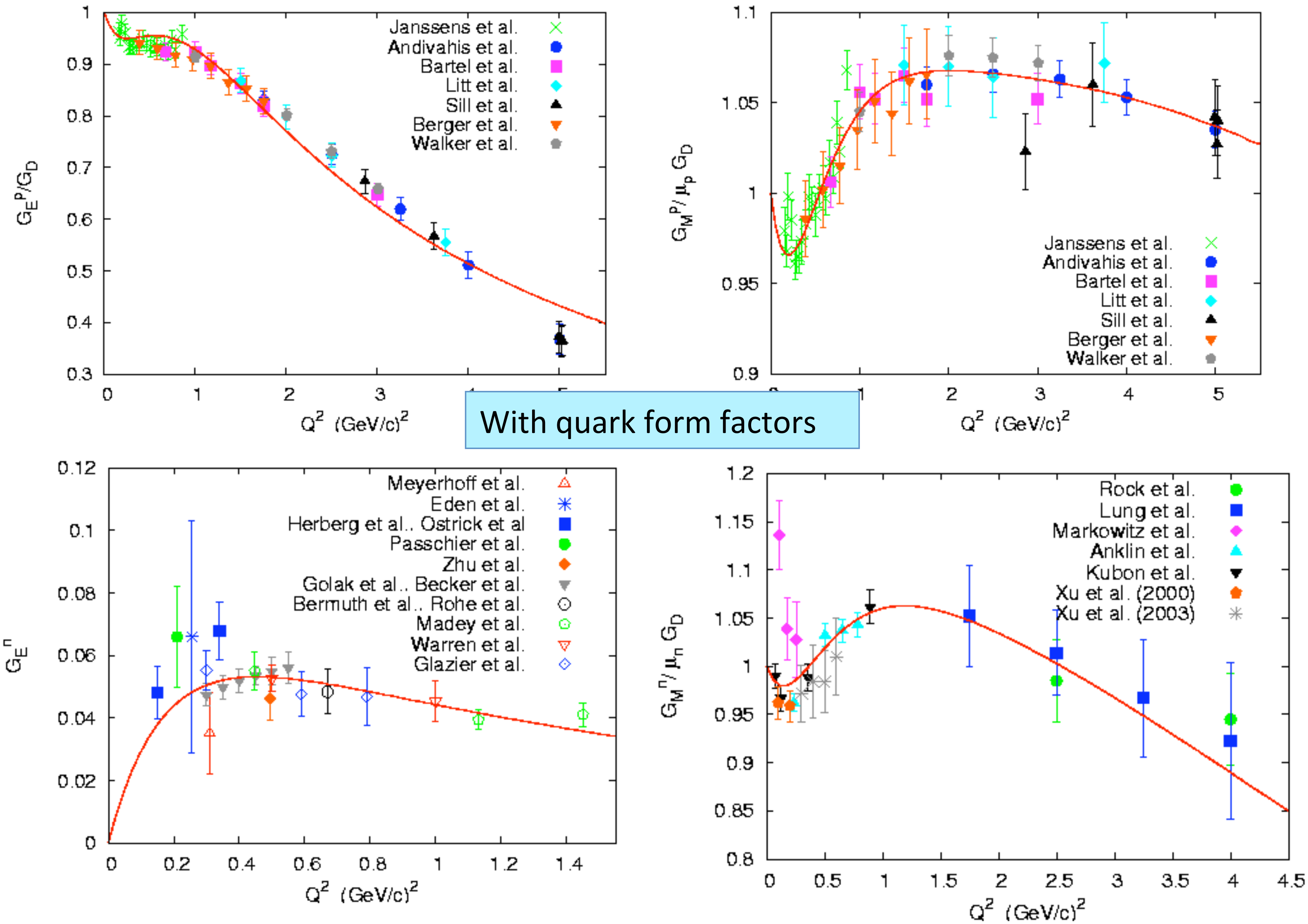
Inelastic proton scattering as elastic scattering on CQ

