Neutral Pion Photo-Production at High Energies

~~Summer School~~ on Reaction Theory
Spring Workshop

Vincent Mathieu
Indiana University

Jefferson Lab
Thomas Jefferson National Accelerator Facility

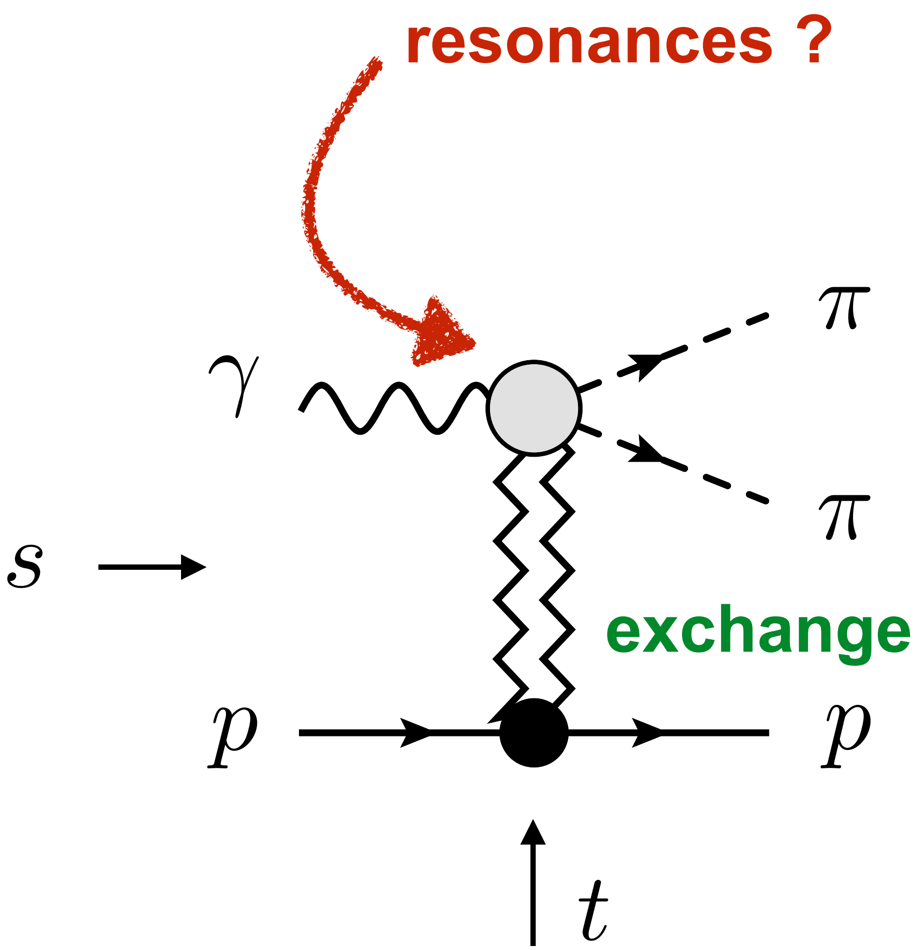
INDIANA UNIVERSITY

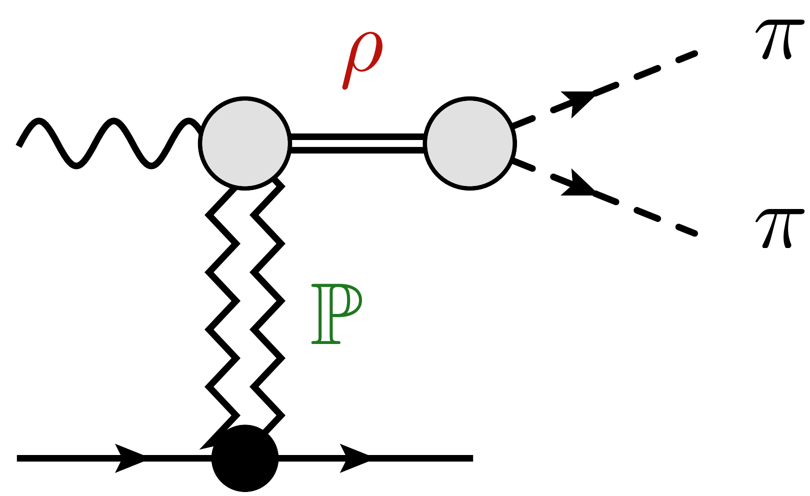
NSF

Motivations: Meson Production @JLab

Mesons produced by beam dissociation on fixed target at high energies.

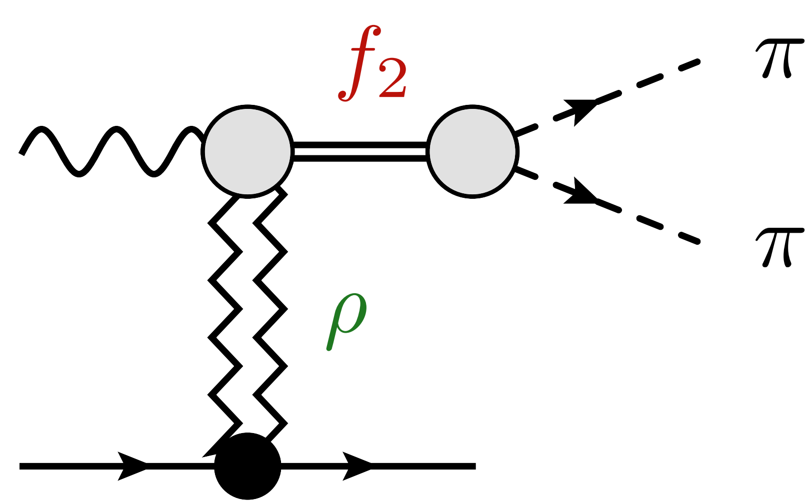
Photon energy @JLab: 8-10 GeV





$\sim s^{1+0.2t}$

Pomeron



$\sim s^{0.5+0.9t}$

vector

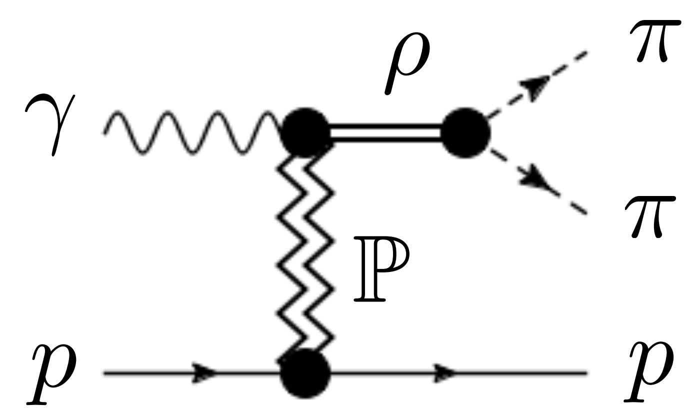
Motivations: Meson Production @JLab

Mesons produced by beam dissociation on fixed target at high energies.

Photon energy @JLab: 8-10 GeV

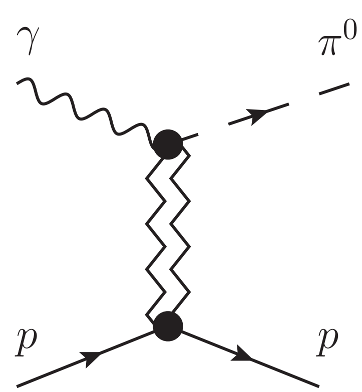
Production parameters can be, in principle, determined with other reactions

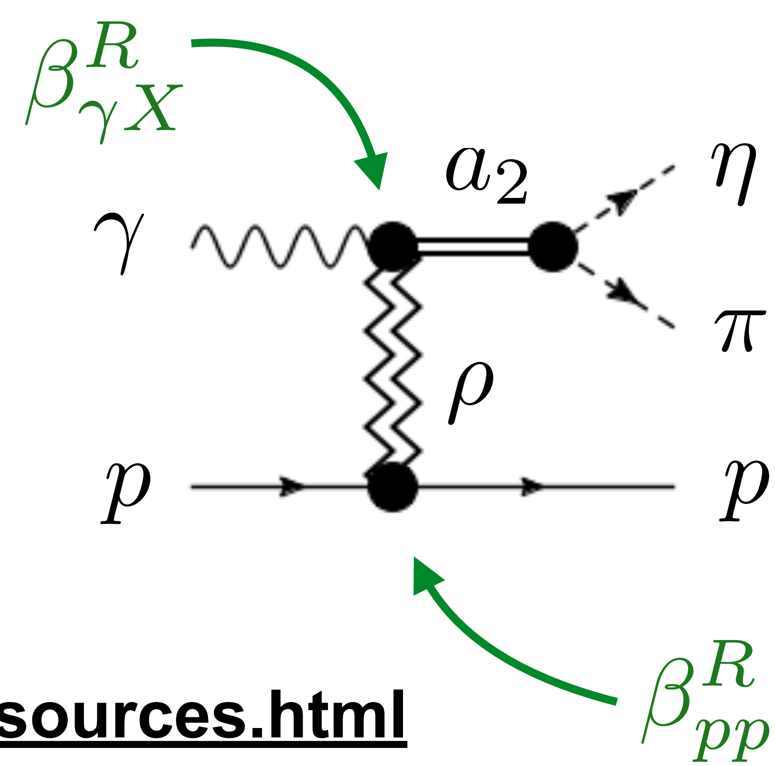
Known energy dependence
Need to extract angular (or t) dependence



$$A_X \propto \beta^R_{pp}(t)\beta^R_{\gamma X}(t)s^{\alpha_R(t)}$$

Simplest case:



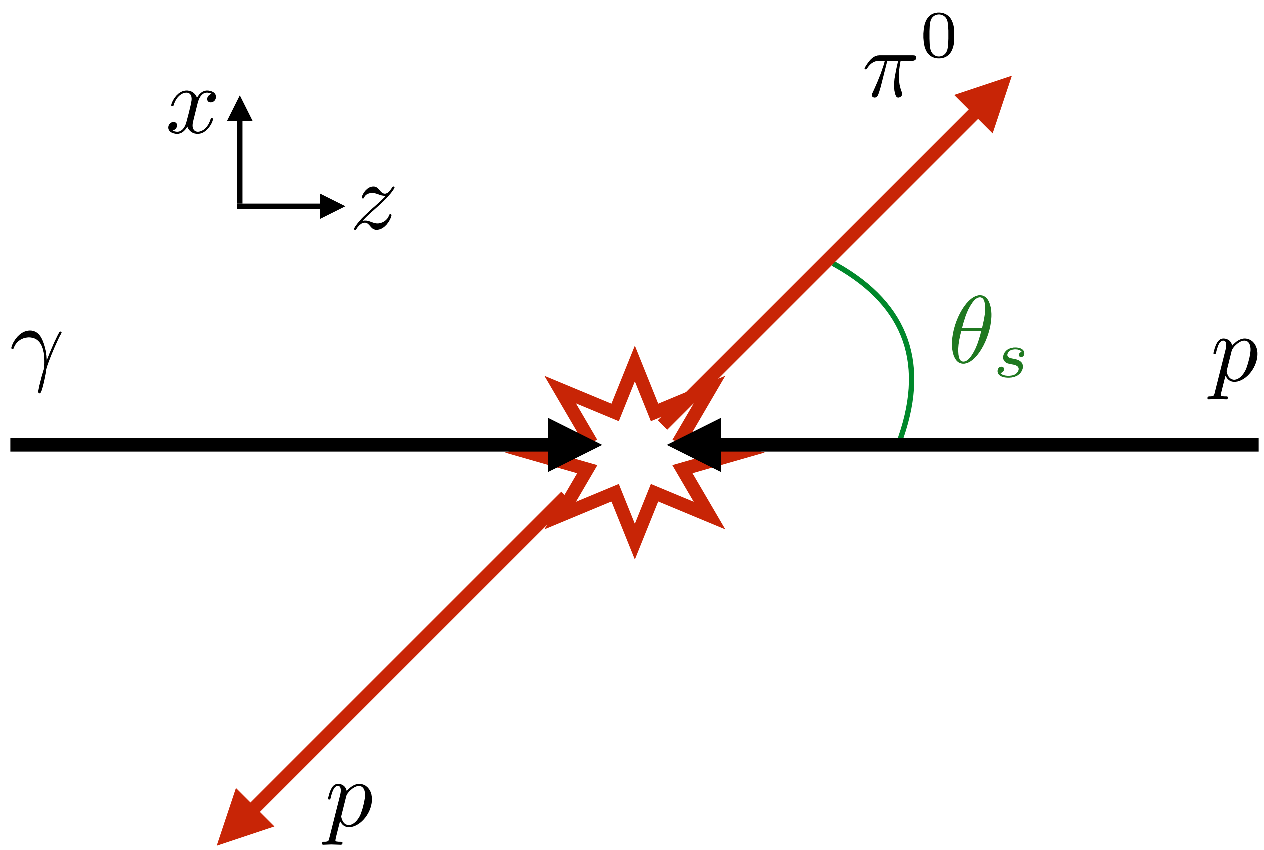


Slides, data, etc: http://www.indiana.edu/~jpac/Resources.html

S-Channel Kinematics

$$\gamma(p_1,\mu_1) + p(p_2,\mu_2) \to \pi^0(p_3,0) + p(p_4,\mu_4)$$

pion mass: μ
nucleon mass: m



$$s = (p_1+p_2)^2 = (p_3+p_4)^2$$
$$t = (p_1-p_3)^2 = (p_2-p_4)^2$$
$$u = (p_1-p_4)^2 = (p_2-p_3)^2$$
$$s+t+u = 2m^2+\mu^2$$

