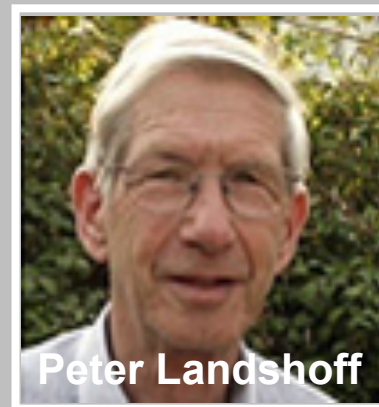
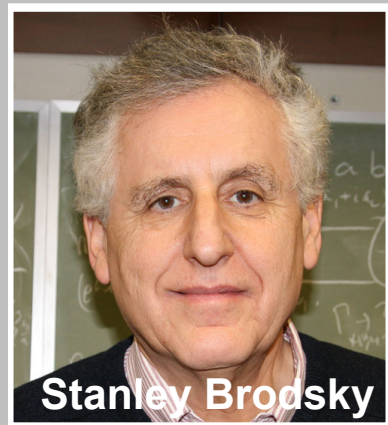




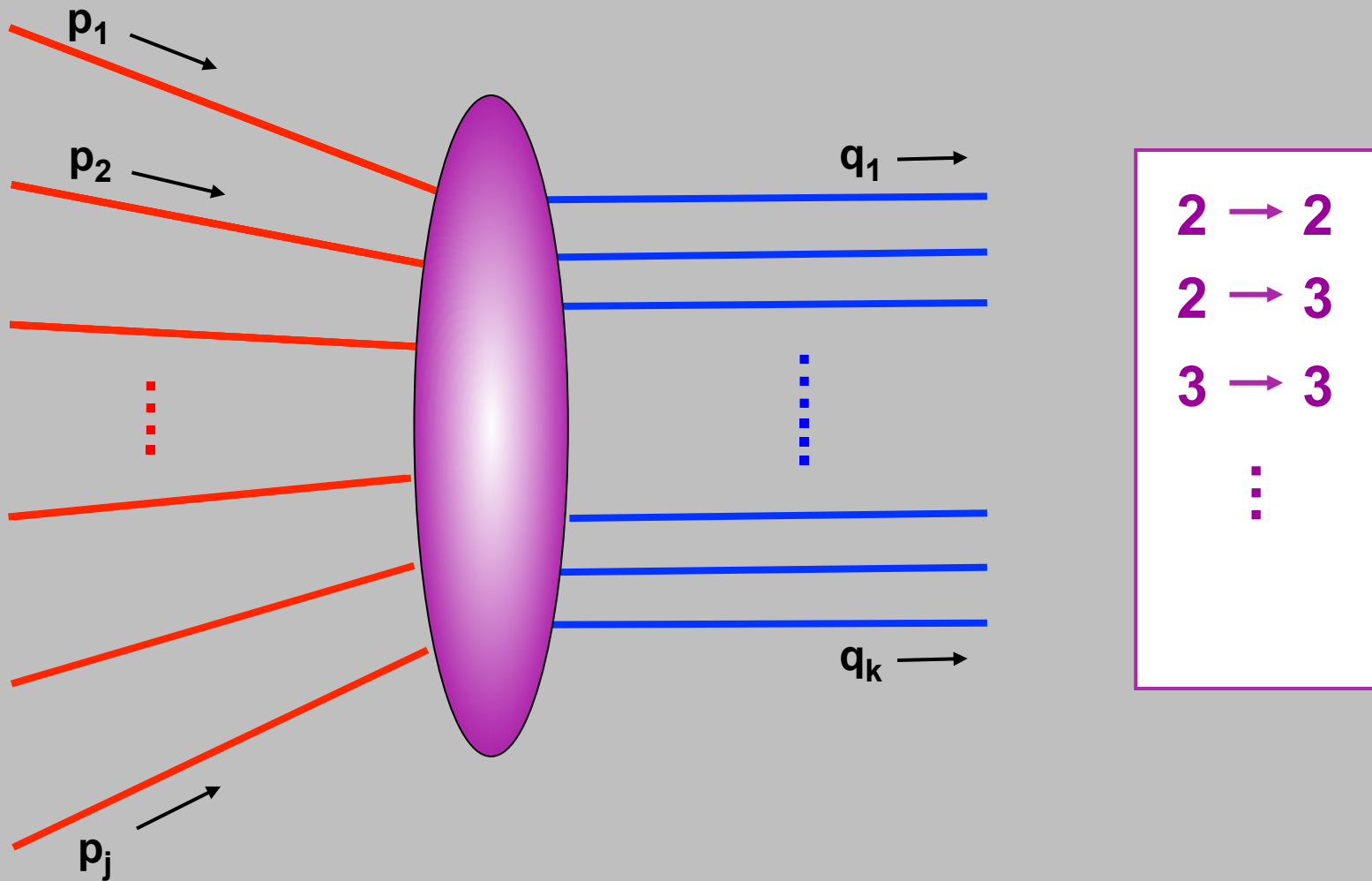
Summer School on Reaction Theory

 INDIANA UNIVERSITY

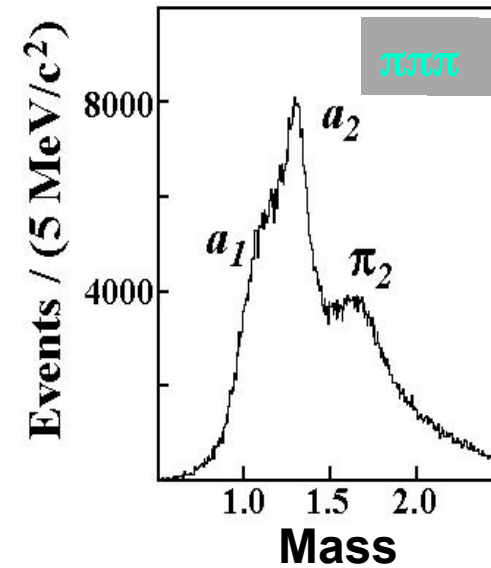
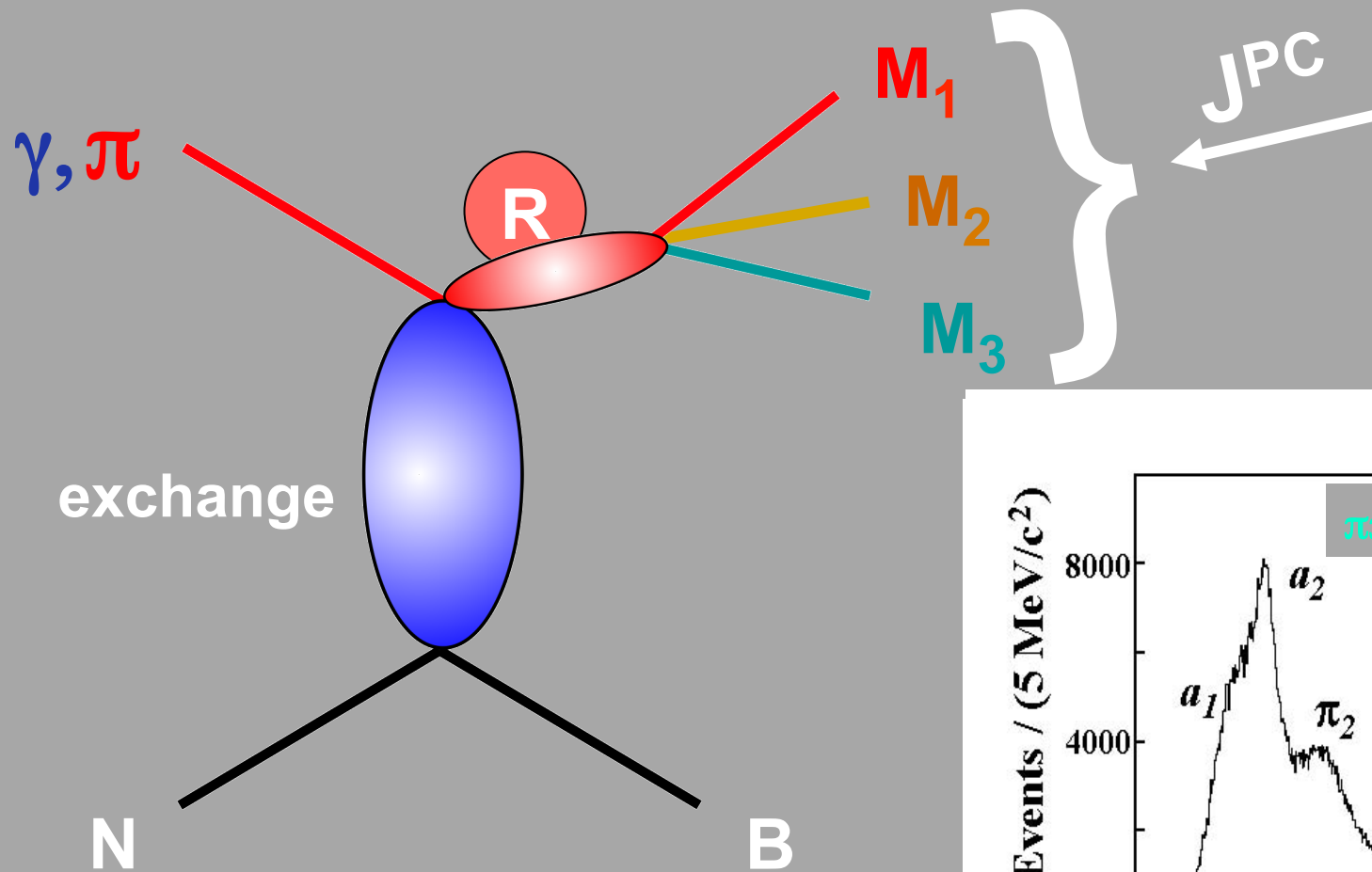
Michael Pennington
Jefferson Lab



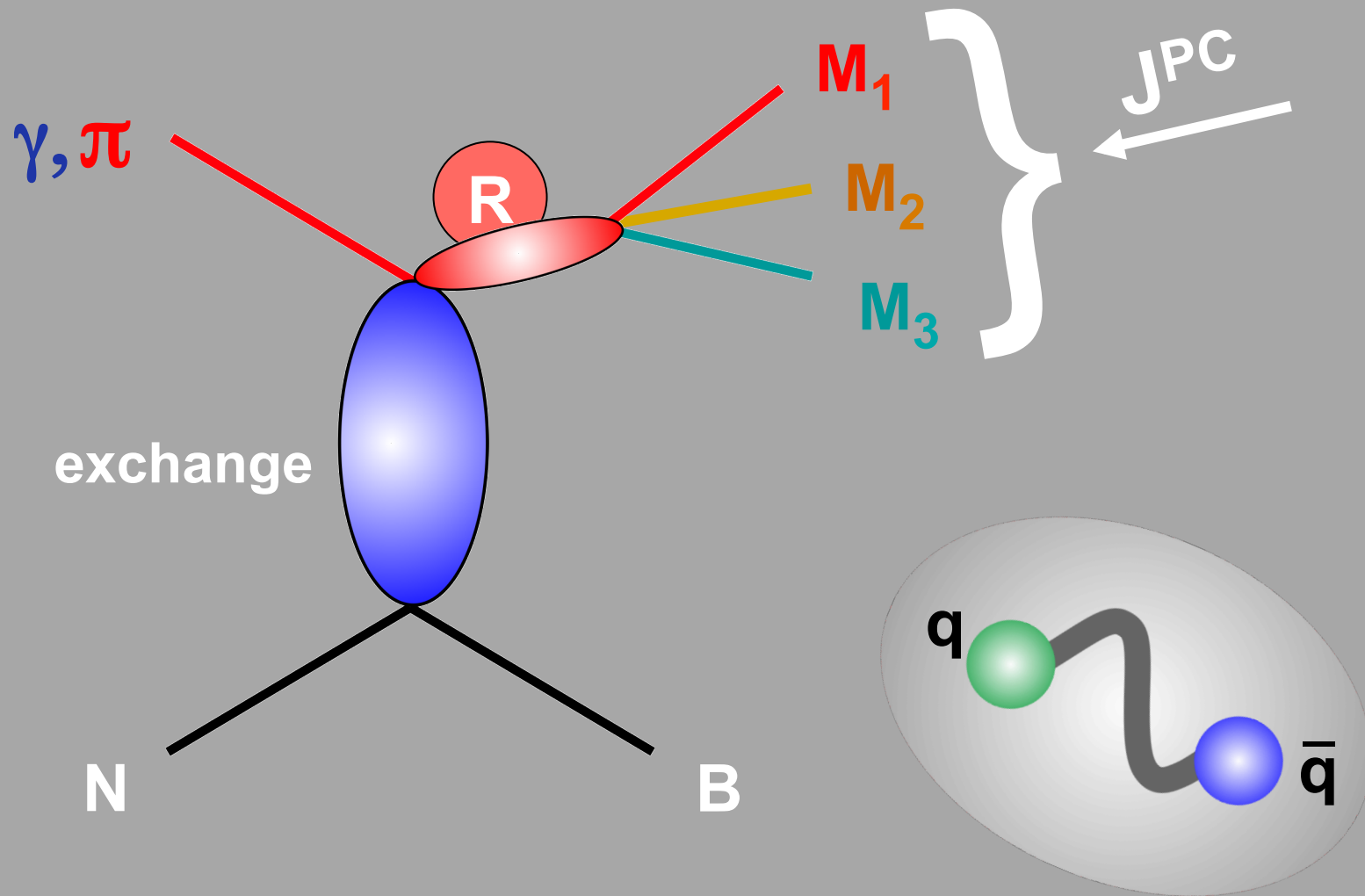
$$S(p_1, p_2, \dots, p_j; \sigma_1, \sigma_2, \dots, \sigma_j; q_1, \dots, q_k; \tau_1, \dots, \tau_k)$$

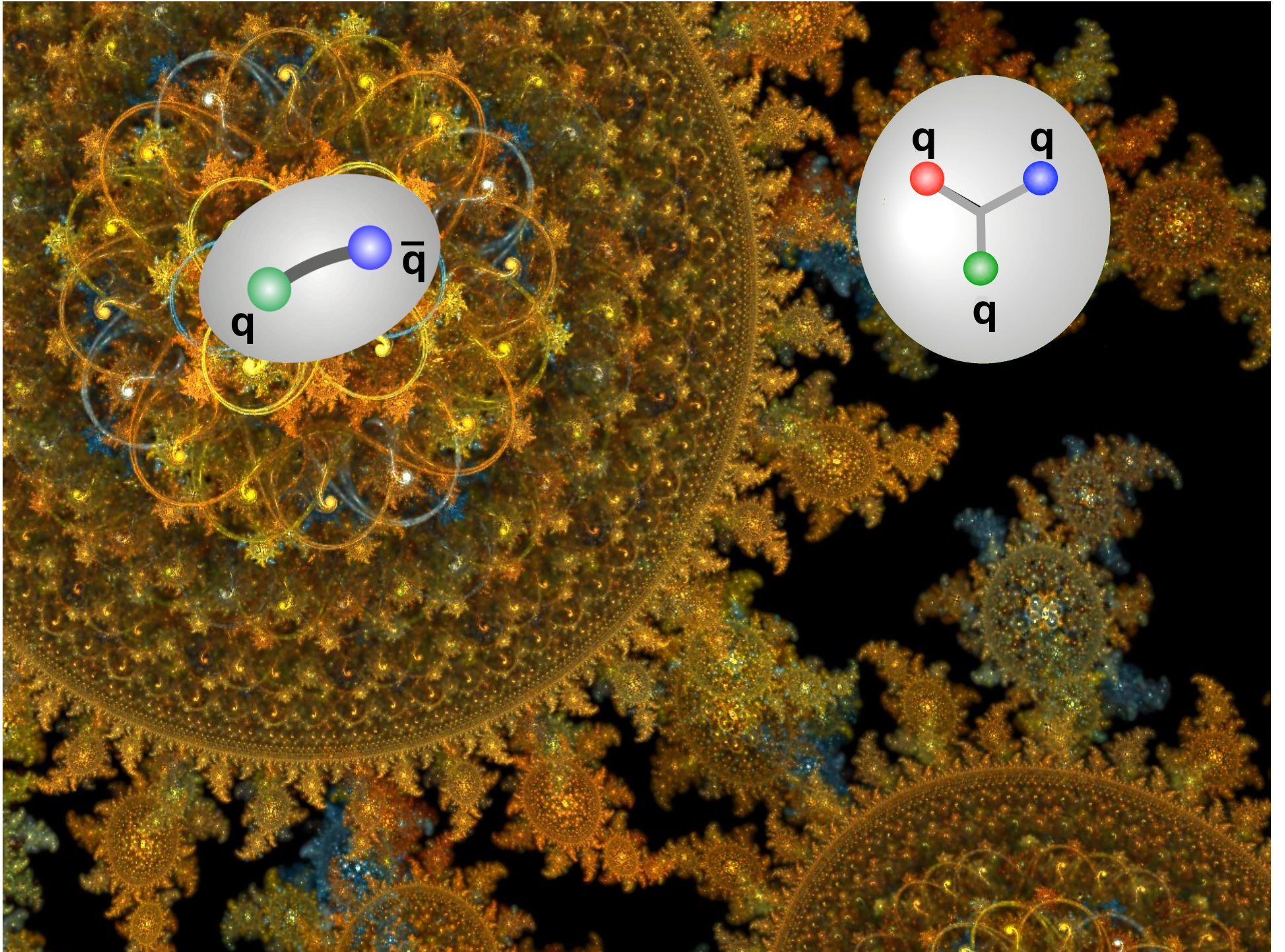


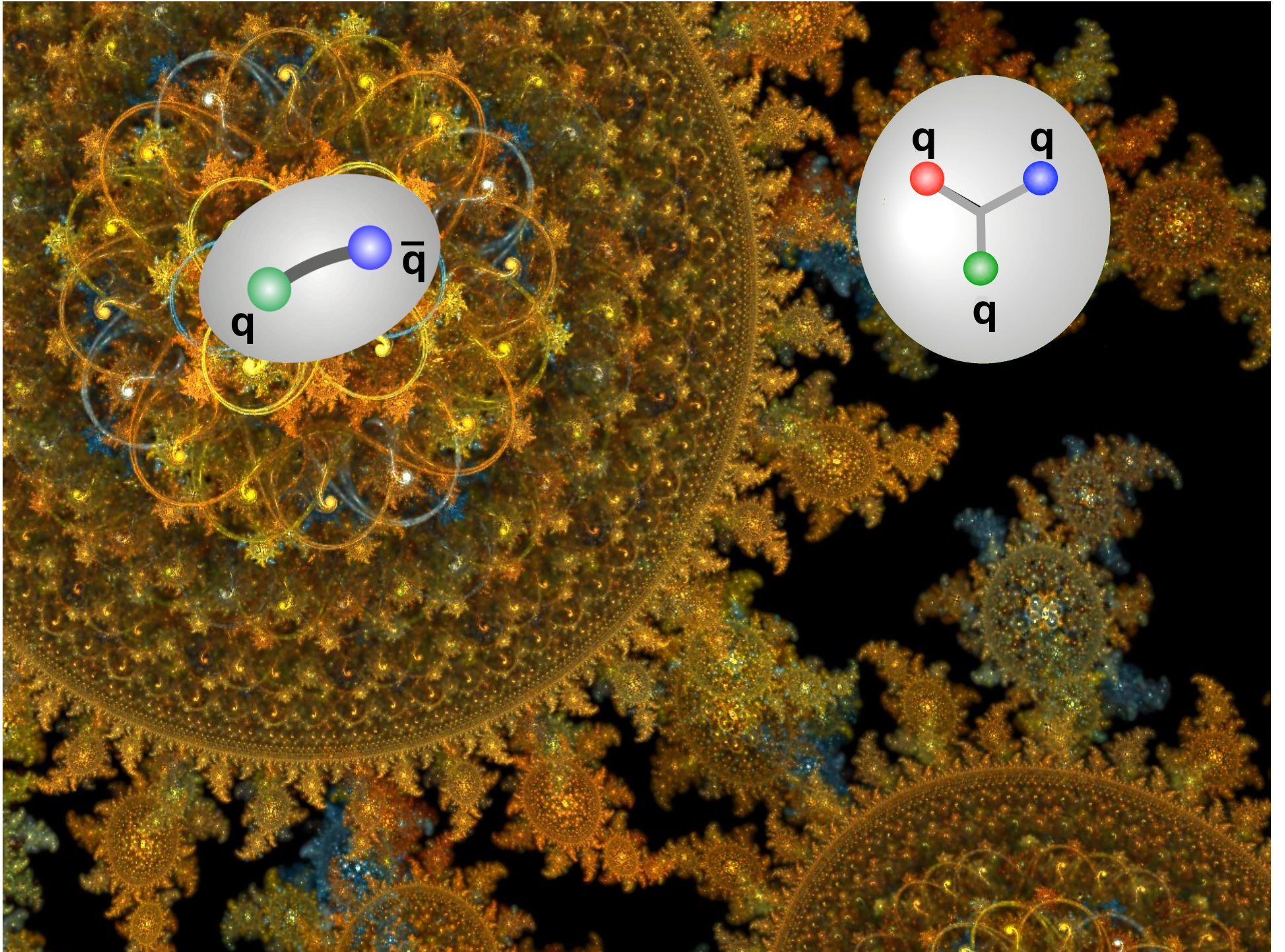
Amplitude Analysis

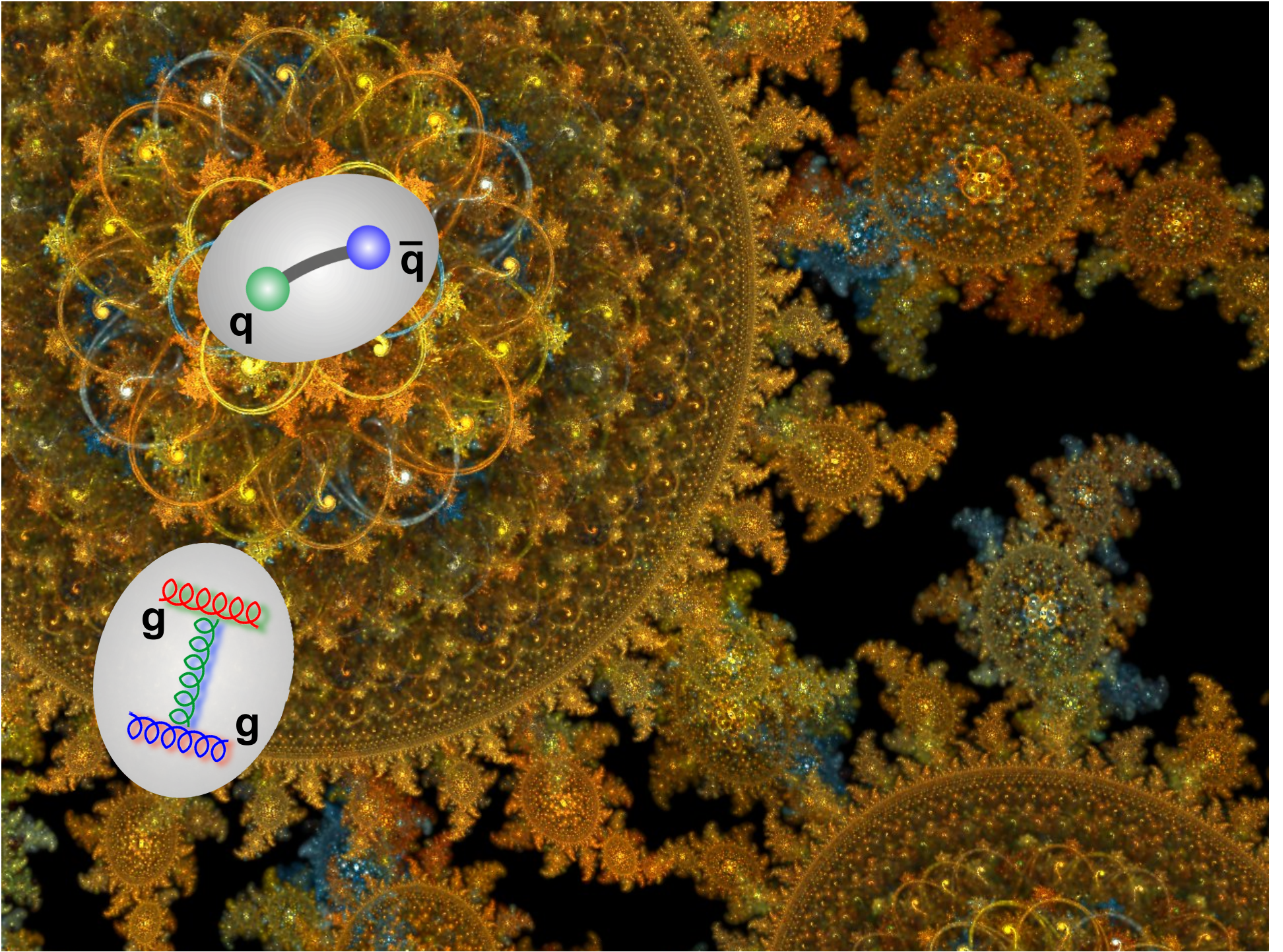


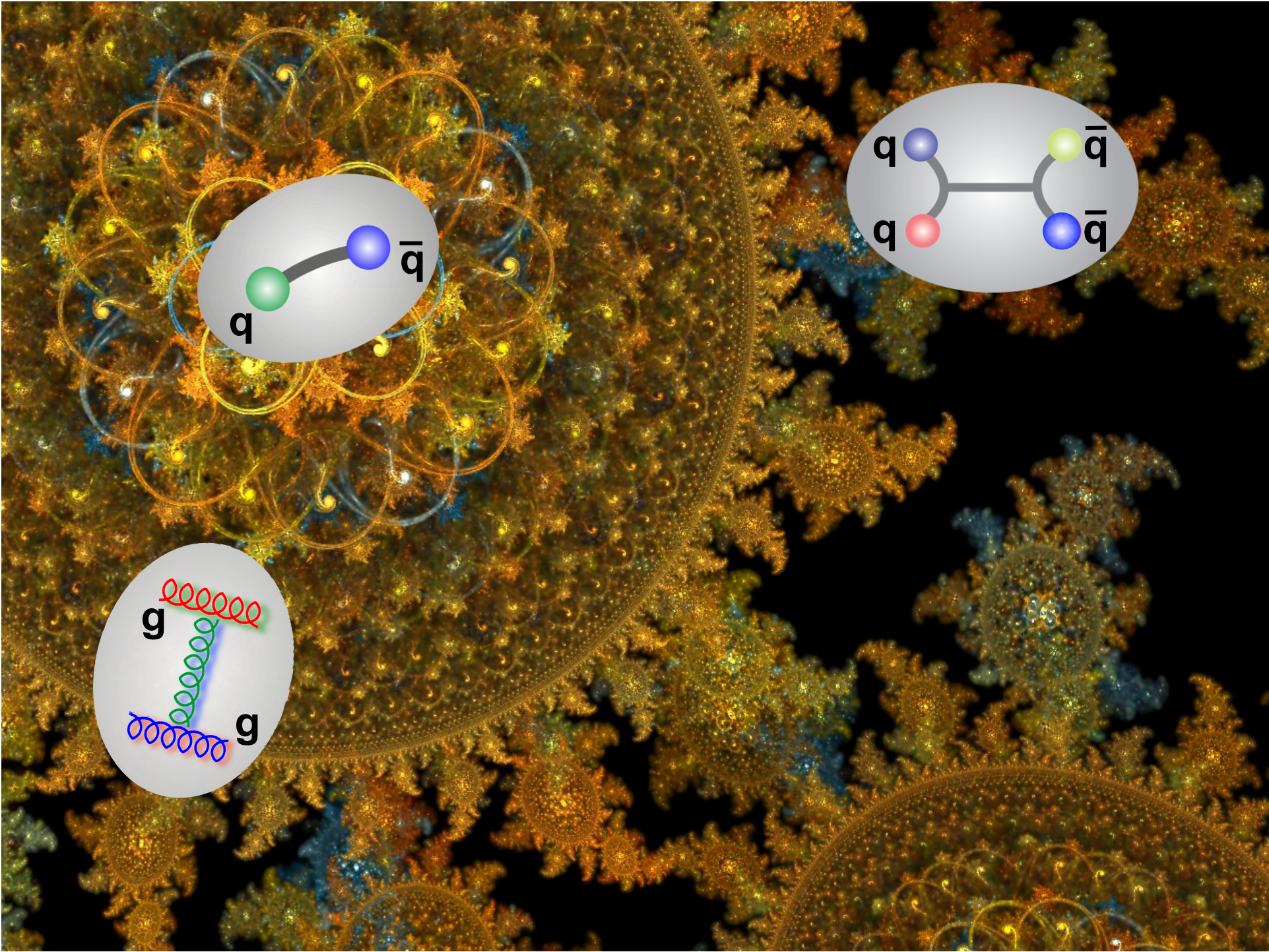
Amplitude Analysis

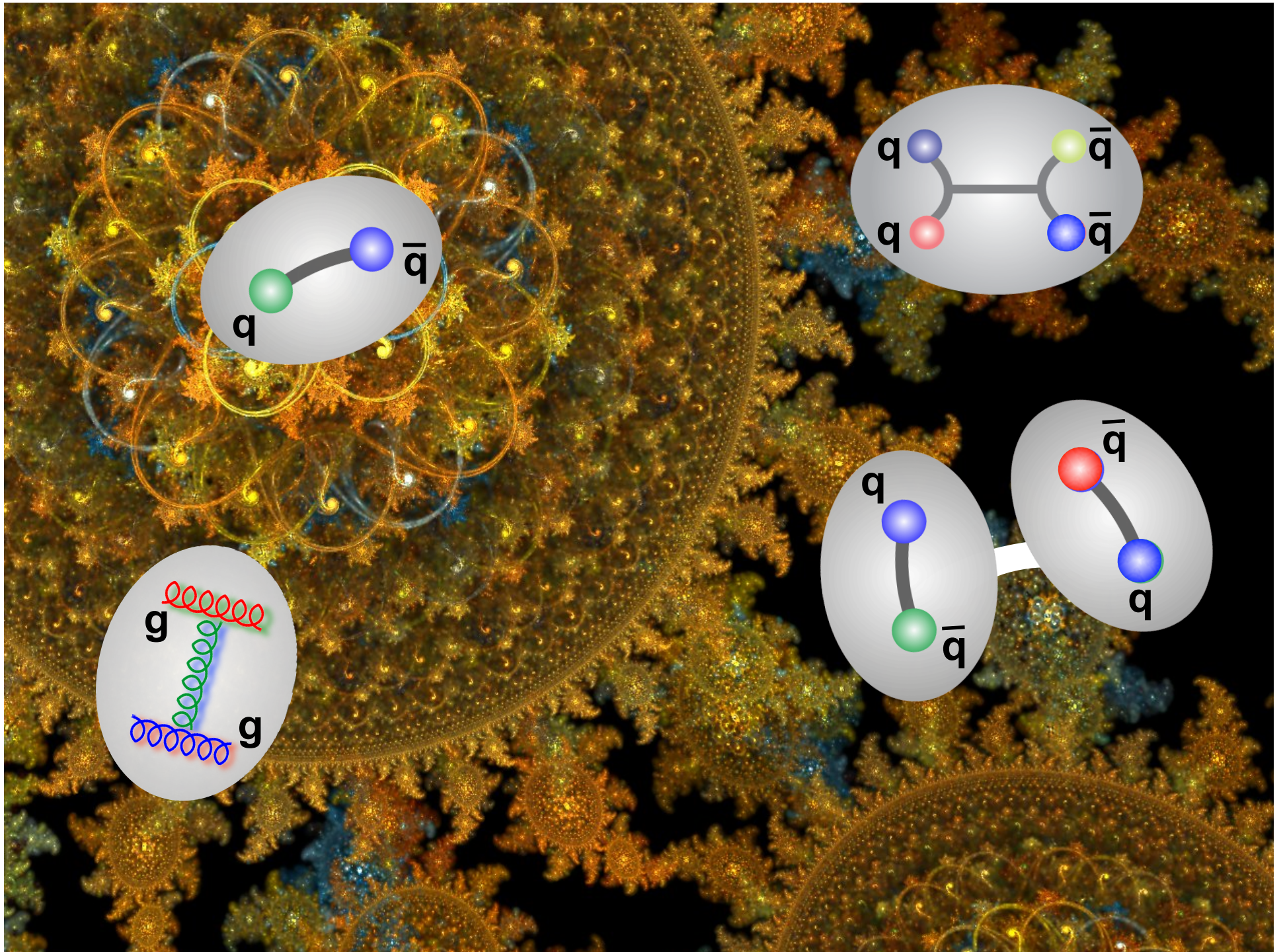


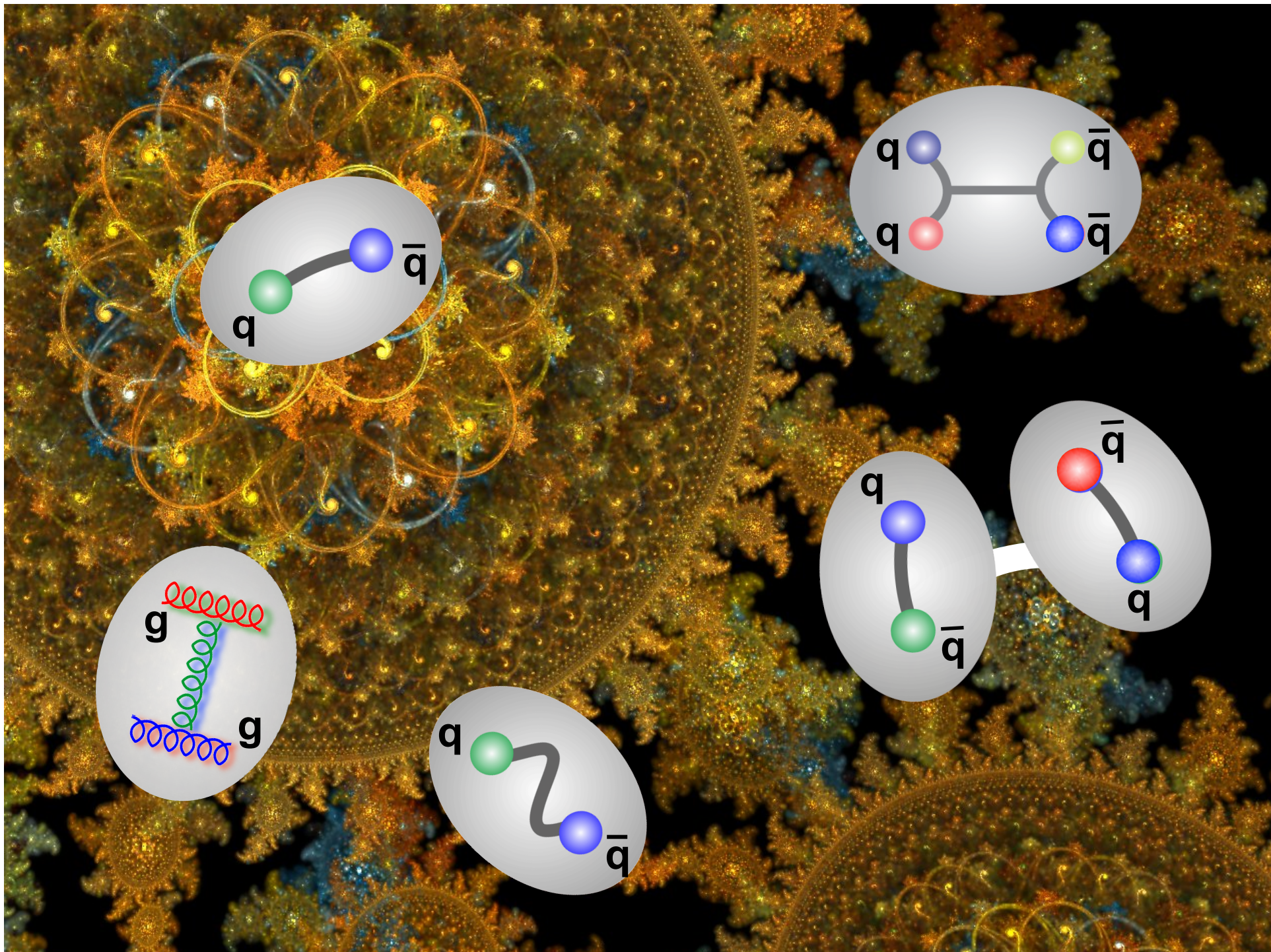




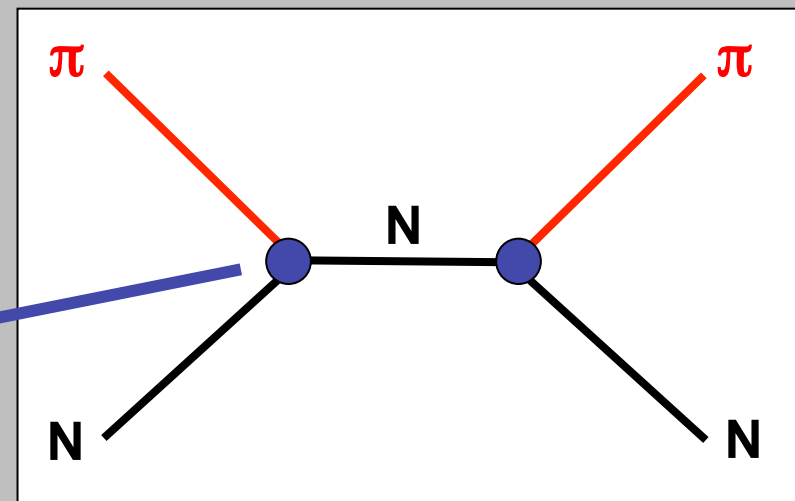
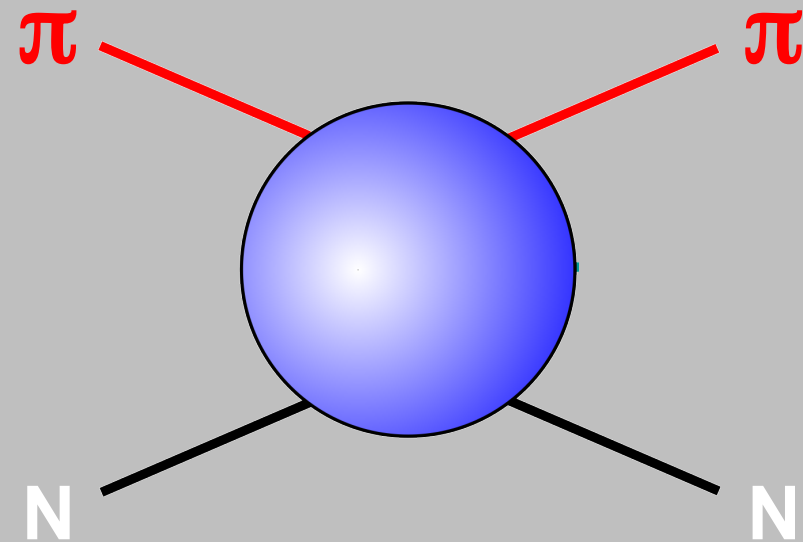
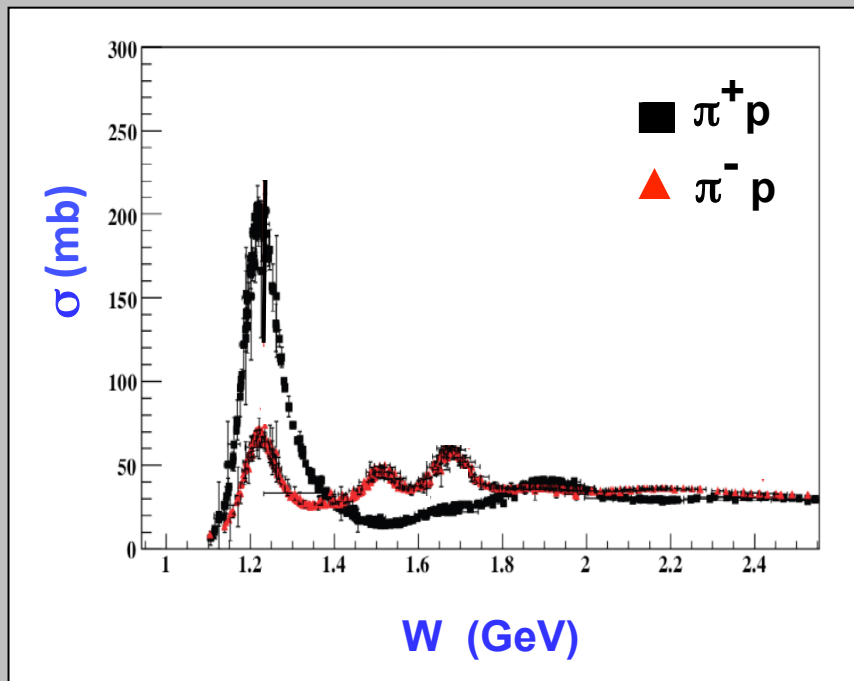