(approximate) scaling function $\longrightarrow$ square of CQ ff

$F(Q^2) = 1/(1 + 1/6\, r_{CQ}^{\,2}\, Q^2)$

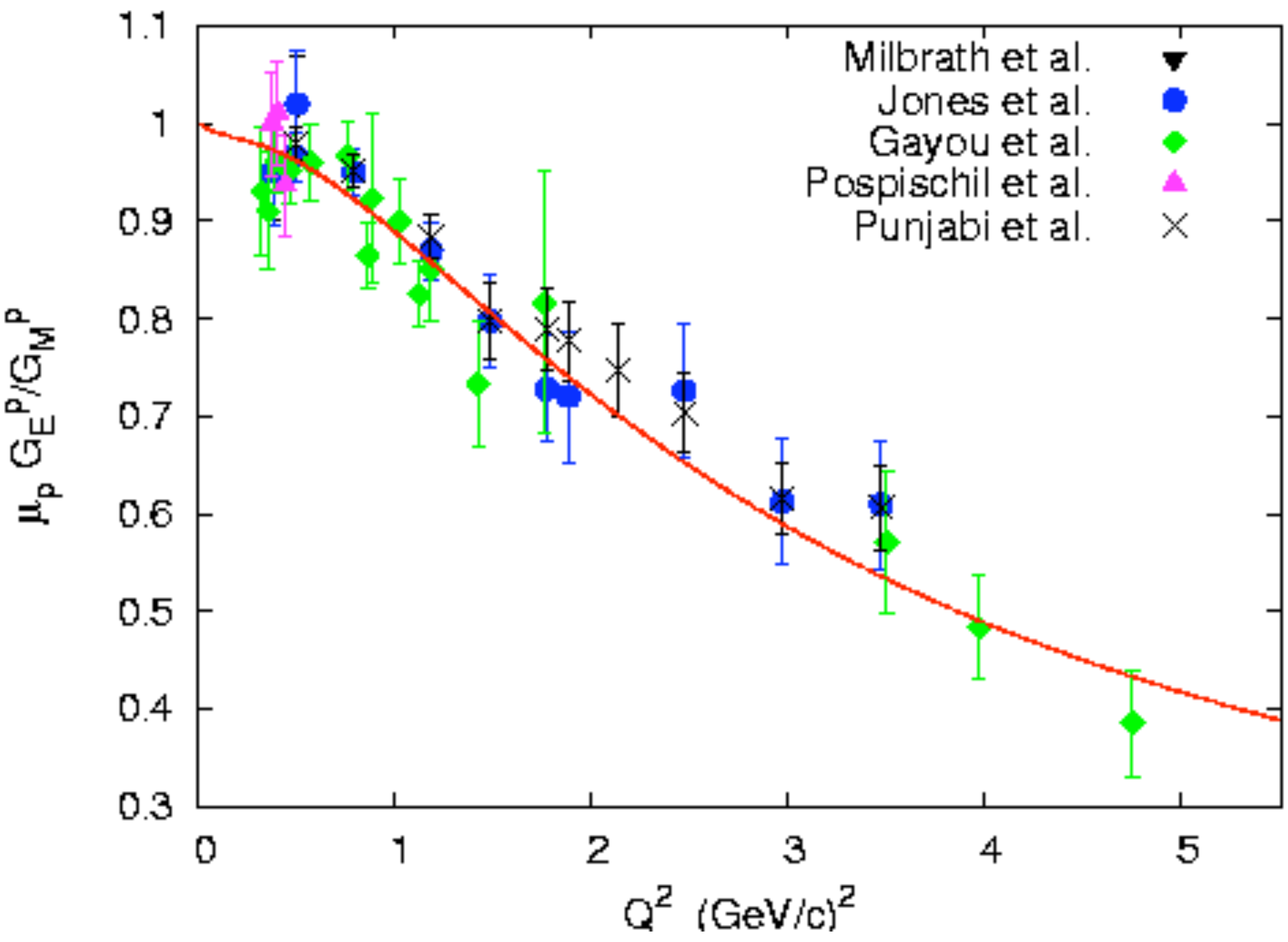
$r_{CQ} \cong 0.2$ fm

Ricco et al., PR **D67,** 094004 (2003)