$$E_\gamma = \frac{s-m^2}{2\sqrt{s}}$$

$$E_p = \frac{s+m^2}{2\sqrt{s}}$$

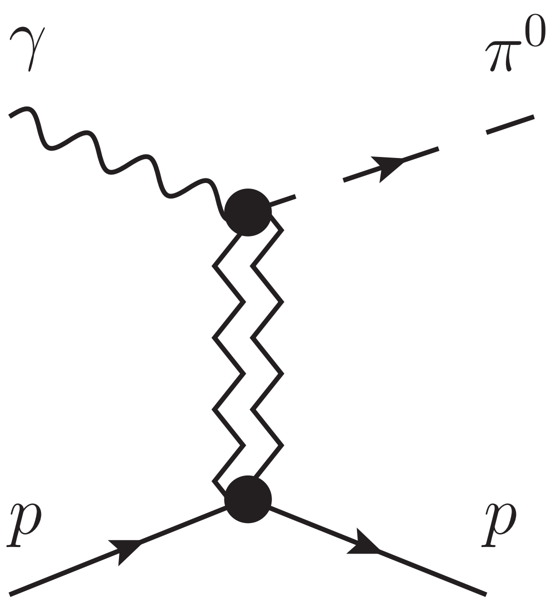
$$\cos\theta_s = \frac{s(t-u)+m^2(m^2-\mu^2)}{4sq_sq_s'}$$

$$E_\pi = \frac{s+\mu^2-m^2}{2\sqrt{s}}$$

$$E_{p'} = \frac{s-\mu^2+m^2}{2\sqrt{s}}$$

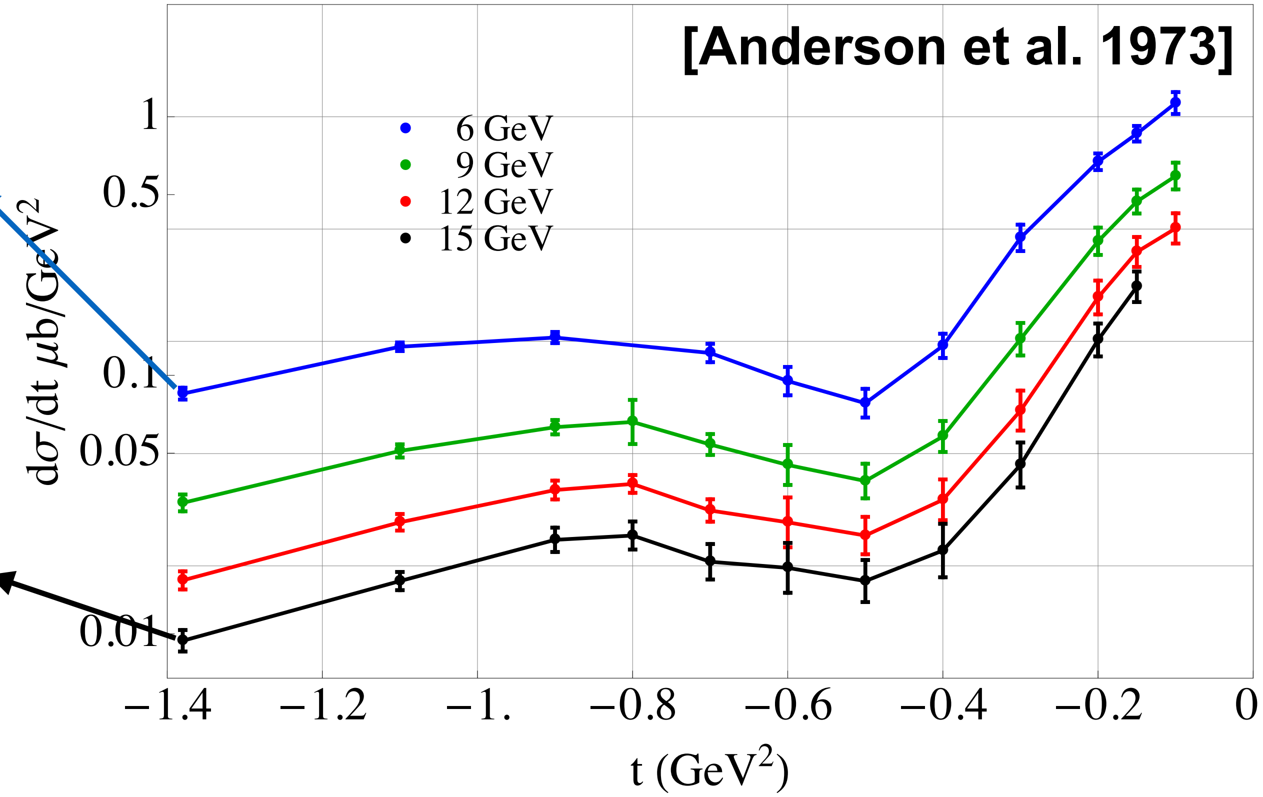
initial momentum: $q_s^2 = E_\gamma^2 = E_p^2 - m^2$
final momentum: $q_s'^2 = E_\pi^2 - \mu^2 = E_{p'}^2 - m^2$

T-Channel Quantum Numbers

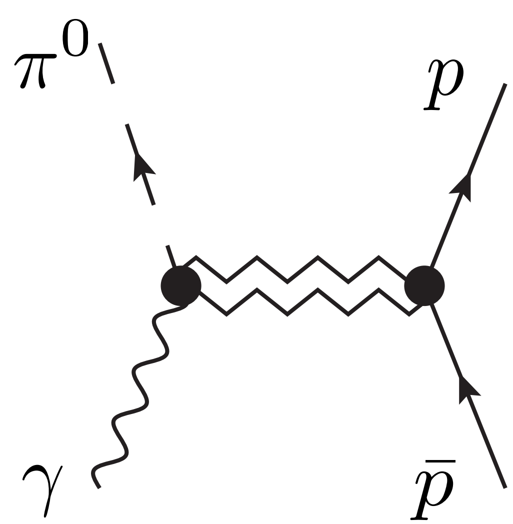


$\cos\theta_s = 0.73$

$\cos\theta_s = 0.90$



What can be exchanged in the t-channel ?



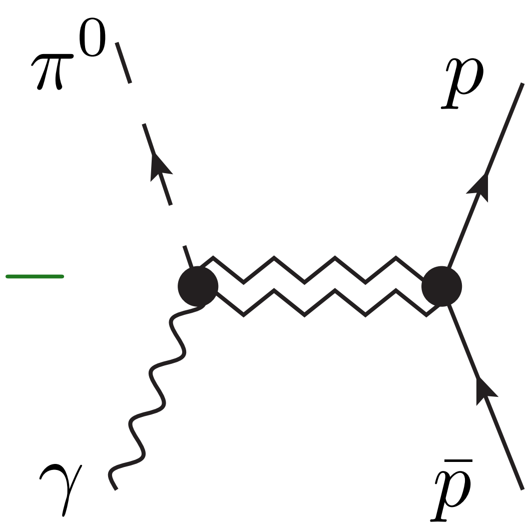
$$I^G J^{PC}$$

$$G = C(-1)^I$$

T-Channel Quantum Numbers

What can be exchanged in the t-channel ?