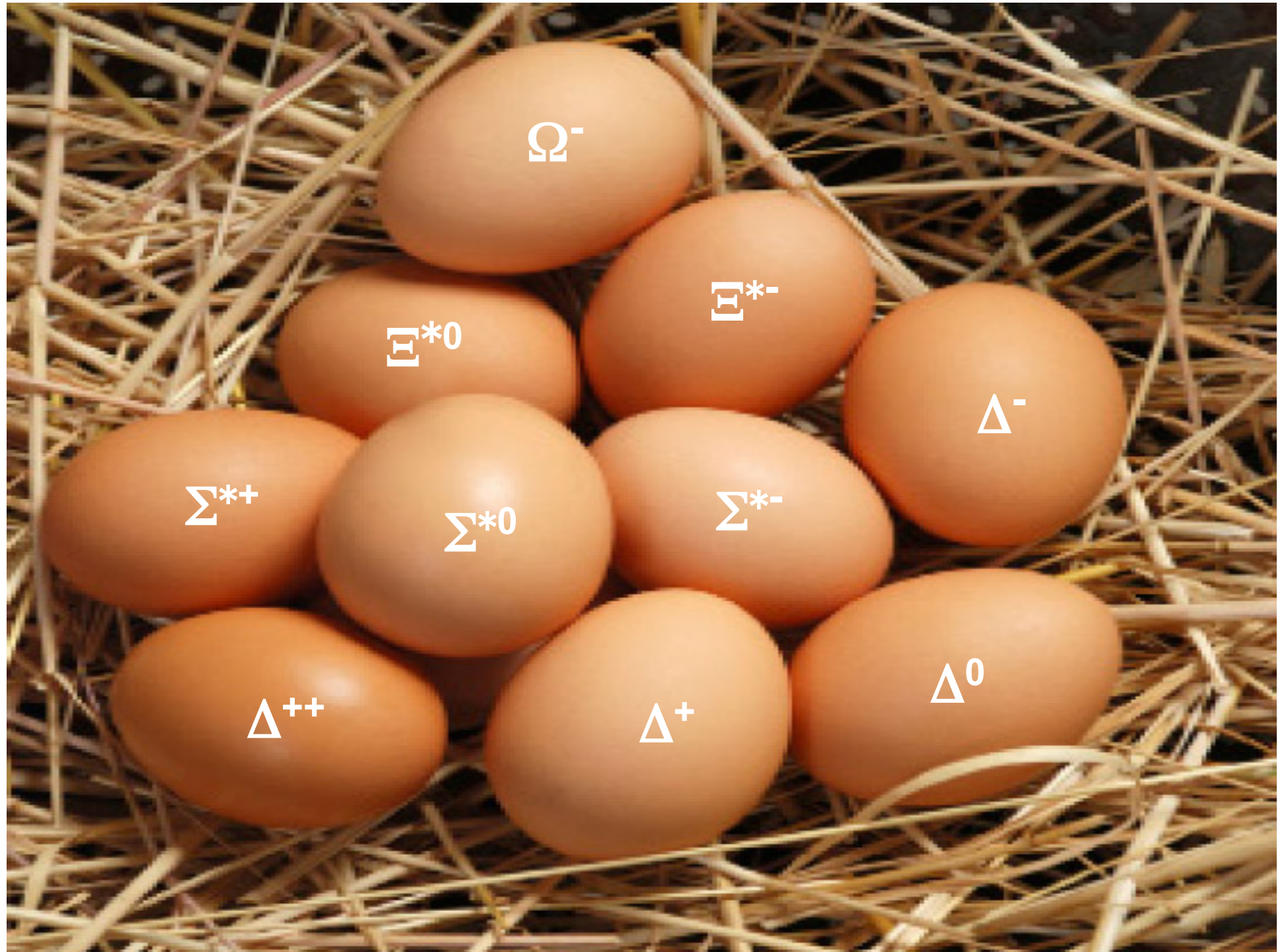




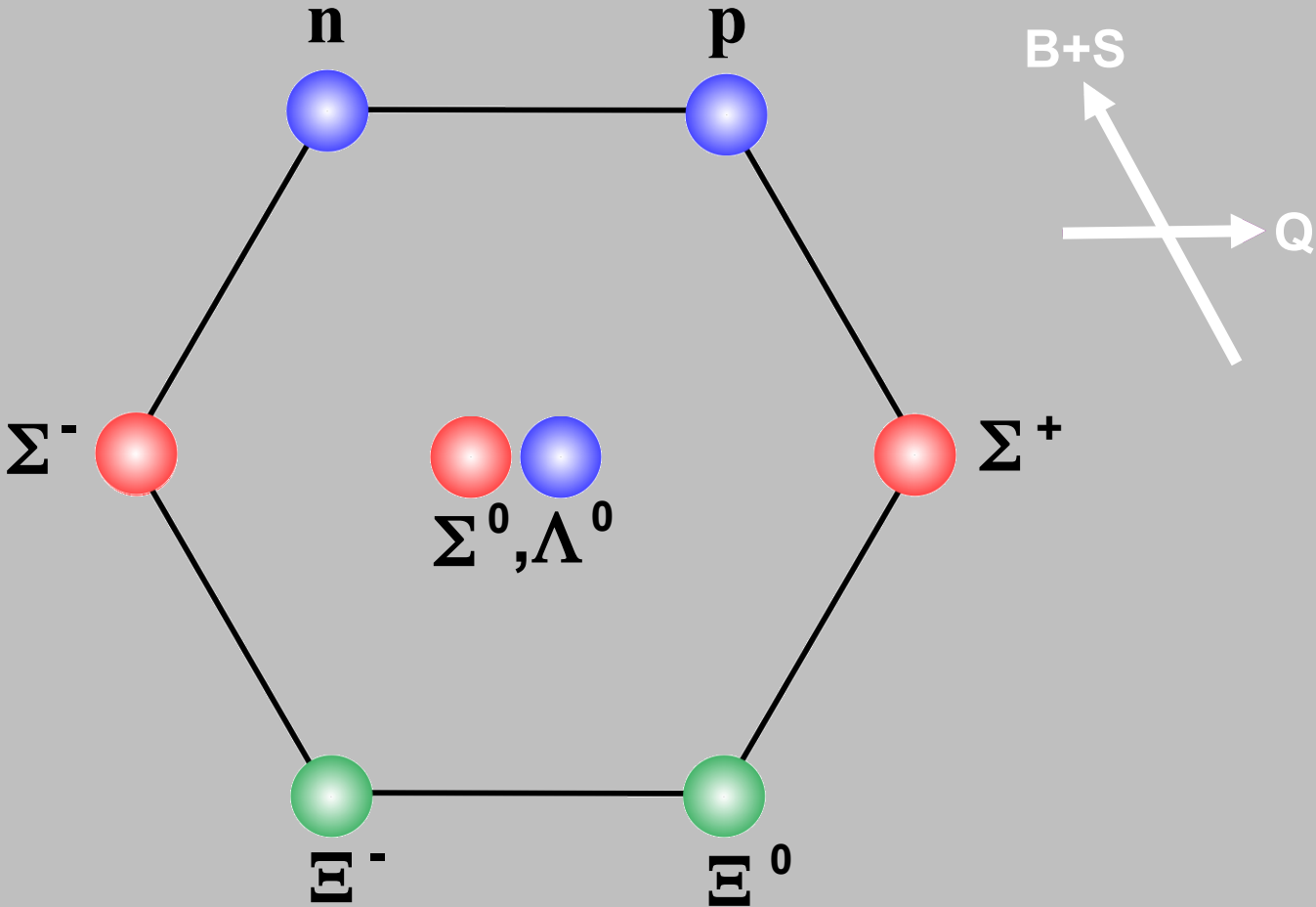
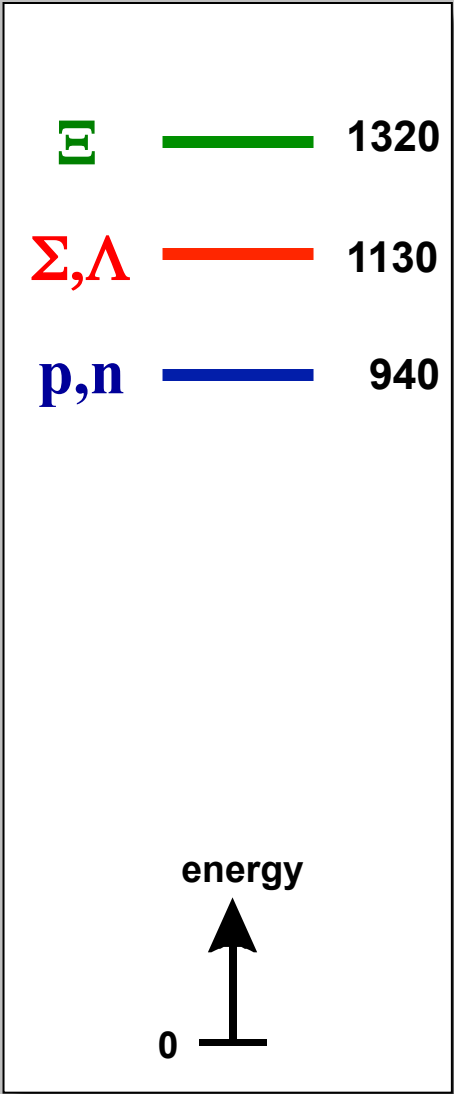
πN scattering



$$\frac{g^2_{\pi NN}}{4\pi} \simeq 14$$

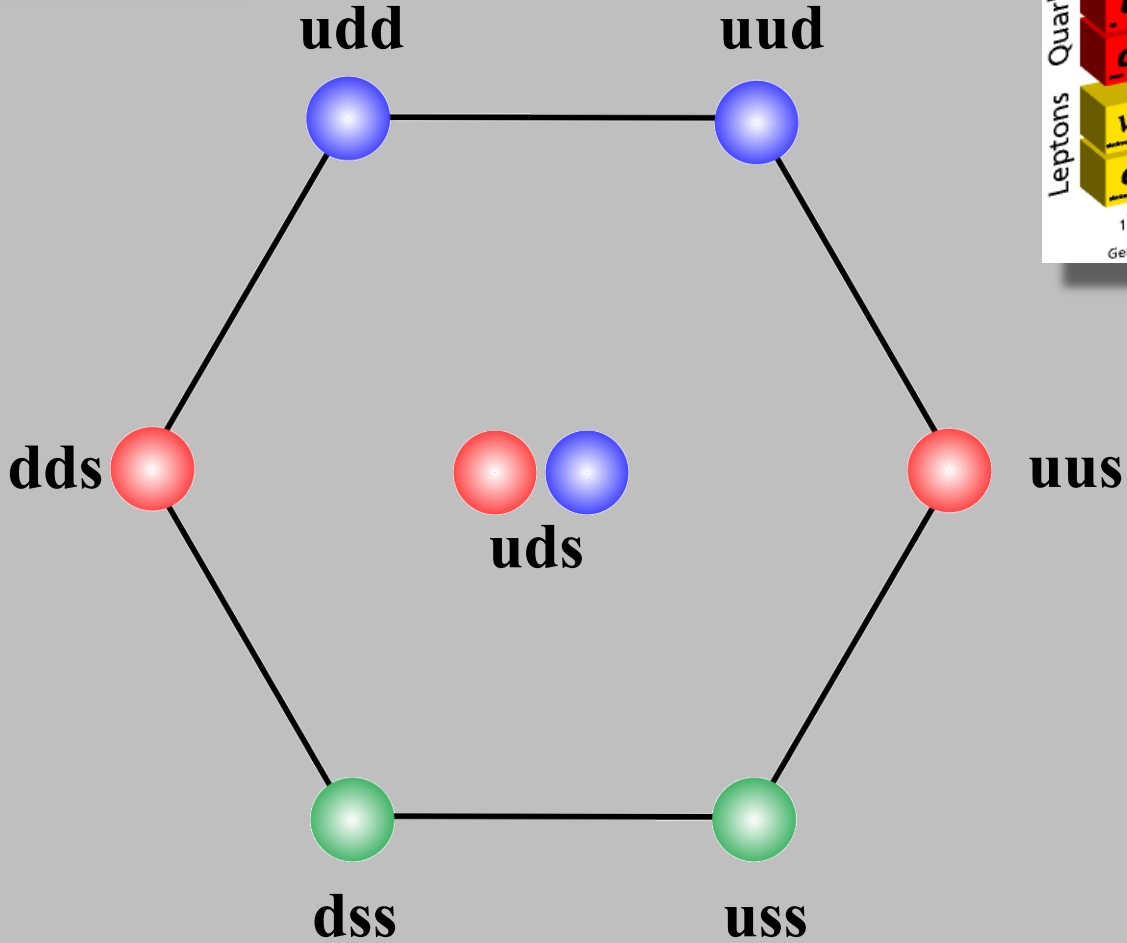
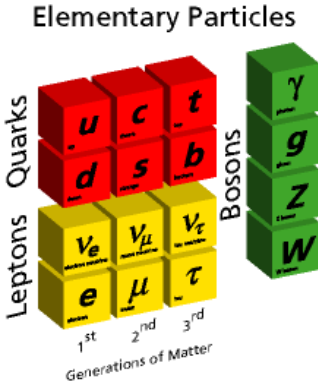


Baryon octet



Ground States

Baryon octet



Legend for quark content:

- s** (green line)
- u,d** (blue line)

Energy scale:

energy ↑

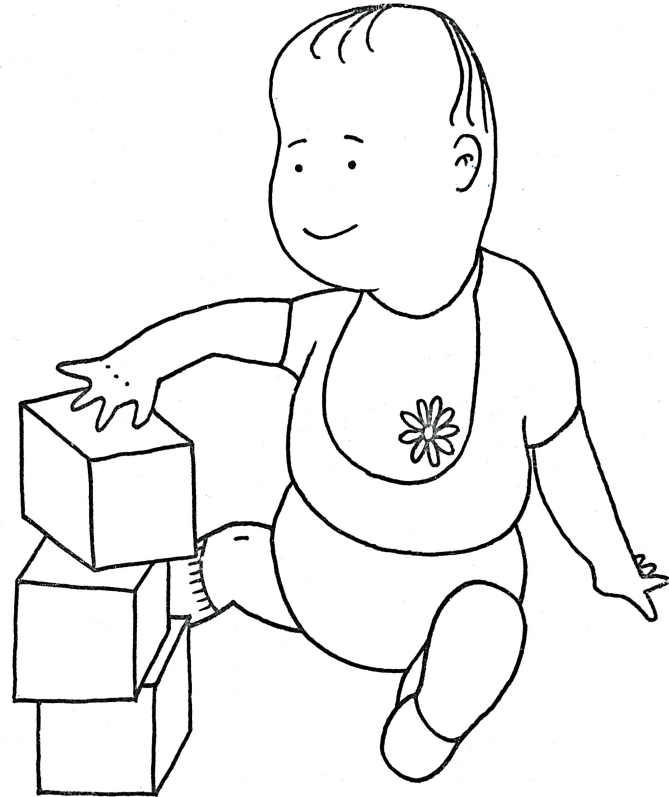
0

Ground States

Quark model

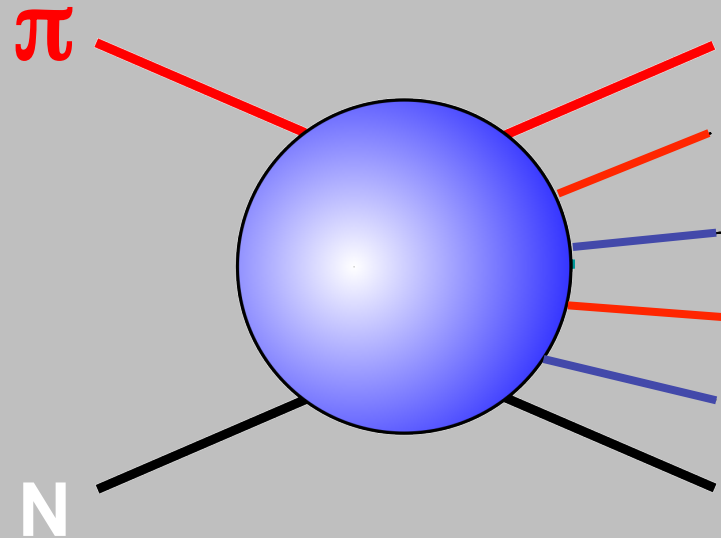
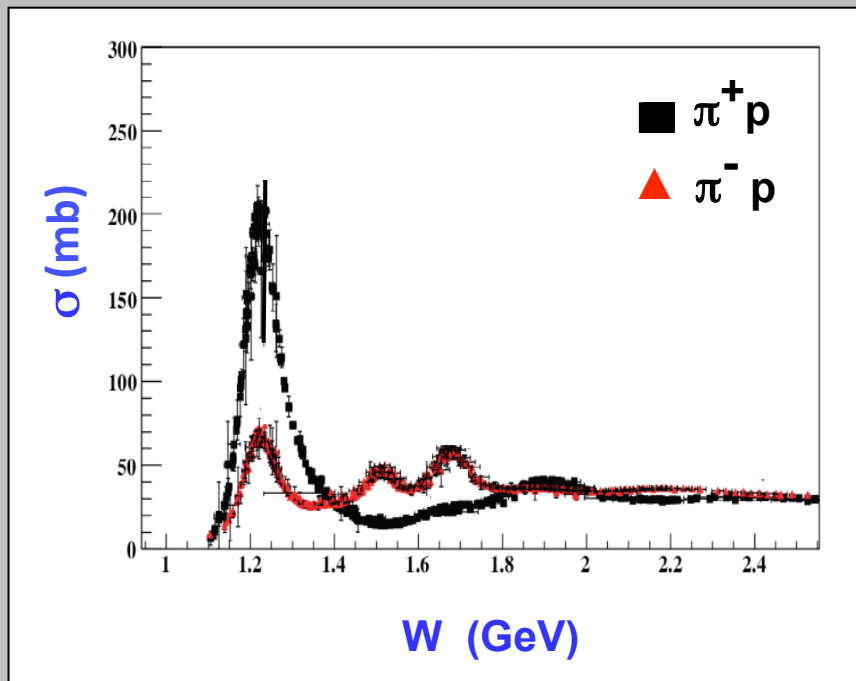


Dalitz



Infant Quark Model

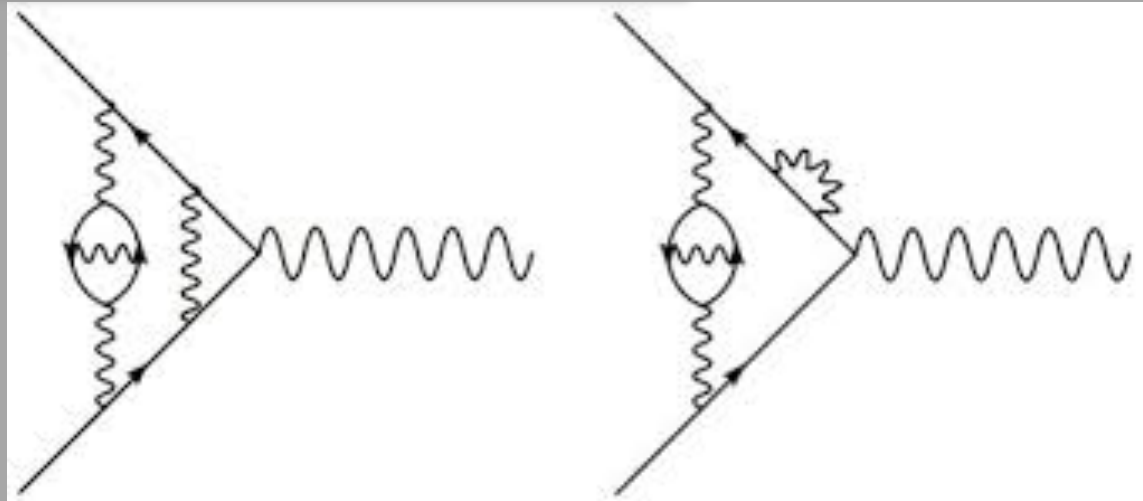
Strong Nuclear Force : 10



$$\frac{g^2_{\pi NN}}{4\pi} \simeq 14$$

Electromagnetism: QED 10^{-2}

$$\frac{e^2}{4\pi} = \frac{1}{137}$$



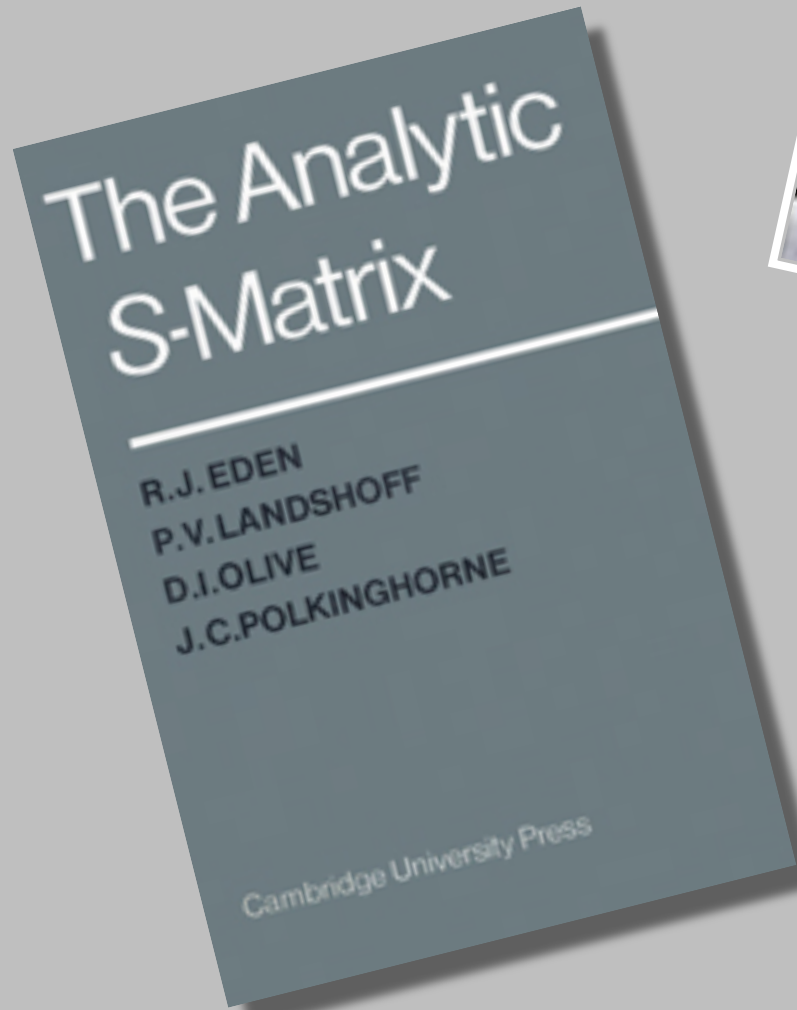
$$\mathcal{L}_{\text{QED}} = \bar{\psi} (i \gamma_{\mu} D^{\mu} - m) \psi - \frac{1}{4} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}$$



Richard Eden



Peter Landshoff



The Analytic S-Matrix

R.J. EDEN
P.V. LANDSHOFF
D.I. OLIVE
J.C. POLKINGHORNE

Cambridge University Press



David Olive



John Polkinghorne

STANDARD MODEL OF ELEMENTARY PARTICLES






Q
U
A
R
K
S

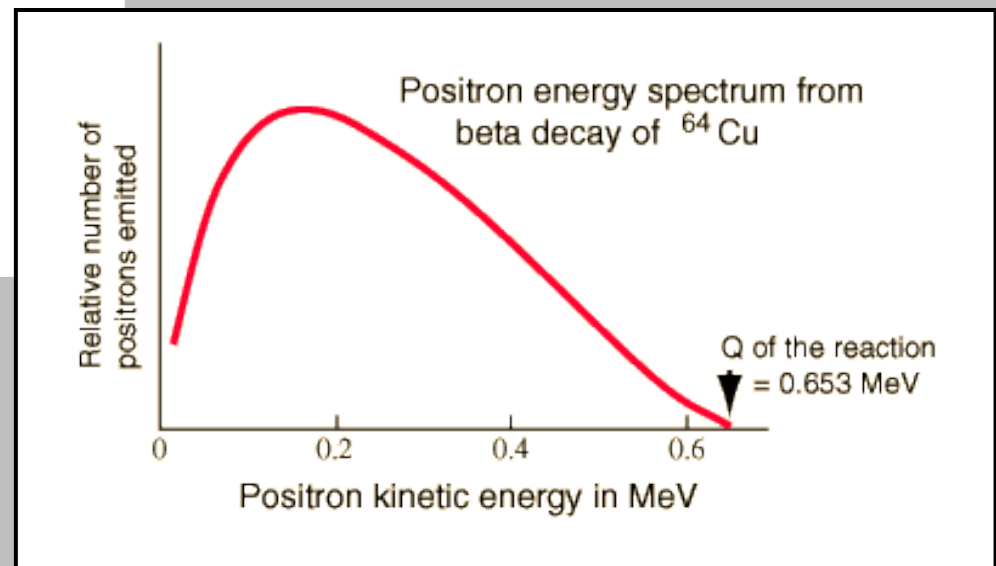
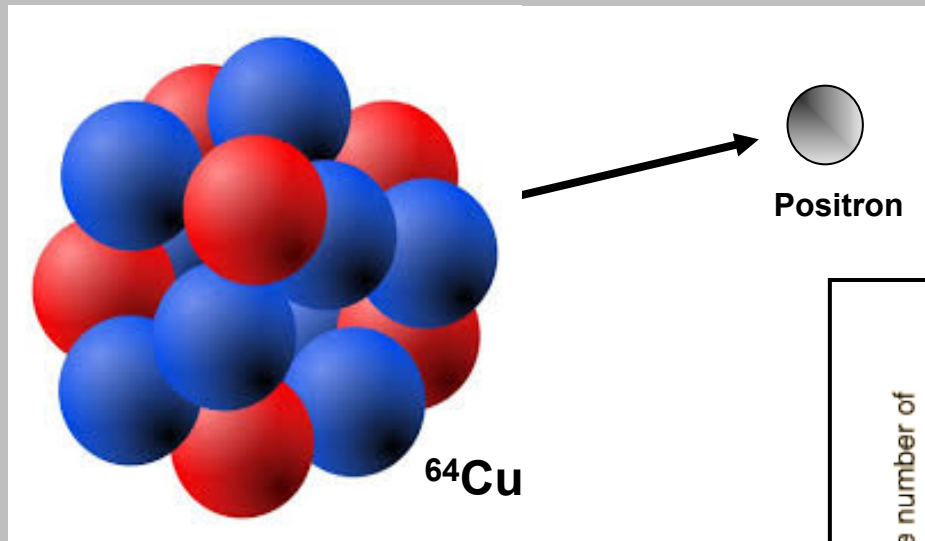
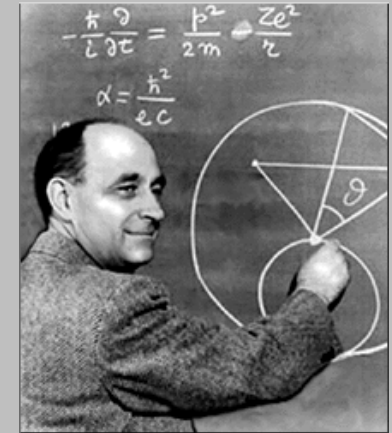
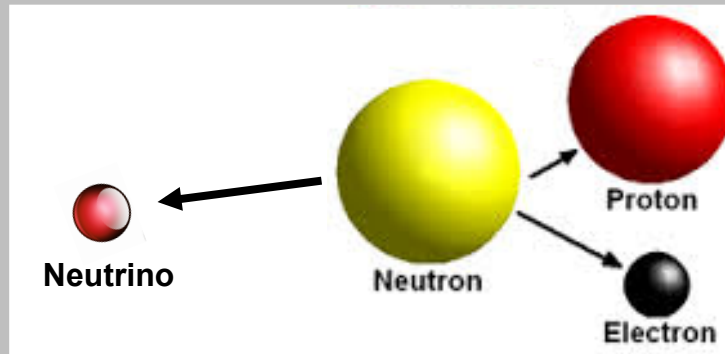
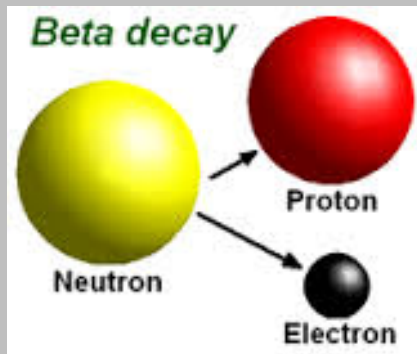
UP mass $2,3 \text{ MeV}/c^2$ charge $\frac{2}{3}$ spin $\frac{1}{2}$ 	CHARM mass $1,275 \text{ GeV}/c^2$ charge $\frac{2}{3}$ spin $\frac{1}{2}$ 	TOP mass $173,07 \text{ GeV}/c^2$ charge $\frac{2}{3}$ spin $\frac{1}{2}$ 
DOWN mass $4,8 \text{ MeV}/c^2$ charge $-\frac{1}{3}$ spin $\frac{1}{2}$ 	STRANGE mass $95 \text{ MeV}/c^2$ charge $-\frac{1}{3}$ spin $\frac{1}{2}$ 	BOTTOM mass $4,18 \text{ GeV}/c^2$ charge $-\frac{1}{3}$ spin $\frac{1}{2}$ 

L
E
P
T
O
N
S

ELECTRON mass $0,511 \text{ MeV}/c^2$ charge -1 spin $\frac{1}{2}$ 	MUON mass $105,7 \text{ MeV}/c^2$ charge -1 spin $\frac{1}{2}$ 	TAU mass $1,777 \text{ GeV}/c^2$ charge -1 spin $\frac{1}{2}$ 
ELECTRON NEUTRINO mass $<2,2 \text{ eV}/c^2$ charge 0 spin $\frac{1}{2}$ 	MUON NEUTRINO mass $<0,17 \text{ MeV}/c^2$ charge 0 spin $\frac{1}{2}$ 	TAU NEUTRINO mass $<15,5 \text{ MeV}/c^2$ charge 0 spin $\frac{1}{2}$ 

G
A
U
G
E
B
O
S
O
N
S

GLUON mass 0 charge 0 spin 1 	HIGGS BOSON mass $126 \text{ GeV}/c^2$ charge 0 spin 0 
PHOTON mass 0 charge 0 spin 1 	
Z BOSON mass $91,2 \text{ GeV}/c^2$ charge 0 spin 1 	
W BOSON mass $80,4 \text{ GeV}/c^2$ charge ± 1 spin 1 	



$G_F \sim 10^{-5}$

Non-Abelian gauge theories

1999

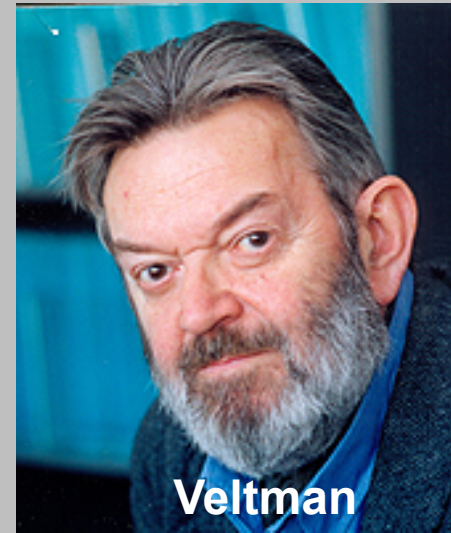


CN Yang

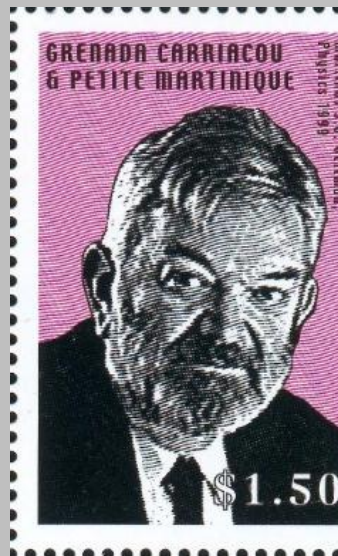
Mills



't Hooft



Veltman





Electroweak Lagrangian

SU(2) x U(1)

$$\mathcal{L}_{\text{GWS}} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

$$\mathcal{L}_1 = -\frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_2 = i\bar{R}\gamma^\mu D_\mu R + i\bar{L}\gamma^\mu D_\mu L$$

$$\mathcal{L}_3 = D_\mu\phi^\dagger D^\mu\phi - m^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \\ + G_c(\bar{L}\phi R + \bar{R}\phi^\dagger L)$$

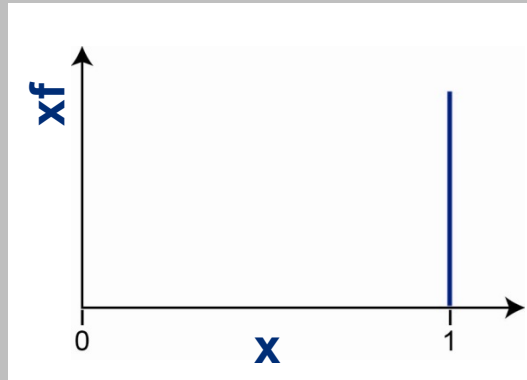
$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + gf^{abc}W_\mu^b W_\nu^c$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