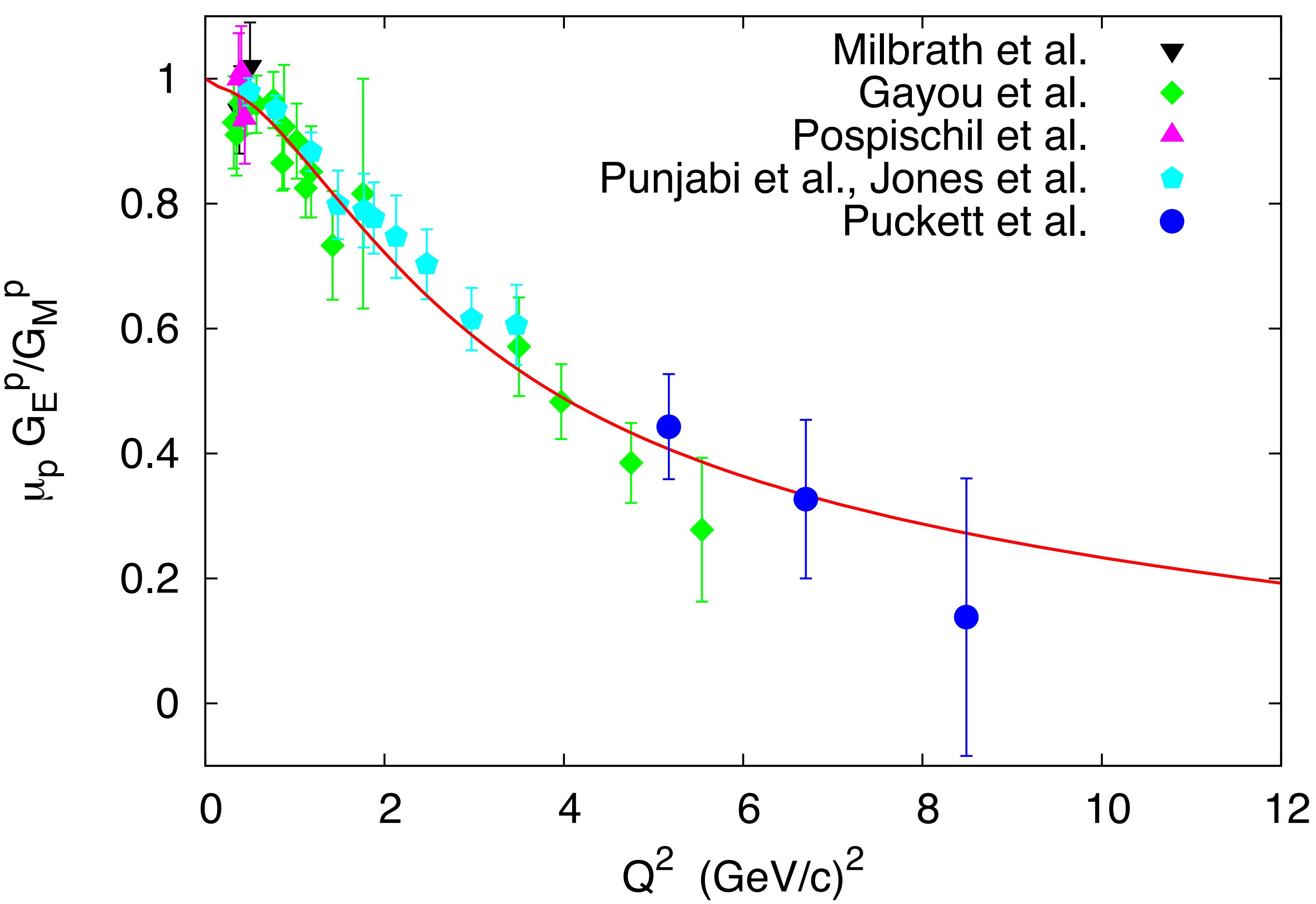


With quark form factors