$C = -$



$I = 0, 1$

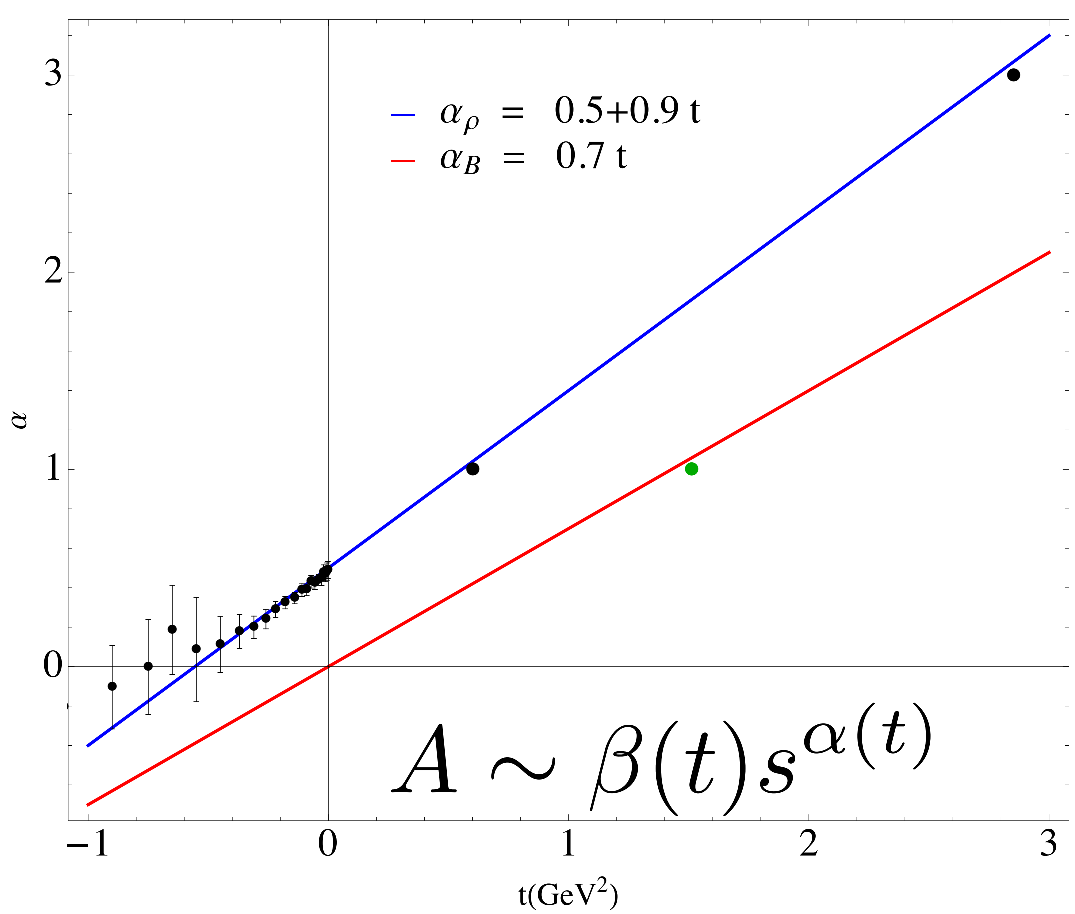
dominant

$(1, 3, 5, \ldots)^{--}$ → $I^G = 0^- : \omega$, $I^G = 1^+ : \rho$

$(1, 3, 5, \ldots)^{+-}$ → $I^G = 0^- : h$, $I^G = 1^+ : b$

exotic $(0, 2, 4, \ldots)^{--}$ **no observation**

$(0, 2, 4, \ldots)^{+-}$

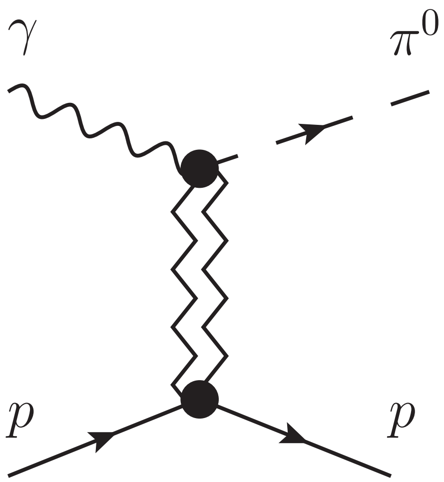


$P = (-)^{L+1} \to L$ odd

$C = (-)^{L+S} \to S = 0$

$\vec{J} = \vec{L} + \vec{S} \to J$odd

Effective Trajectory

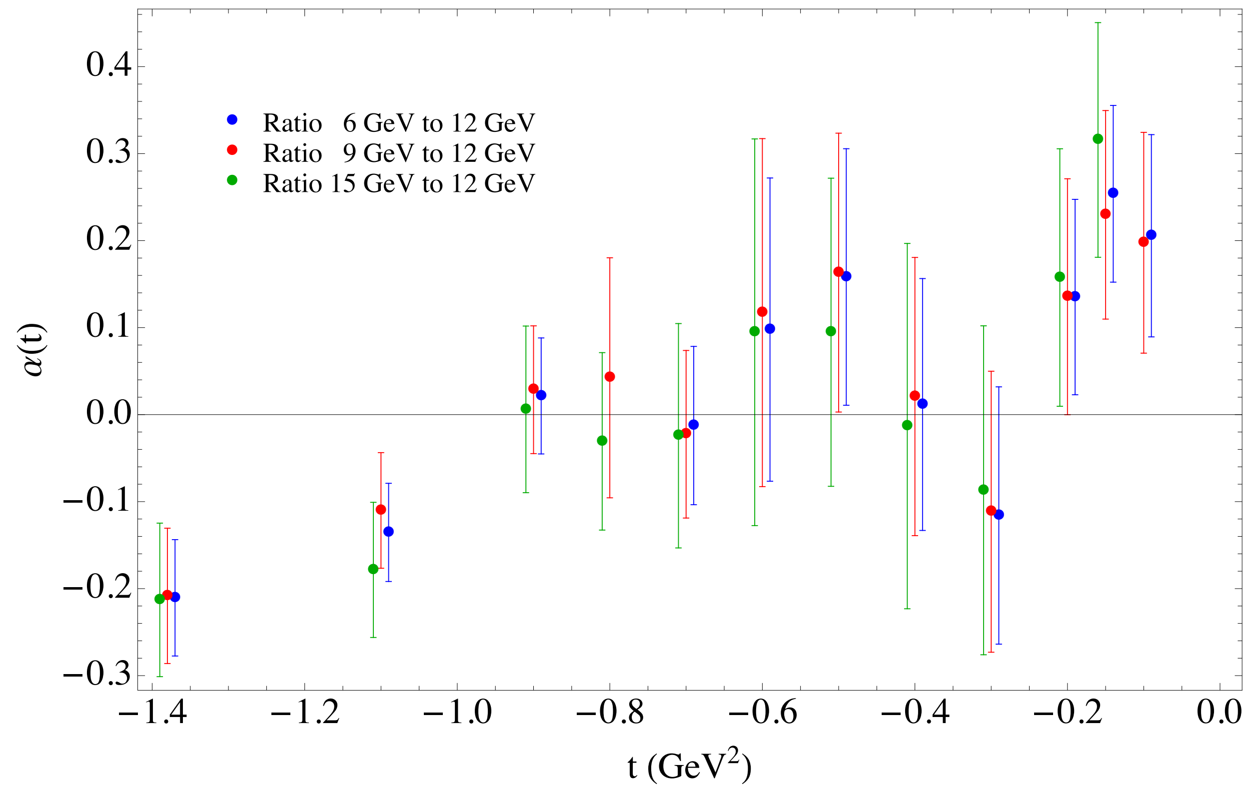


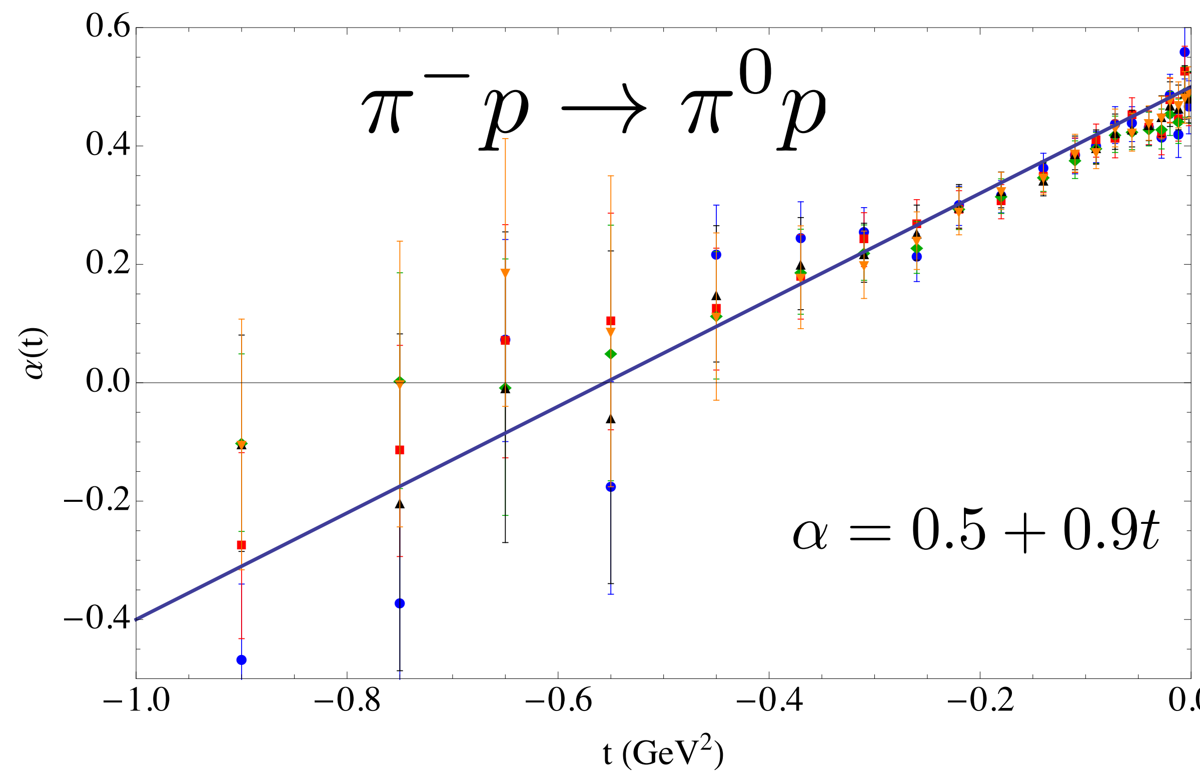
$$A \propto \beta(t) s^{\alpha(t)}$$

$$\frac{d\sigma}{dt} \propto \frac{1}{p^2} \beta^2(t) s^{2\alpha(t)}$$

$$\alpha_{\text{eff}} = \frac{1}{2} \log\left(\frac{p^2 \frac{d\sigma}{dt}}{p_0^2 \frac{d\sigma_0}{dt}}\right) \log^{-1}\left(\frac{s}{s_0}\right)$$

Correct energy dependence
BUT
multiple contributions



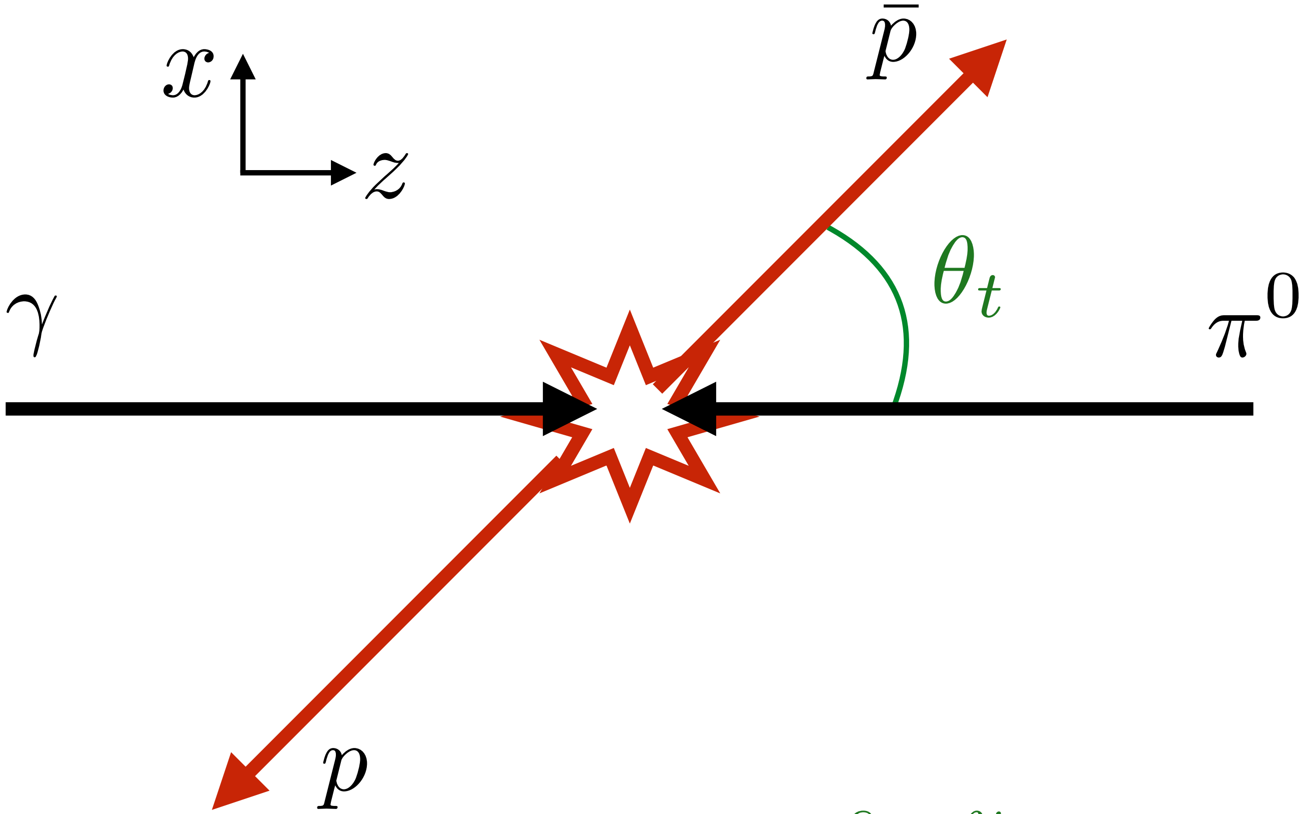


T-Channel Kinematics

$$\gamma(p_1, \lambda_1) + \pi^0(-p_3, 0) \to \bar{p}(-p_2, \lambda_2) + p(p_4, \lambda_4)$$

pion mass: μ
nucleon mass: m

$$t = (p_1 - p_3)^2 > 0$$
$$s = (p_1 + p_2)^2 < 0$$



$$\cos\theta_t = \frac{s-u}{4 q_t q_t'}$$

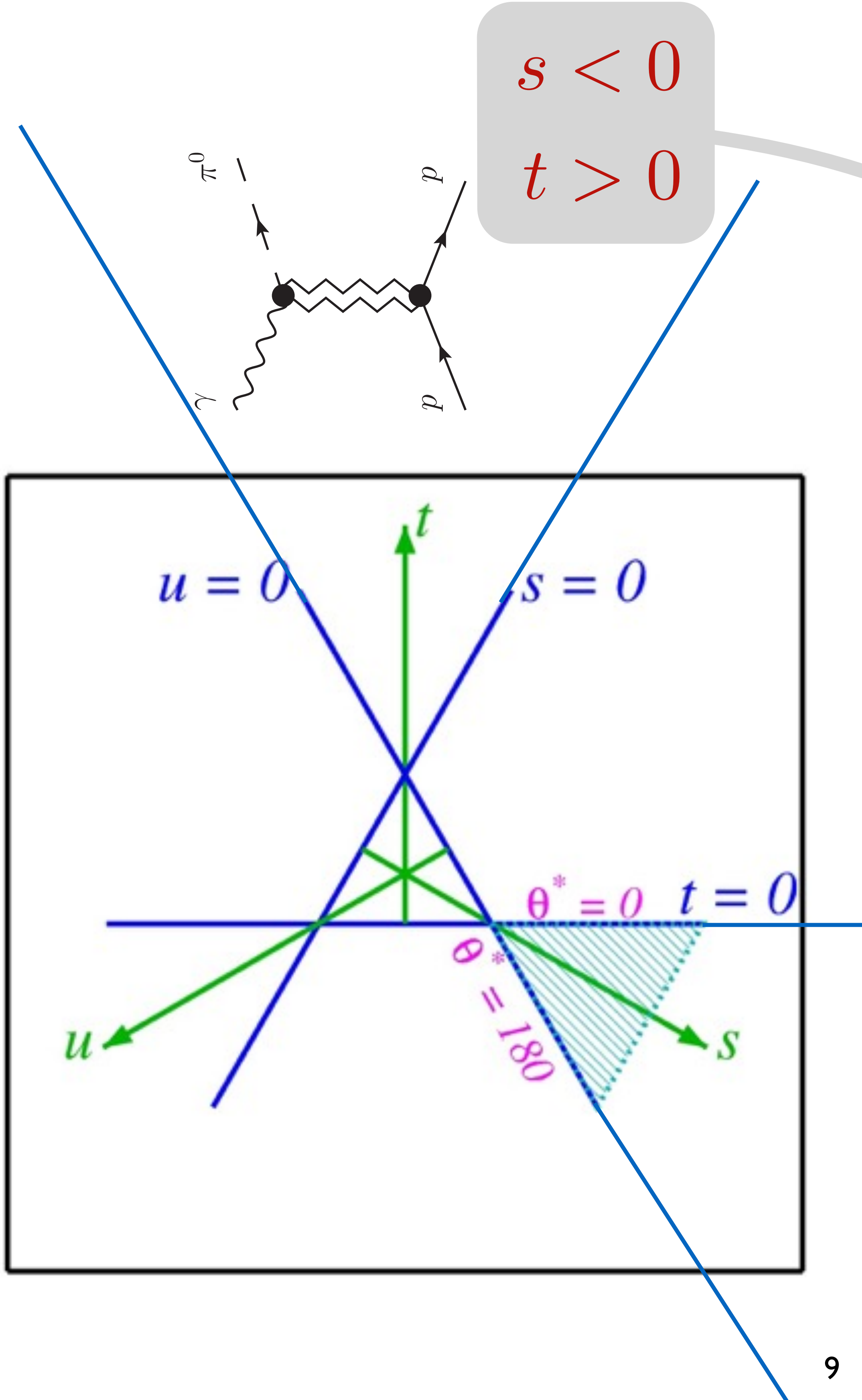
$$E_\gamma^t = \frac{t-\mu^2}{2\sqrt{t}}$$

$$E_\pi^t = \frac{t+\mu^2}{2\sqrt{t}}$$

$$E_p^t = \frac{t}{2}$$

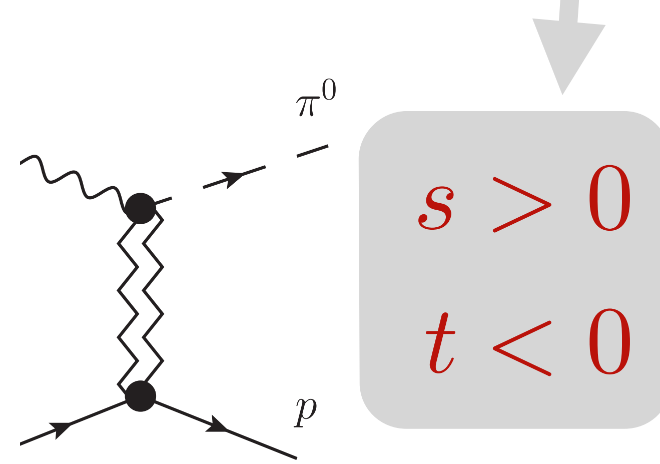
initial momentum: $q_t^2 = E_\gamma^{t2} = E_\pi^{t2} - \mu^2$
final momentum: $q_t'^2 = t/4 - m^2 = E_p^{t2} - m^2$