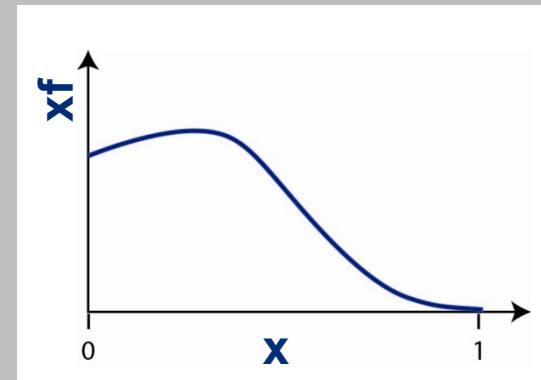
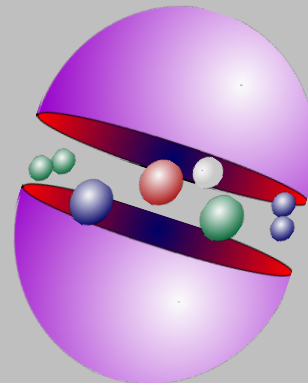
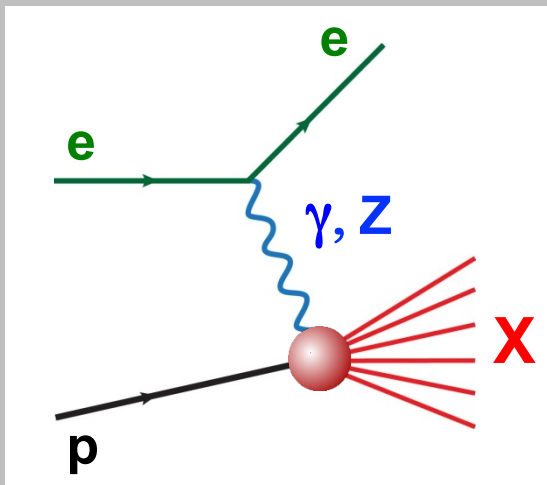
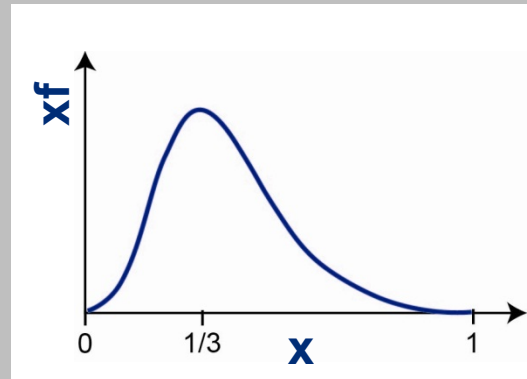
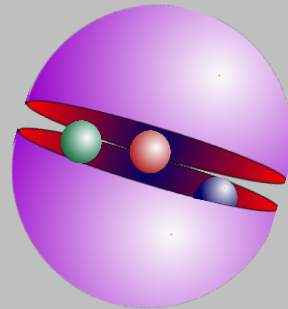
$$D_\mu R = (\partial_\mu + ig'B_\mu)R$$

$$D_\mu L = [\partial_\mu + (i/2)g'B_\mu - (i/2)g\sigma_i W_\mu^i]L$$

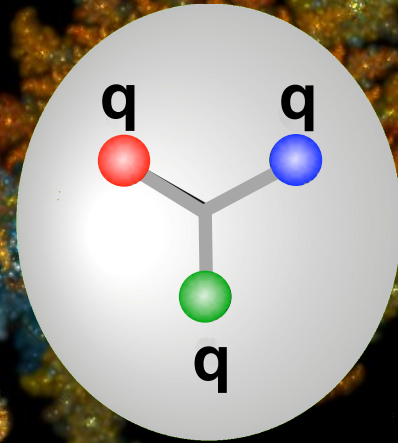
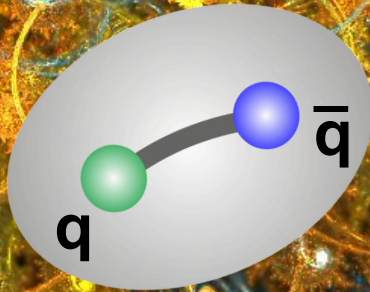
$$D_\mu\phi = [\partial_\mu - (i/2)g\sigma_i W_\mu^i - (i/2)g'B_\mu]\phi$$



parton structure
of the nucleon



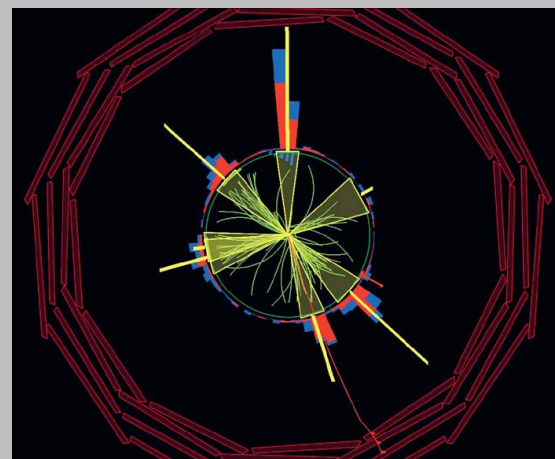
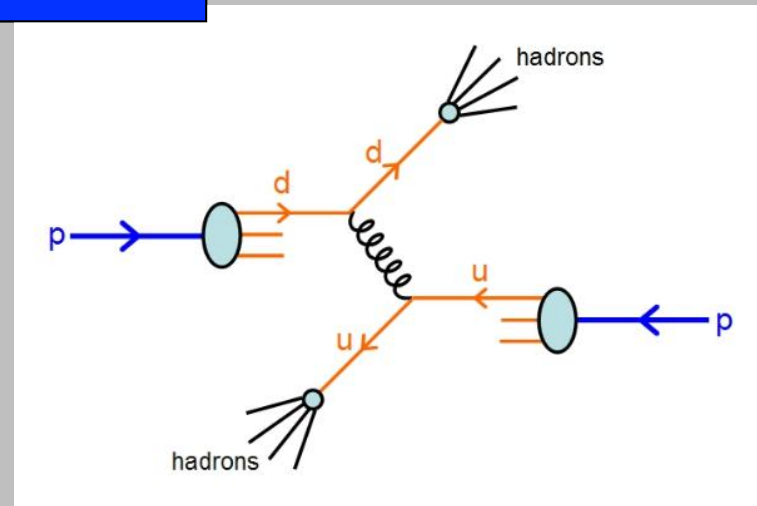
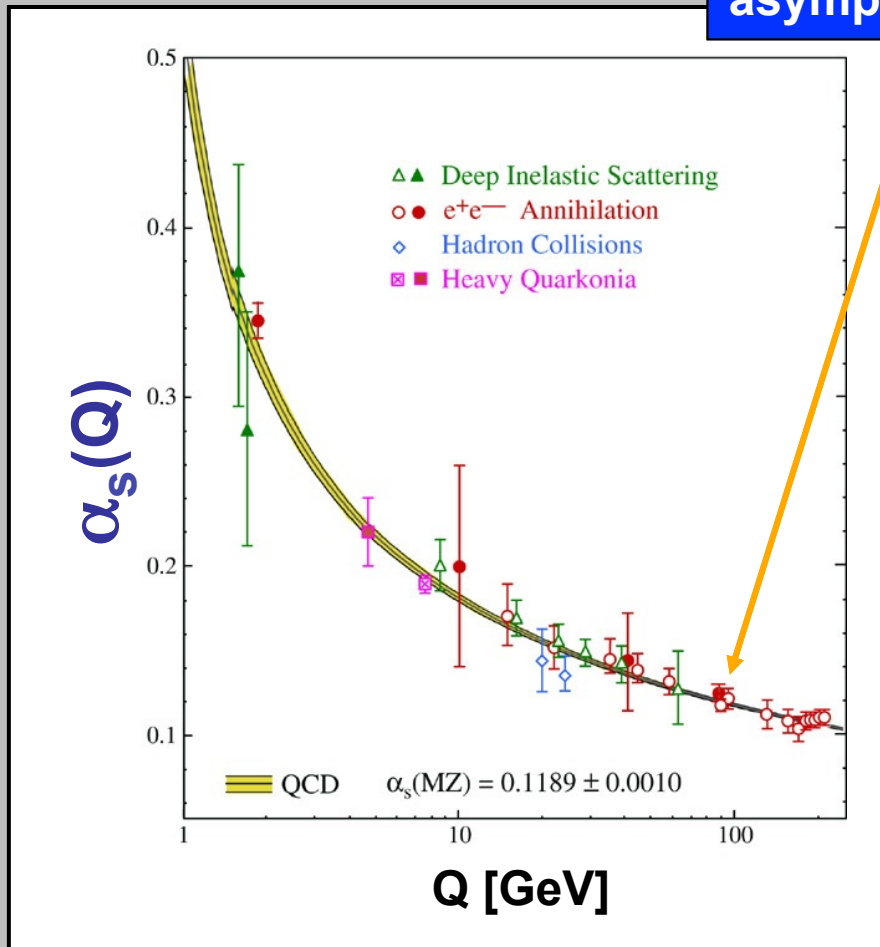
QCD Lagrangian



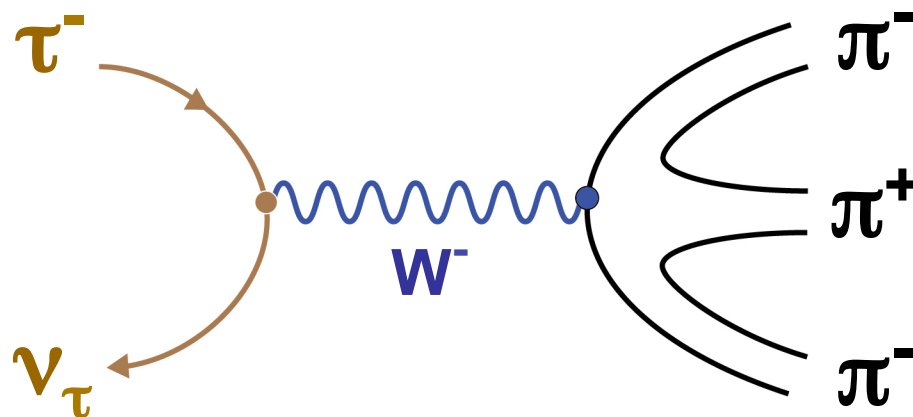
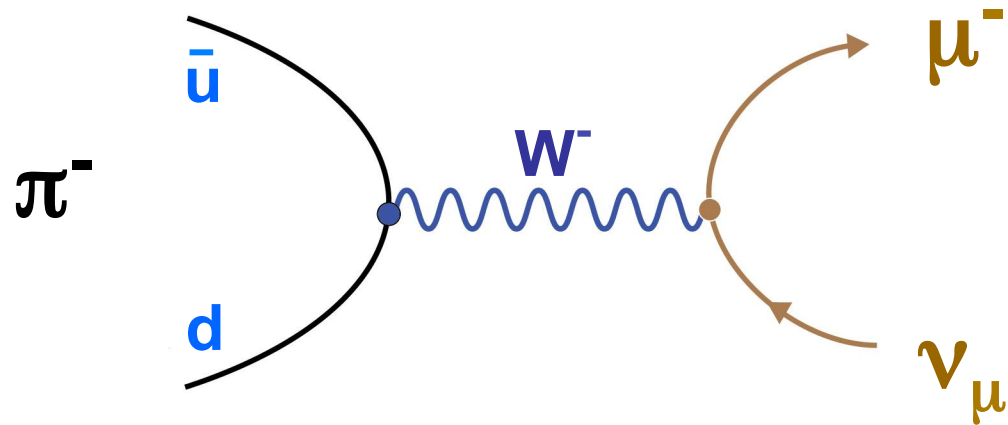
$$\mathcal{L}_{\text{QCD}} = \sum_{q=u,d,s,c,b,t} \bar{q} (i\gamma_{\mu} D^{\mu} - m_q) q - \frac{1}{4} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}$$

QCD coupling

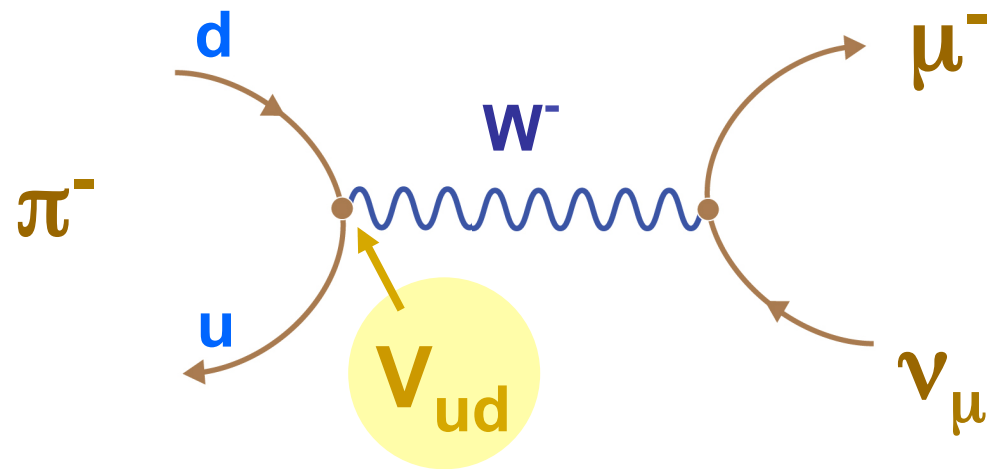
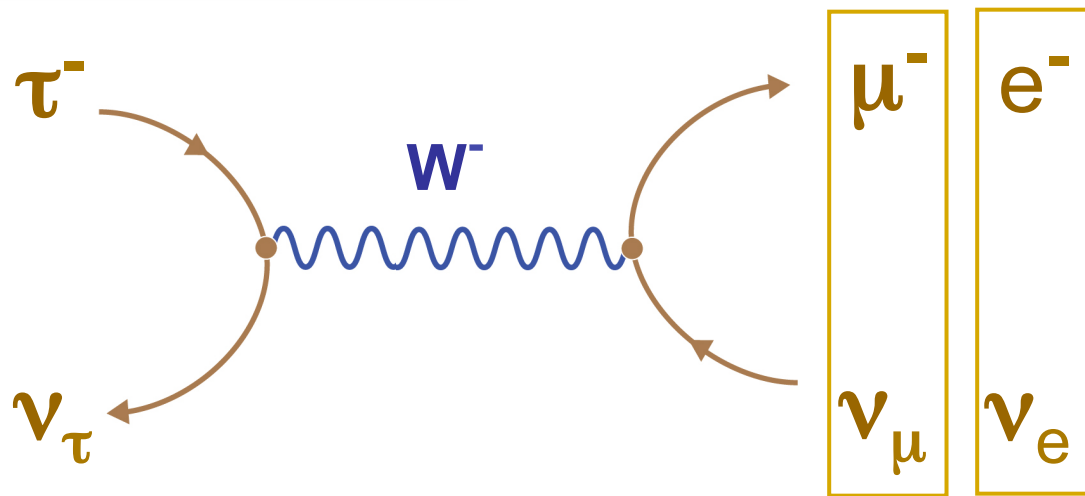
asymptotic freedom



$\tau^- \rightarrow \nu_\tau \pi\pi\pi$ decay

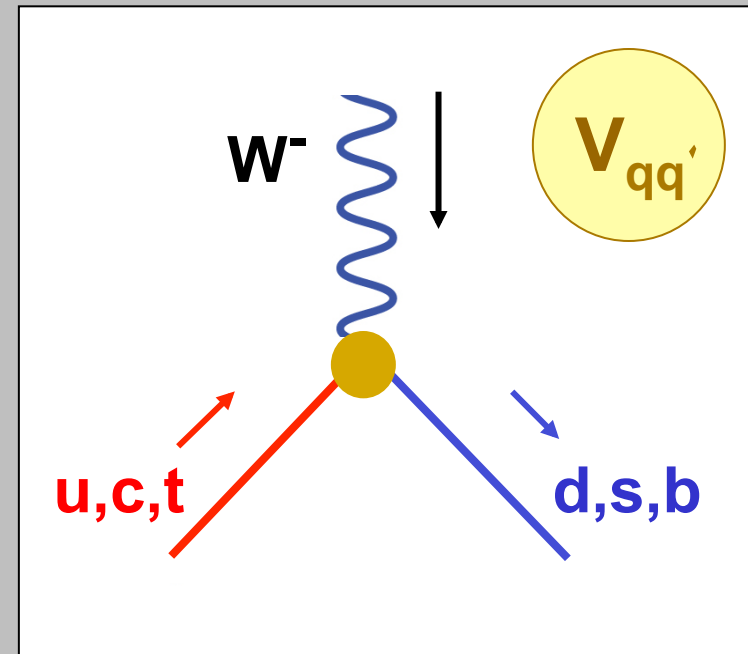
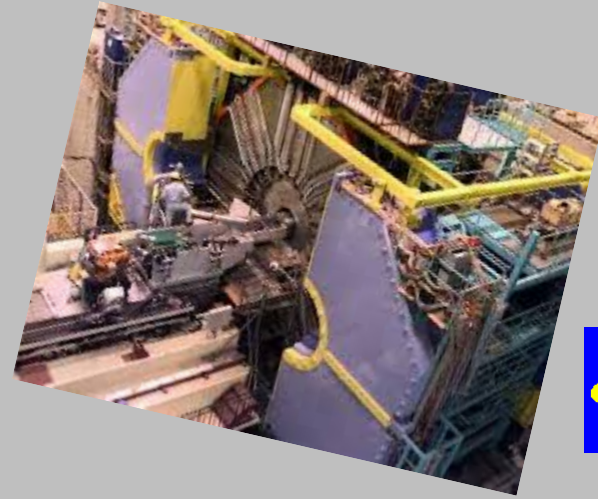


weak decays

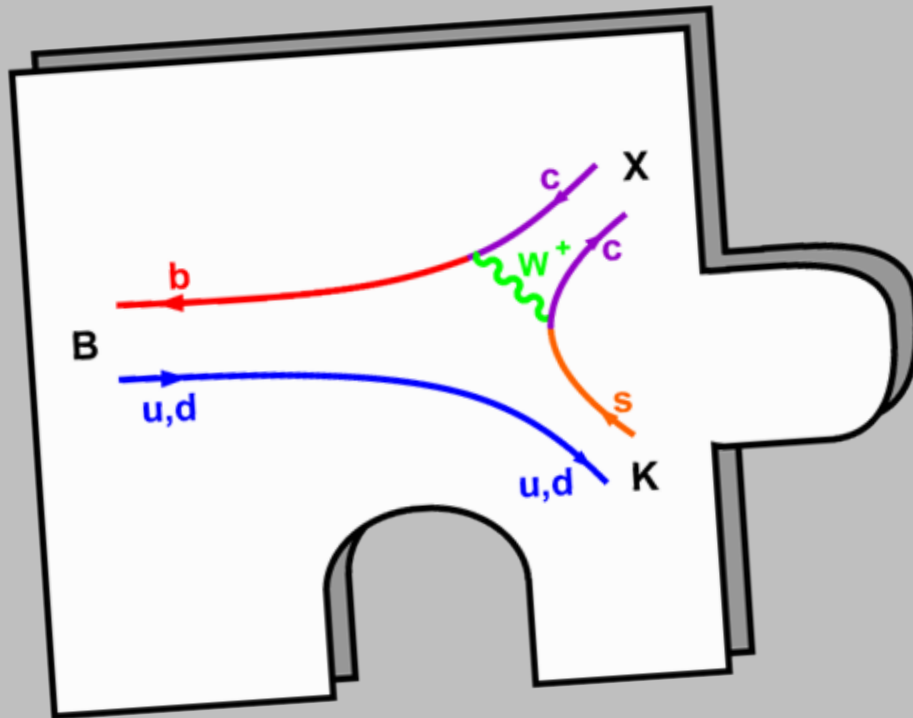


CKM matrix elements

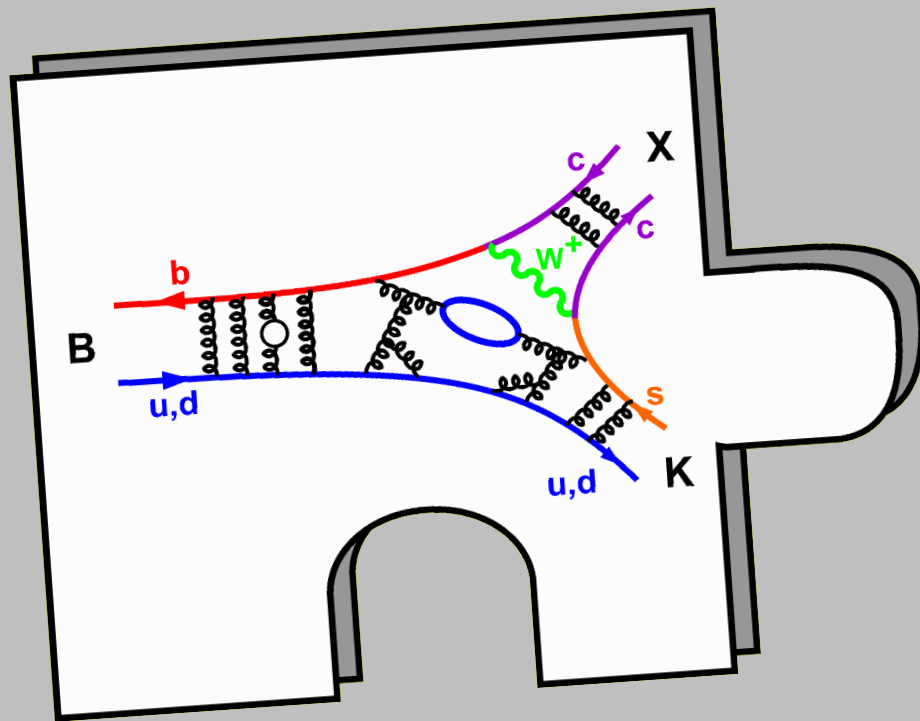
$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$



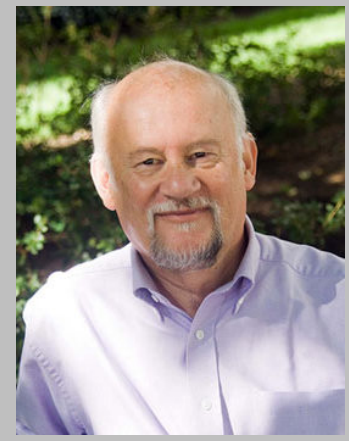
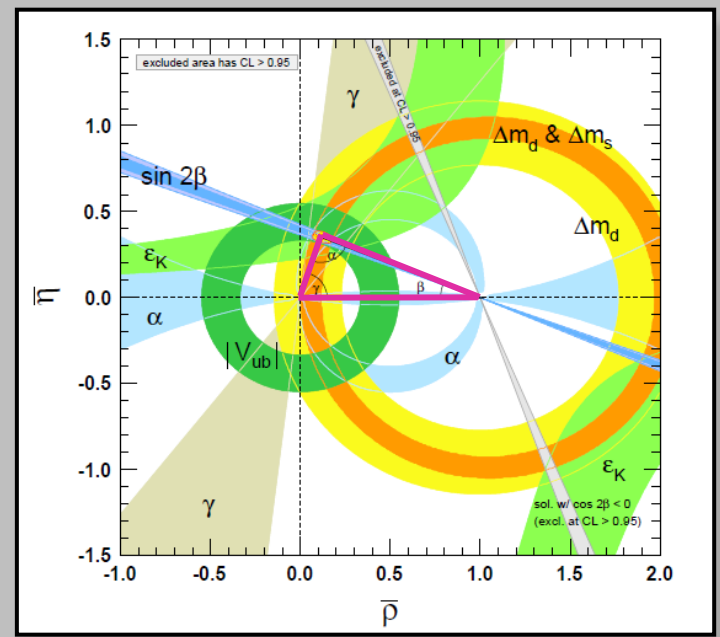
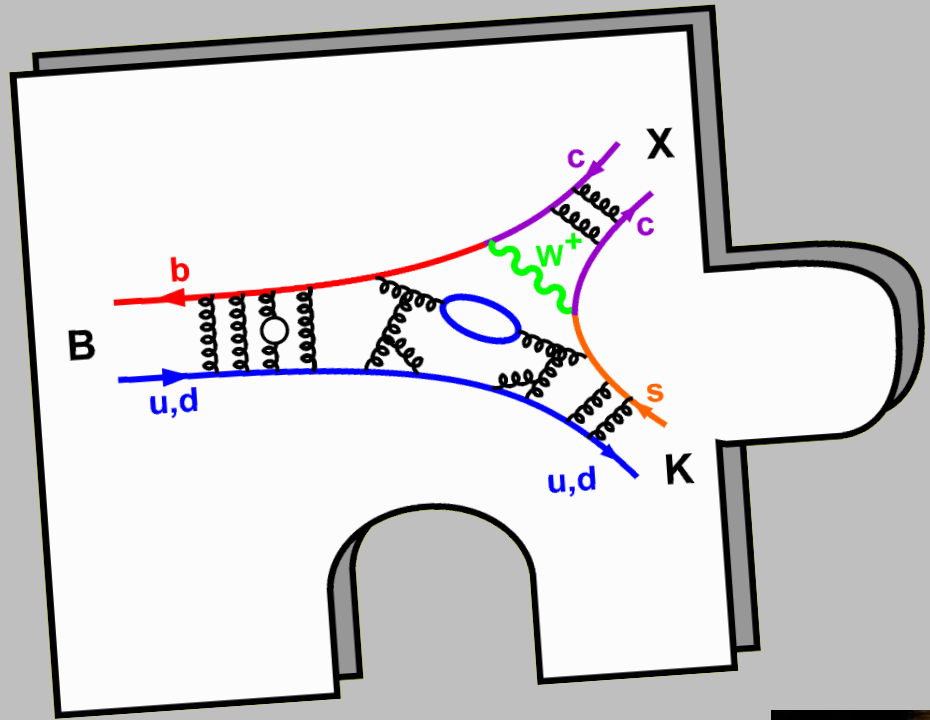
Physics Puzzle



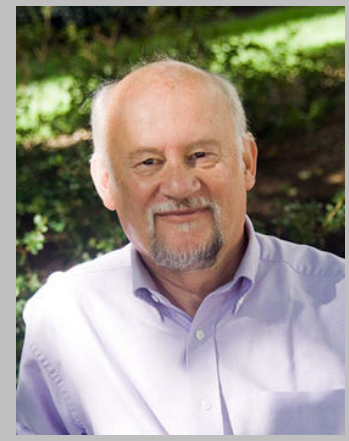
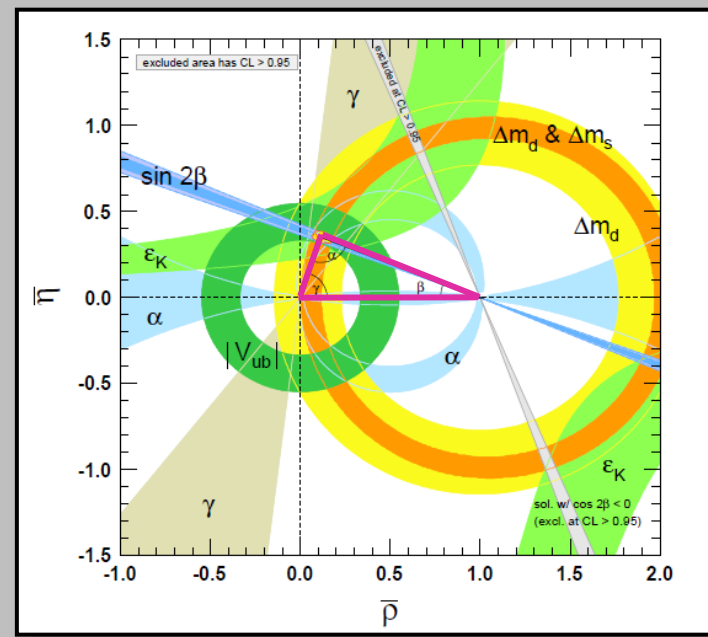
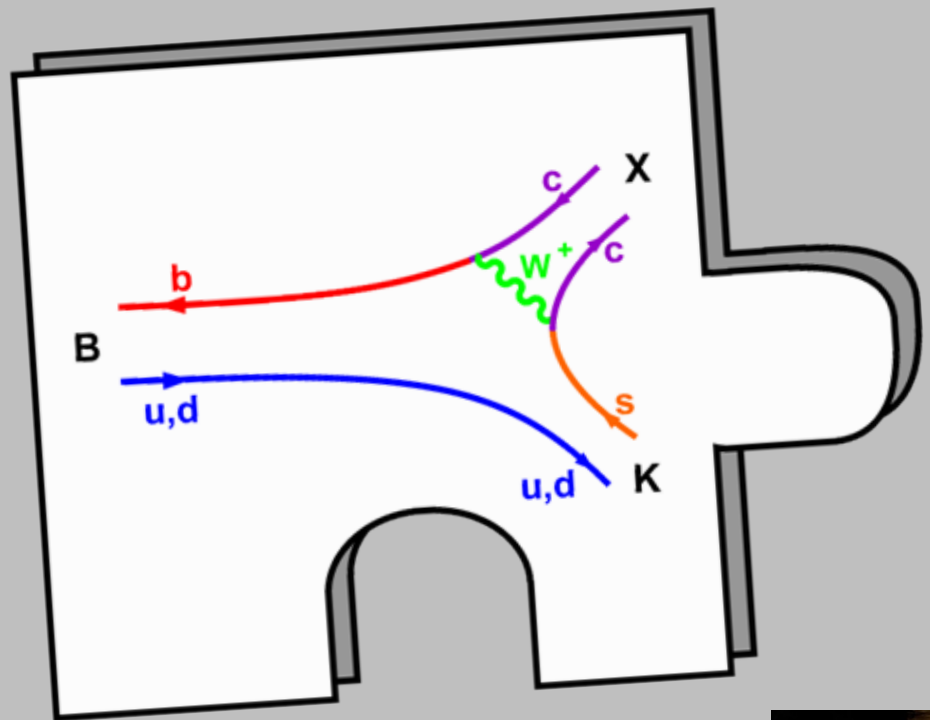
Physics Puzzle



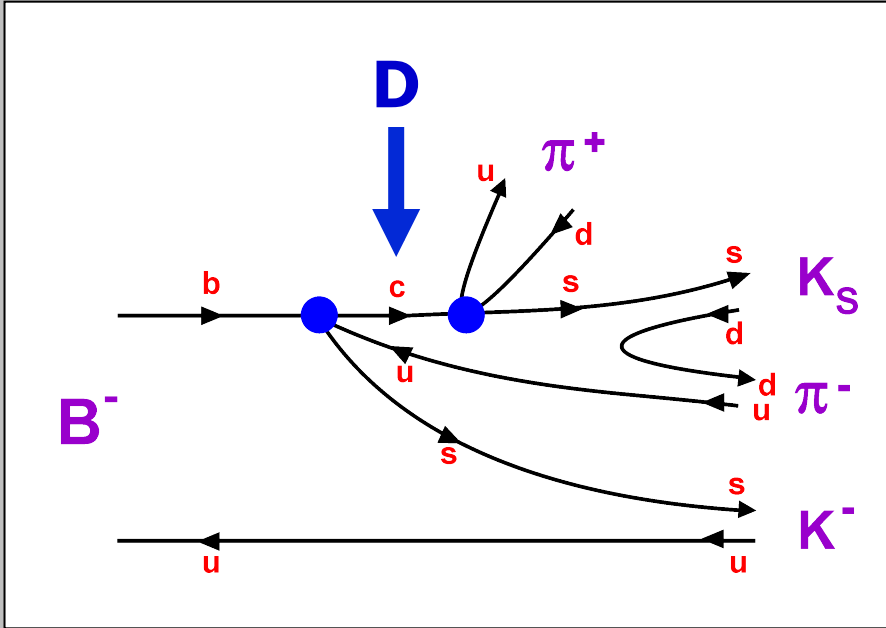
CKM matrix elements



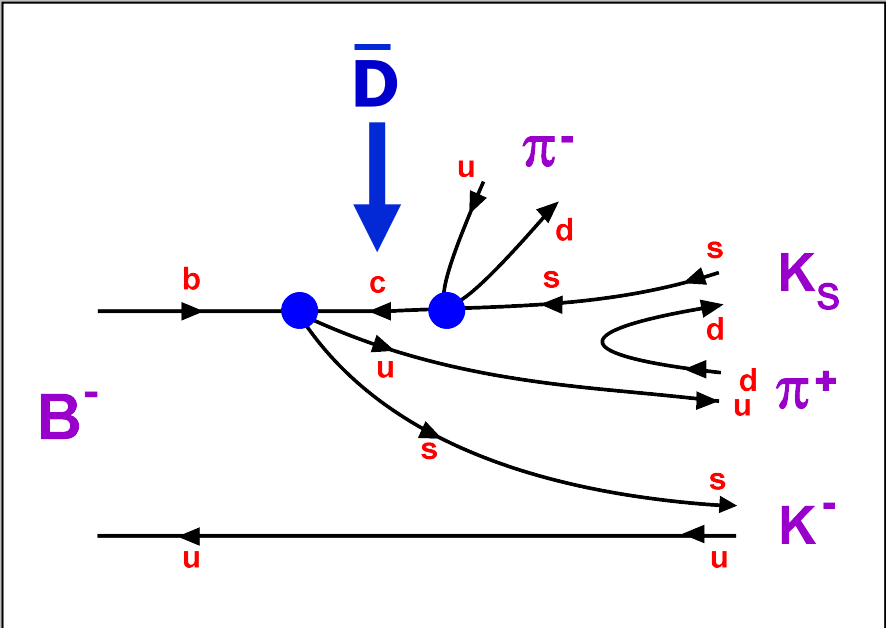
CKM matrix elements



$B \rightarrow D\bar{K} \rightarrow \bar{K}K\pi\pi$



$B \rightarrow \bar{D}K \rightarrow \bar{K}K\pi\pi$



~~CP~~



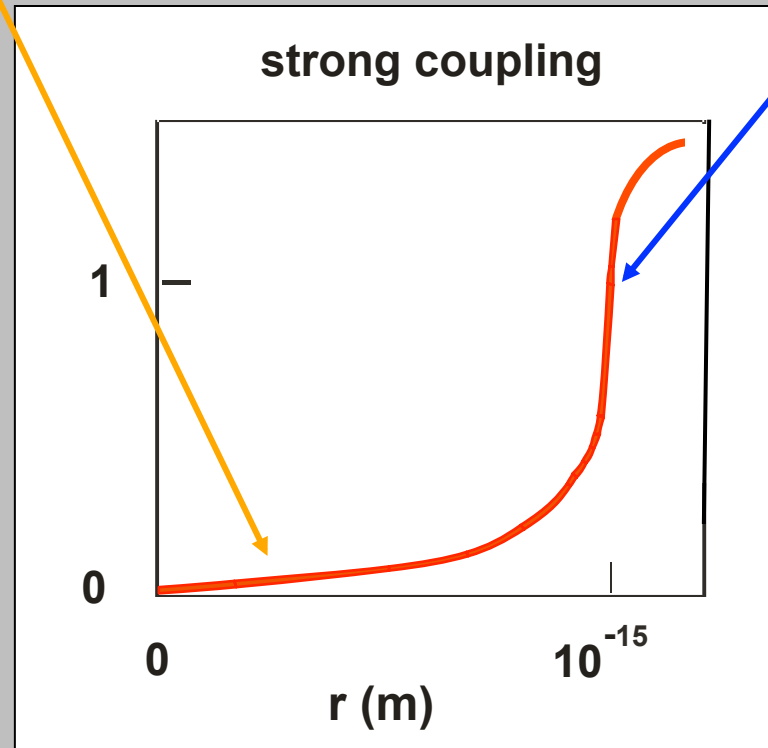
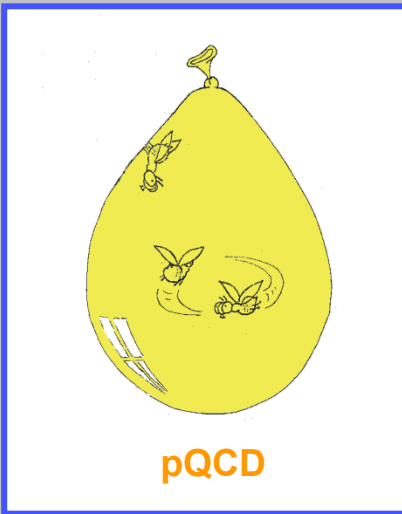






QCD

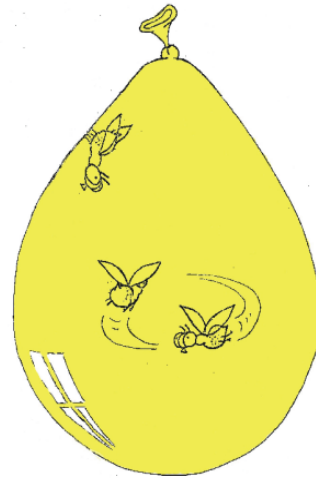
asymptotic freedom



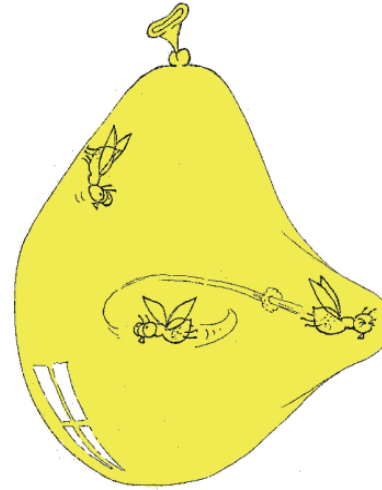
confinement



Strong physics problems

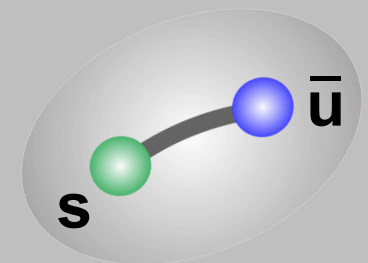
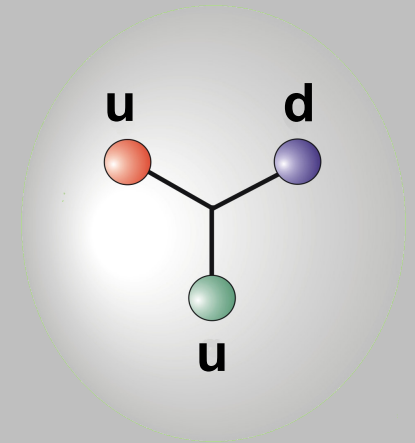
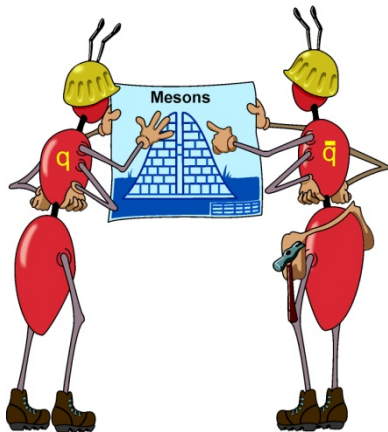


pQCD

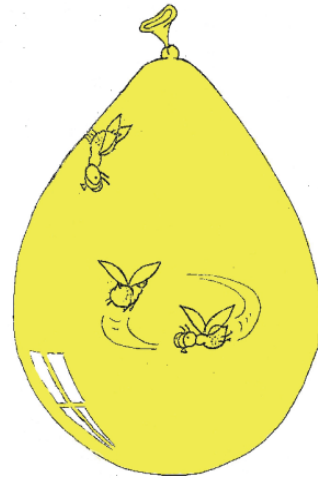


strong coupling

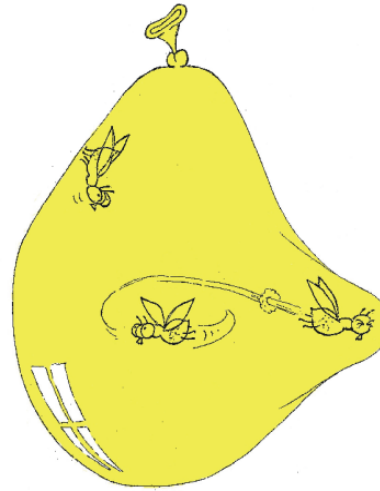
bound states



Strong physics problems

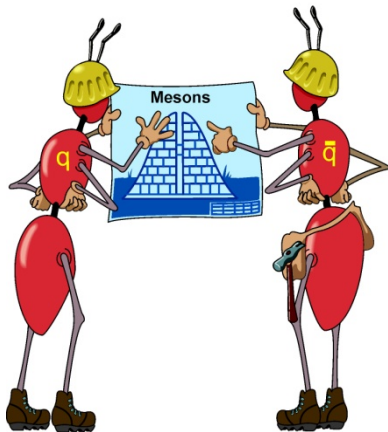


pQCD

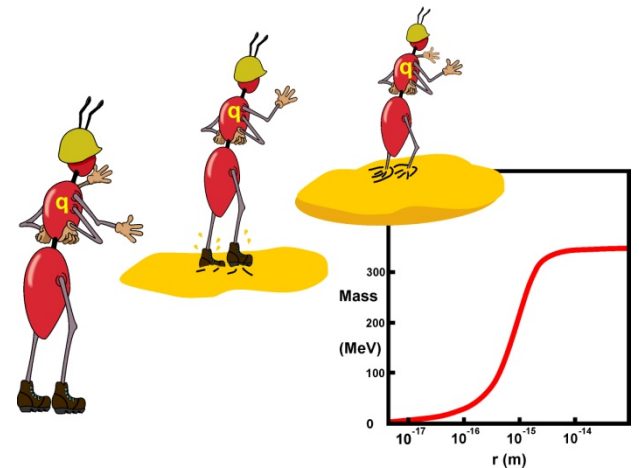


strong coupling

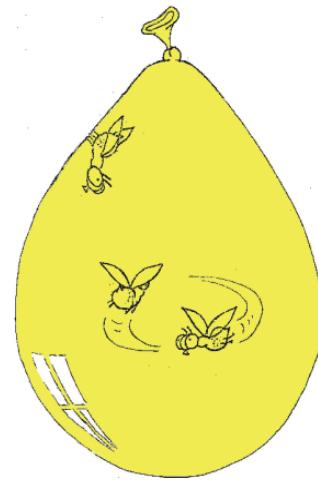
bound states



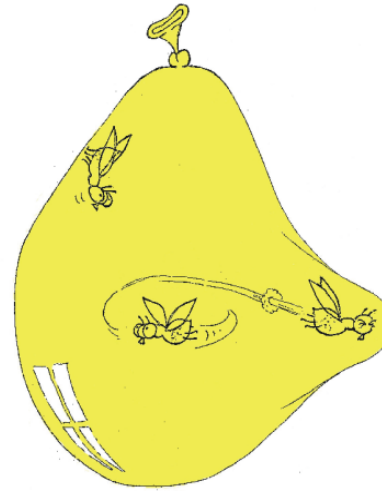
mass generation



Strong physics problems

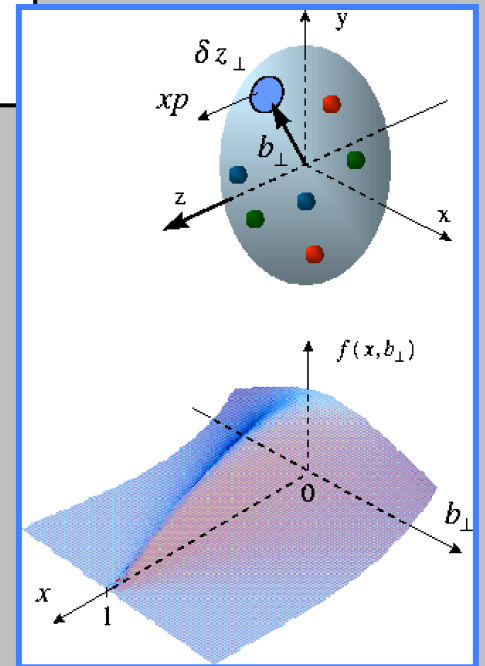
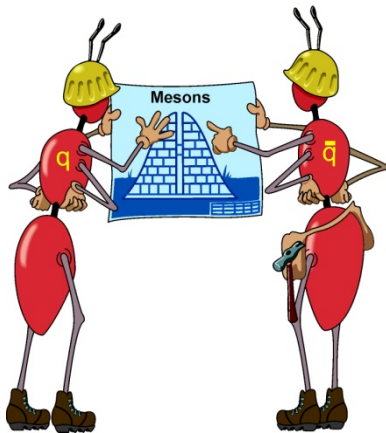