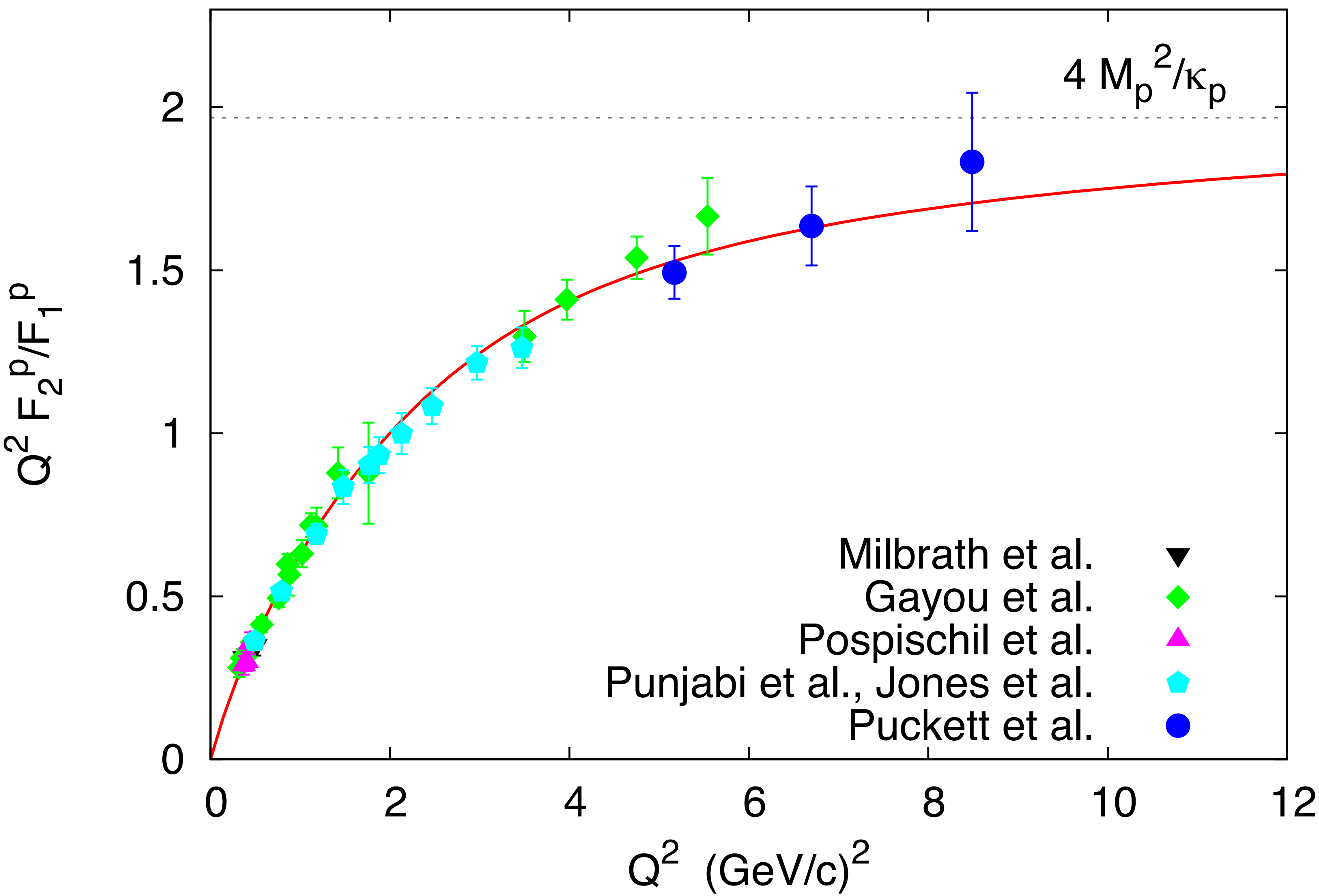
Genoa group, Phys. Rev. C76, 062201 (2007)