Mandelstam Plane

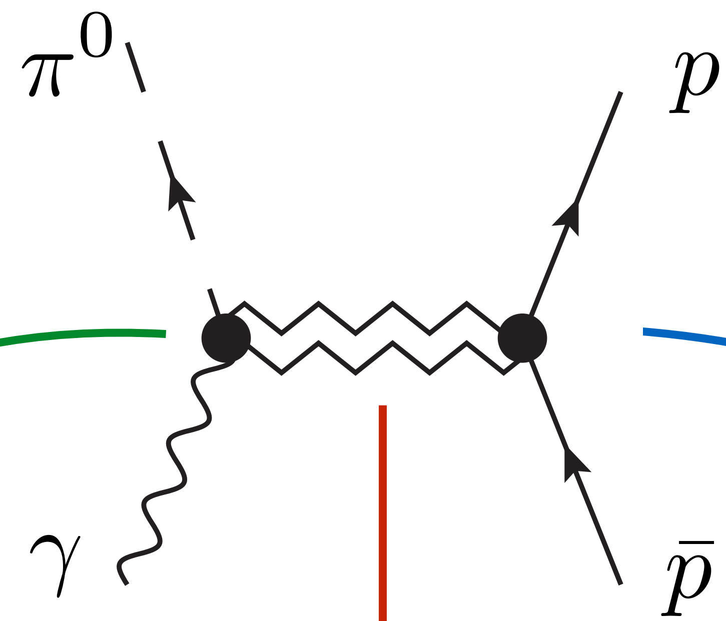


Build a model for vector production in the t-channel

Continue it to the s-channel



Vector Pole Model



$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

$$i\epsilon_{\alpha\beta\mu\nu}\, p_3^\beta\; p_1^\mu \varepsilon^\nu(p_1, \lambda_1)$$

$$\frac{-g^{\alpha\bar{\alpha}} + (p_1 - p_3)^\alpha (p_1 - p_3)^{\bar{\alpha}}}{m_V^2 - t}$$

$$\bar{u}(p_4, \lambda_4)\;\left[g_4 \gamma^{\bar{\alpha}} + g_1 \gamma^{[\bar{\alpha}}\gamma^{\sigma]}(p_2 - p_4)_\sigma\right]\; v(-p_2, \lambda_2)$$

CGLN Basis

[CGLN Phys. Rev. 106 1345]

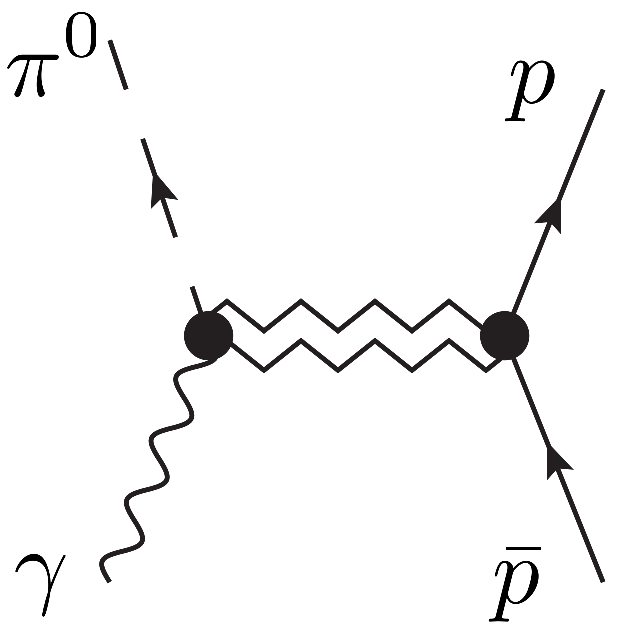
$$M_1 = \frac{1}{2}\gamma_5\,\gamma^\mu\gamma^\nu\,F_{\mu\nu}$$

$$F^{\mu\nu} = \varepsilon^\mu(p_1,\lambda_1)p_1^\nu - \varepsilon^\nu(p_1,\lambda_1)p_1^\mu$$

$$M_2 = \gamma_5\,p_3^\mu\,(p_2+p_4)^\nu\,F_{\mu\nu}$$

$$M_3 = -\gamma_5\,\gamma^\mu\,p_3^\nu\,F_{\mu\nu}$$

$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\,\gamma^\alpha\,p_3^\beta\,F_{\mu\nu}$$



$$= \bar{u}(p_4,\lambda_4)\;[g_1(tM_1 - M_2) + g_4 M_4]\;v(-p_2,\lambda_2)\quad \times \frac{1}{t-m_V^2}$$

$$= A^t_{\lambda_2\lambda_4;\lambda_1}(s,t)$$

Crossing from t- to s-channel

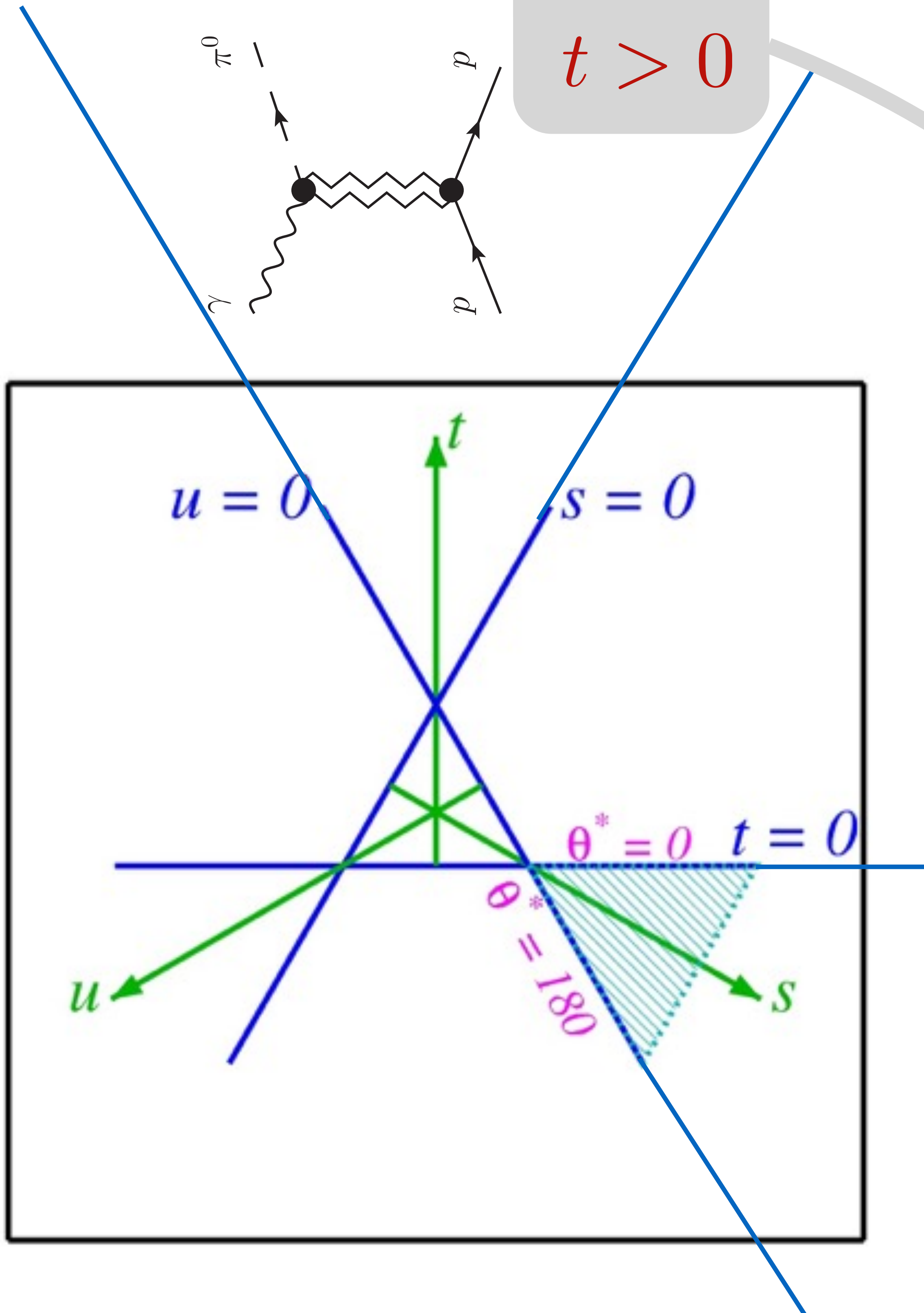
$s < 0$

$t > 0$

$$\bar{u}(p_4, \lambda_4)\,[g_1(tM_1 - M_2) + g_4 M_4]\,v(-p_2, \lambda_2)\frac{1}{t - m_V^2}$$

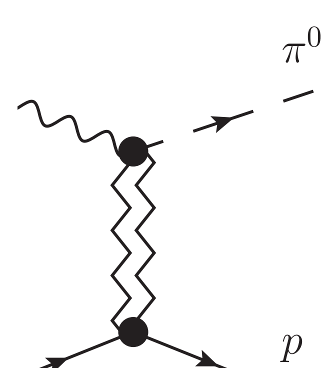
$$\sim \frac{s}{t - m_V^2}$$

$$\frac{s}{t - m_{J=1}^2} + \frac{s^3}{t - m_{J=3}^2} + \ldots$$



$$\bar{u}(p_4, \mu_4)\,[g_1(tM_1 - M_2) + g_4 M_4]\,u(p_2, \mu_2)$$

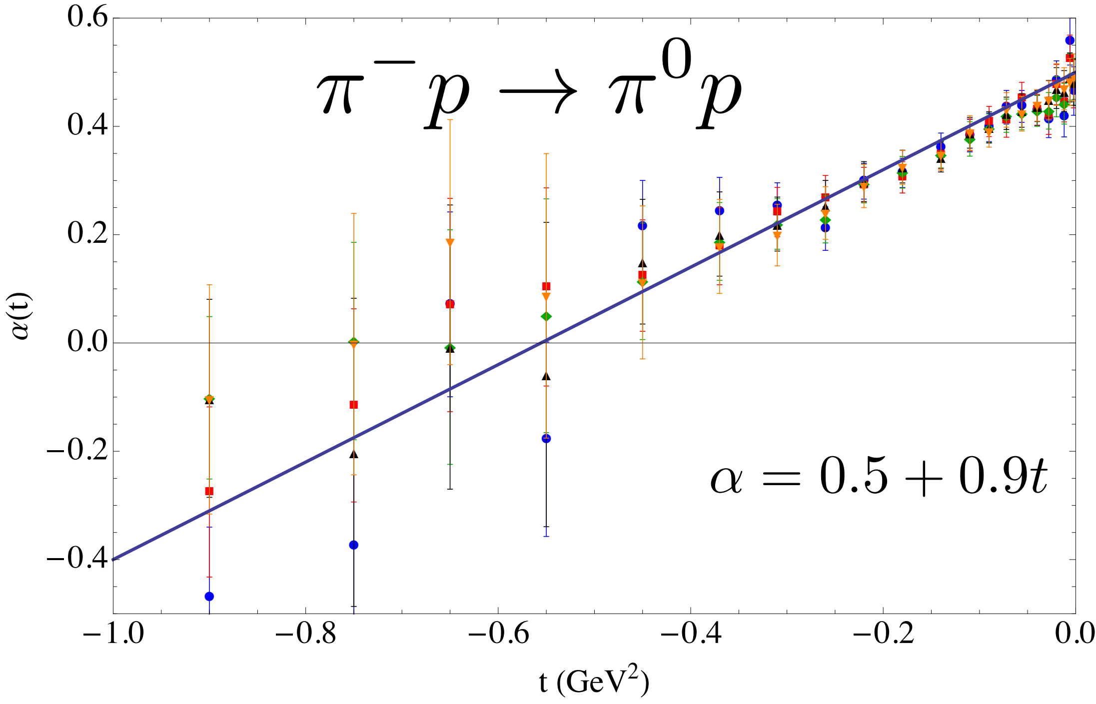
$$\times \frac{\beta(t)}{s}\,\frac{(s)^{\alpha(t)} - (-s)^{\alpha(t)}}{2 \sin \pi\alpha(t)}$$