pQCD

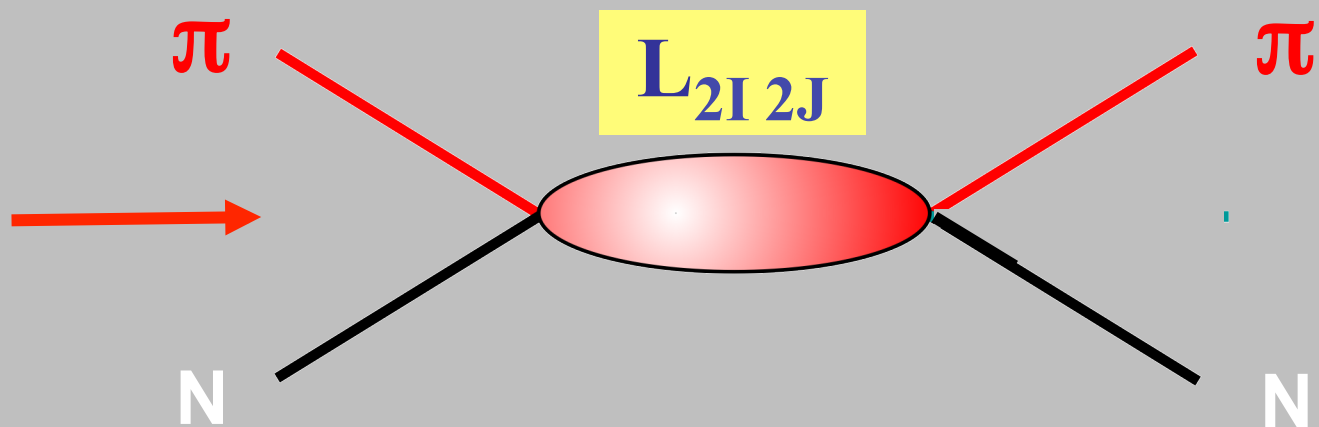
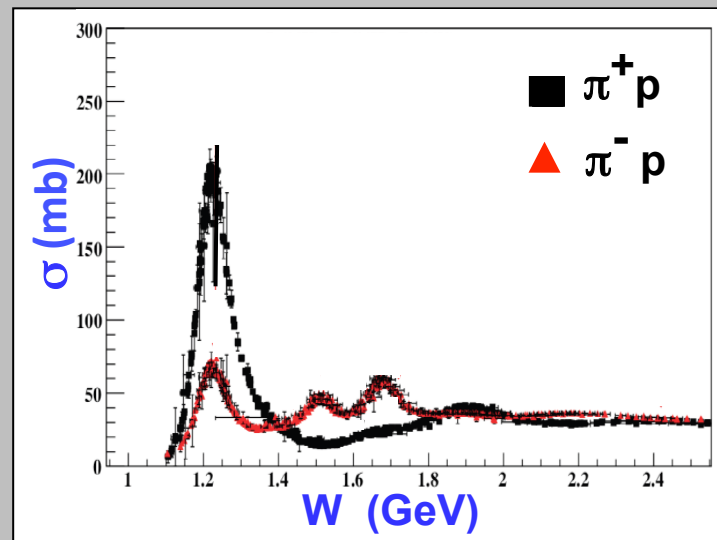
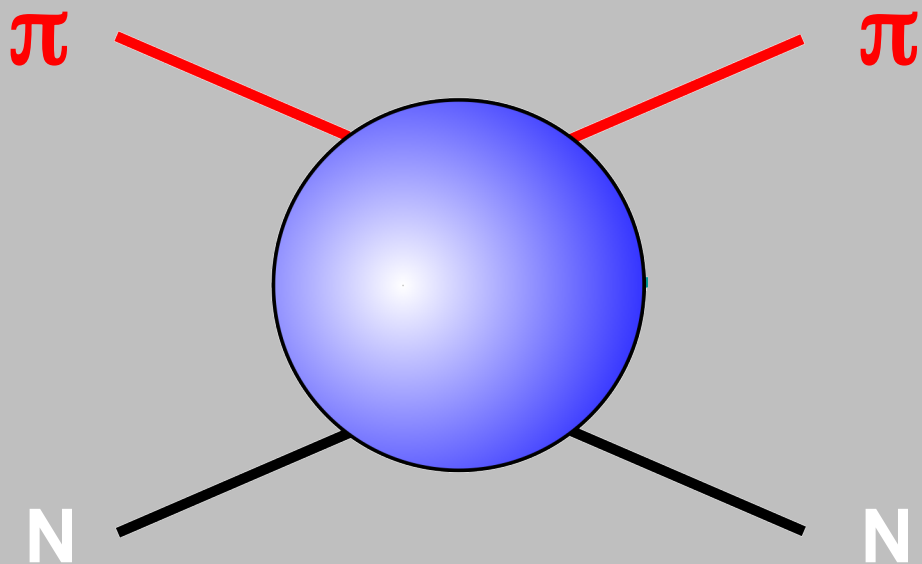


strong coupling

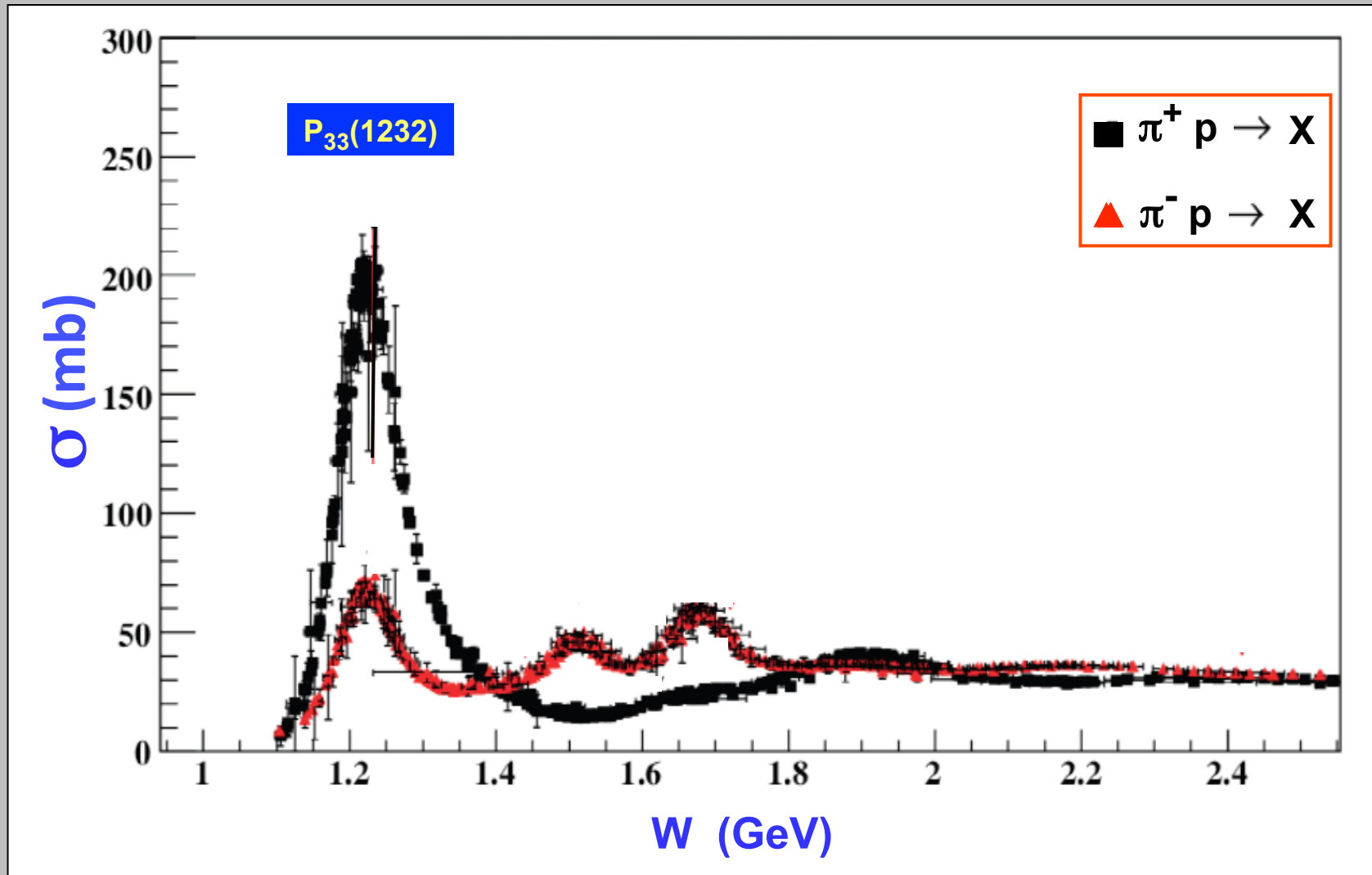
bound states



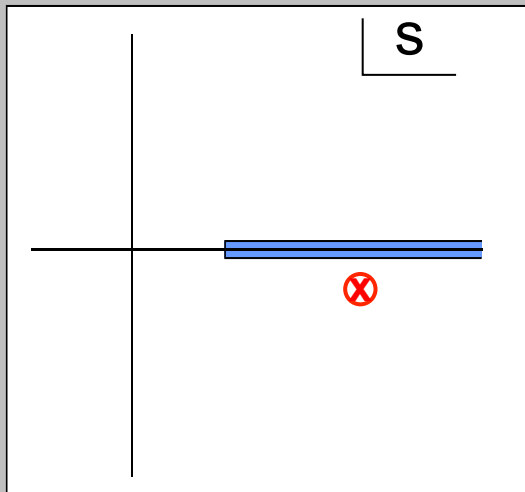
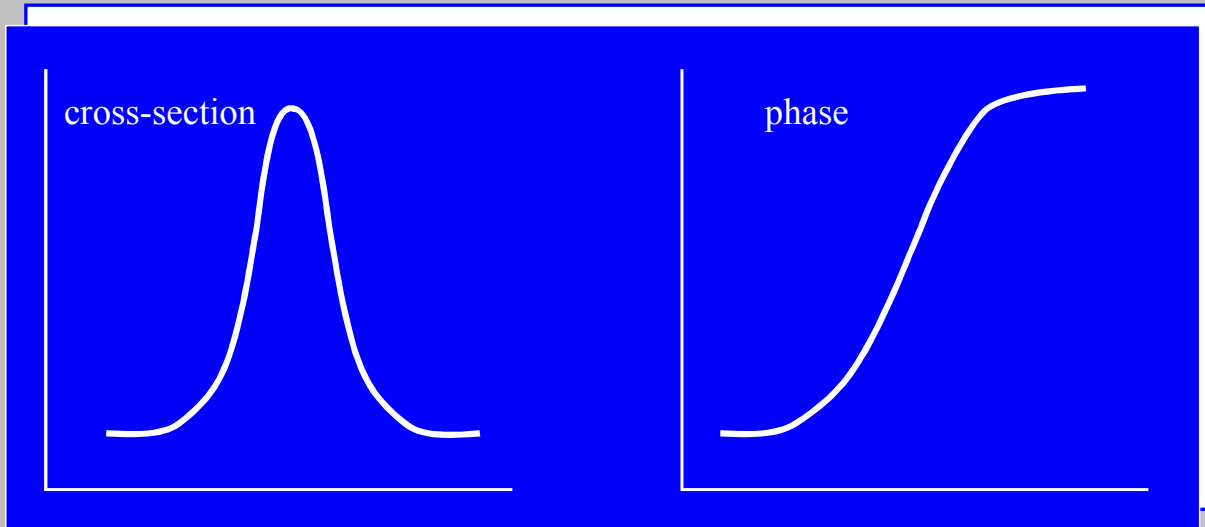
πN scattering



Baryon resonances (N^* s and Δ^* s)



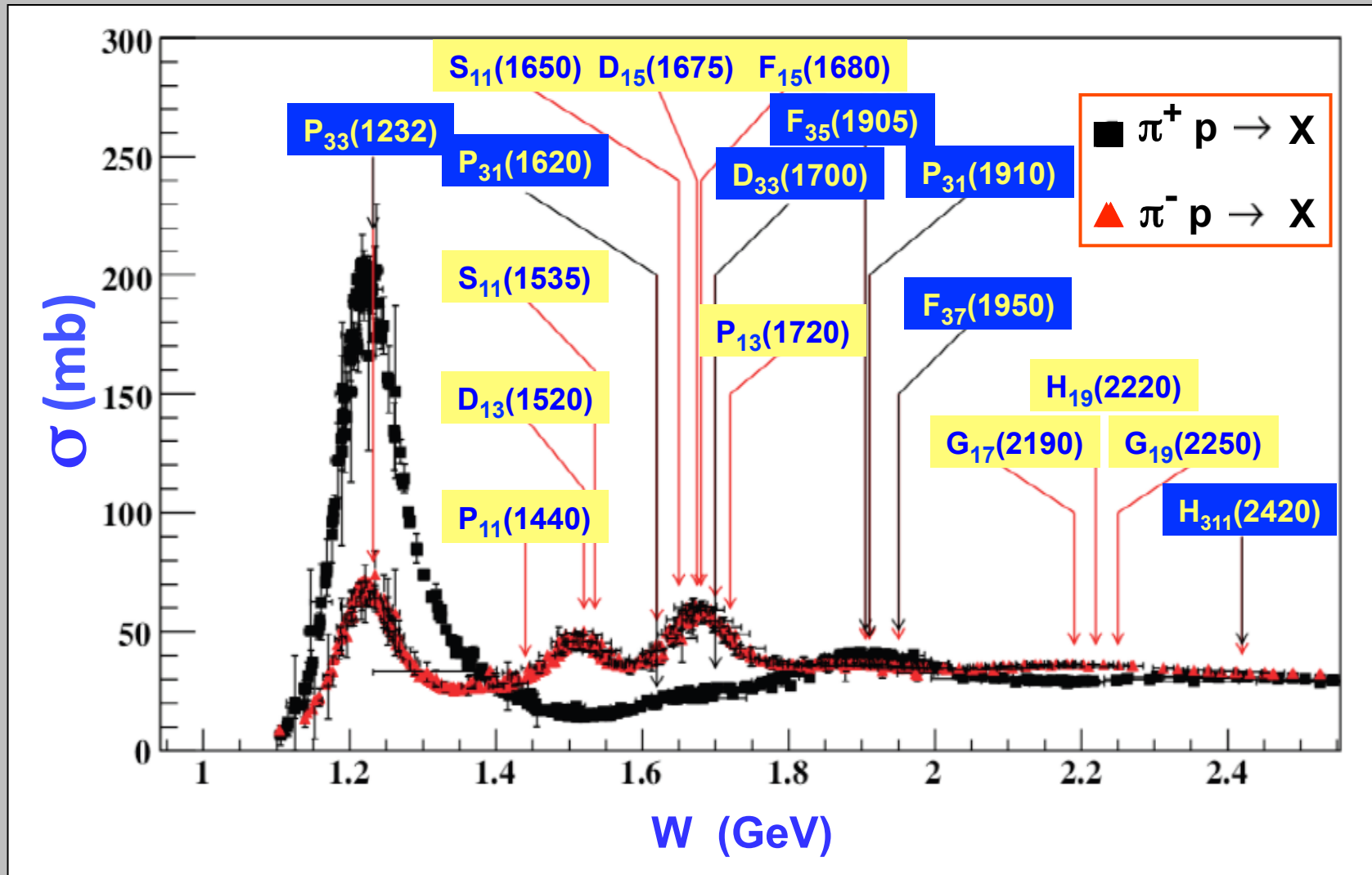
hadron states



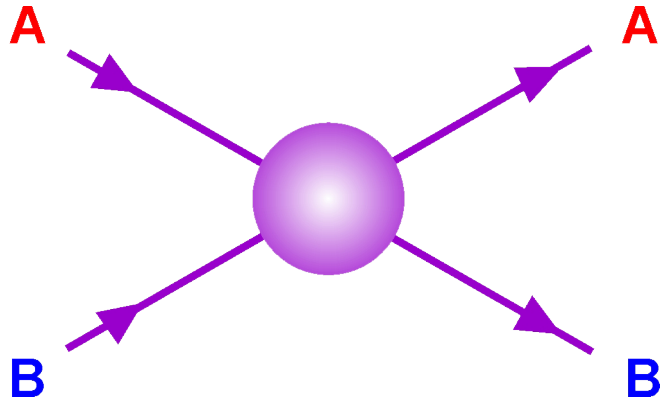
$$\frac{1}{M^2 - s - iM\Gamma}$$

$$s = E^2$$

Baryon resonances (N^* s and Δ^* s)

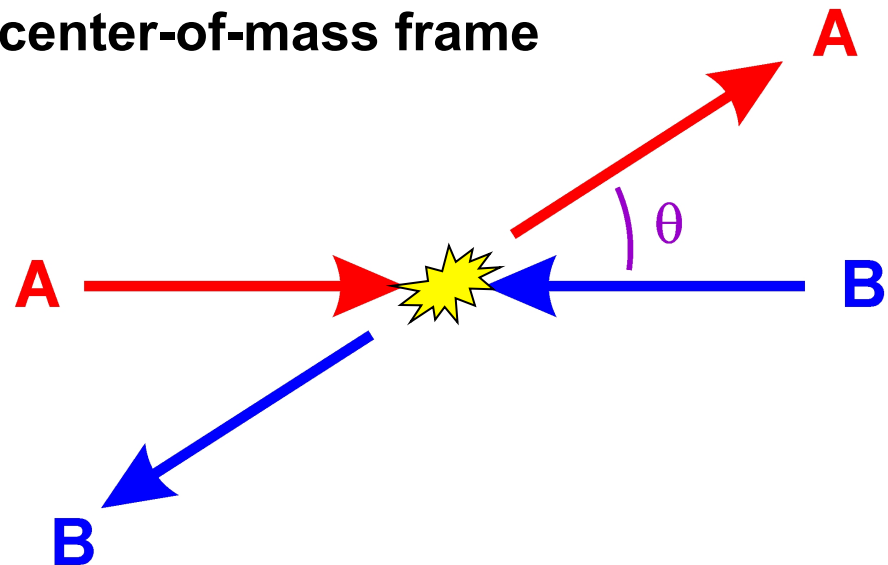


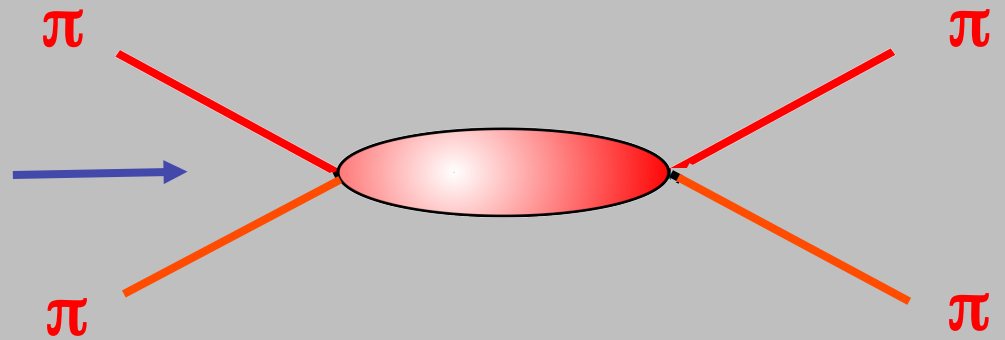
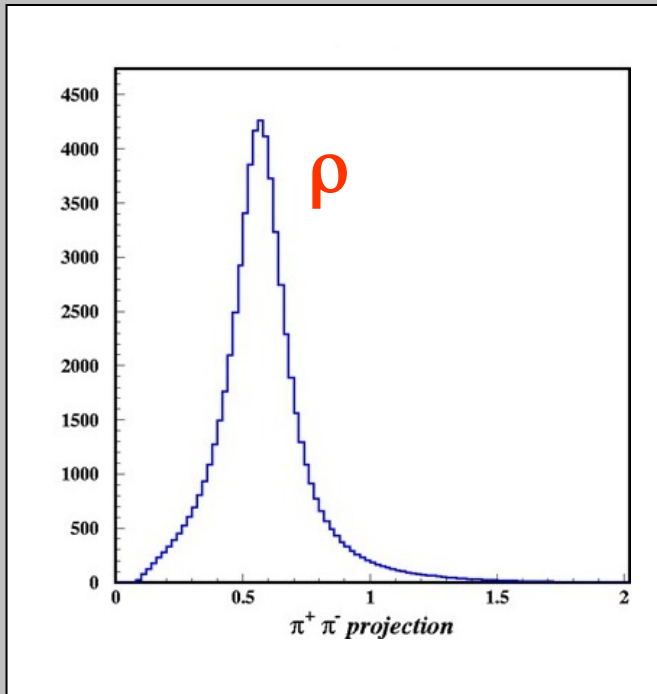
spinless particle scattering



variables: $E = \sqrt{s}$, ϑ

center-of-mass frame

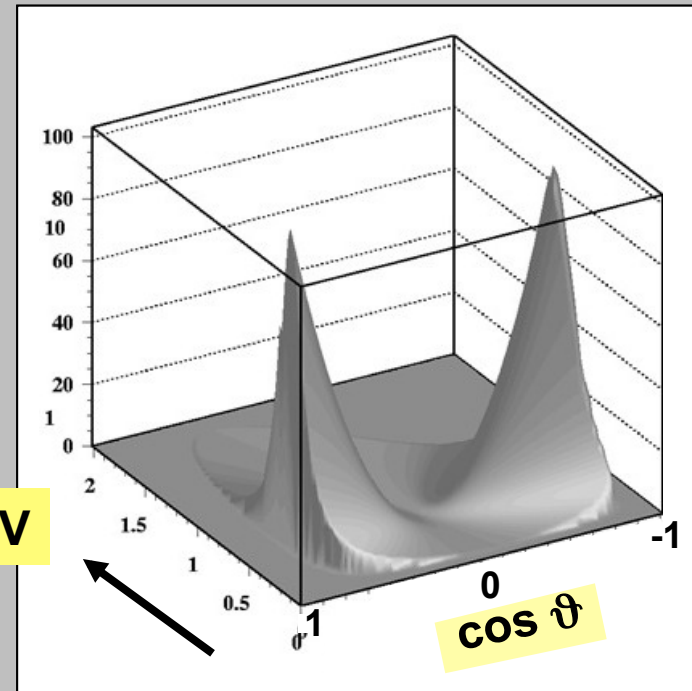




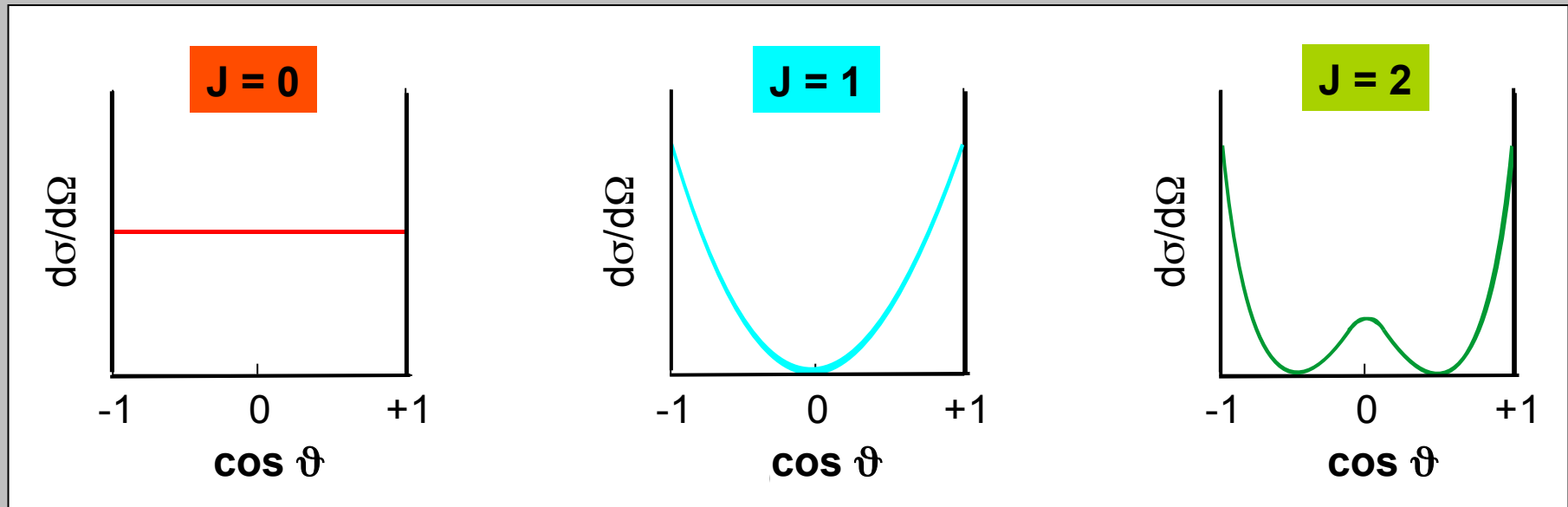
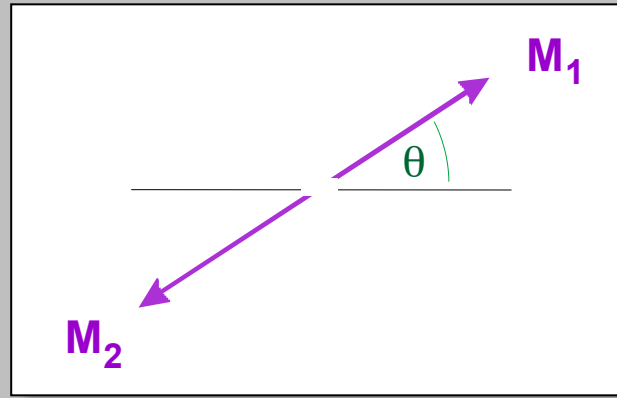
$$F(s, \vartheta) = 3 f_1(s) \cos \vartheta$$

$$s = M^2(\pi\pi)$$

$M(\pi\pi)$ GeV

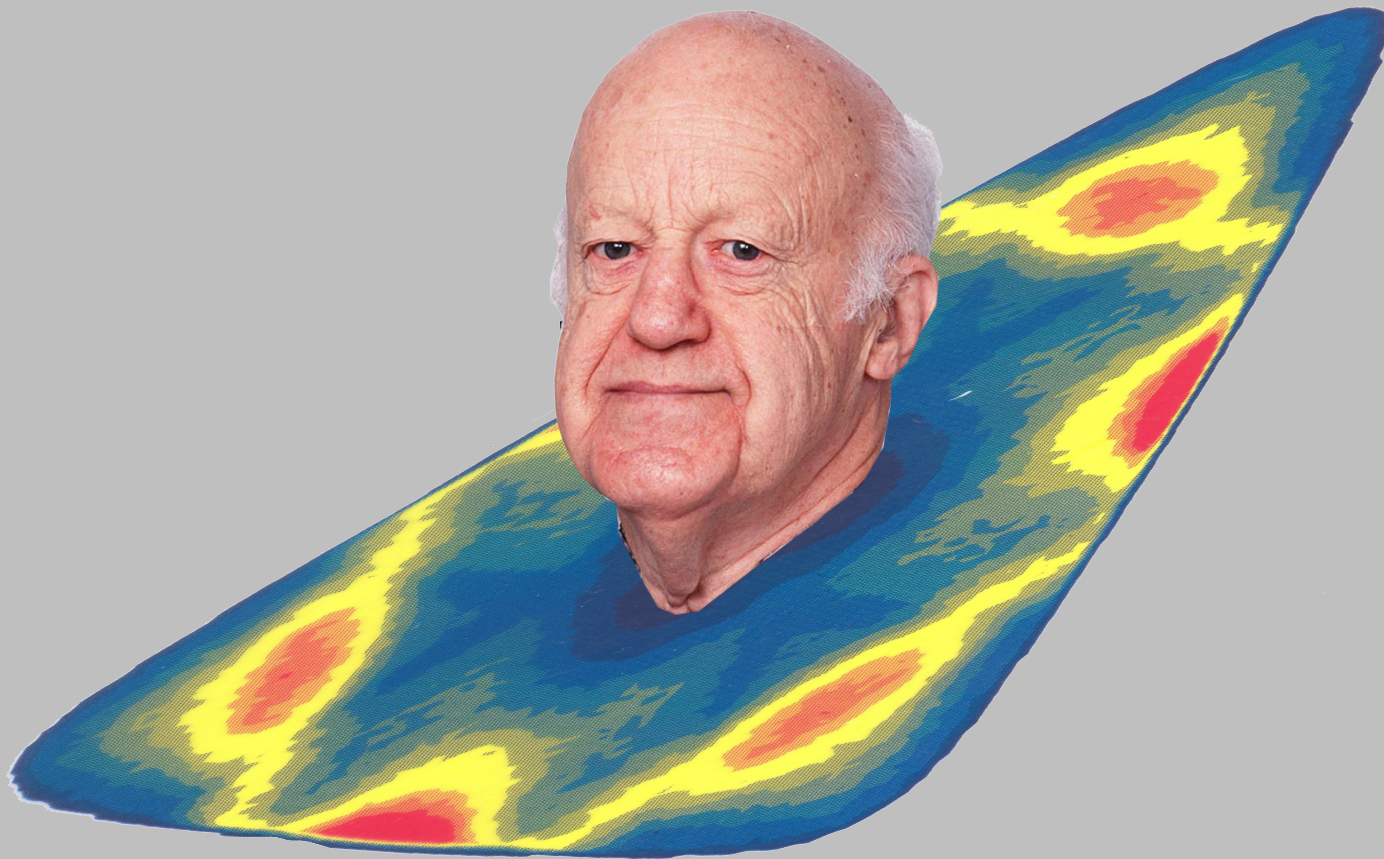


Spin analysis

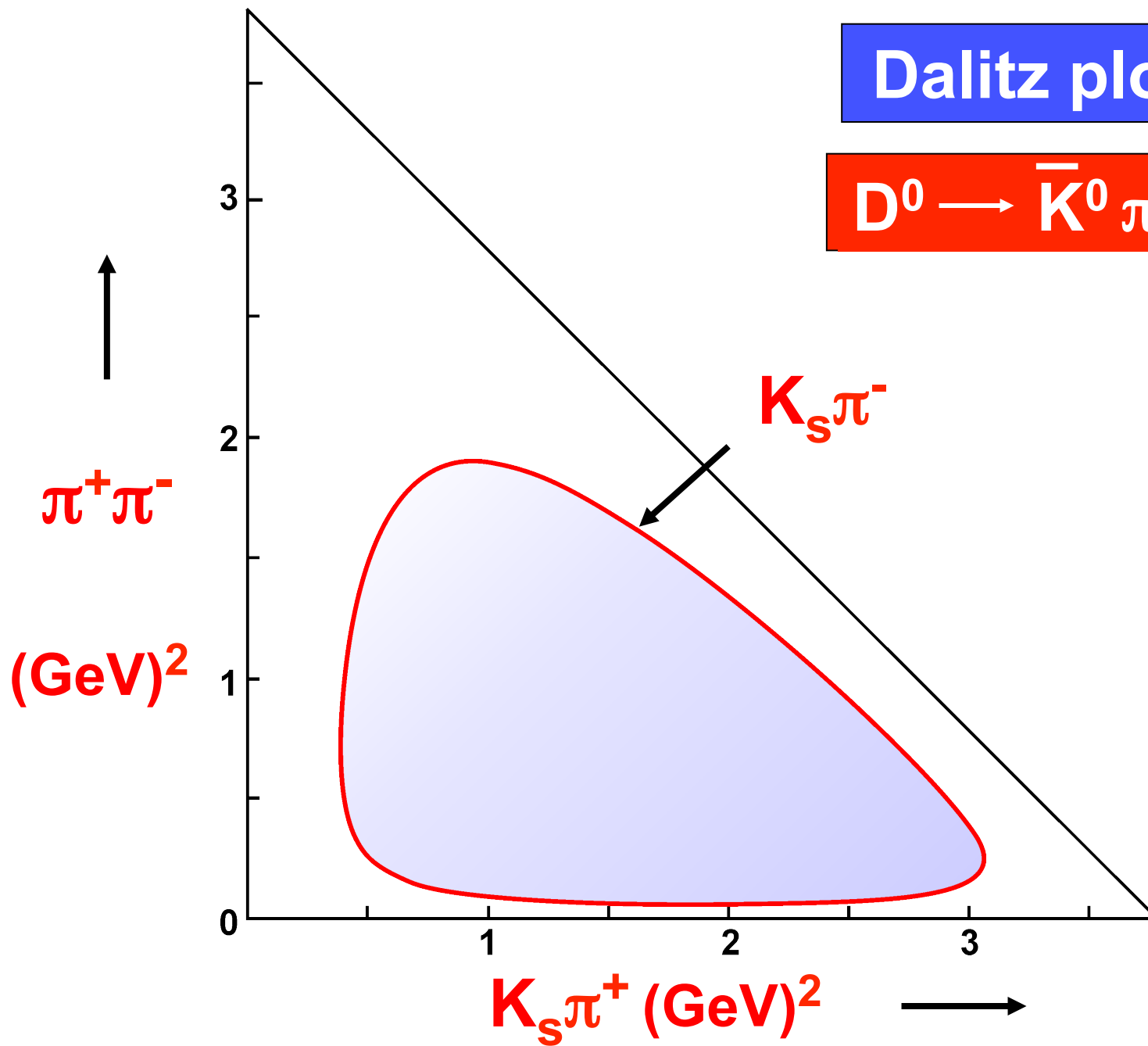


Spectroscopy: interplay of **poles & zeros**

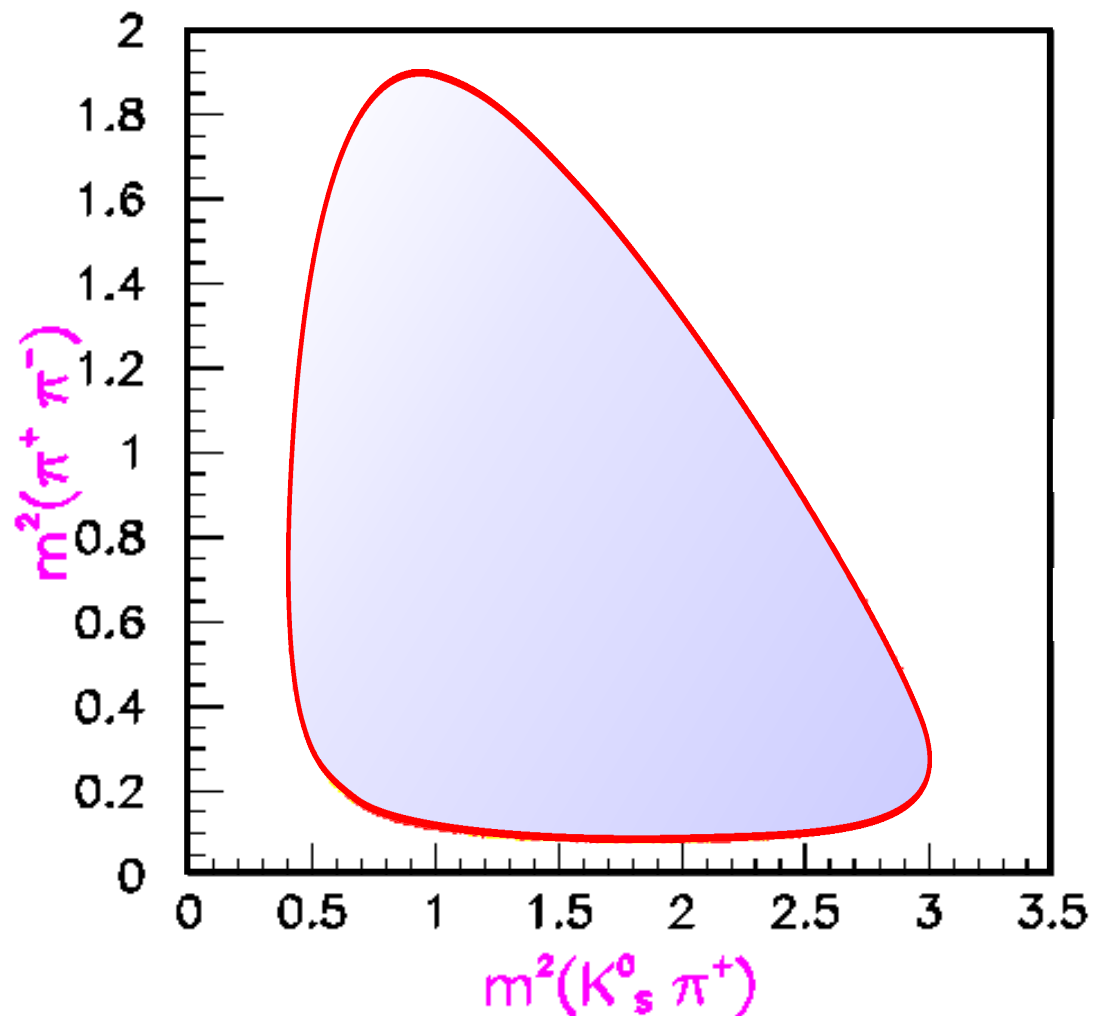
Dalitz Analysis



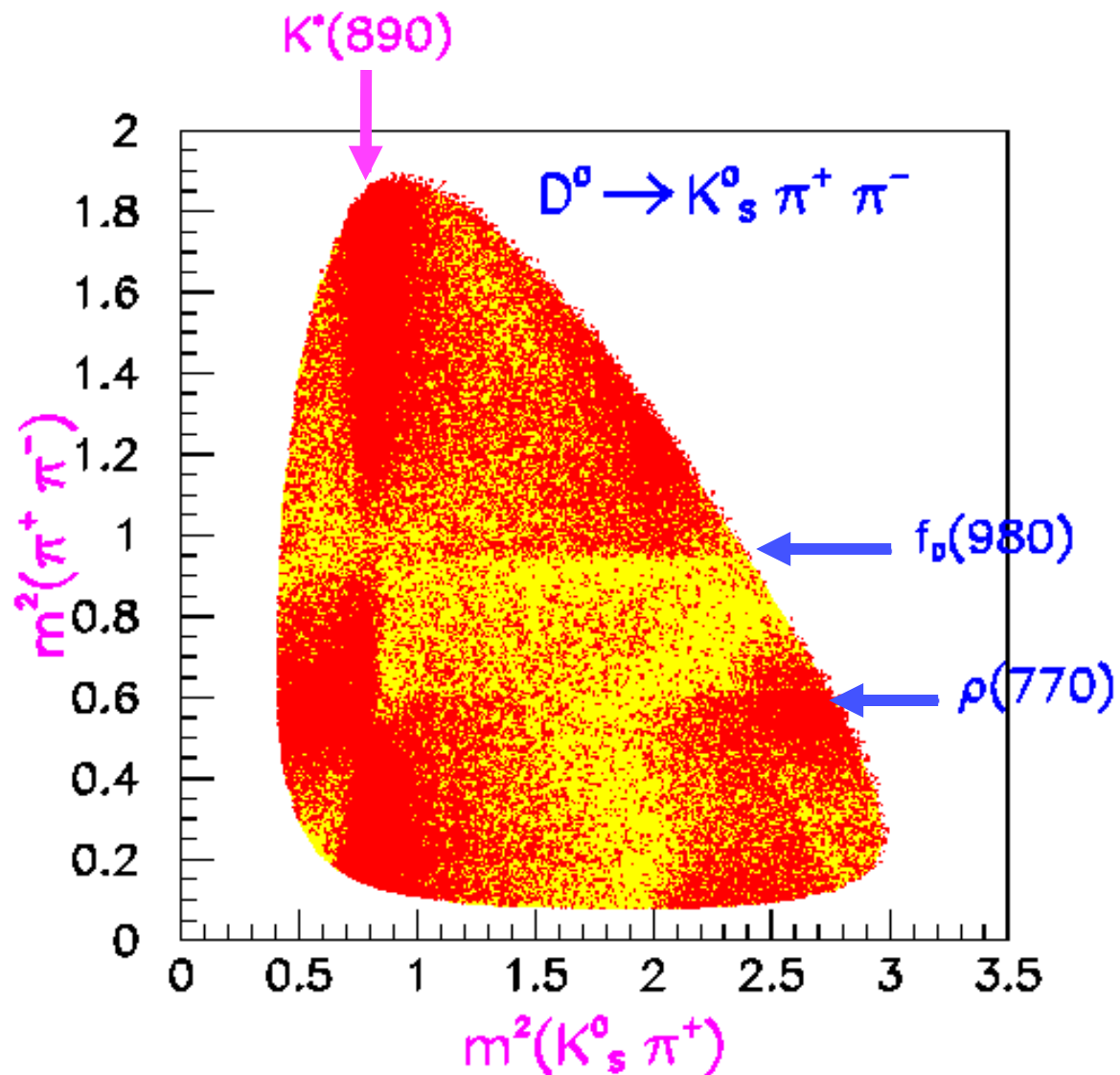
Dalitz plot



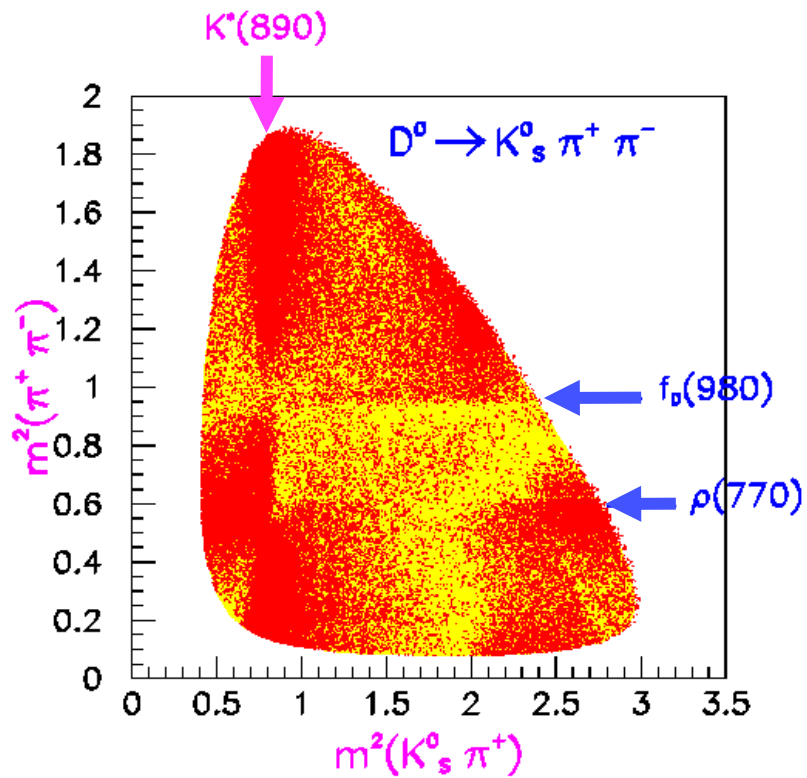
Dalitz plot of $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$.