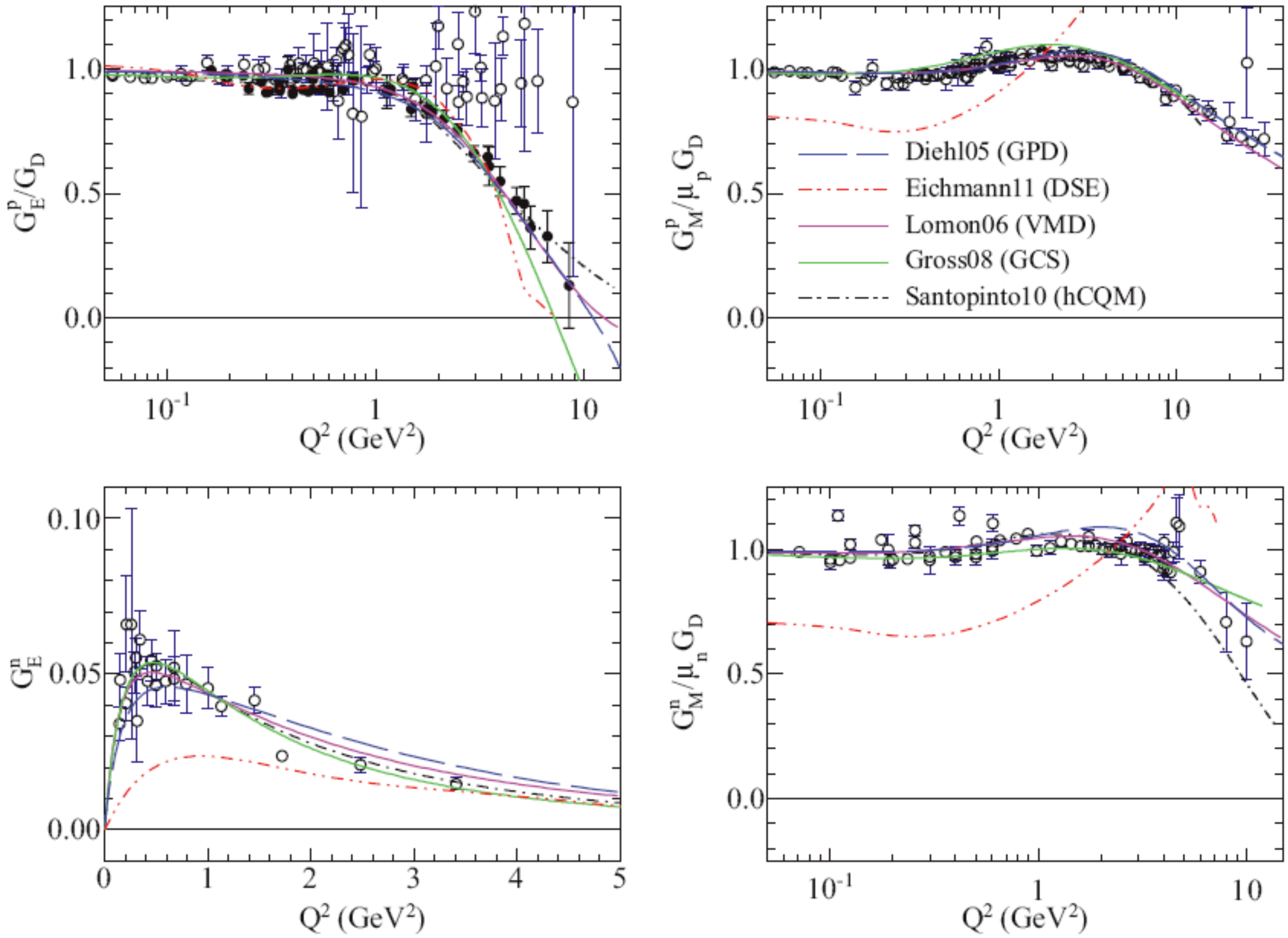
Milbrath et al.
Jones et al.
Gayou et al.
Pospischil et al.
Punjabi et al.
$\mu_p G_E^p/G_M^p$
Q^2 (GeV/c)^2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
1
2
3
4
5



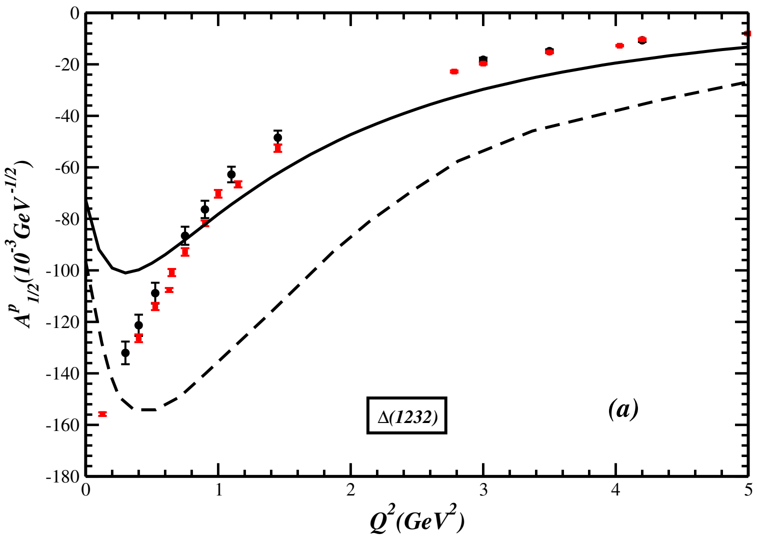
Milbrath et al.
Gayou et al.
Pospischil et al.
Punjabi et al., Jones et al.
Puckett et al.
$\mu_p G_E^p/G_M^p$
Q^2 (GeV/c)^2
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
12