$s > 0$

$t < 0$

Residues

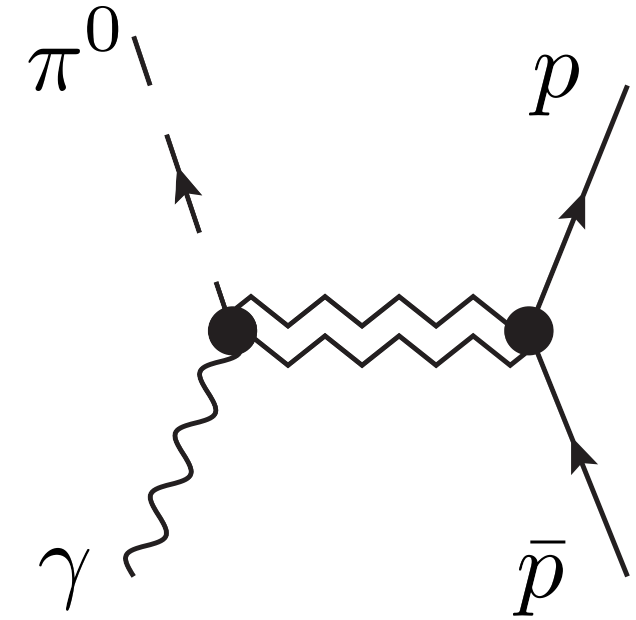


$$\bar{u}(p_4,\mu_4)\left[g_1(tM_1 - M_2) + g_4 M_4\right] u(p_2,\mu_2)$$

$$\times \beta(t) \frac{1 - e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)} s^{\alpha(t)-1}$$

Sinus produces poles at $\alpha = 0, \pm 1, \pm 2, \ldots$

Signature produces zeros at $\alpha = 0, \pm 2, \pm 4, \ldots$



Amplitudes has poles at $\alpha = \pm 1, \pm 3, \ldots$

Amplitudes is finite at $\alpha = 0, \pm 2, \pm 4, \ldots$

Trajectory is the spin of the particle exchanged

$$\alpha(0.75^2) = 1$$
$$\alpha(-0.55) = 0$$
$$\alpha(-1.66) = -1$$

$$M = 1 \to \alpha \neq 0$$
$$\to \beta(-0.55) = 0$$
$$\to \beta(t) \propto \alpha(t)$$

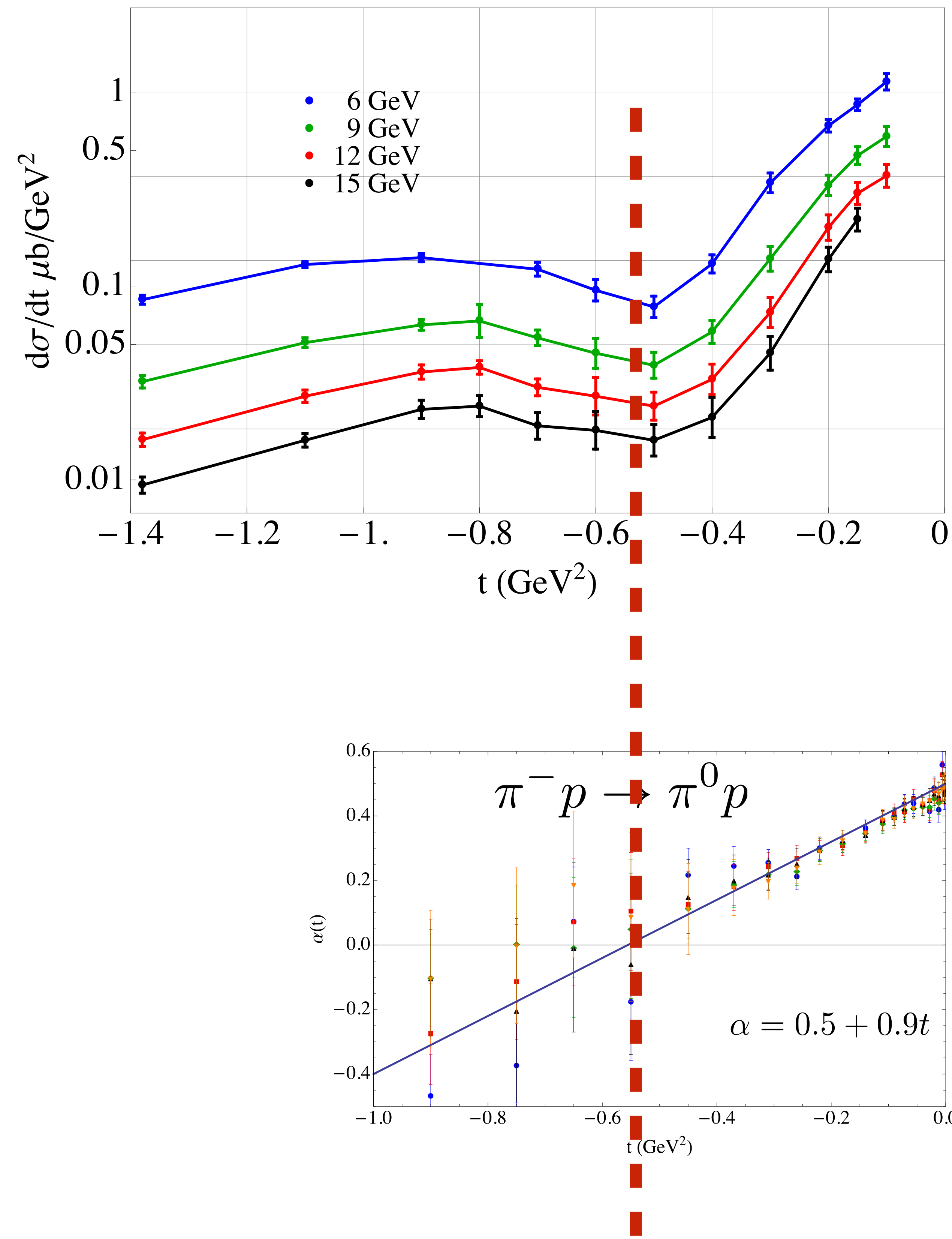
$$\beta(t) \propto \alpha(t)(\alpha(t)+1)$$

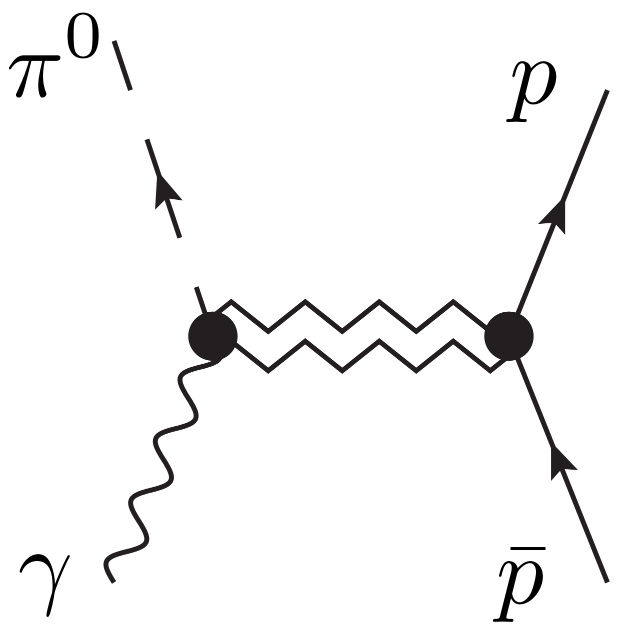
Residues

$$\bar{u}(p_4,\mu_4)\left[g_1(tM_1 - M_2) + g_4 M_4\right] u(p_2,\mu_2)$$

$$\times\beta(t)\frac{1-e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}s^{\alpha(t)-1}$$

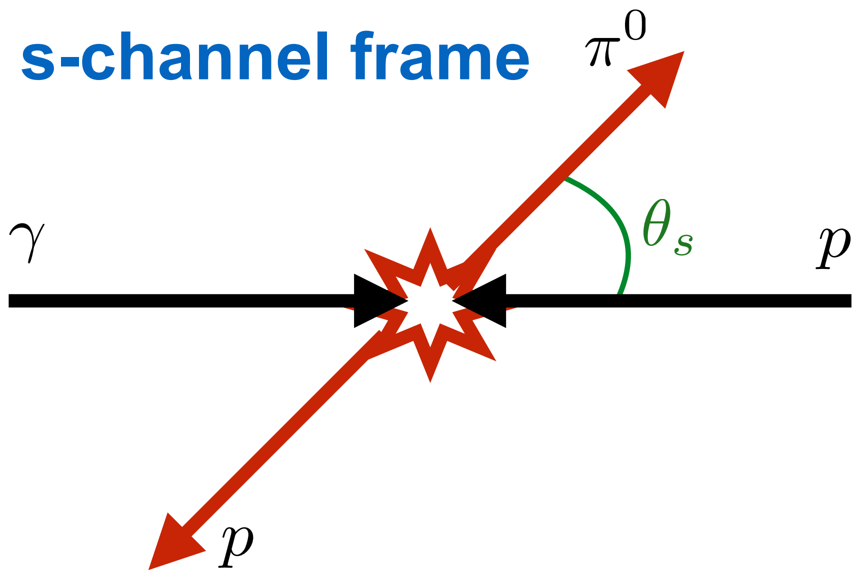
$$\beta(t) \propto \alpha(t)(\alpha(t)+1)$$



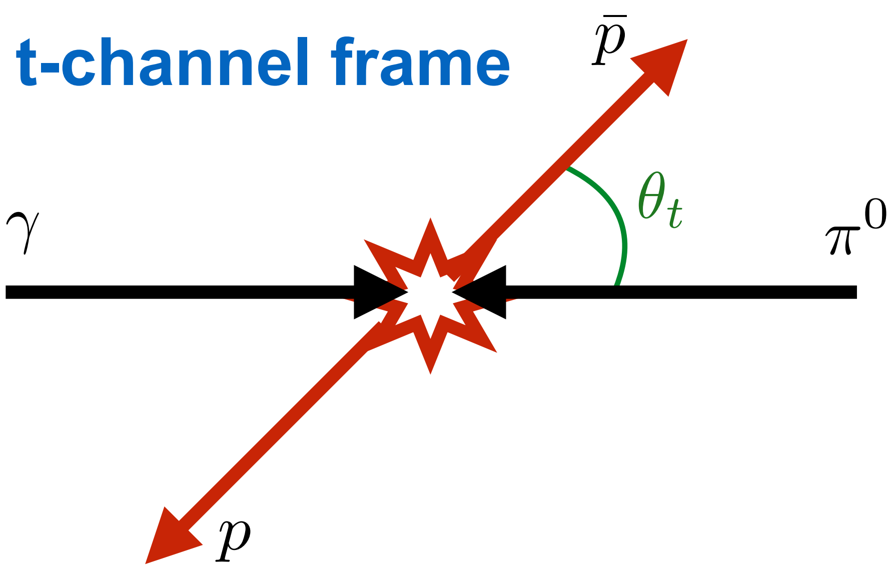


Helicity Amplitudes

s-channel frame



t-channel frame



$$A^s_{\mu_4;\mu_2\mu_1} = \bar{u}(p_4,\mu_4)\left[\sum_{i=1}^{4} A_i M_i\right] u(p_2,\mu_2)$$

$$A^t_{\lambda_4\lambda_2;\lambda_1} = \bar{u}(p_4,\lambda_4)\left[\sum_{i=1}^{4} A_i M_i\right] v(-p_2,\lambda_2)$$

Observables can be expressed in either frame.
Choose the t-channel for simplicity

model

$$A_1 = -tA_2$$

$$A_2 = -g_1\alpha(t)\left(\alpha(t)+1\right)\frac{1-e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}s^{\alpha(t)-1}$$

$$A_3 = 0$$

$$A_4 = g_4\alpha(t)\left(\alpha(t)+1\right)\frac{1-e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}s^{\alpha(t)-1}$$