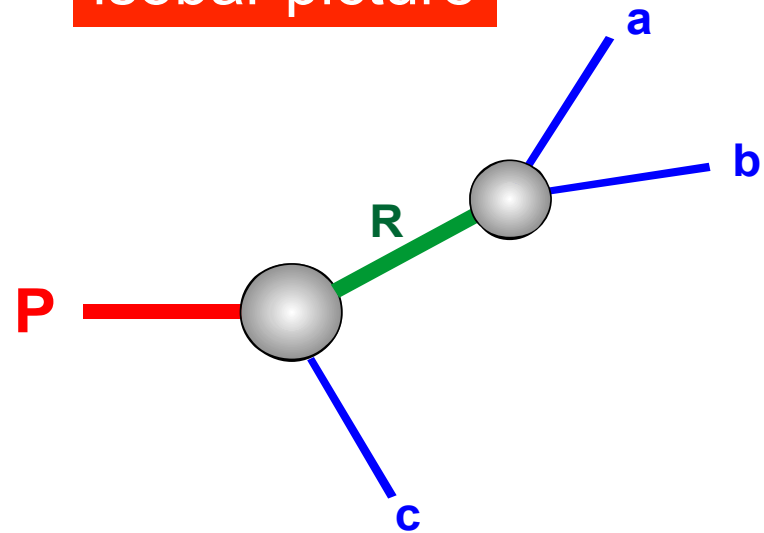
Dalitz plot of $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$.



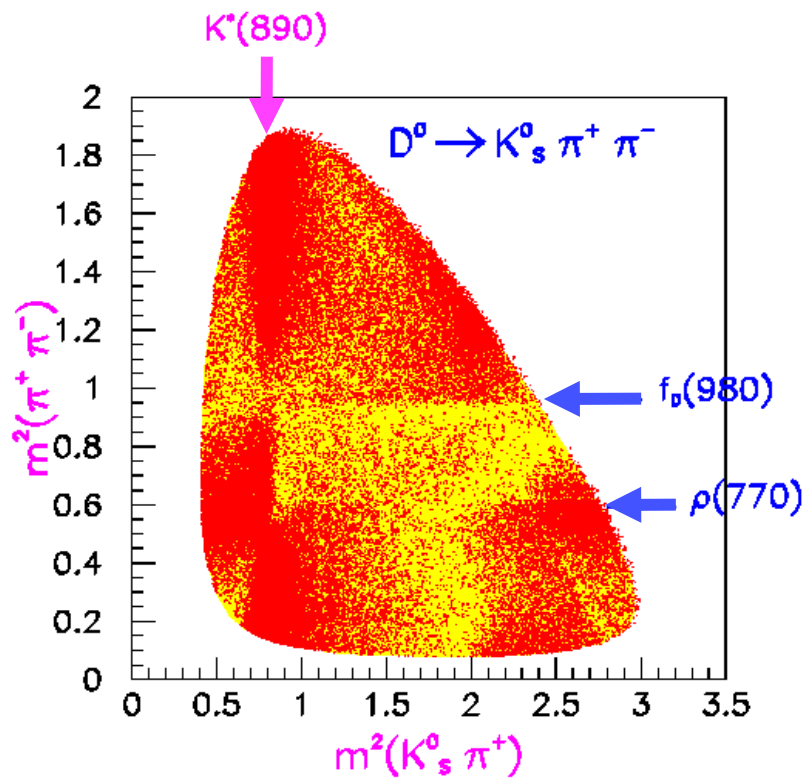
Dalitz plot of $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$.



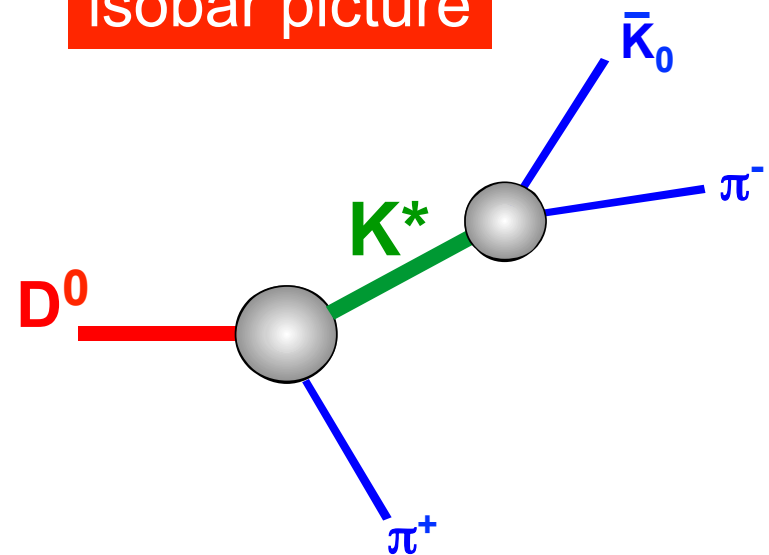
isobar picture



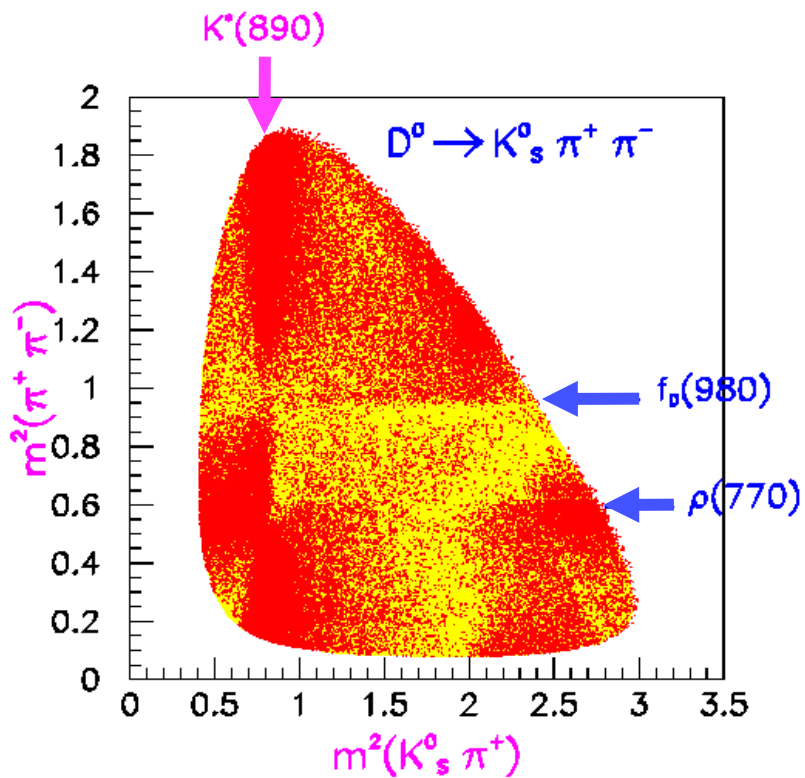
Dalitz plot of $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$.



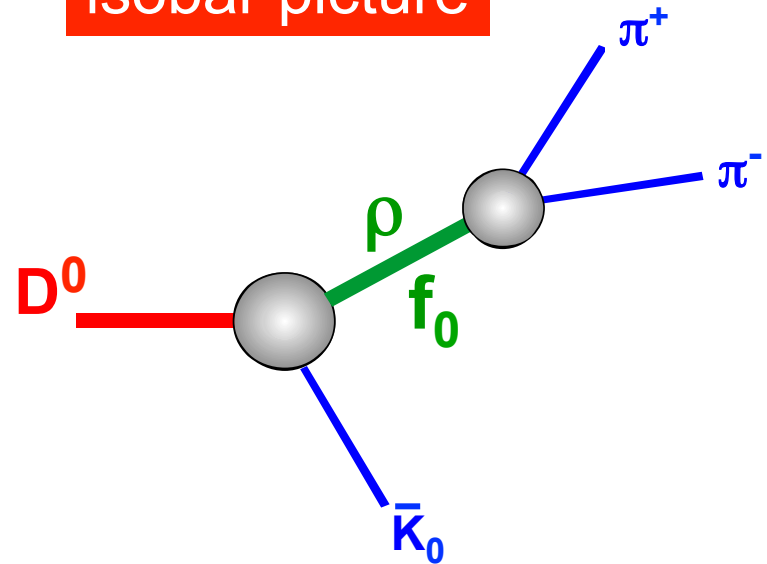
isobar picture

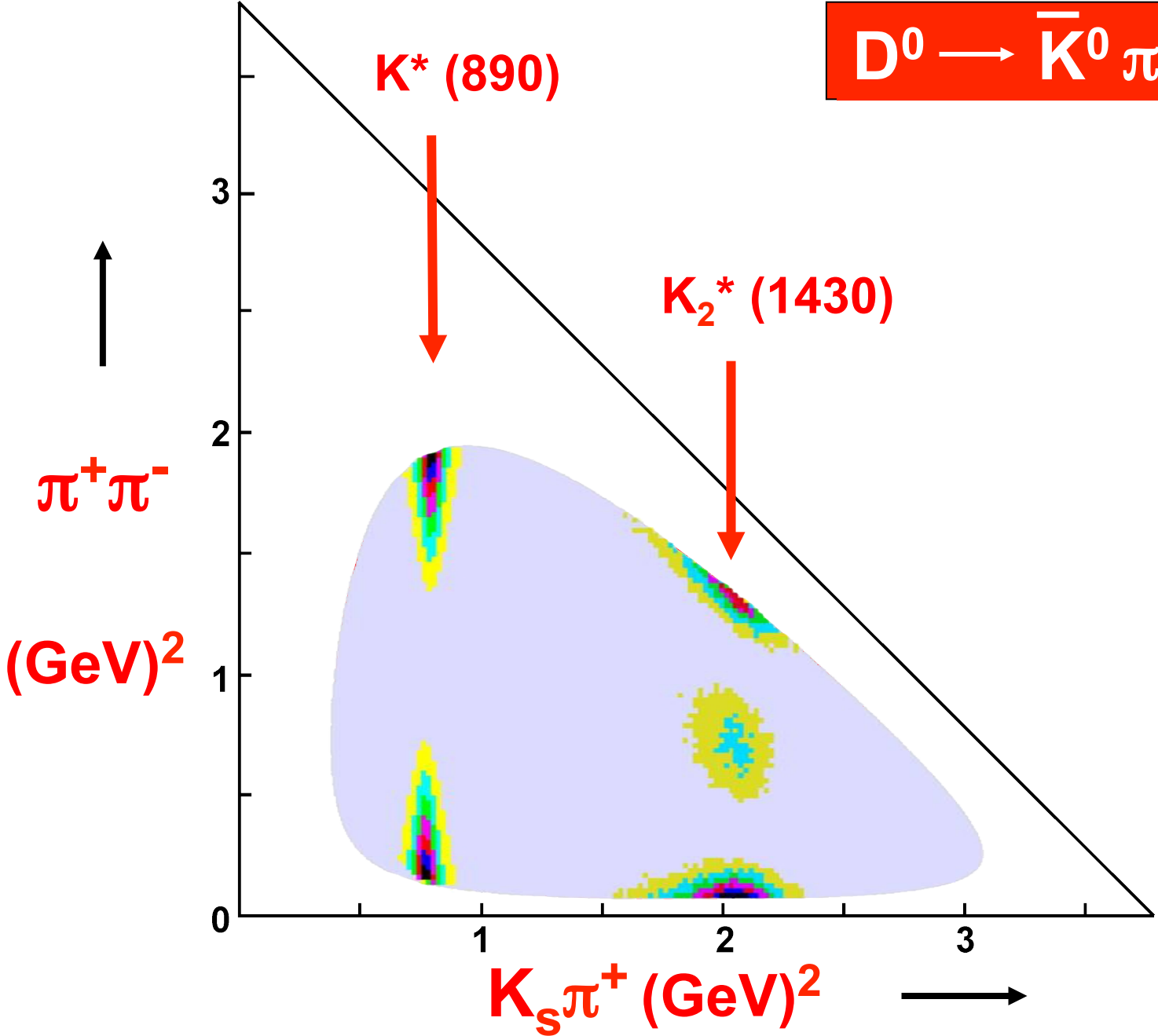


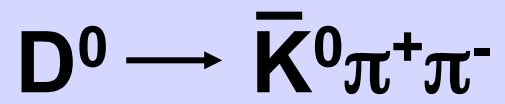
Dalitz plot of $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$.



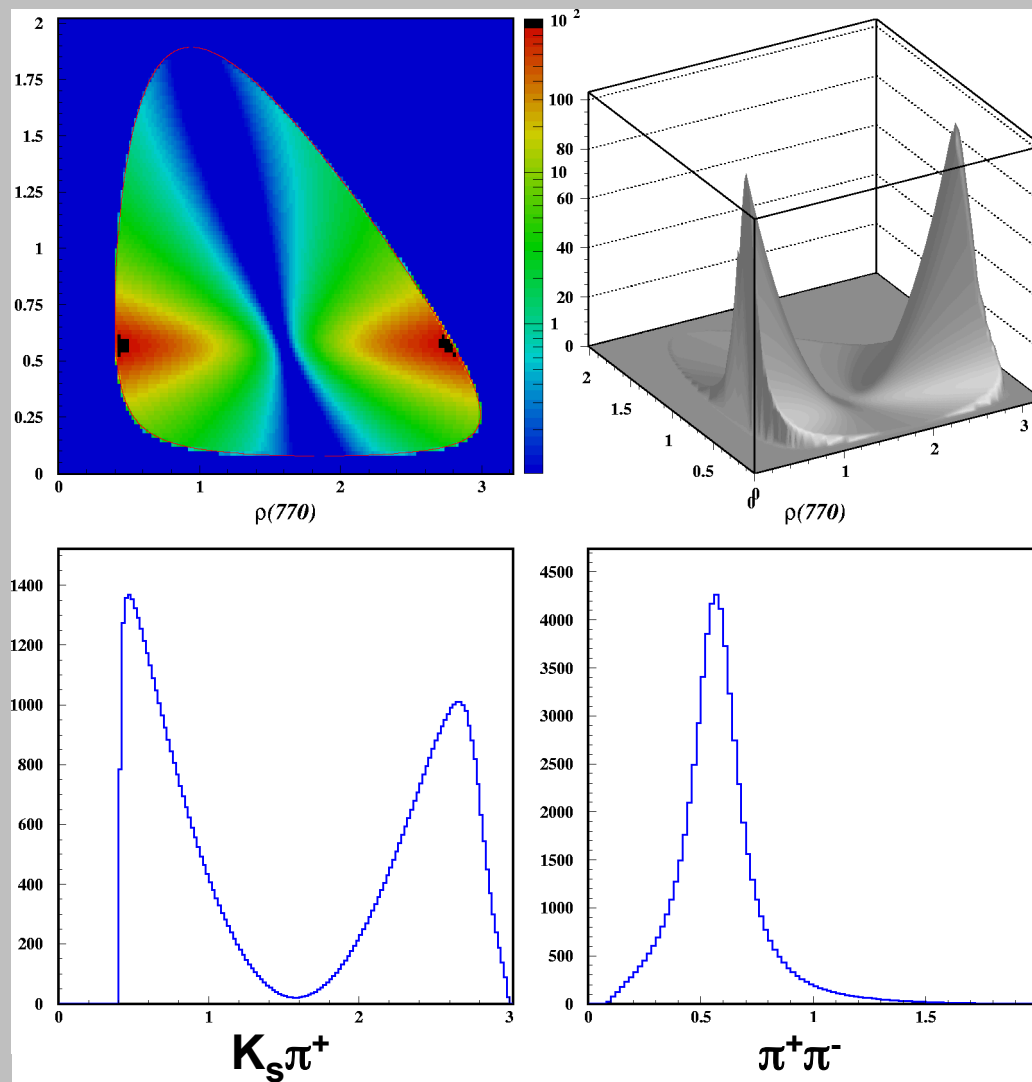
isobar picture

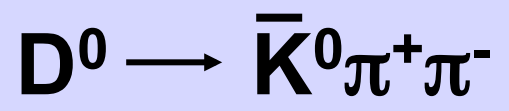




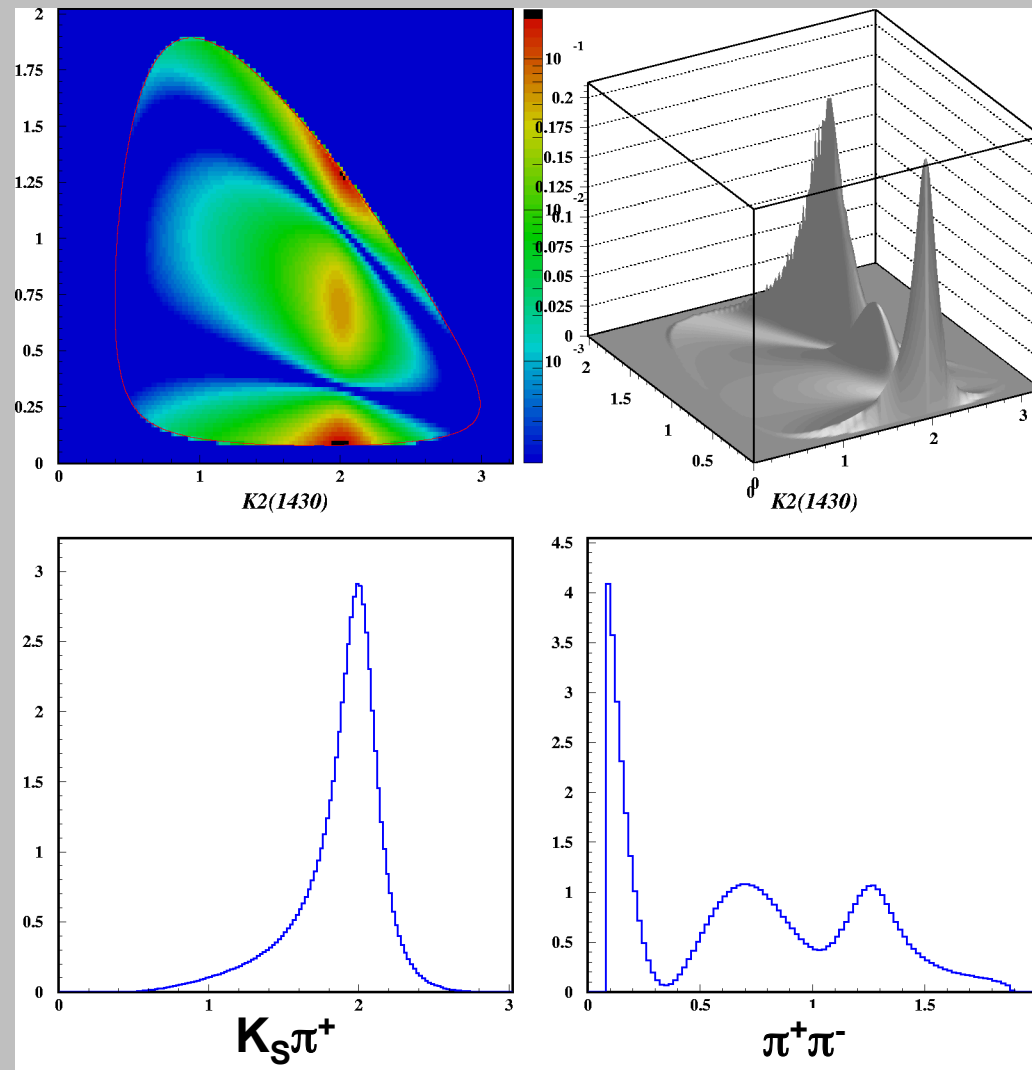


$\rho(770)$



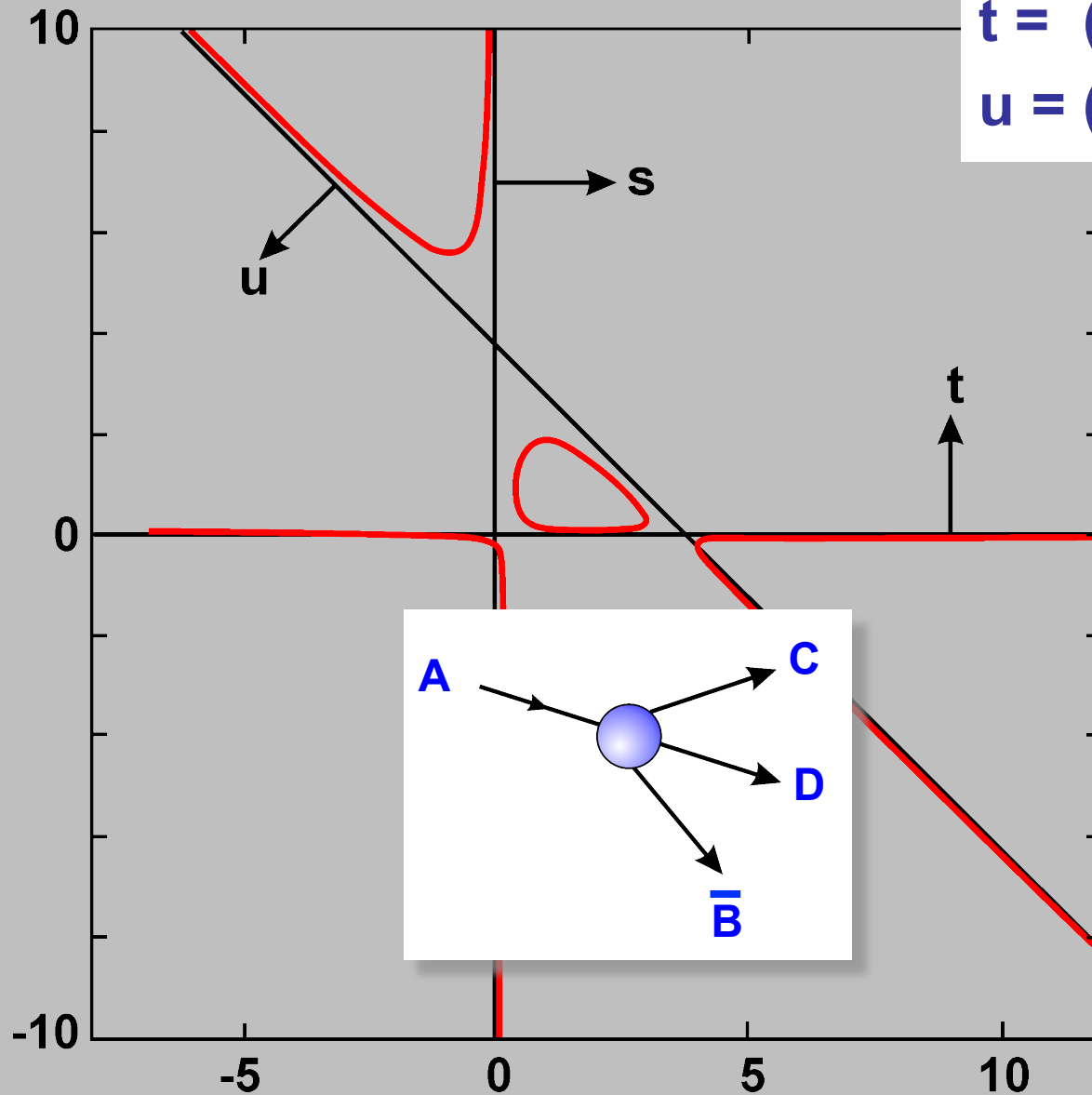


$K_2^*(1430)$

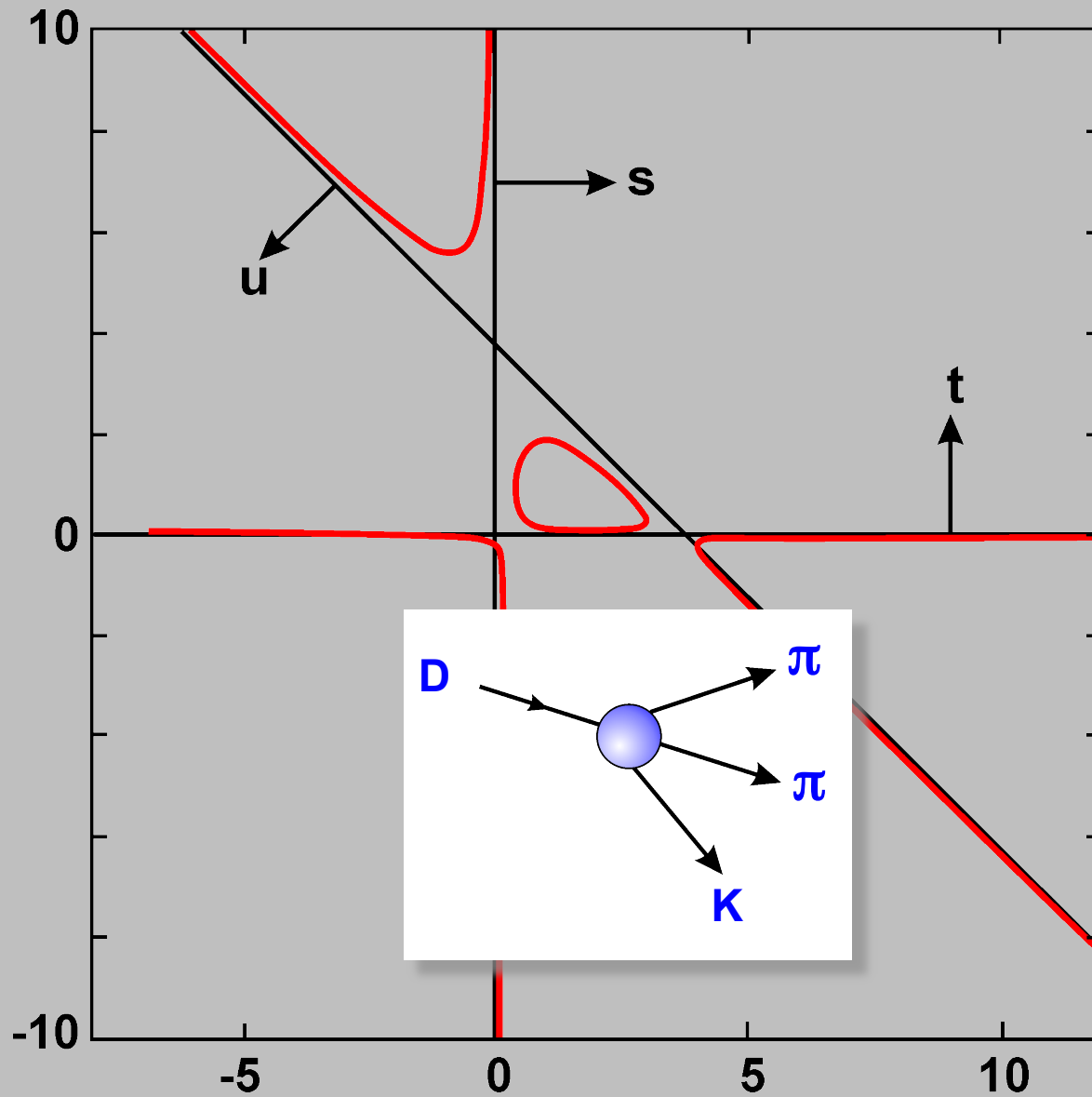


decay region

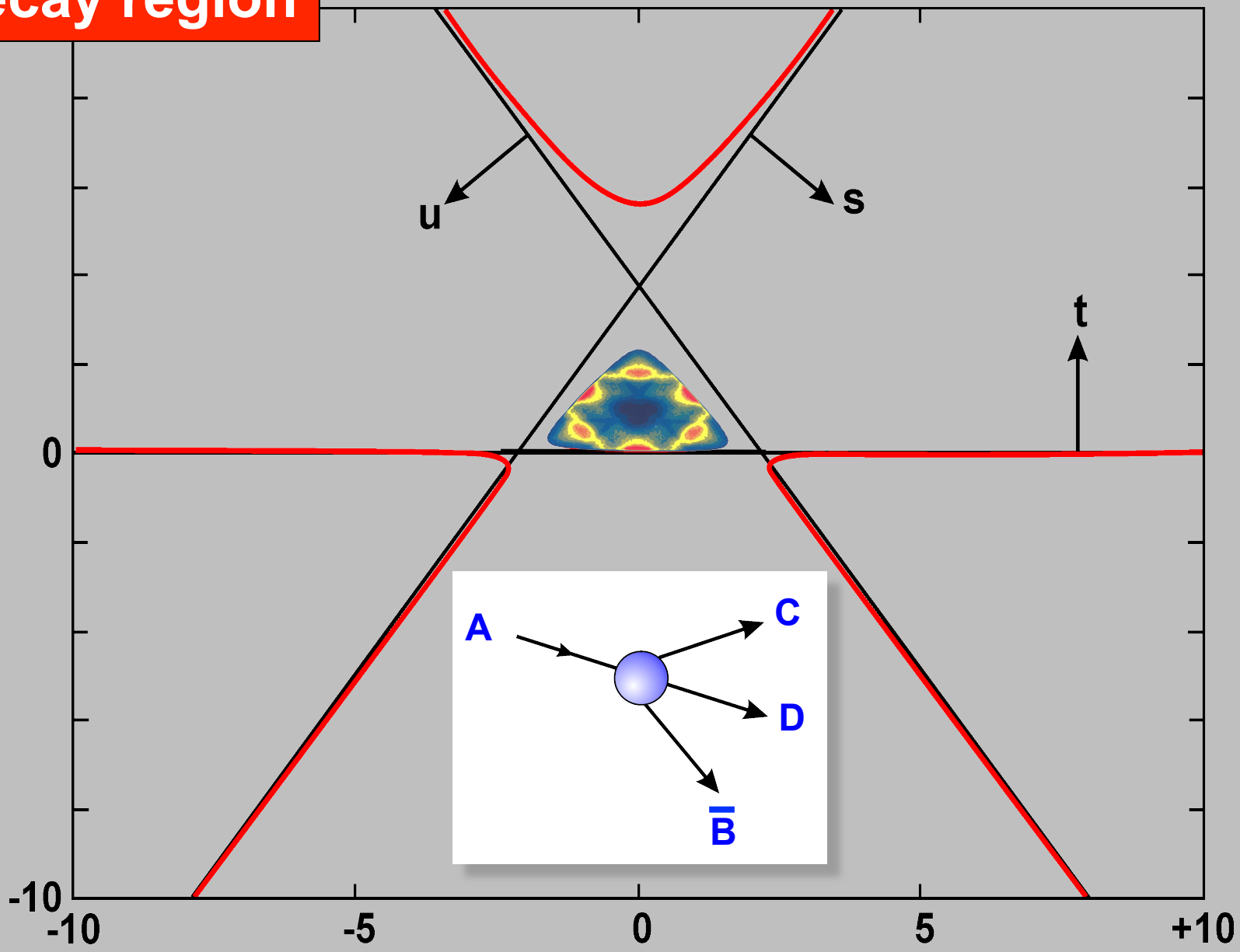
$$s = (p_A + p_B)^2$$
$$t = (p_A - p_C)^2$$
$$u = (p_A - p_D)^2$$