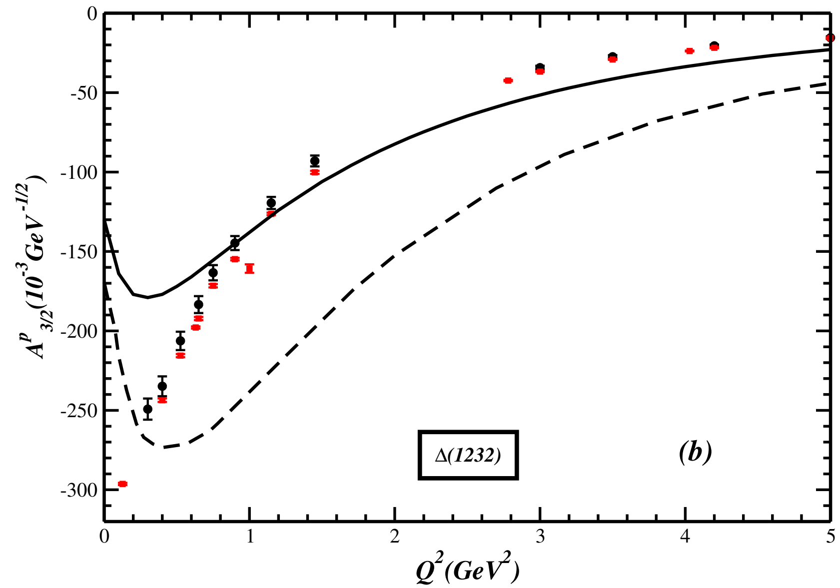
$4\,M_p^2/\kappa_p$
$Q^2\,F_2^p/F_1^p$
Q^2 (GeV/c)^2
0
0.5
1
1.5
2
0
2
4
6
8
10
12
Milbrath et al.
Gayou et al.
Pospischil et al.
Punjabi et al., Jones et al.
Puckett et al.



G_E^p/G_D
1.0
0.5
0.0
10^{-1}
1
10
Q^2 (GeV^2)
$G_M^p/\mu_p G_D$
Diehl05 (GPD)
Eichmann11 (DSE)
Lomon06 (VMD)
Gross08 (GCS)
Santopinto10 (hCQM)
G_E^n
0.10
0.05
0.00
0
2
3
4
5
$G_M^n/\mu_n G_D$



Relativistic hCQM
In Point Form

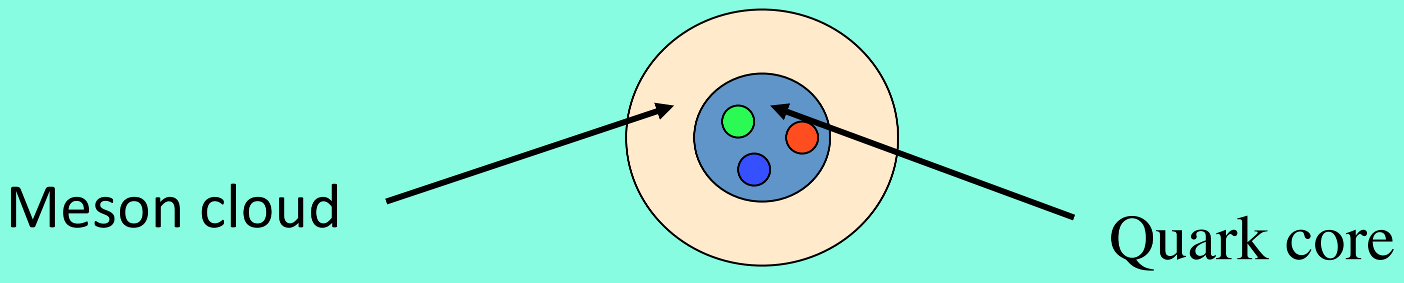


Y.B. Dong, M.Giannini., E. Santopinto,
A. Vassallo,
Few-Body Syst. **55 (2014) 873-876**

please note

- the medium Q^2 behaviour is fairly well reproduced
- there is lack of strength at low Q^2 (outer region) in the e.m. transitions
- emerging picture:

  quark core plus (meson or sea-quark) **cloud**



Conclusions First Part

- CQM provide a good systematic frame for baryon studies
- fair description of e.m. properties (specially N-N* transitions)
- possibility of understanding missing mechanisms
- quark antiquark pairs effects

  unquenching: important break through