Helicity Amplitudes

$$A_1 = -tA_2$$

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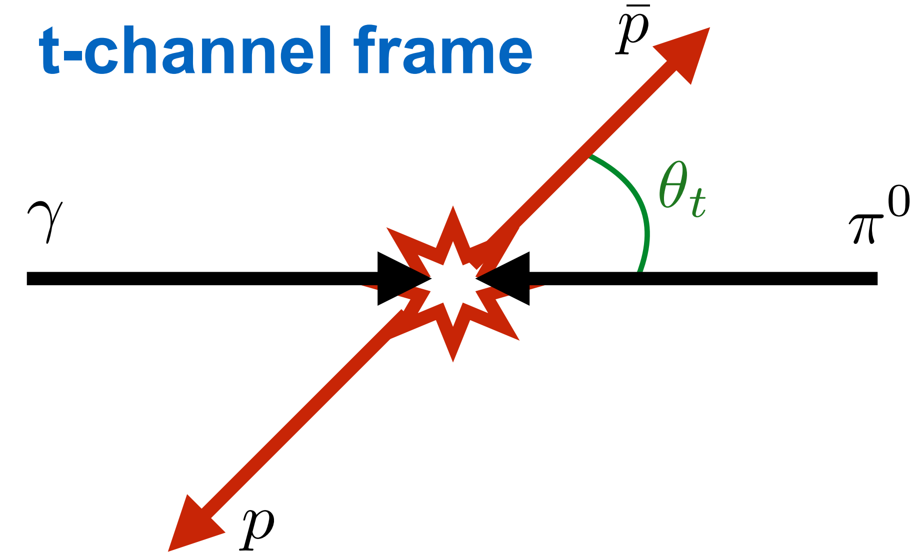
t-channel frame



$$A^t_{\lambda_4\lambda_2;\lambda_1} = \bar{u}(p_4,\lambda_4)\left[\sum_{i=1}^{4} A_i M_i\right] v(-p_2,\lambda_2)$$

correct angular dependence → $d^1_{\lambda_2-\lambda_4,\lambda_1}(\theta_t)$

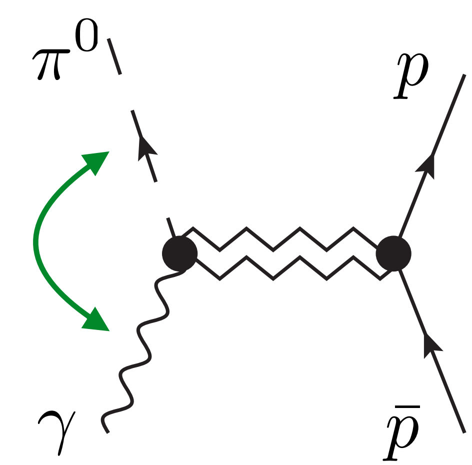
$$A^t_{++,1} = \sqrt{2}q_t\frac{\sin\theta_t}{2}\left[\sqrt{t}\left(A_1-2MA_4\right)-2q_t'\left(A_1+tA_2\right)\right]$$

$$A^t_{--,1} = \sqrt{2}q_t\frac{\sin\theta_t}{2}\left[\sqrt{t}\left(A_1-2MA_4\right)+2q_t'\left(A_1+tA_2\right)\right]$$

$$A^t_{+-,1} = \sqrt{2}q_t\sin^2\frac{\theta_t}{2}\left[-\left(2MA_1-tA_4\right)+2q_t'\sqrt{t}A_3\right]$$

$$A^t_{-+,1} = \sqrt{2}q_t\cos^2\frac{\theta_t}{2}\left[-\left(2MA_1-tA_4\right)-2q_t'\sqrt{t}A_3\right]$$

not in this model (vector pole)



helicity flip at the gamma-pion vertex

Differential Cross Section: Model I

$$\frac{d\sigma}{dt} = \frac{1}{64\pi m^2 E_\gamma^2} \sum_\lambda \left|A_\lambda^t\right|^2$$

model

$$A_1 = -tA_2$$

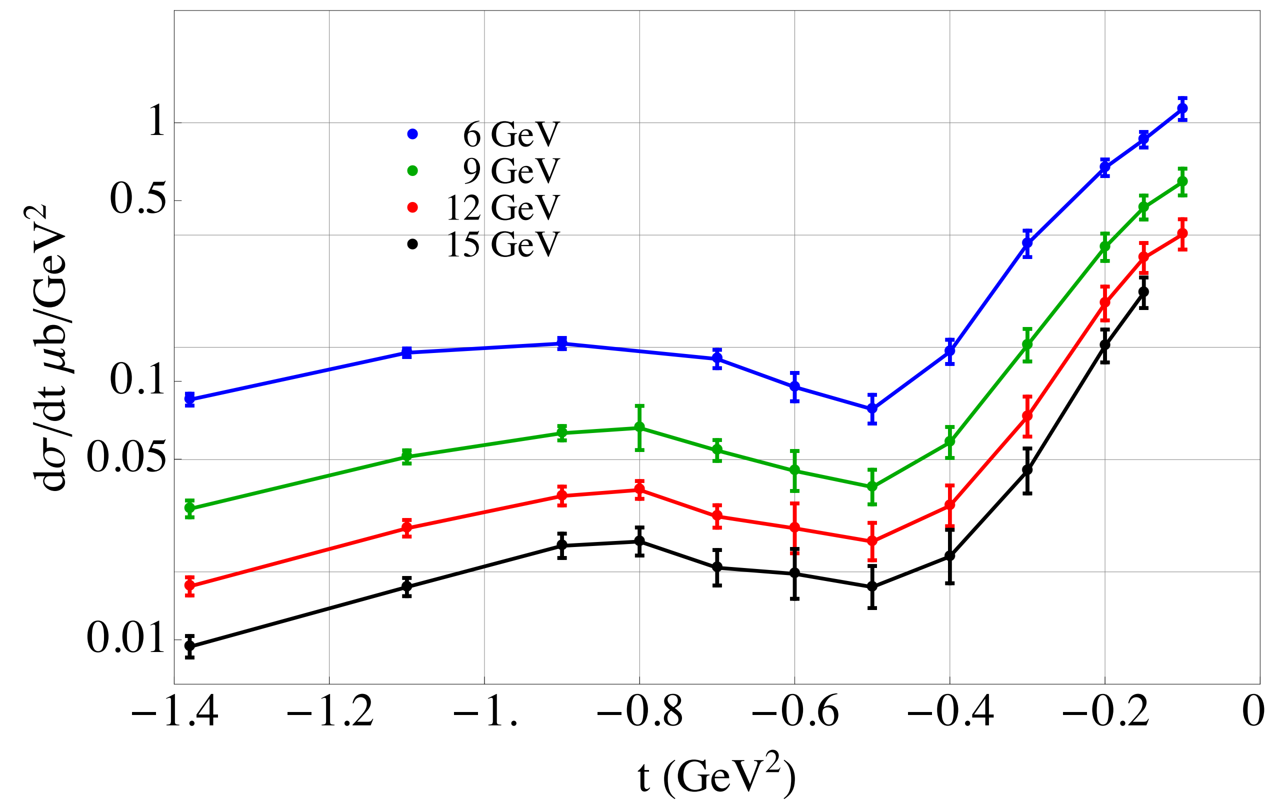
$$A_2 = -g_1\alpha(t)\left(\alpha(t)+1\right)\frac{1-e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)} s^{\alpha(t)-1}$$

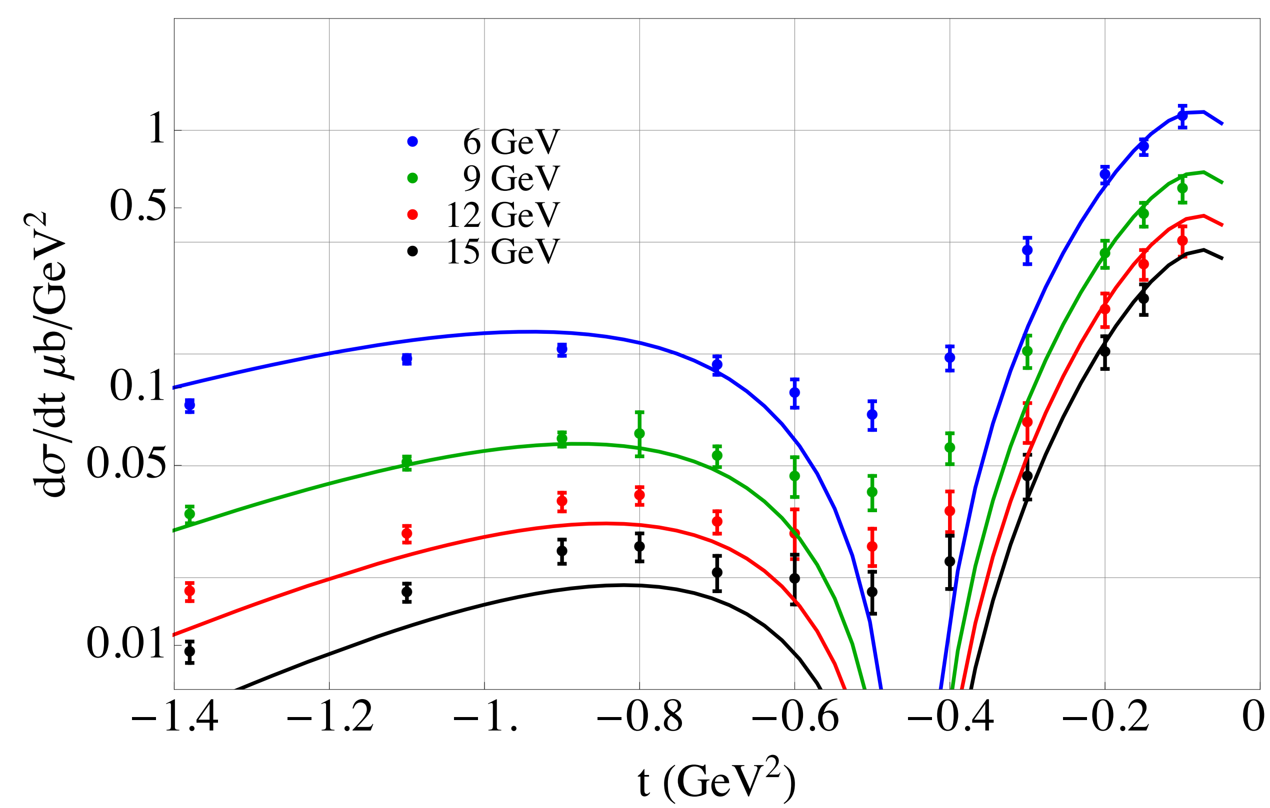
$$A_3 = 0$$

$$A_4 = g_4\alpha(t)\left(\alpha(t)+1\right)\frac{1-e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)} s^{\alpha(t)-1}$$

$$\alpha(t) = \alpha_0 + \alpha' t$$

	Estimate	Standard Error
g4	42.0091	3.55691
g1	13.7525	16.6151
α0	0.345466	0.0092382
αp	0.772804	0.0161112





Differential Cross Section: Model II

model

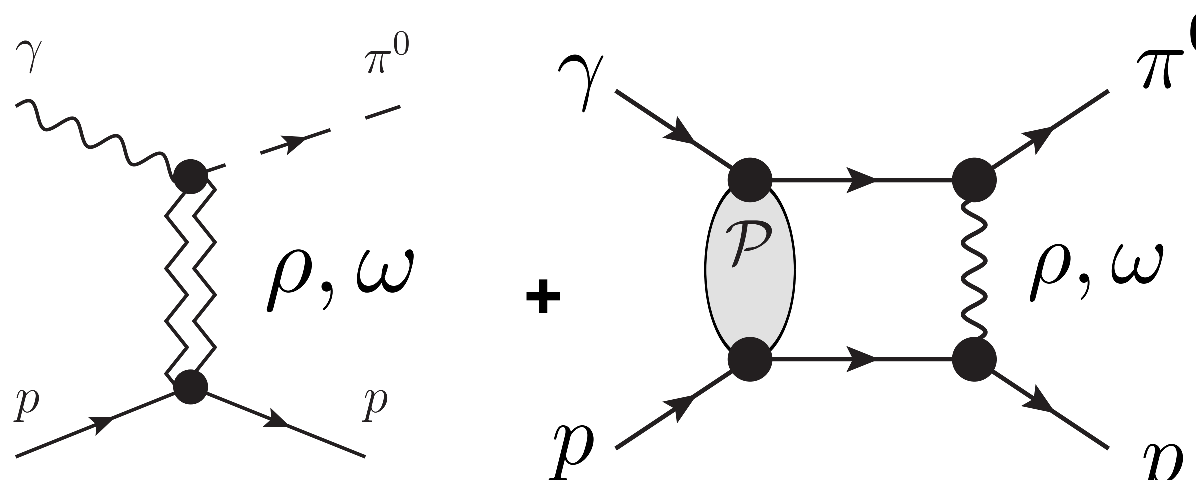
$$A_1 = -tA_2$$

$$A_2 = -g_1\alpha(t)\left(\alpha(t)+1\right)\frac{1-e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}s^{\alpha(t)-1} - \frac{g_{c1}}{\log(s)}\alpha_c(t)\left(\alpha_c(t)+1\right)\frac{1-e^{-i\pi\alpha_c(t)}}{2\sin\pi\alpha_c(t)}s^{\alpha_c(t)-1}$$