decay region

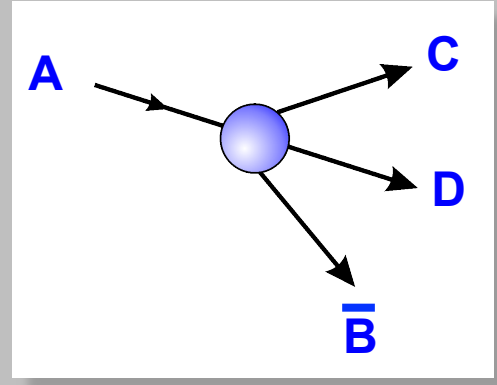


decay region

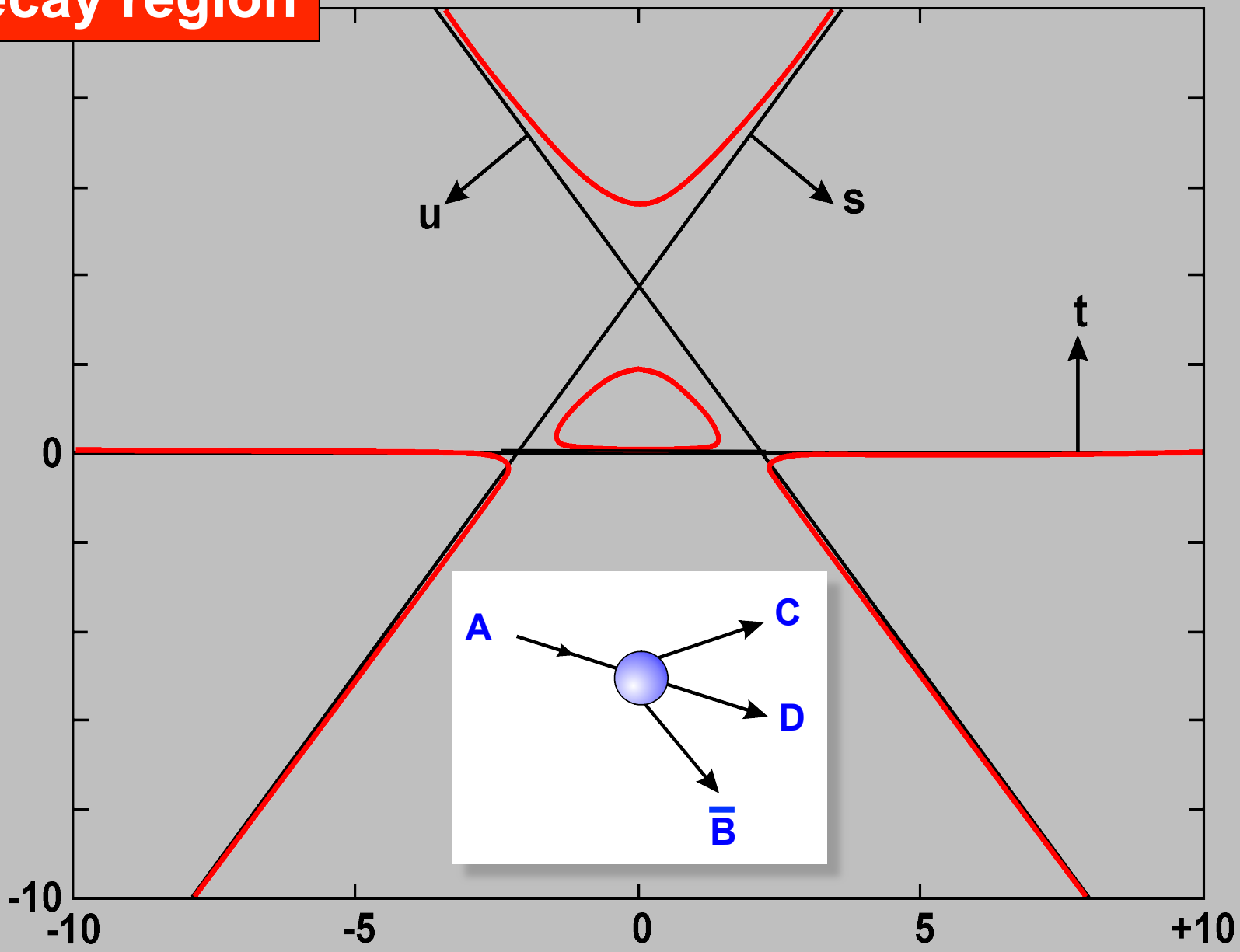


u s

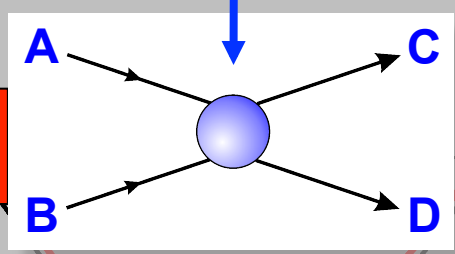
t



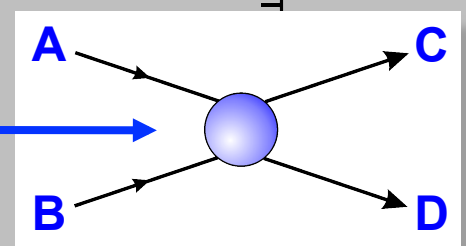
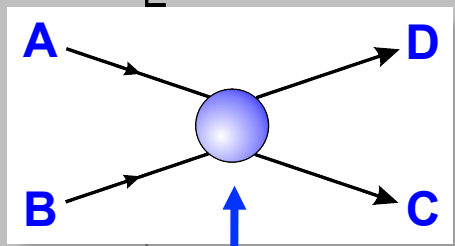
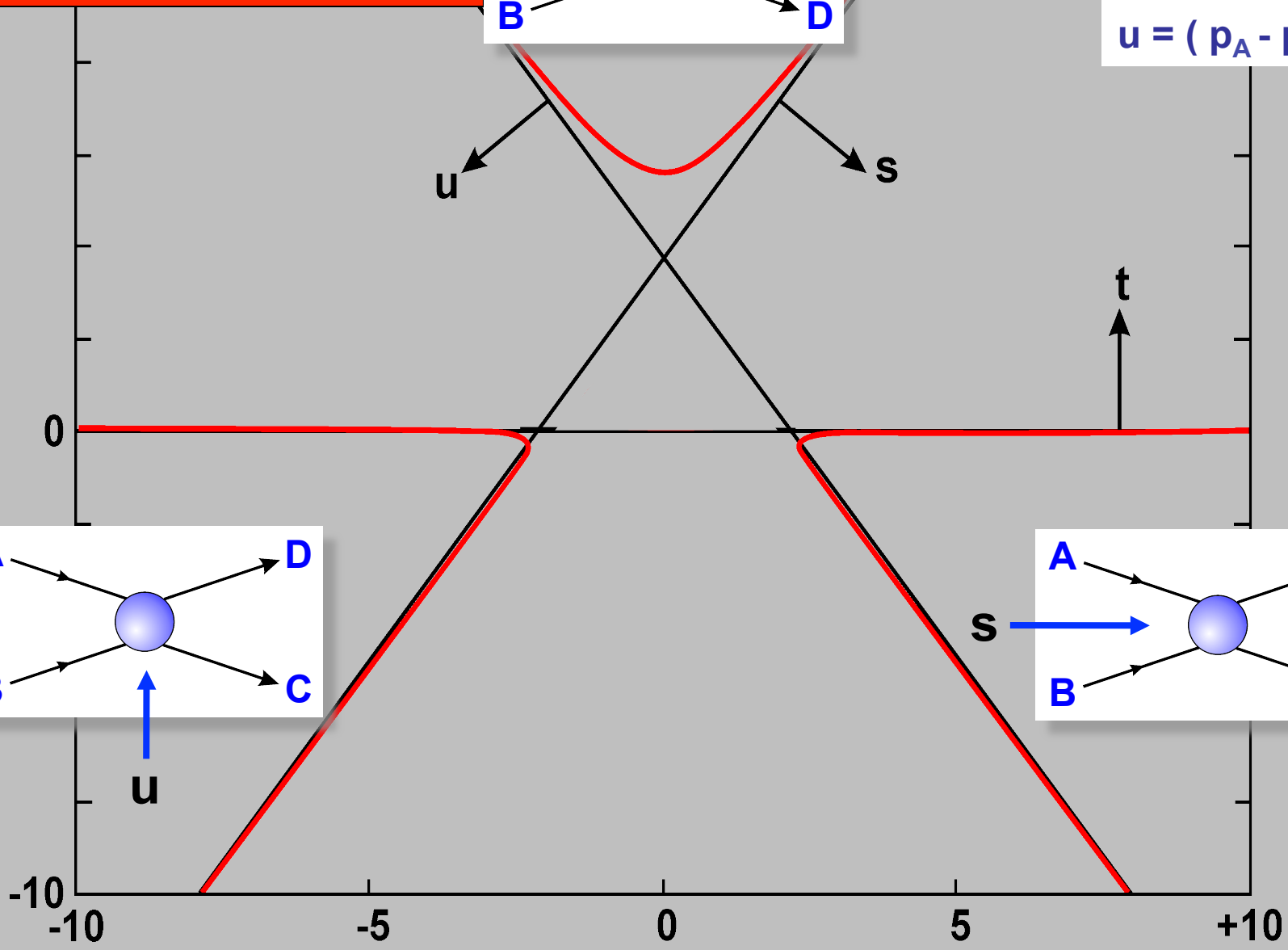
decay region



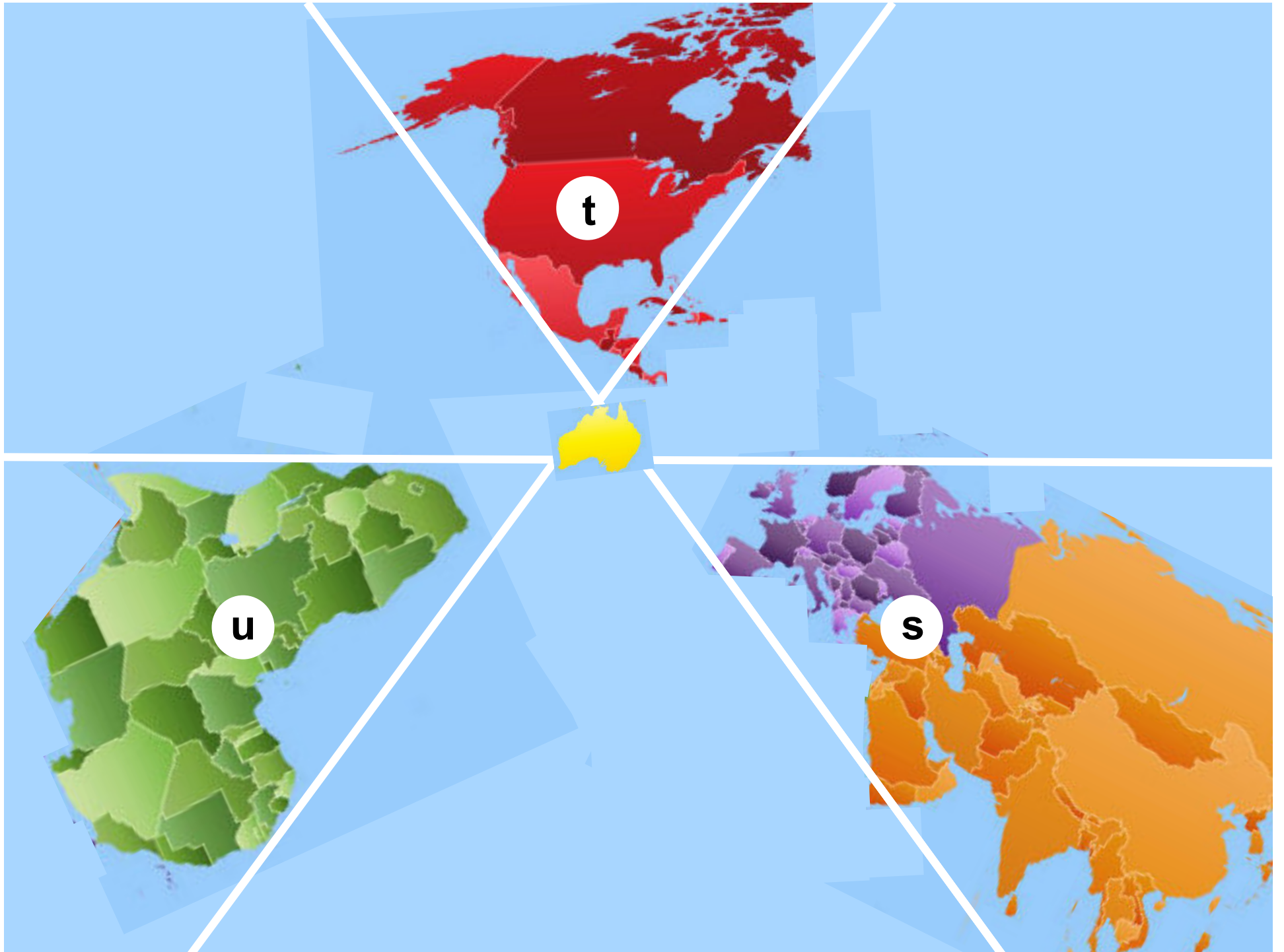
scattering regions



$$s = (p_A + p_B)^2$$
$$t = (p_A - p_C)^2$$
$$u = (p_A - p_D)^2$$









t



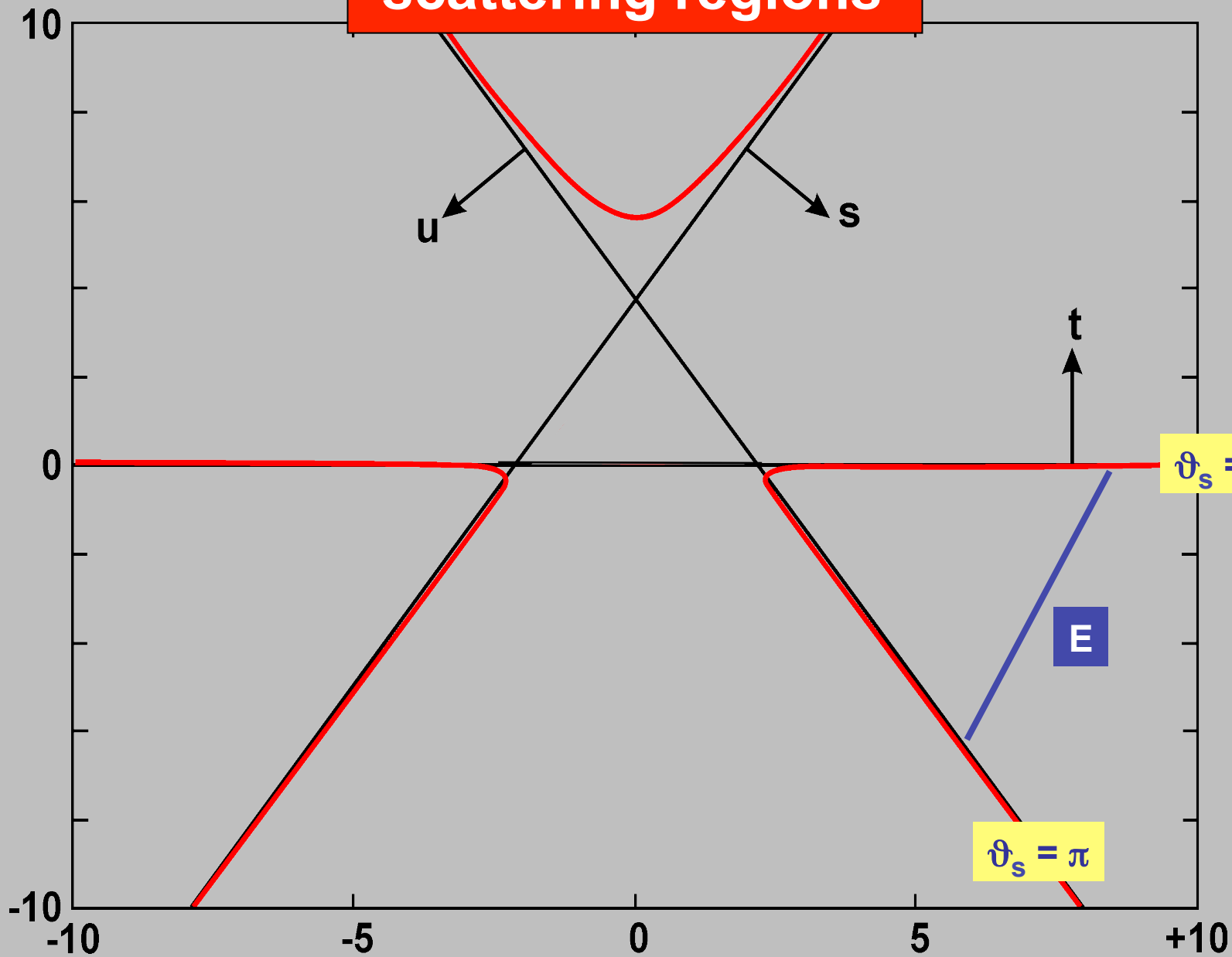
u

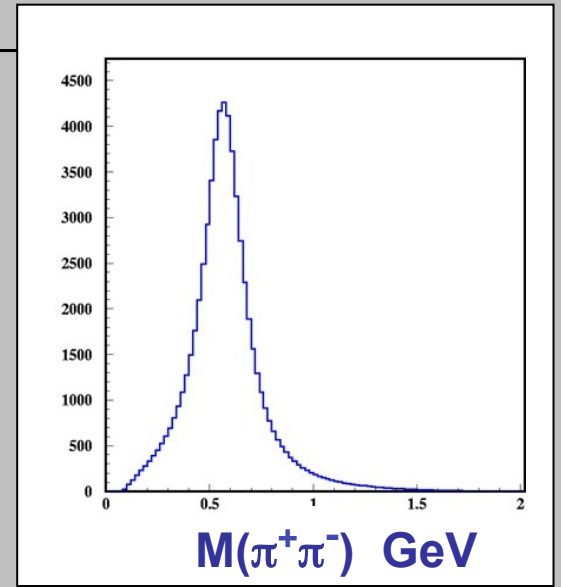
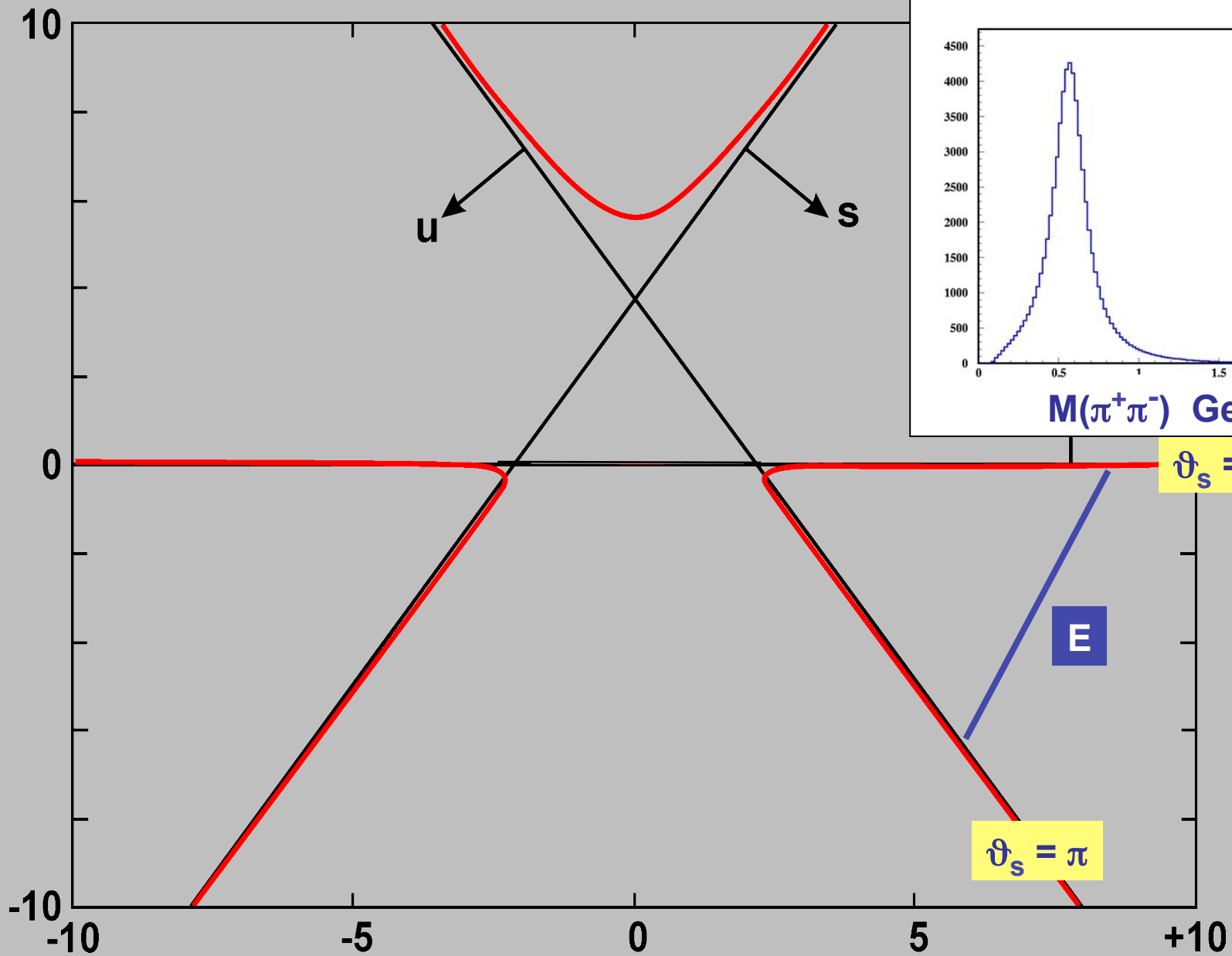


s



scattering regions



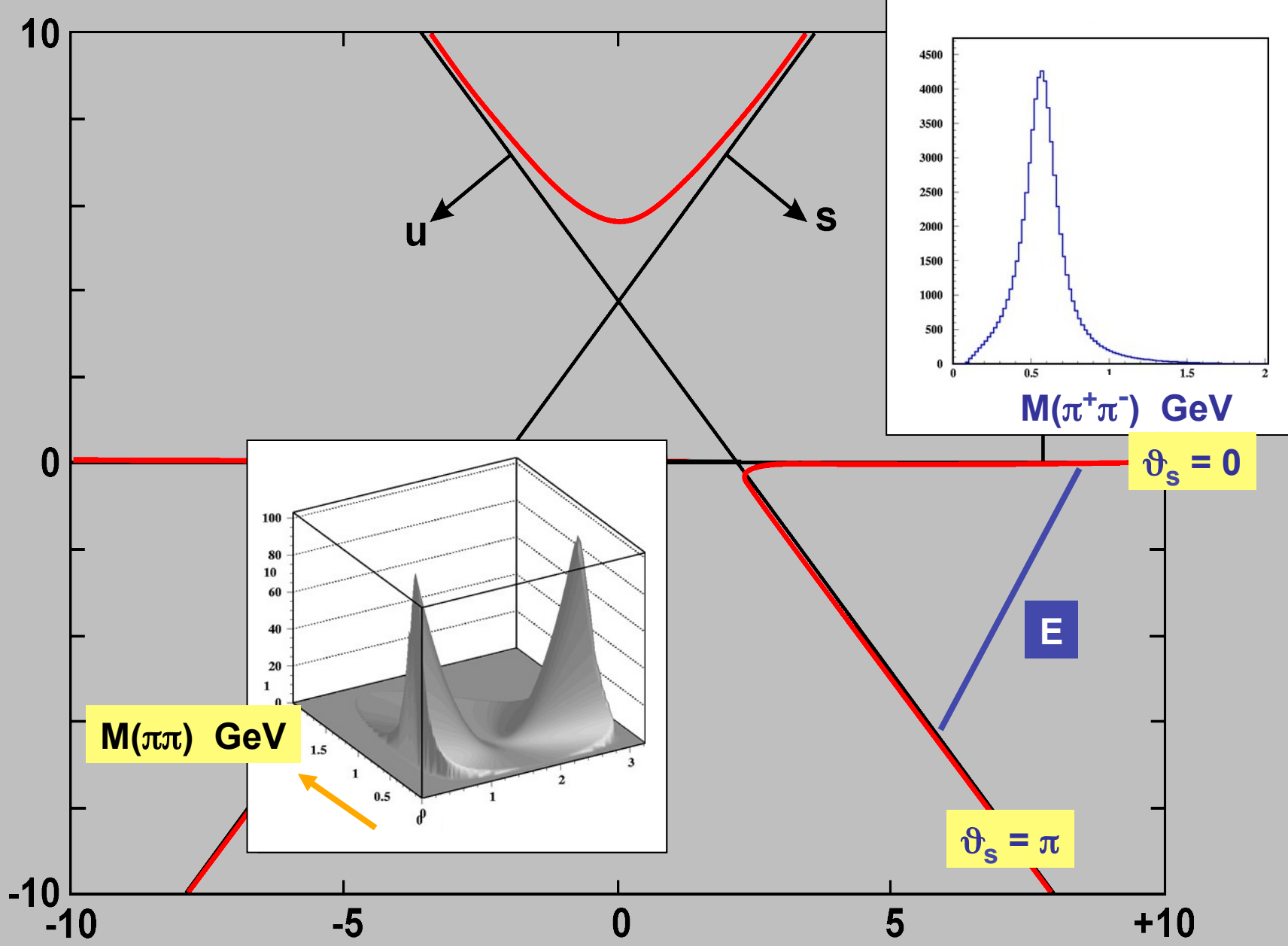


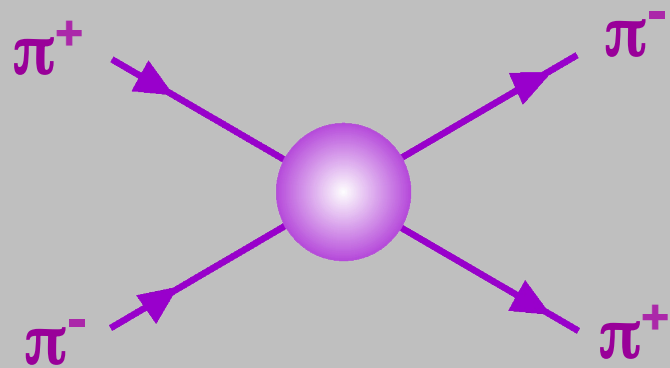
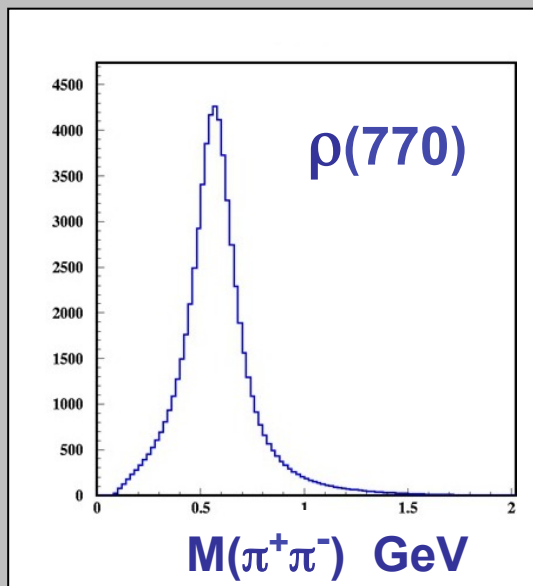
$M(\pi^+\pi^-)$ GeV

$\vartheta_s = 0$

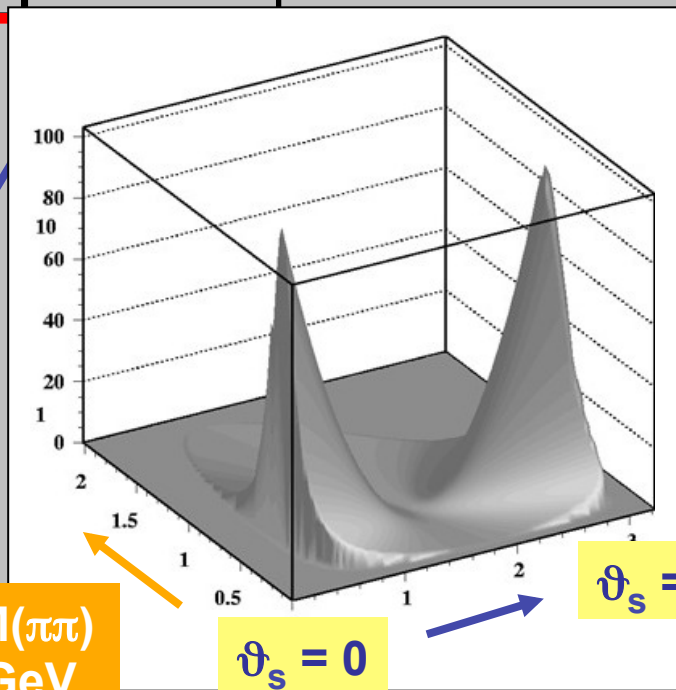
E

$\vartheta_s = \pi$

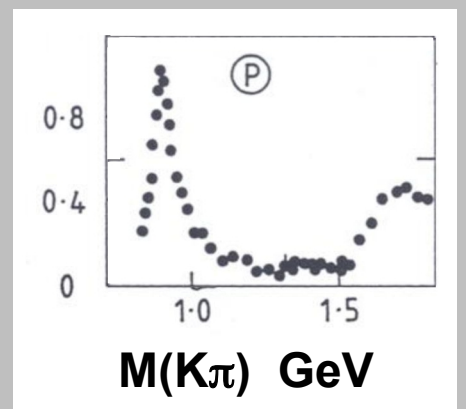
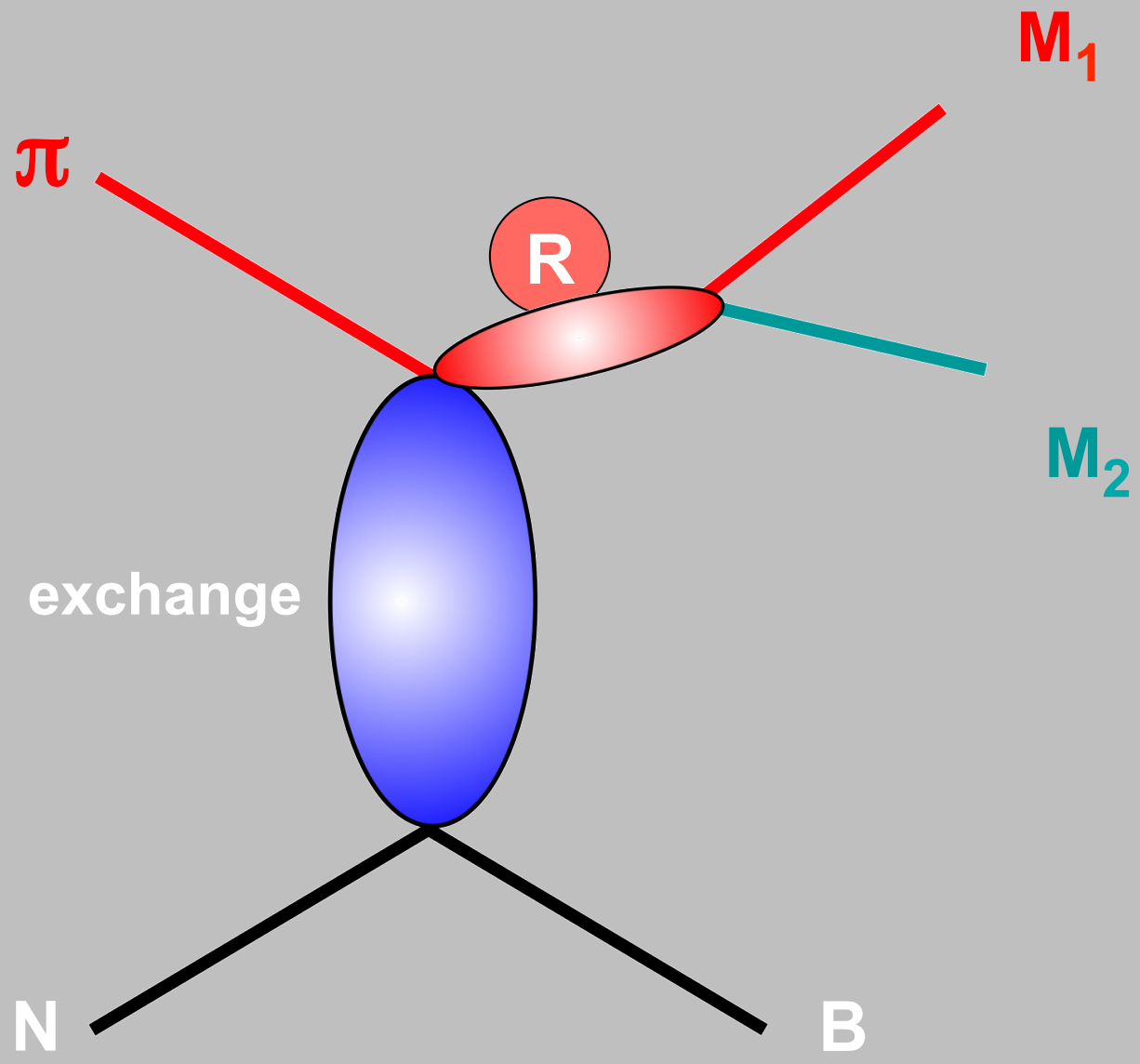




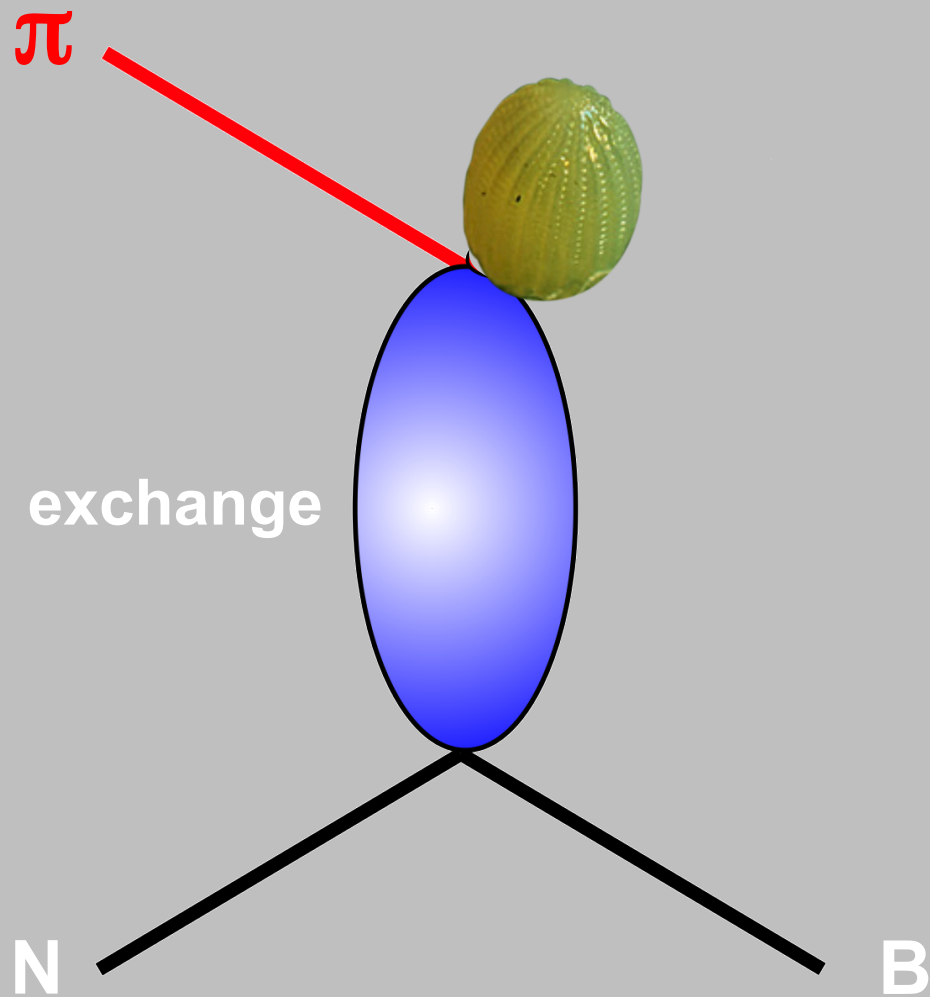
S



Hadroproduction



Hadroproduction



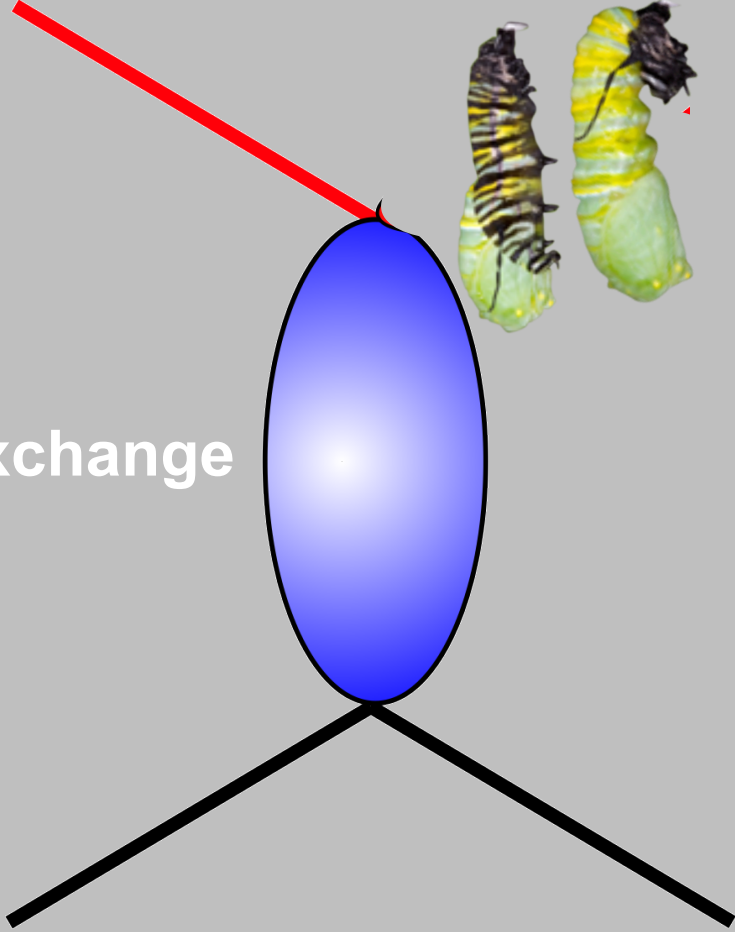
Hadroproduction

π

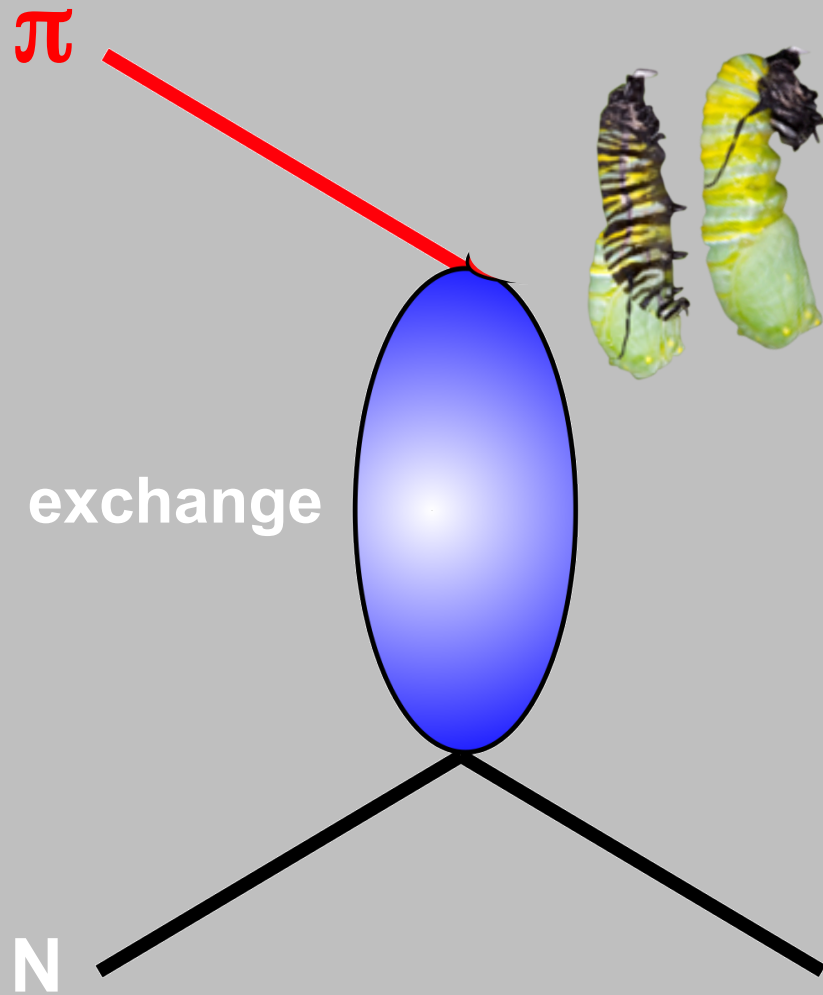
exchange

N

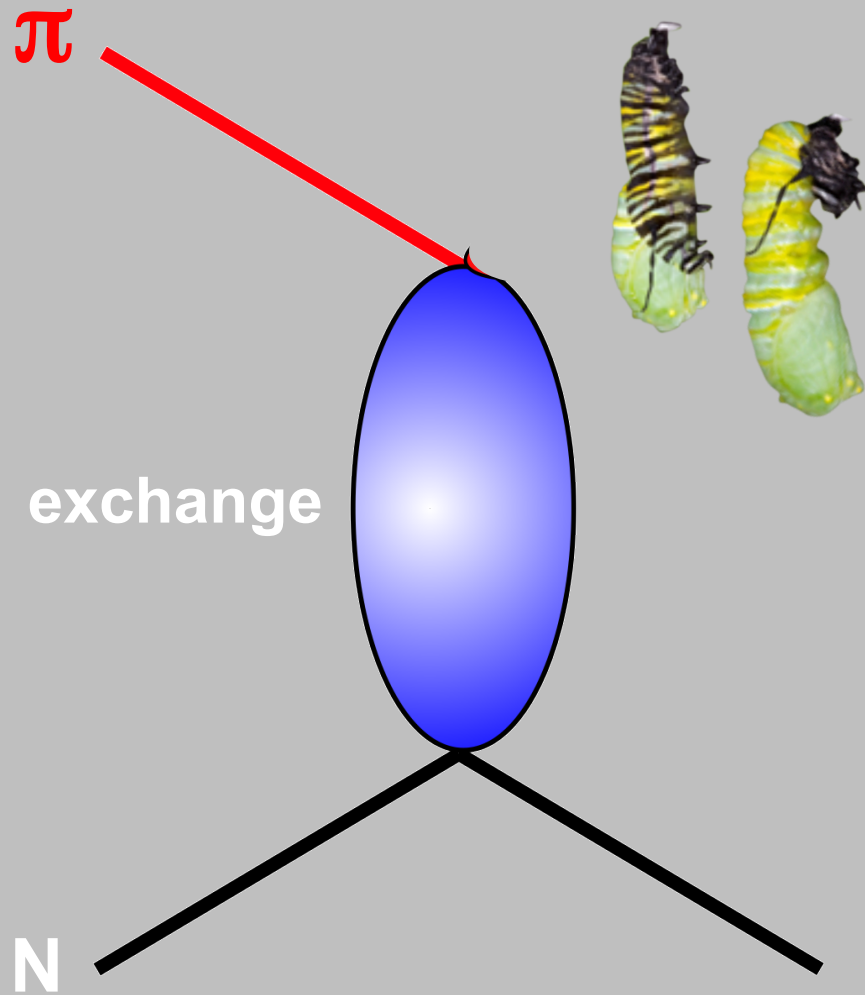
B



Hadroproduction



Hadroproduction



Hadroproduction

π

exchange

N

B



Hadroproduction

π

exchange

N

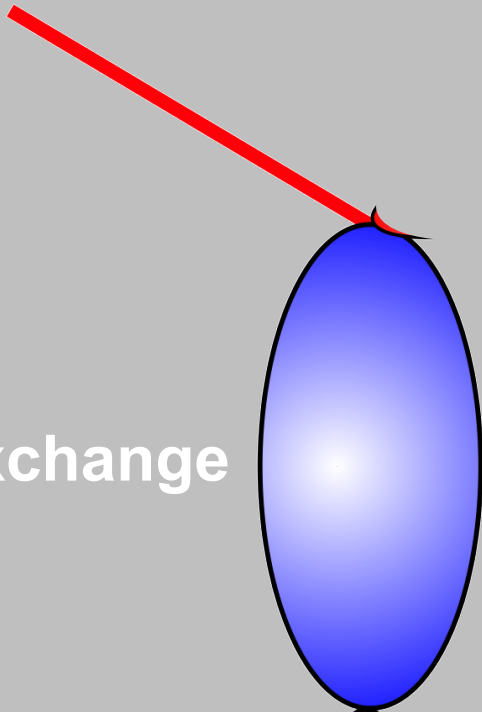
B



Hadroproduction



π



exchange

N

B

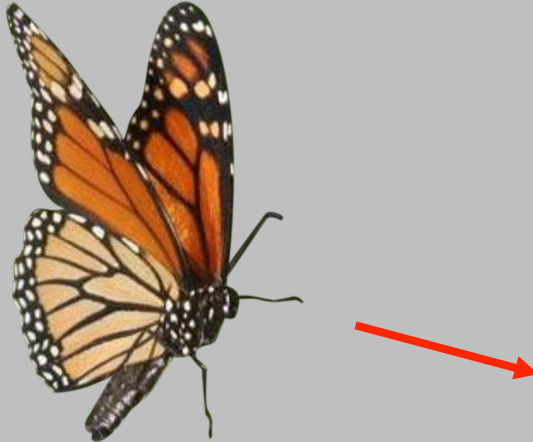
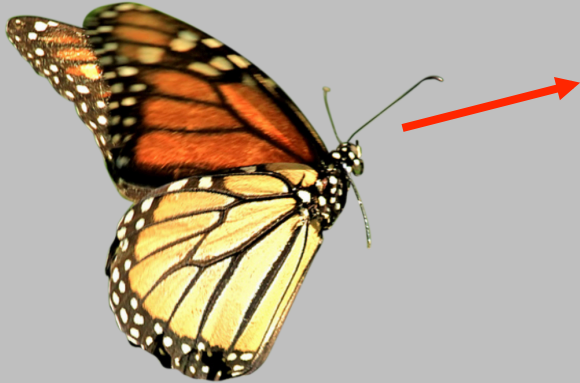
Hadroproduction