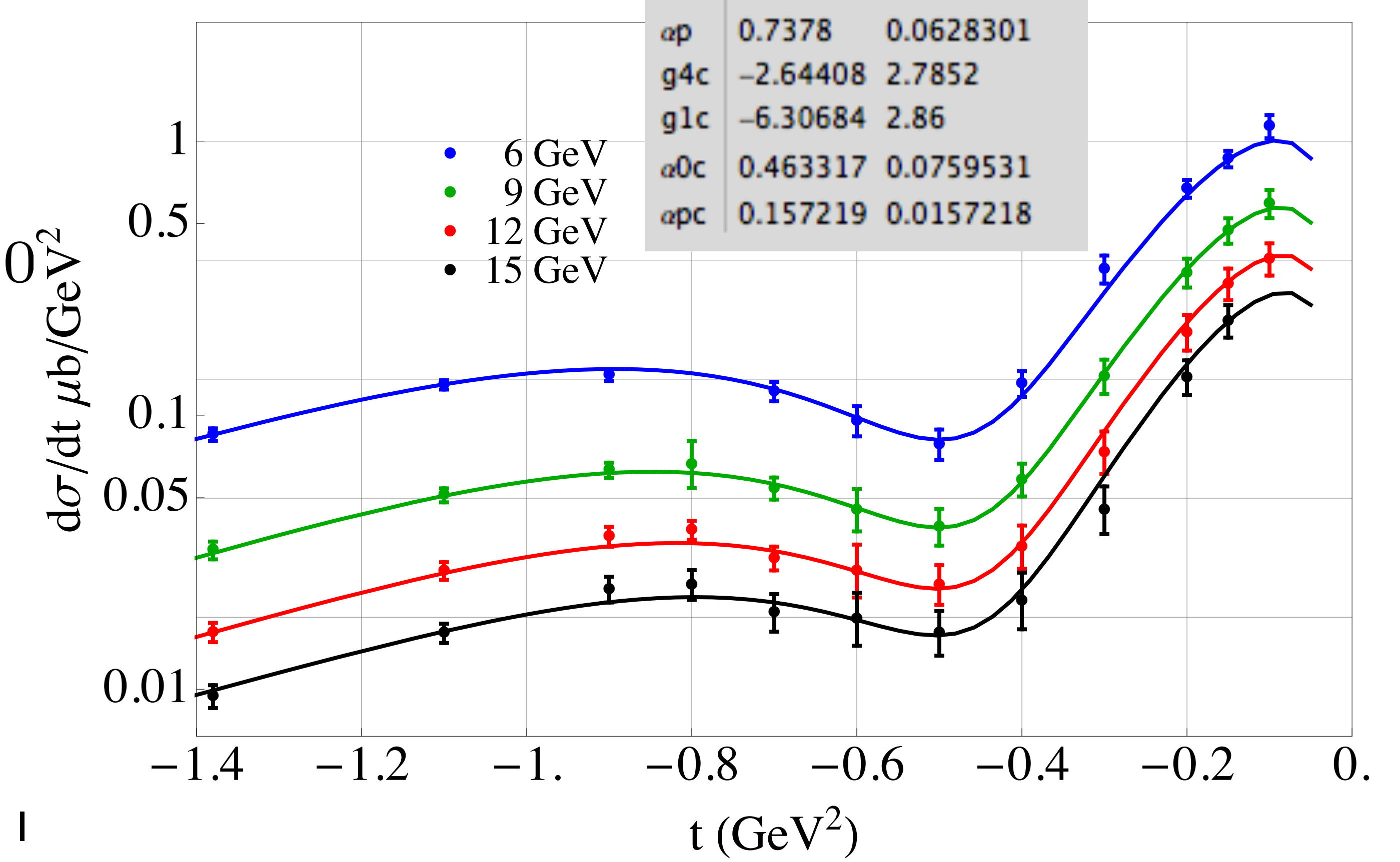
$$A_3 = 0$$

$$A_4 = g_4\alpha(t)\left(\alpha(t)+1\right)\frac{1-e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}s^{\alpha(t)-1} + \frac{g_{c4}}{\log(s)}\alpha_c(t)\left(\alpha_c(t)+1\right)\frac{1-e^{-i\pi\alpha_c(t)}}{2\sin\pi\alpha_c(t)}s^{\alpha_c(t)-1}$$

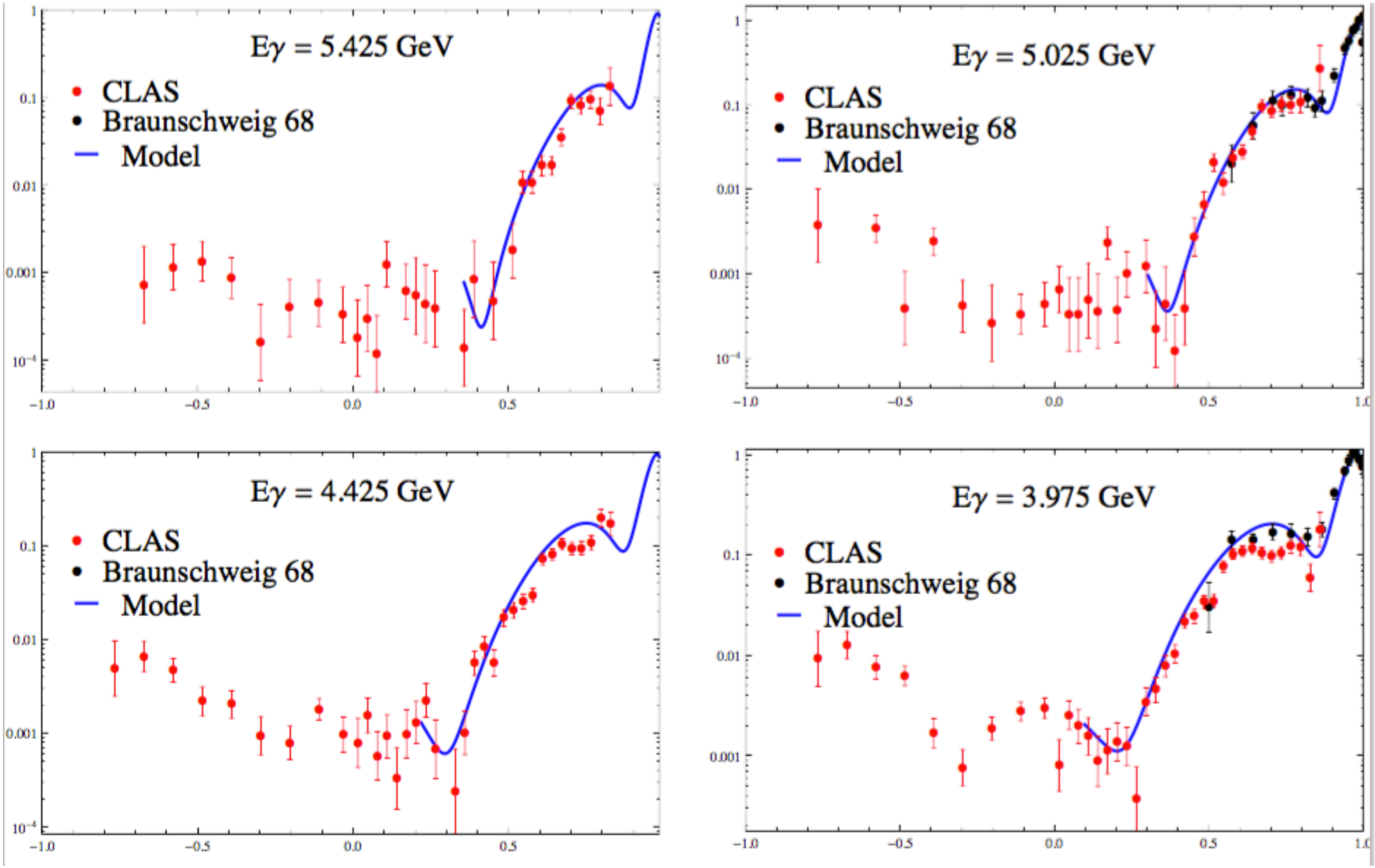
Add another contribution:



	Estimate	Standard Error
g4	-35.5016	4.6483
g1	-9.76152	9.6737
α0	0.341744	0.0179134
αp	0.7378	0.0628301
g4c	-2.64408	2.7852
g1c	-6.30684	2.86
α0c	0.463317	0.0759531
αpc	0.157219	0.0157218

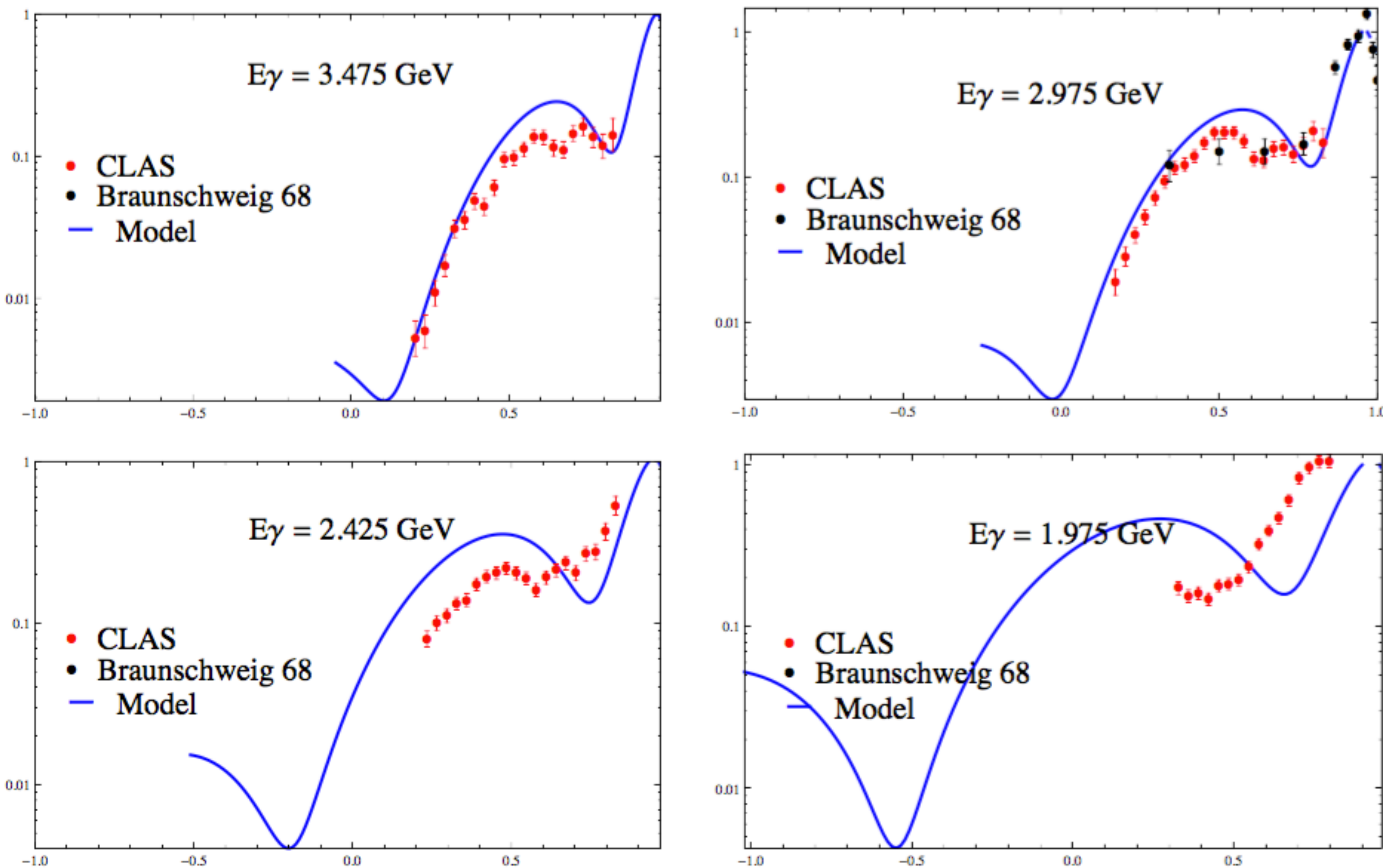


CLAS Preliminary Data



Data from M. Kunkel

CLAS Preliminary Data

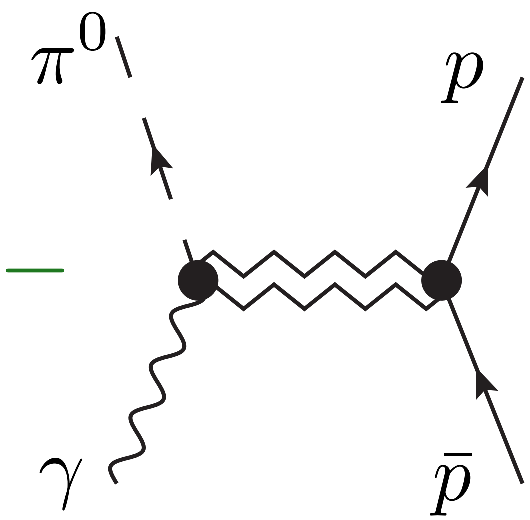


Data from M. Kunkel

T-Channel Quantum Numbers

What can be exchanged in the t-channel ?