π

exchange

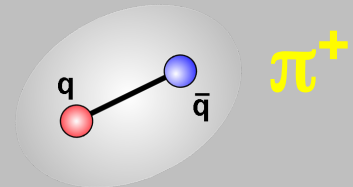
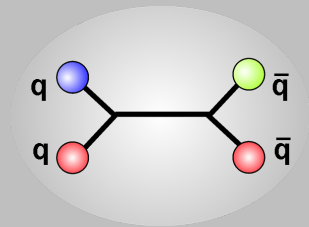
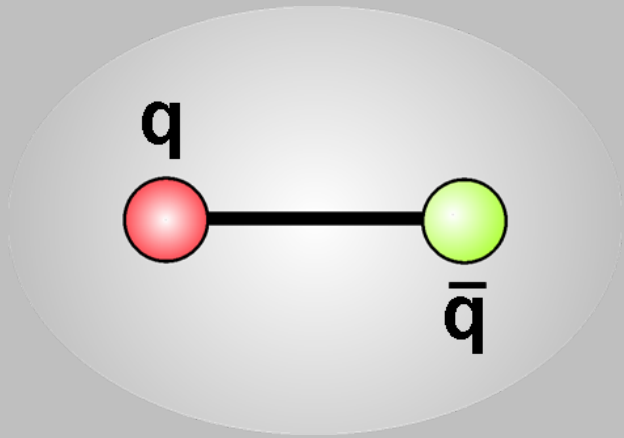
N

B

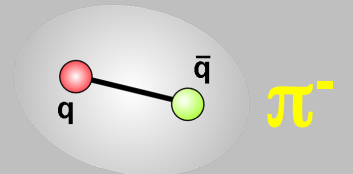


$\rho(770)$

$$J^{PC} = 1^{--}$$



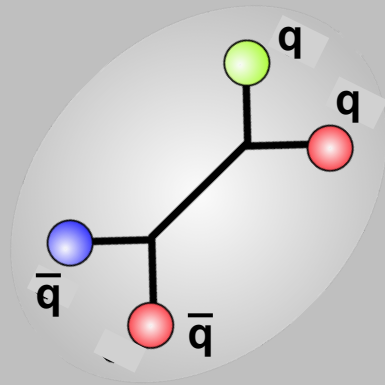
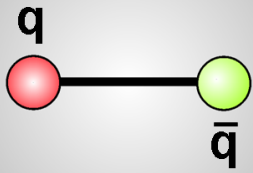
π^+



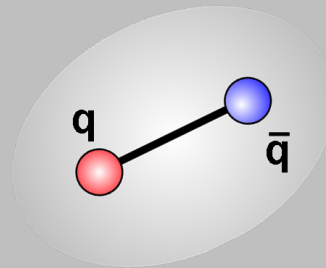
π^-

$a_1(1260)$

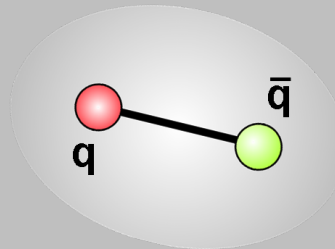
$$J^{PC} = 1^{++}$$



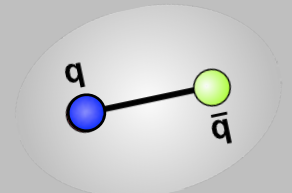
ρ^+



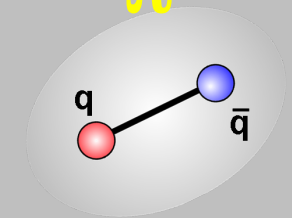
π^-



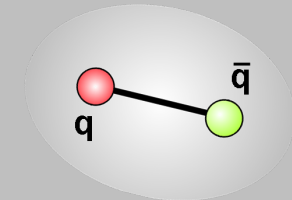
π

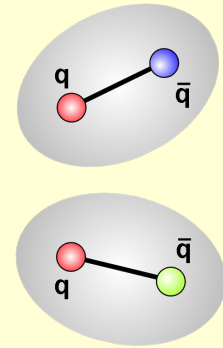
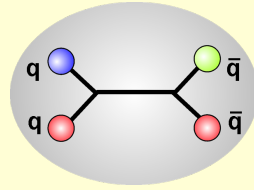
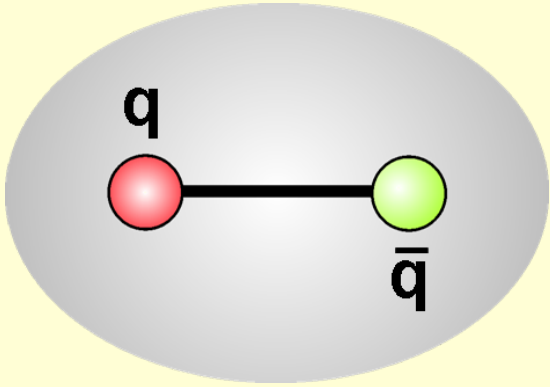


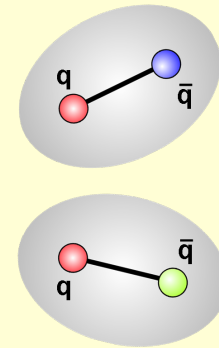
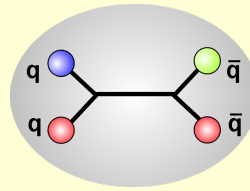
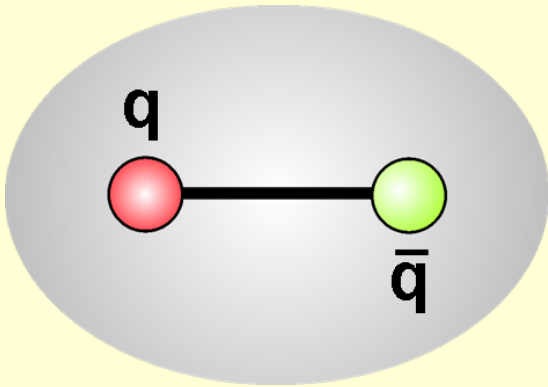
π



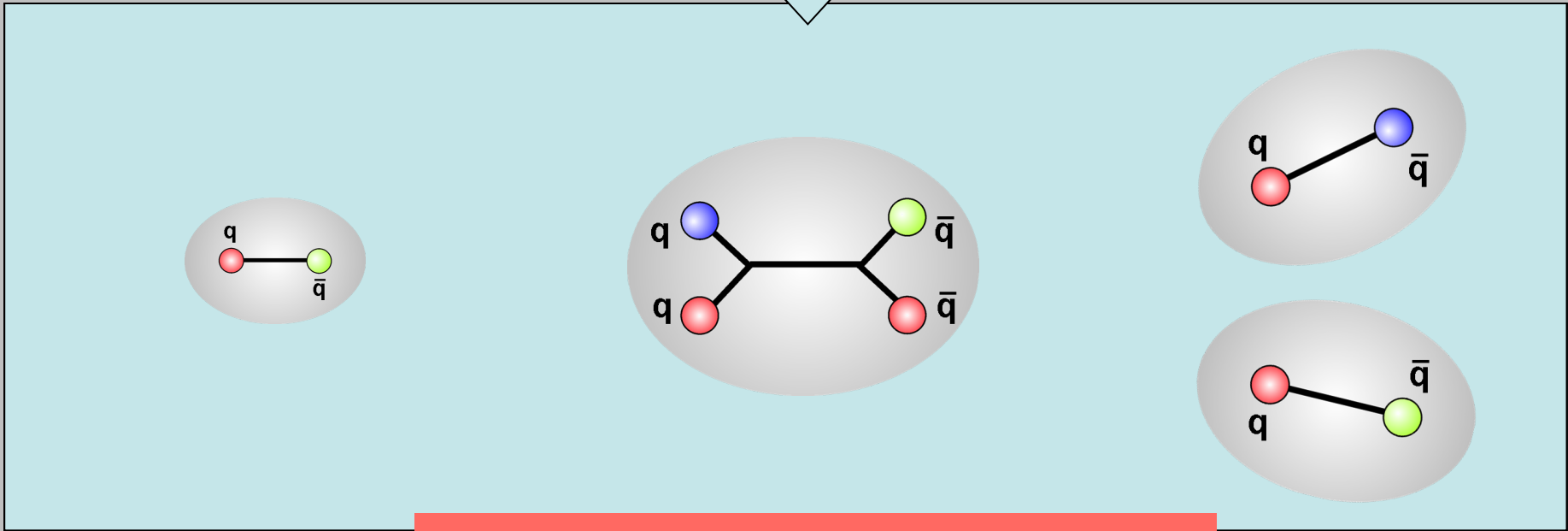
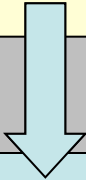
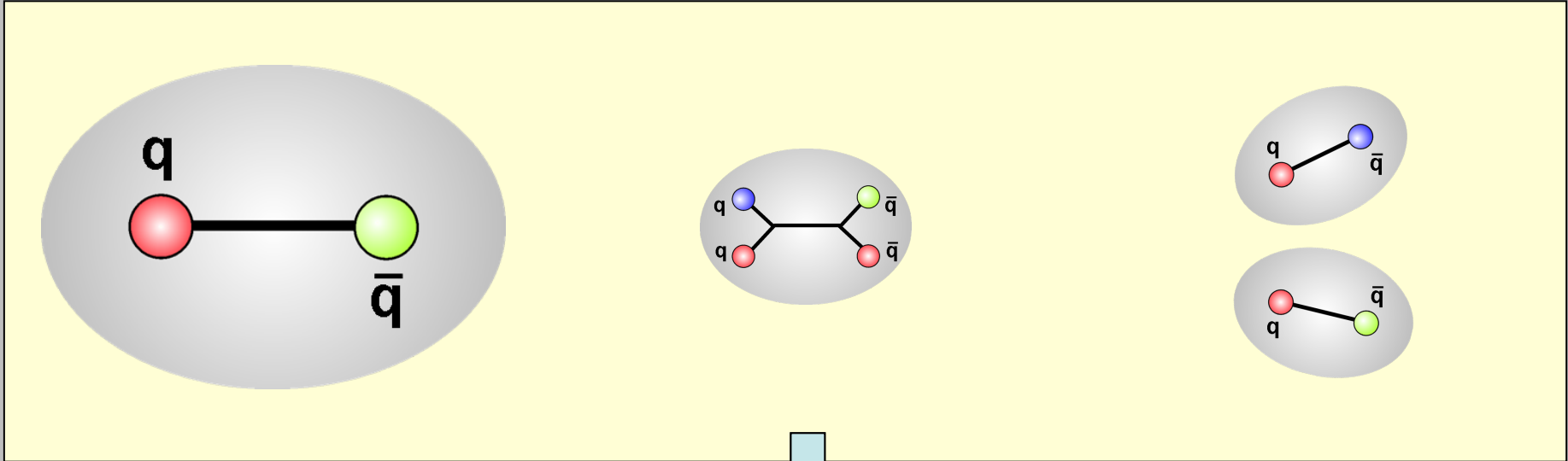
π





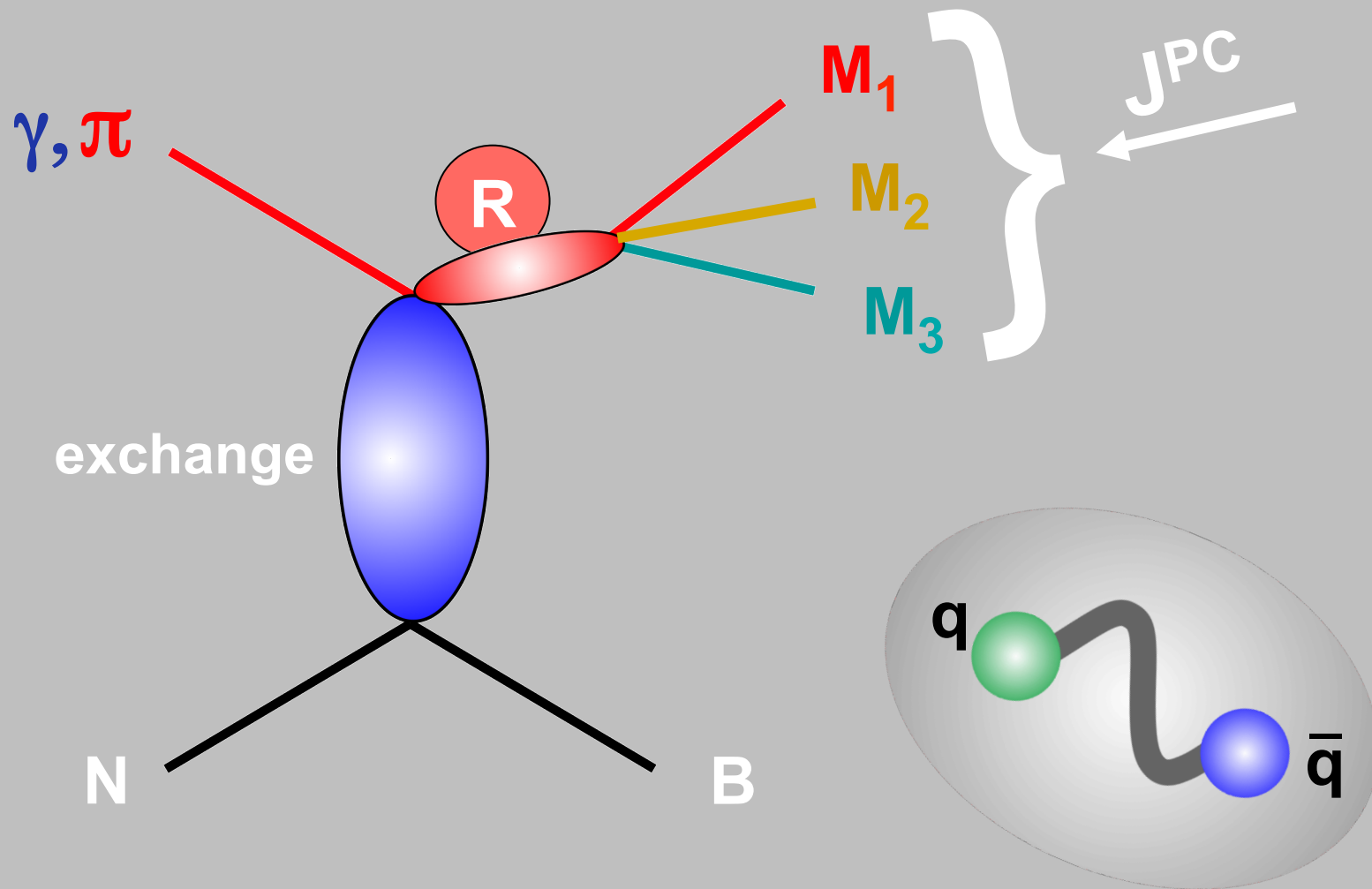


Degrees of freedom

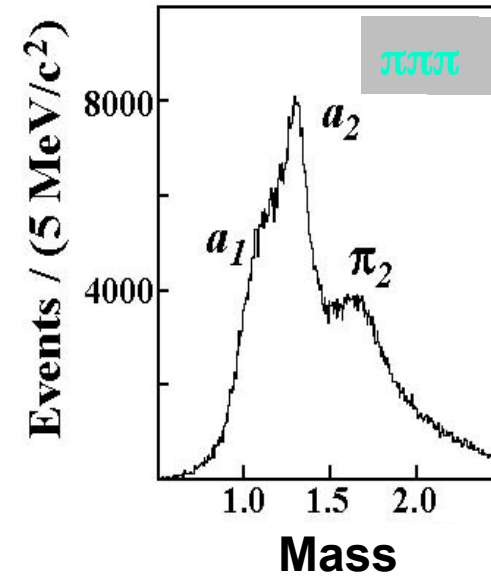
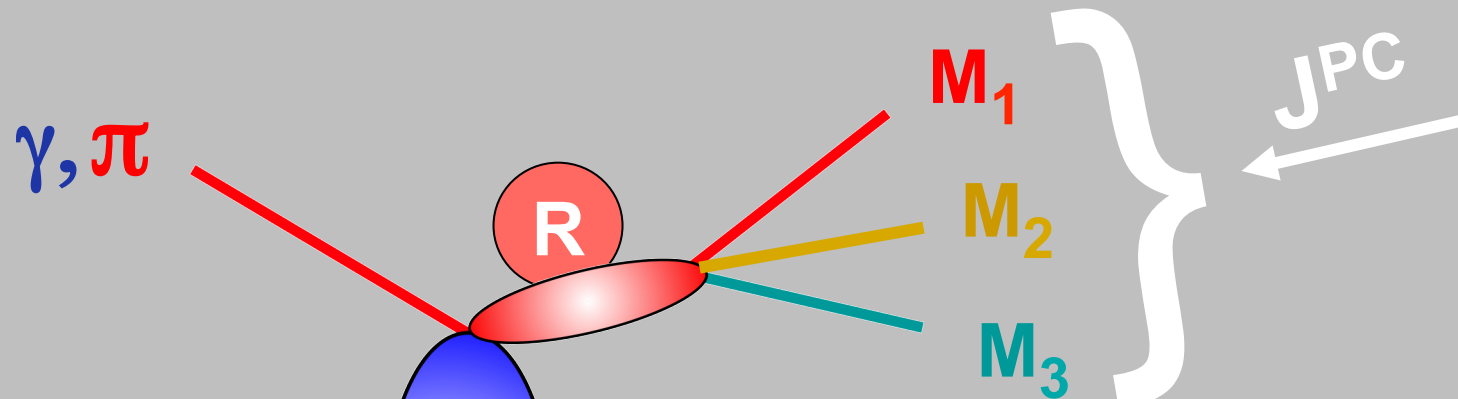


Dynamically generated states

Amplitude Analysis

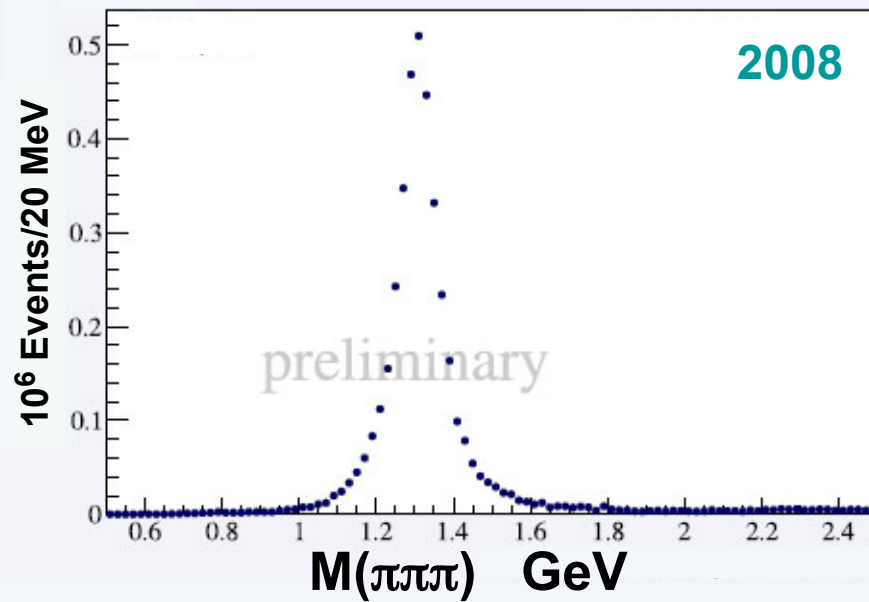


Amplitude Analysis



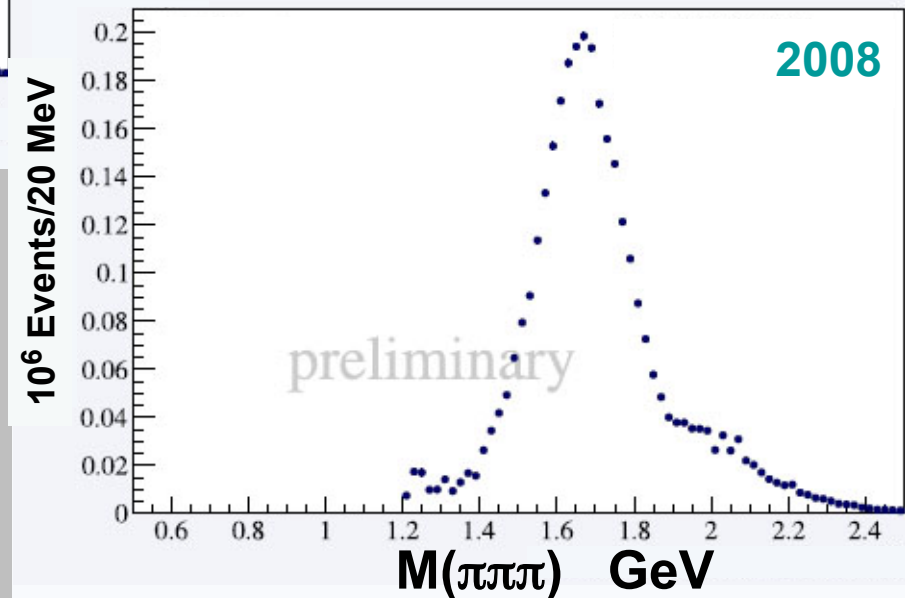
COMPASS @ CERN

$2^{++} 1^+ [\rho\pi]D : a_2(1320)$



PWA: 2 of 88 waves

$2^{-+} 0^+ [f_2\pi]S : \pi_2(1670)$

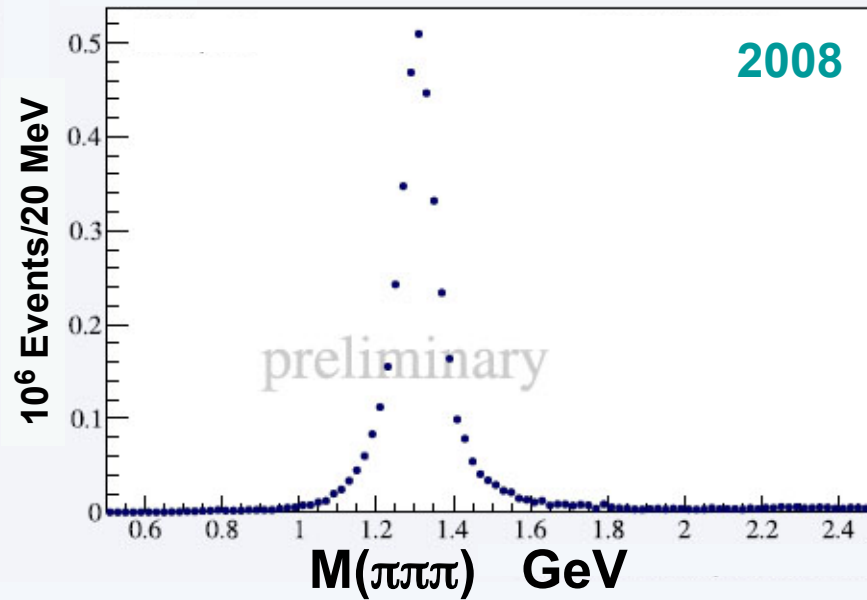


COMPASS @ CERN

Robust 1^{-+} results awaited

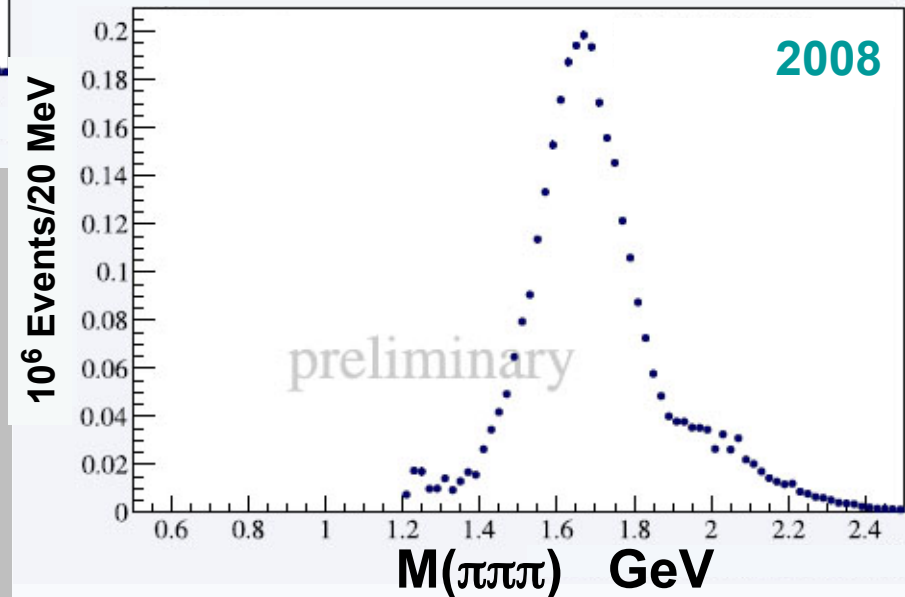
impact of limiting
number of waves 

$2^{++} 1^+ [\rho\pi]D : a_2(1320)$



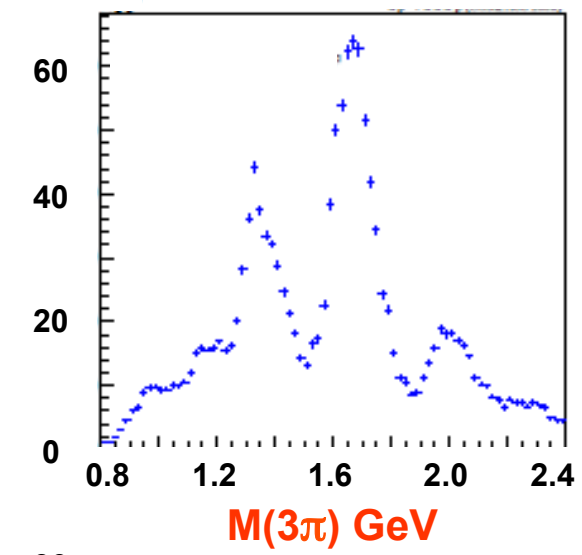
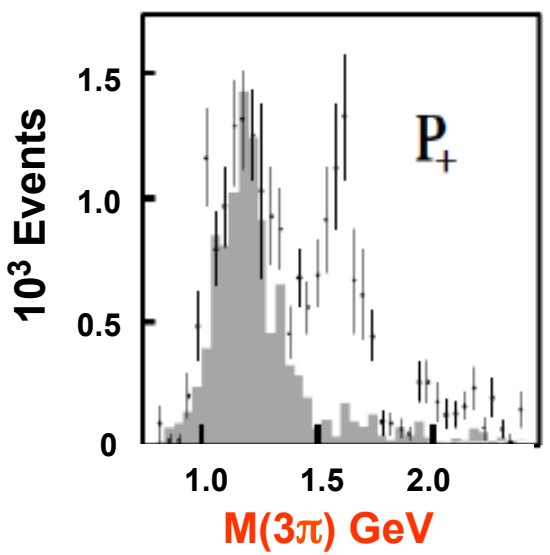
PWA: 2 of 88 waves

$2^{-+} 0^+ [f_2\pi]S : \pi_2(1670)$



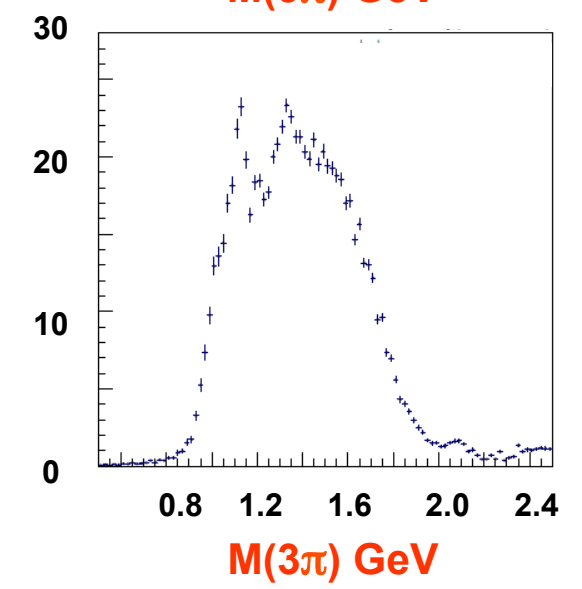
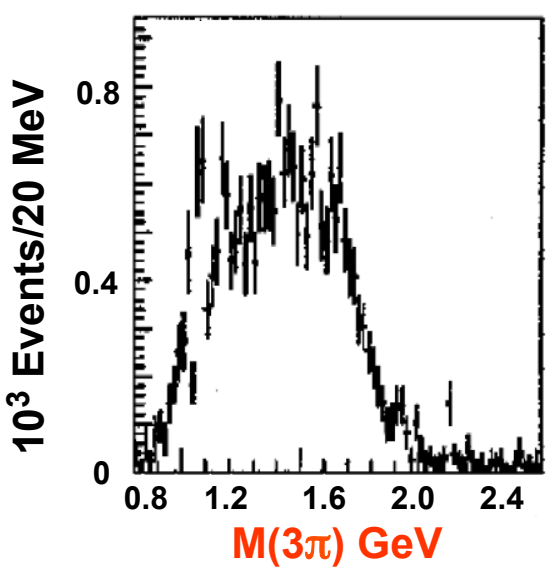
P-wave $\rho\pi$ 1^{-+}

BNL-E852-I
21 waves



COMPASS
21 waves

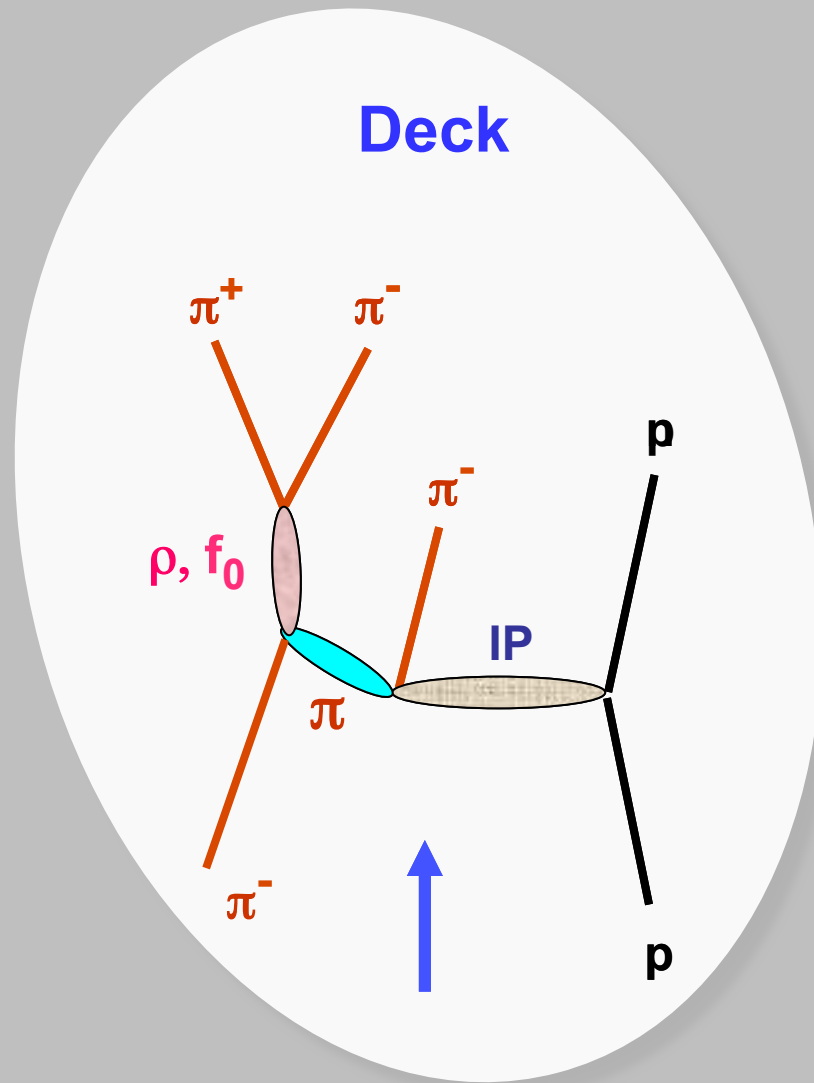
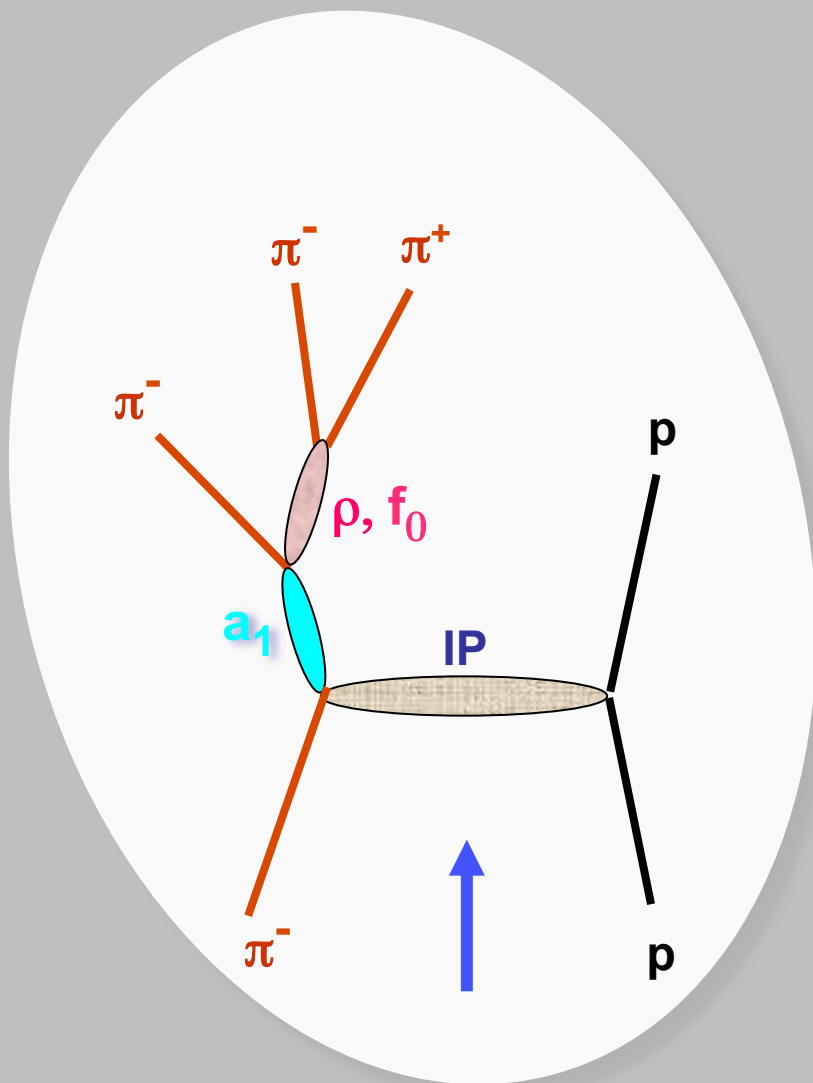
VES
42 waves



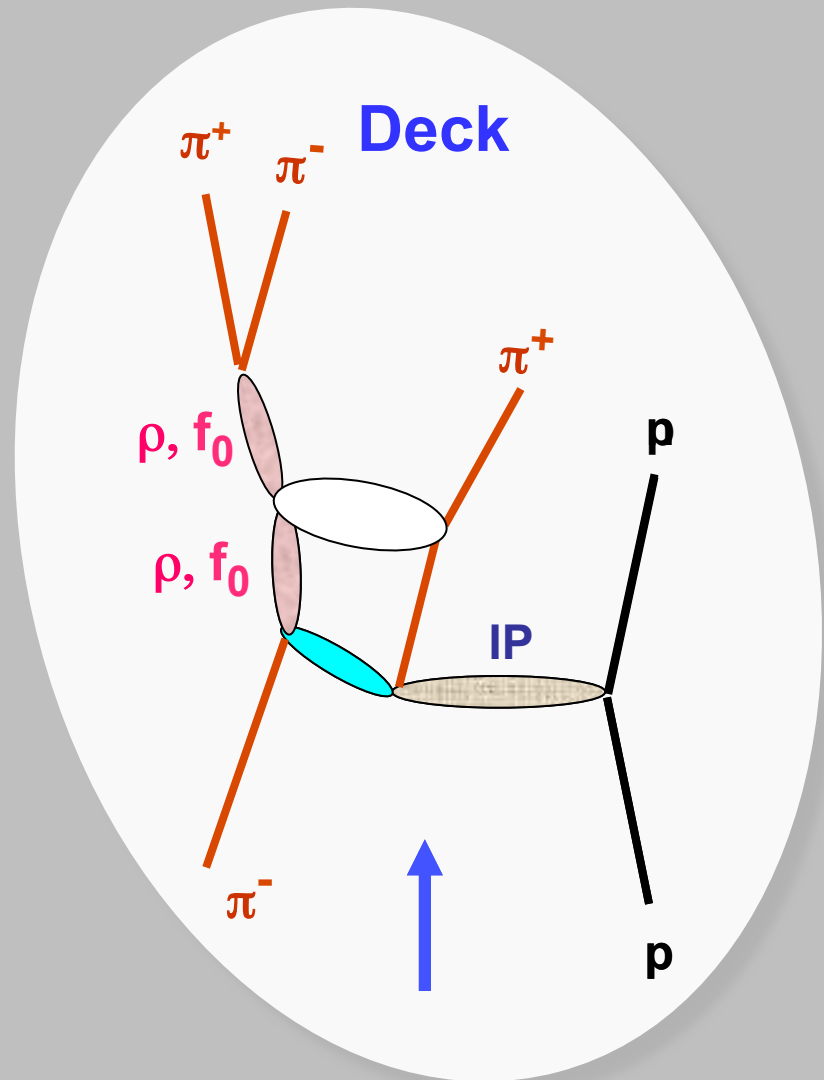
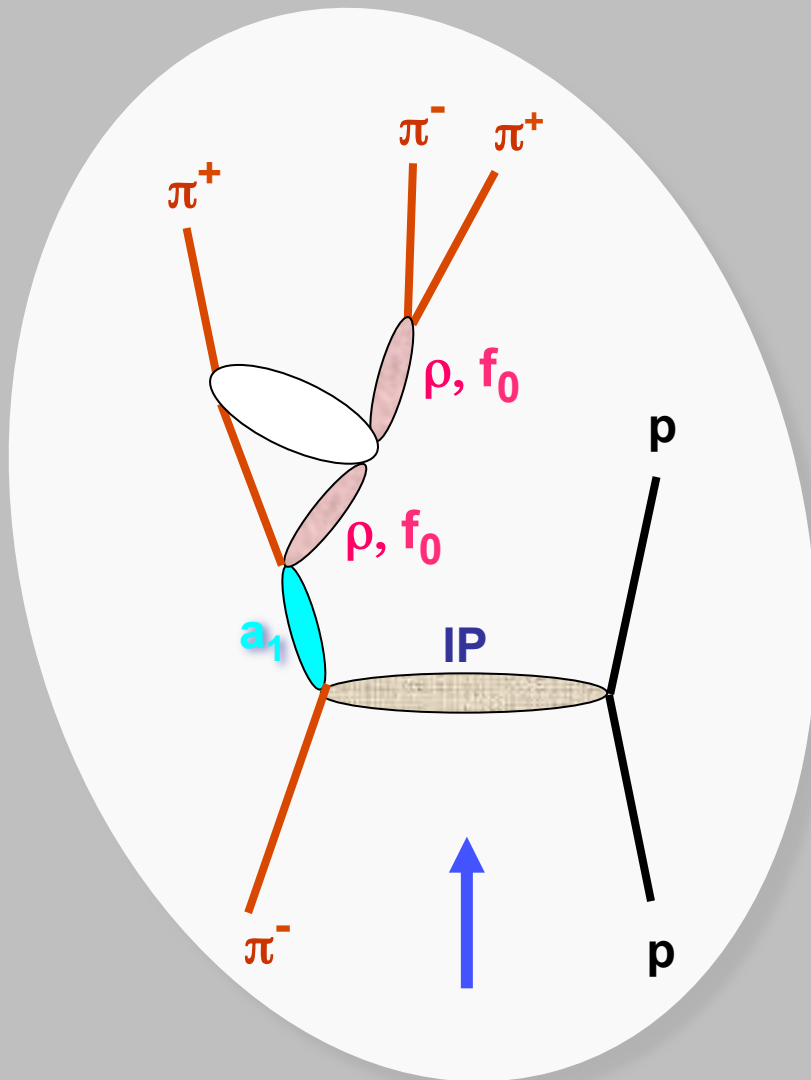
COMPASS
44 waves

● essential to eliminate partial wave truncation

reaction mechanisms



reaction mechanisms

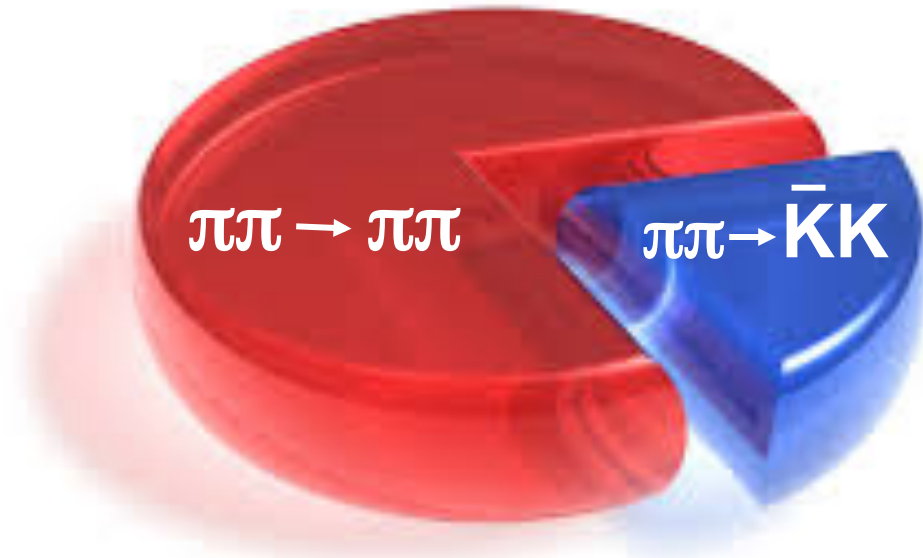


Unitarity



$$\text{Sum of probabilities} = \sum_i P_i = 1$$

Conservation of probability



$$\text{Sum of probabilities} = \sum_i P_i = 1$$



S-MATRIX RELOADED

**The Analytic
S-Matrix**

R.J. EDEN
P.V. LANDSHOFF
D.I. OLIVE
J.C. POLKINGHORNE

Cambridge University Press

**Weapons:
analyticity
unitarity**

S-MATRIX RELOADED

**The Analytic
S-Matrix**

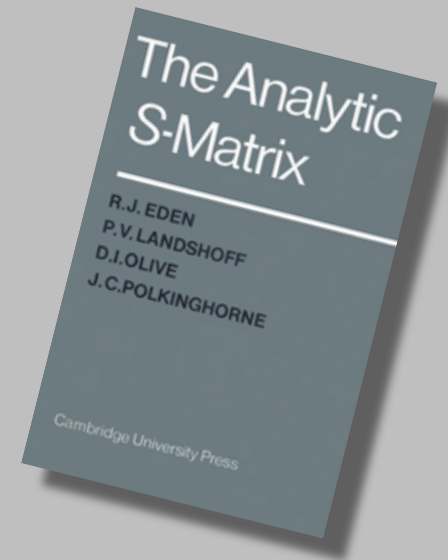
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Outstanding theory issues

- Techniques of **Amplitude Analysis**

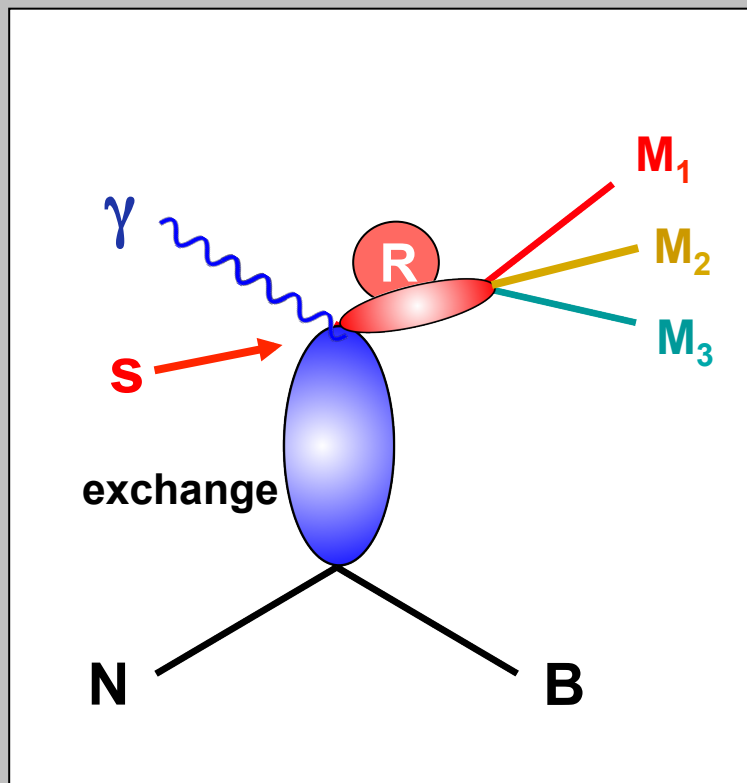
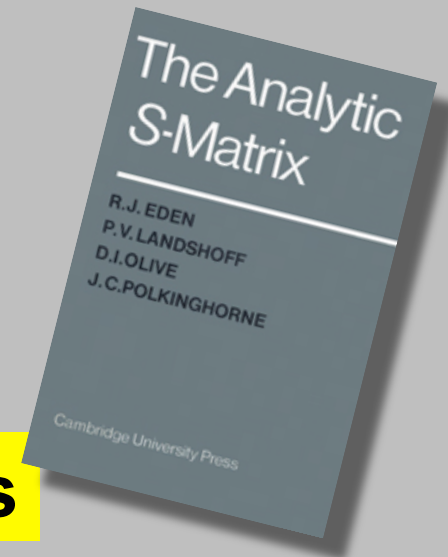
synergy with scattering on the lattice



Outstanding theory issues

- Techniques of **Amplitude Analysis**

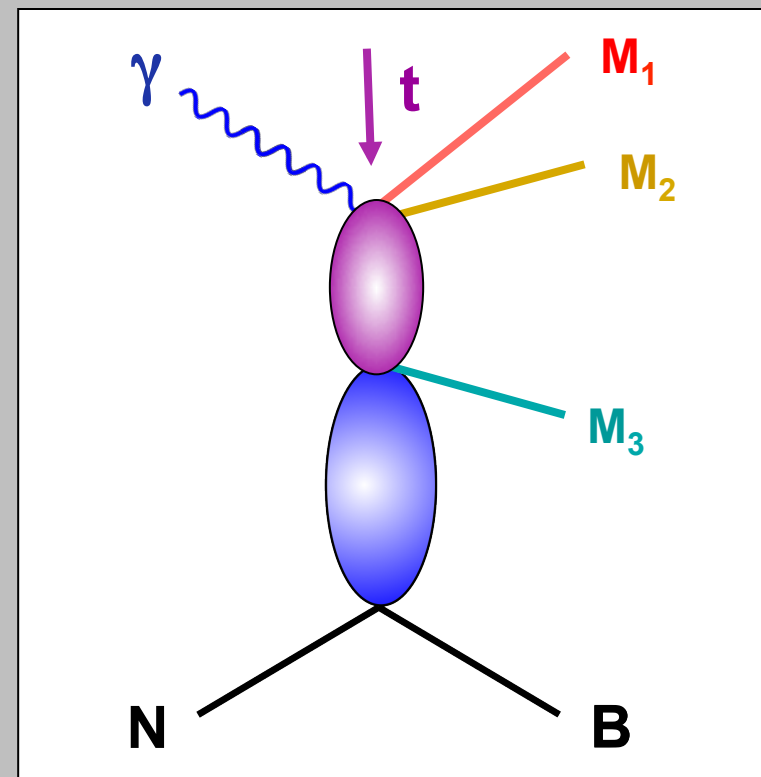
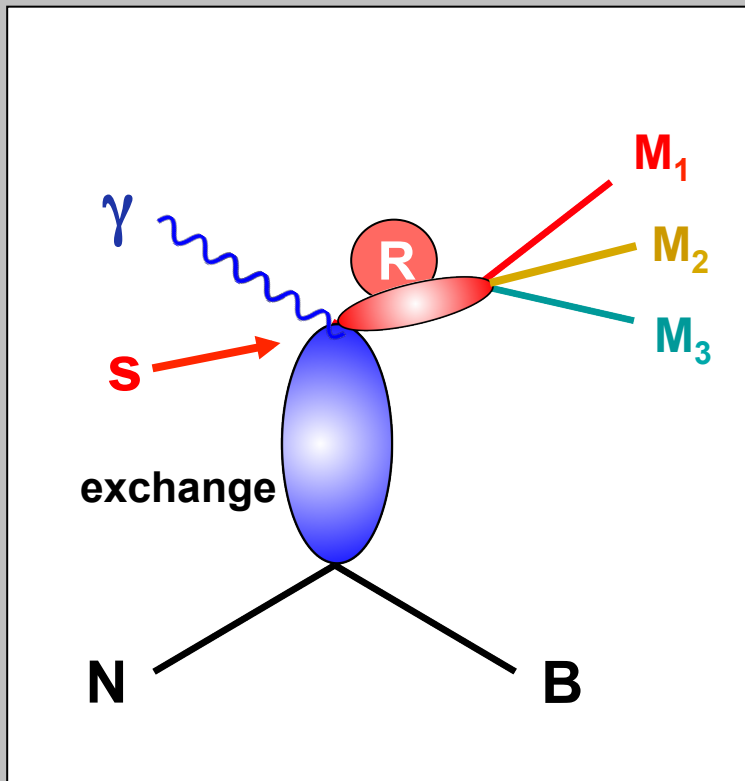
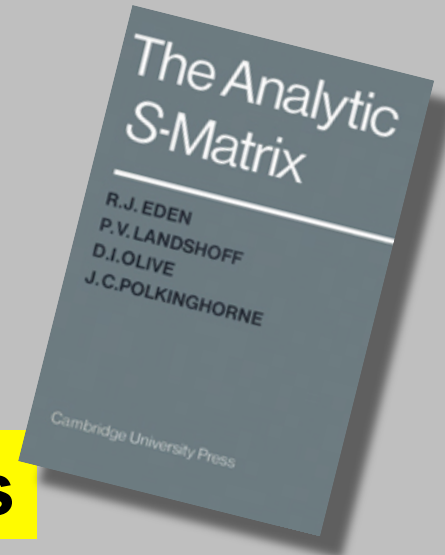
- Understand reaction mechanisms



Outstanding theory issues

- Techniques of **Amplitude Analysis**

- Understand reaction mechanisms





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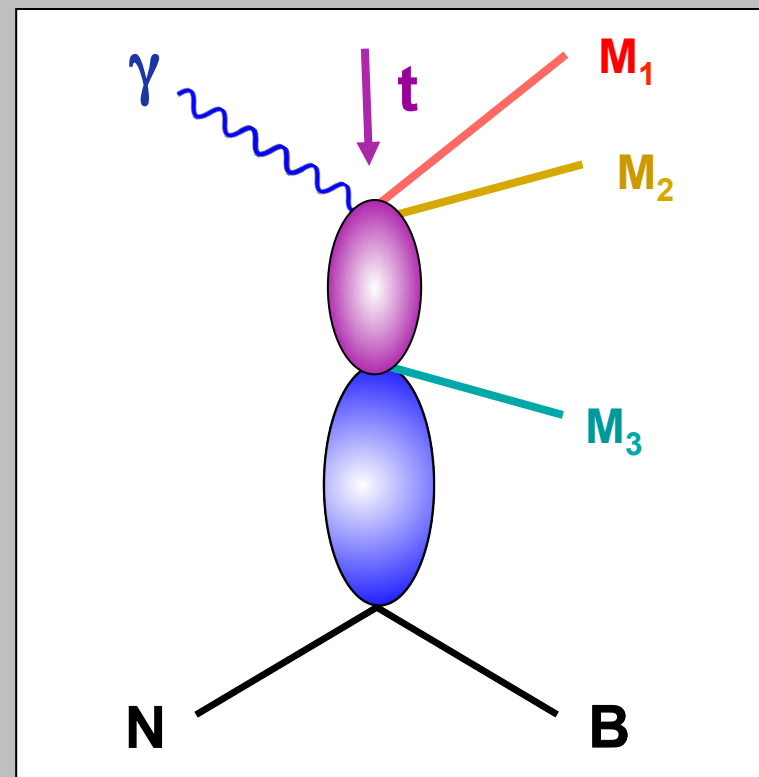
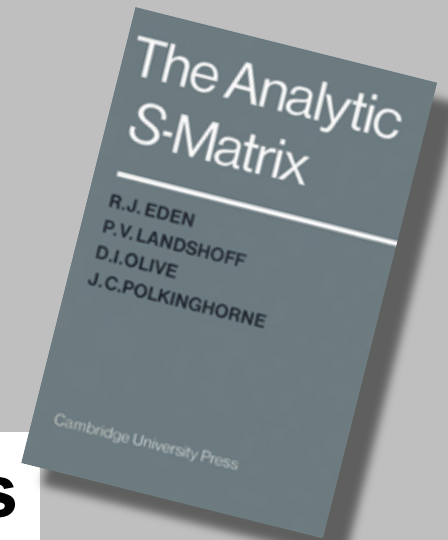
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Outstanding theory issues

- Techniques of **Amplitude Analysis**

- Understand reaction mechanisms

- **Crossing: eliminates partial wave truncation**

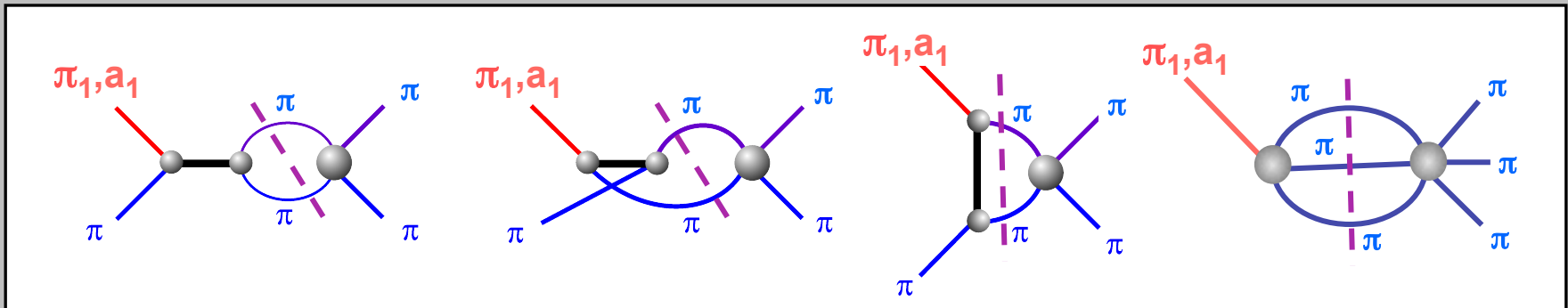
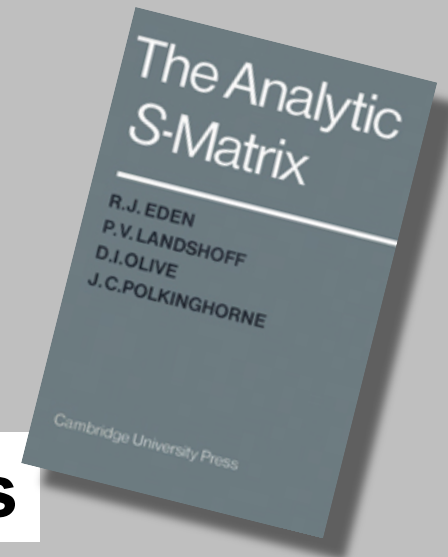


Outstanding theory issues

- Techniques of **Amplitude Analysis**

- Understand reaction mechanisms

- **Unitarity and final state interactions**

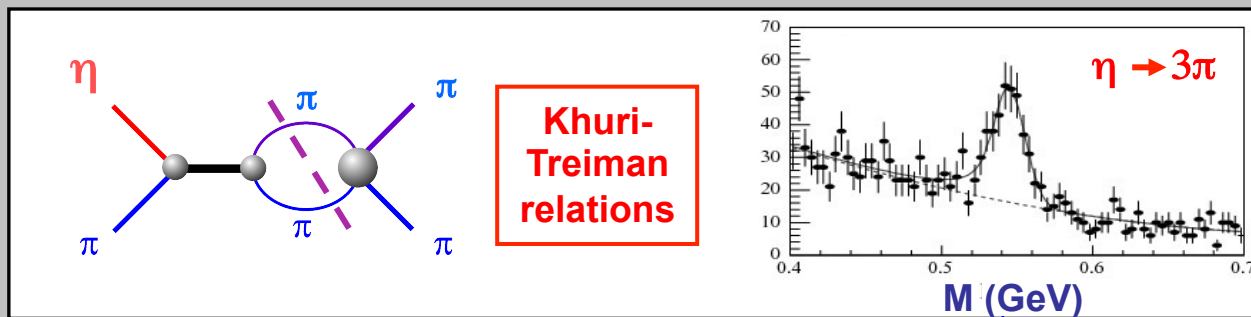
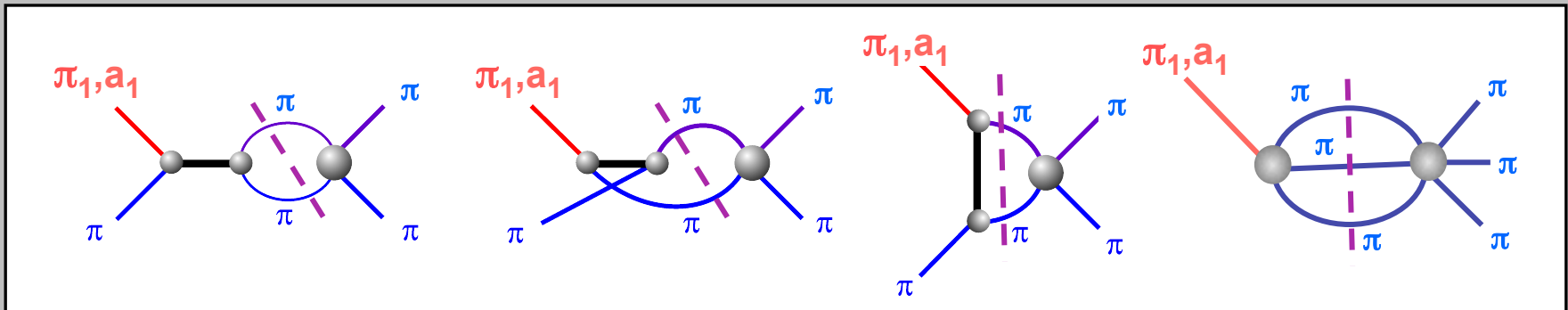
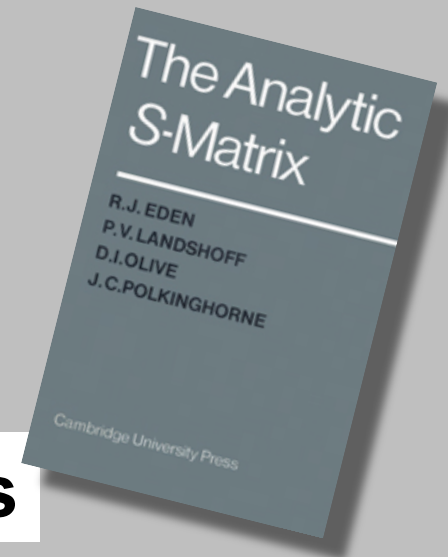


Outstanding theory issues

- Techniques of **Amplitude Analysis**

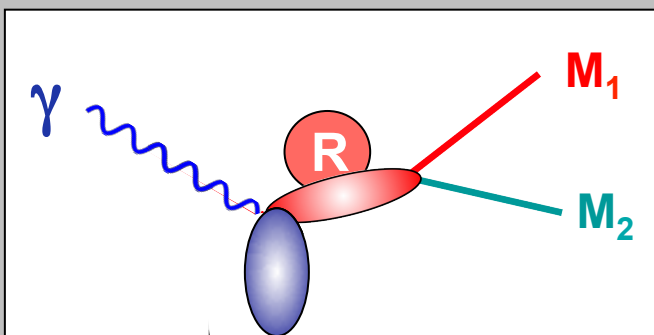
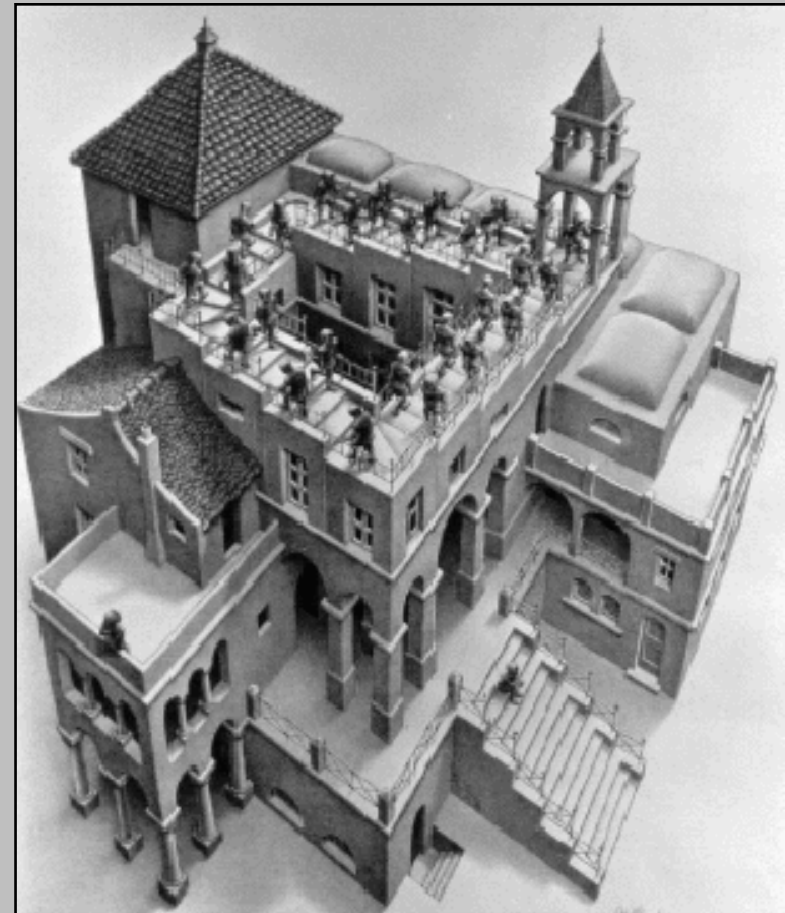
- Understand reaction mechanisms

- **Unitarity and final state interactions**



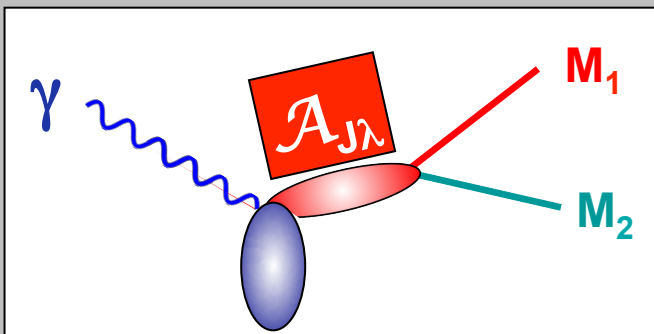
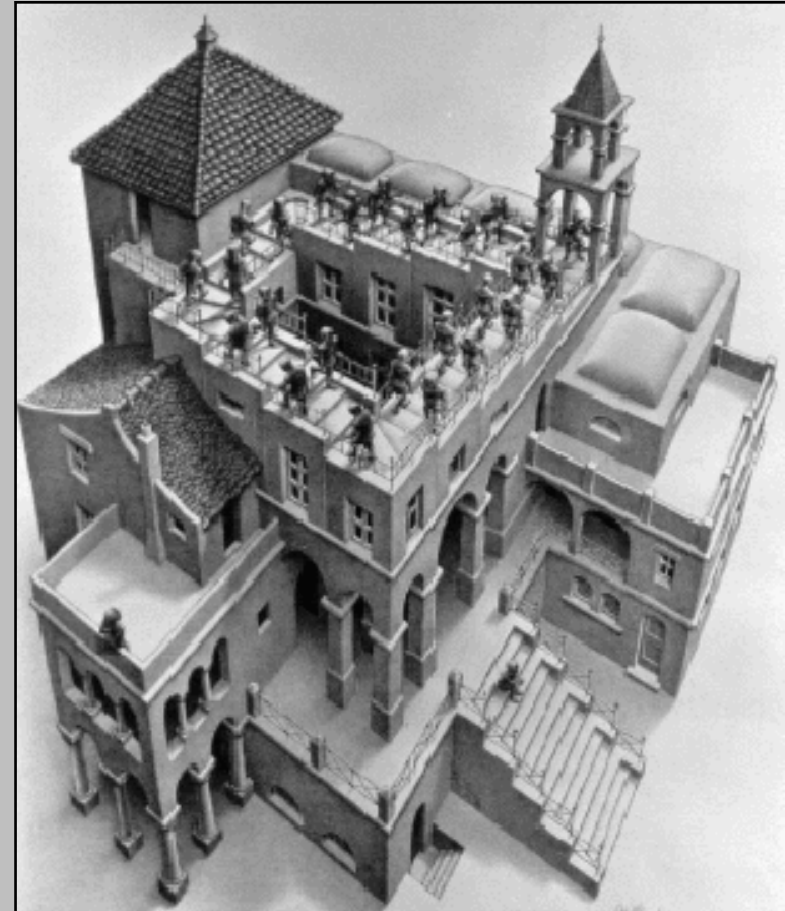
Model

Analyses



- **Isobar Model**

Model-independent Analyses

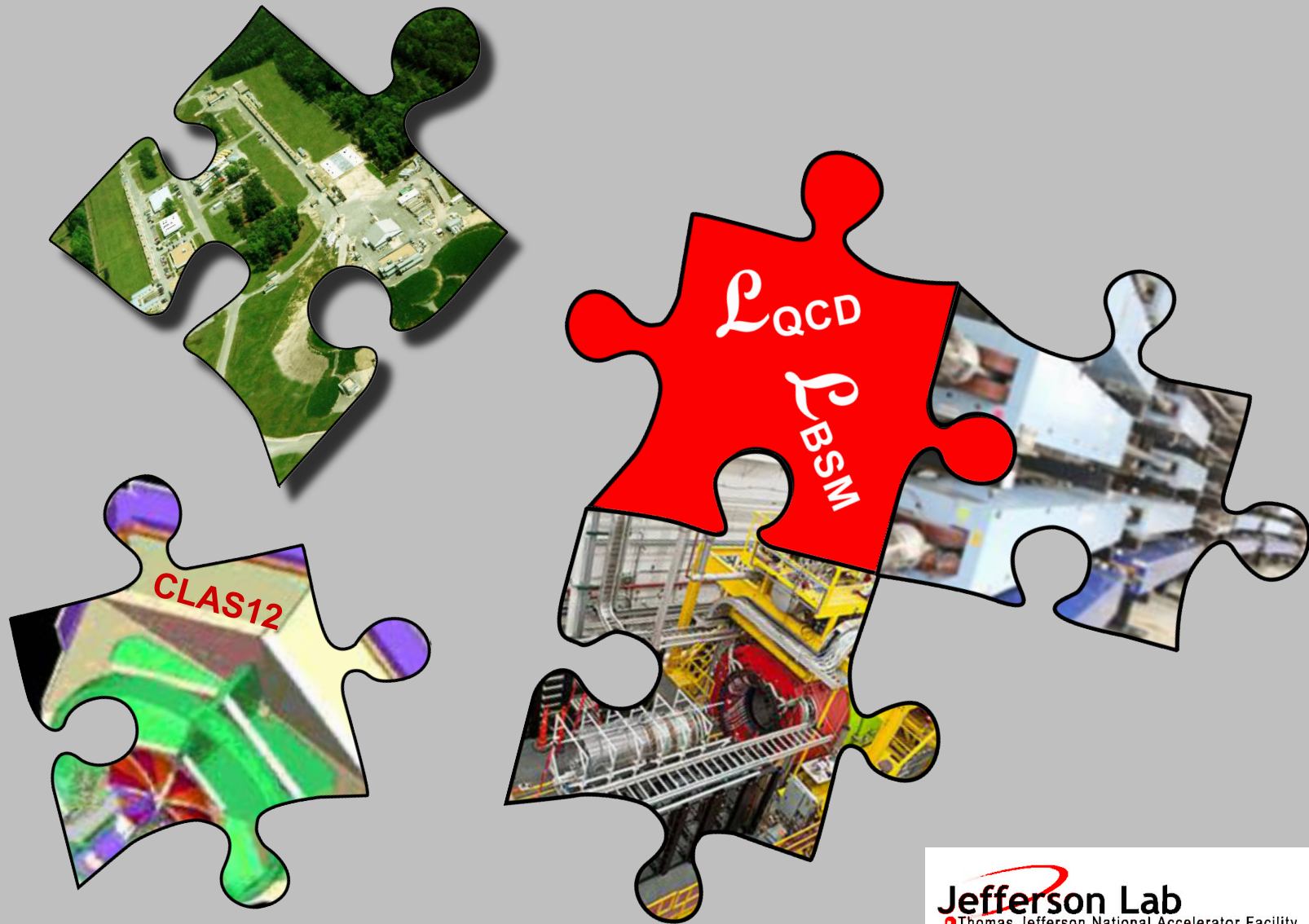


- **Beyond the Isobar**

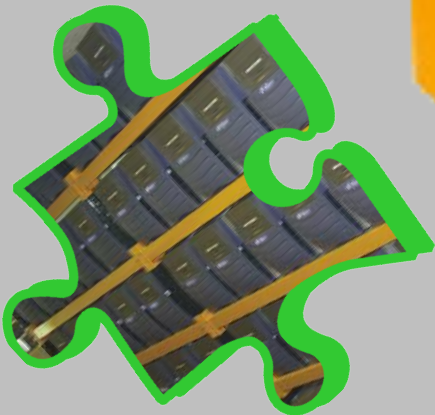
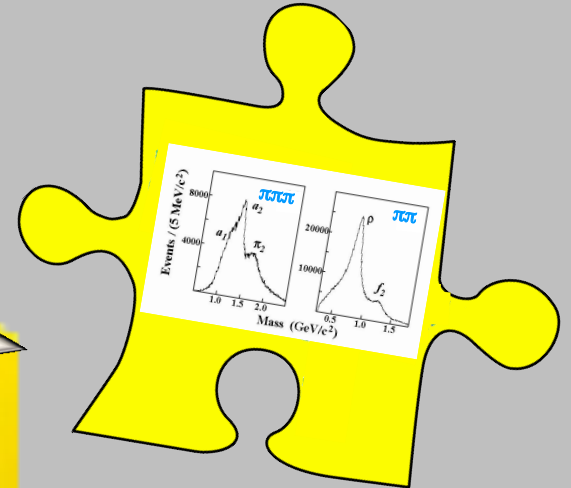
Global working : Amplitude Analyses & Spectroscopy



Joint Physics Analysis Center program



Joint Physics Analysis Center program



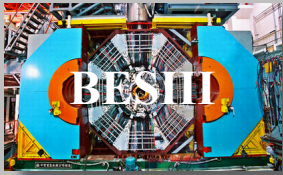
JPAC



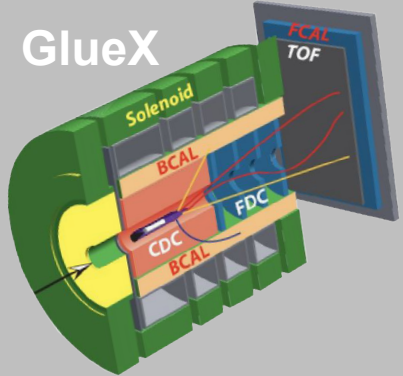
NABIS



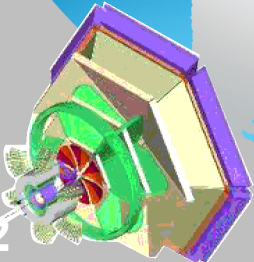
HaSpect



GlueX



CLAS12



cool QCD