$C = -$

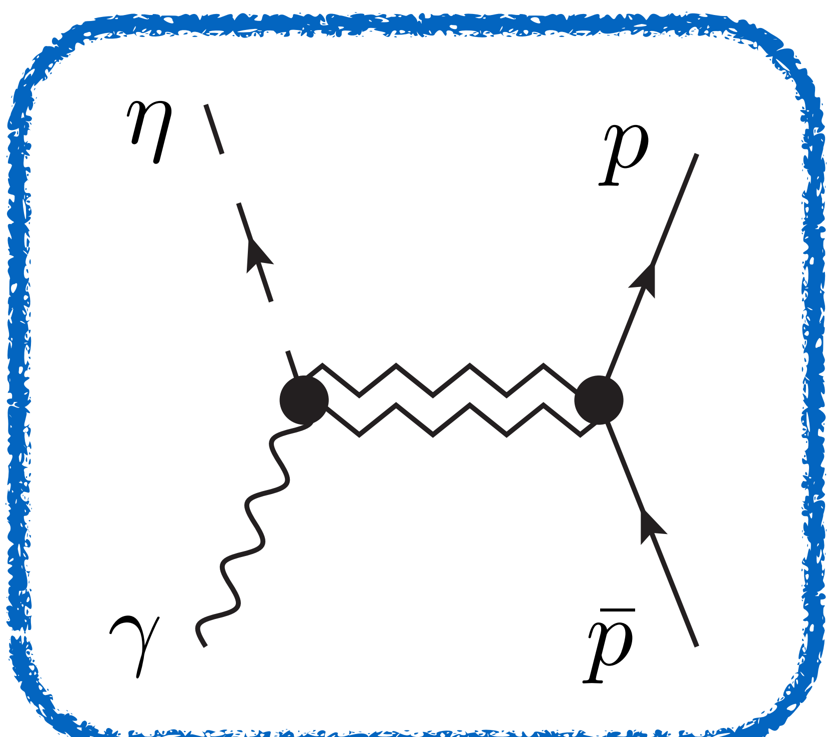


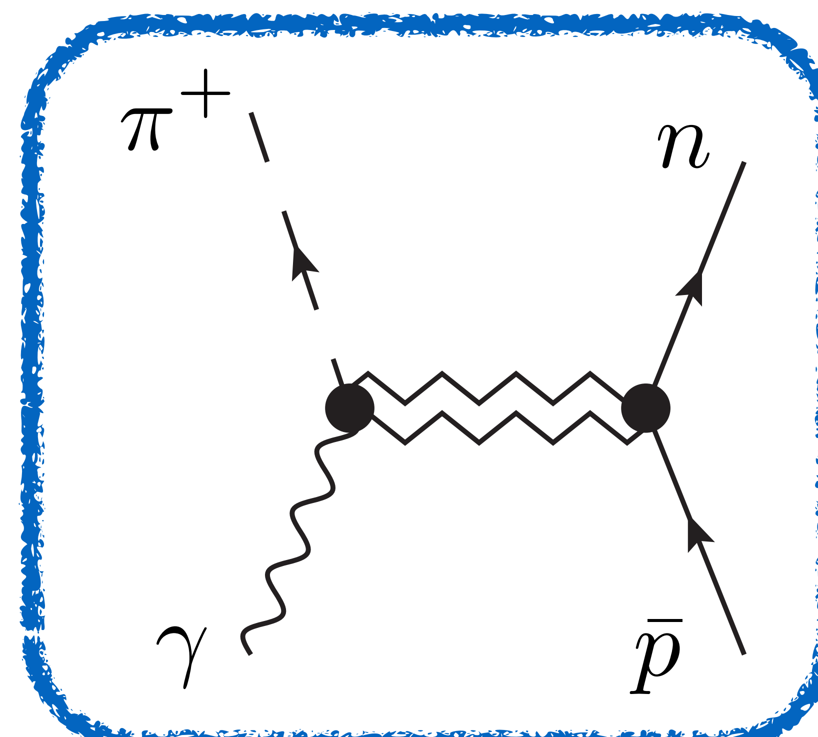
$I = 0, 1$

$(1, 3, 5, \ldots)^{--}$ → $I^G = 0^- : \omega$; $I^G = 1^+ : \rho$

$(1, 3, 5, \ldots)^{+-}$ → $I^G = 0^- : h$; $I^G = 1^+ : b$

$(\cancel{0}, 2, 4, \ldots)^{--}$





T-Channel Quantum Numbers

			$\gamma\pi^0$	$\gamma\eta$	$\gamma\pi^\pm$
$F_1 = -A_1 + 2MA_4,$	$\eta = +1,$	$CP = +1,$	ρ, ω	ρ, ω	a
$F_2 = A_1 + tA_2,$	$\eta = -1,$	$CP = -1,$	b, h	b, h	π, b
$F_3 = 2MA_1 - tA_4,$	$\eta = +1,$	$CP = +1,$	ρ, ω	ρ, ω	a
$F_4 = A_3,$	$\eta = -1,$	$CP = +1.$	$\tilde{\rho}, \tilde{\omega}$	$\tilde{\rho}, \tilde{\omega}$	$\tilde{a}$

$\eta = P(-1)^J$ $\quad G = C(-1)^I$ $\quad I^G J^{PC}$

$\rho : 1^+(1,3,\ldots)^{--}$

$\omega : 0^-(1,3,\ldots)^{--}$

$b : 1^+(1,3,\ldots)^{+-}$

$h : 0^-(1,3,\ldots)^{+-}$

$\pi : 1^-(0,2,\ldots)^{-+}$

$a : 1^-(0,2,\ldots)^{++}$

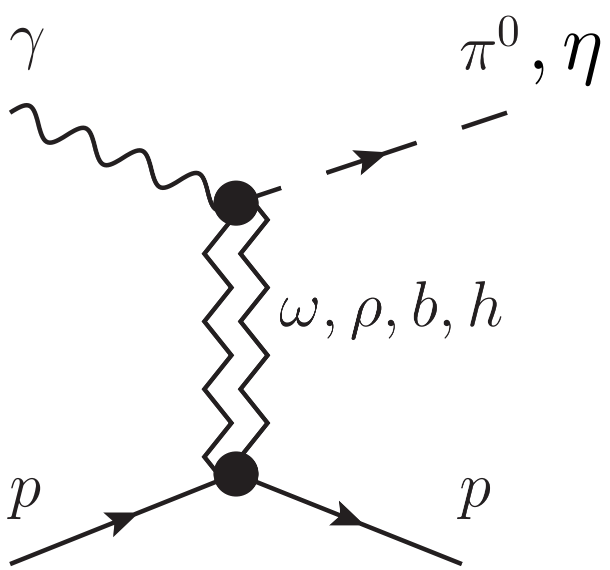
$\tilde{\rho} : 1^+(2,4,\ldots)^{--}$

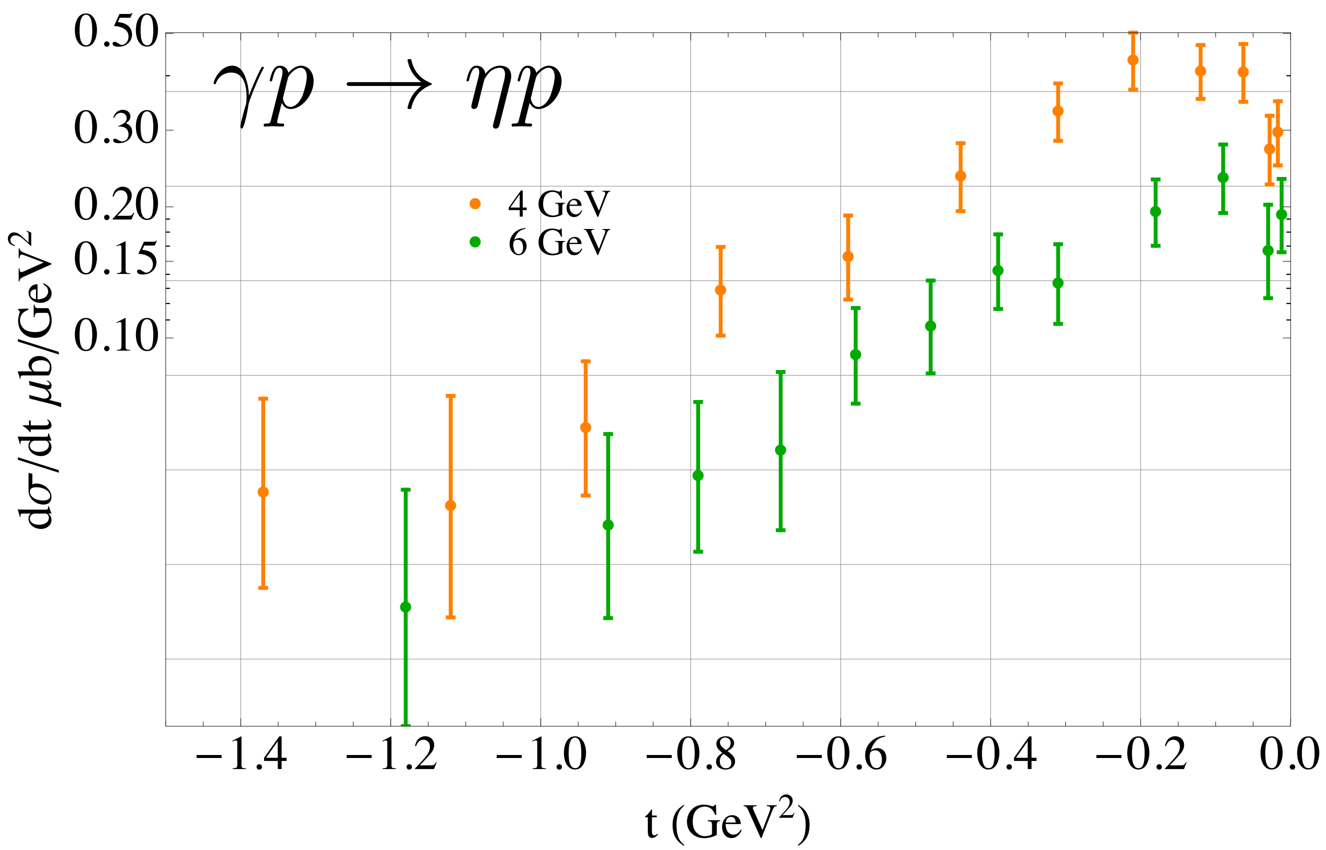
$\tilde{\omega} : 0^-(2,4,\ldots)^{--}$

$\tilde{a} : 1^-(1,3,\ldots)^{--}$

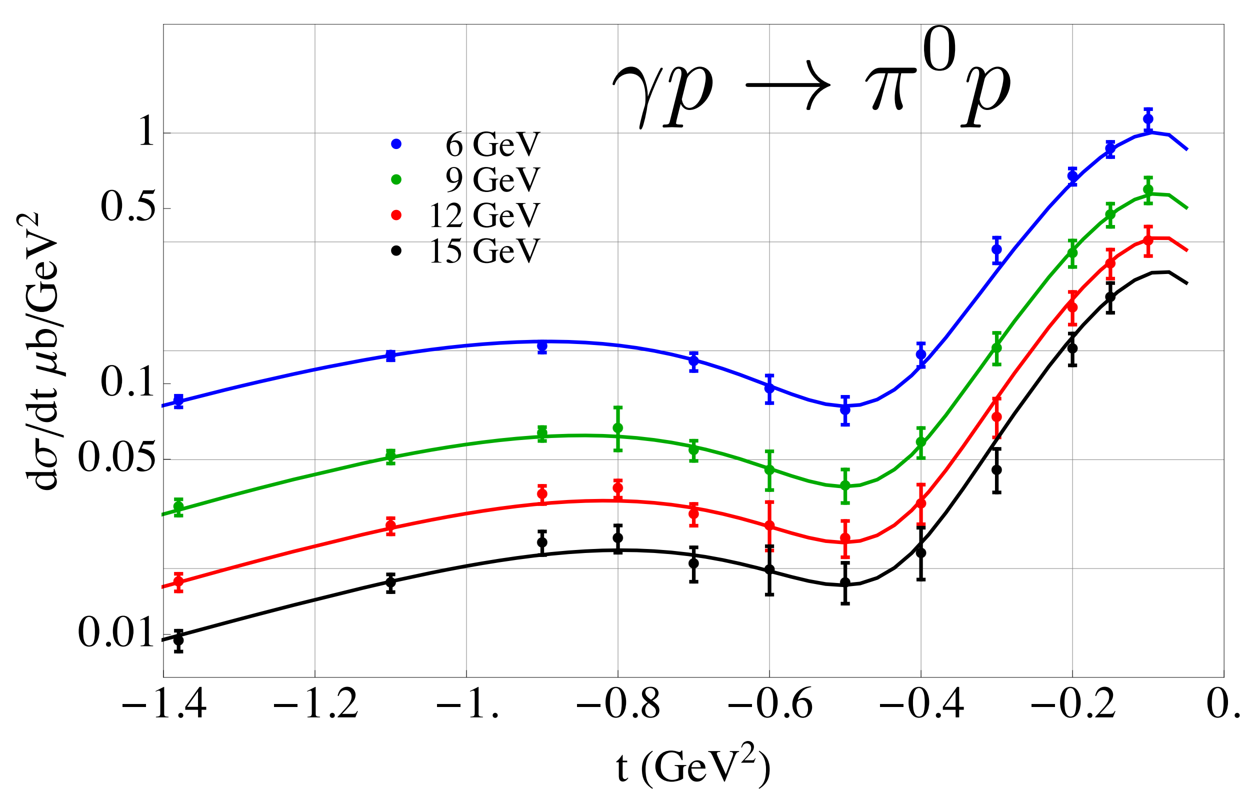
Eta Photoproduction

Same exchanges



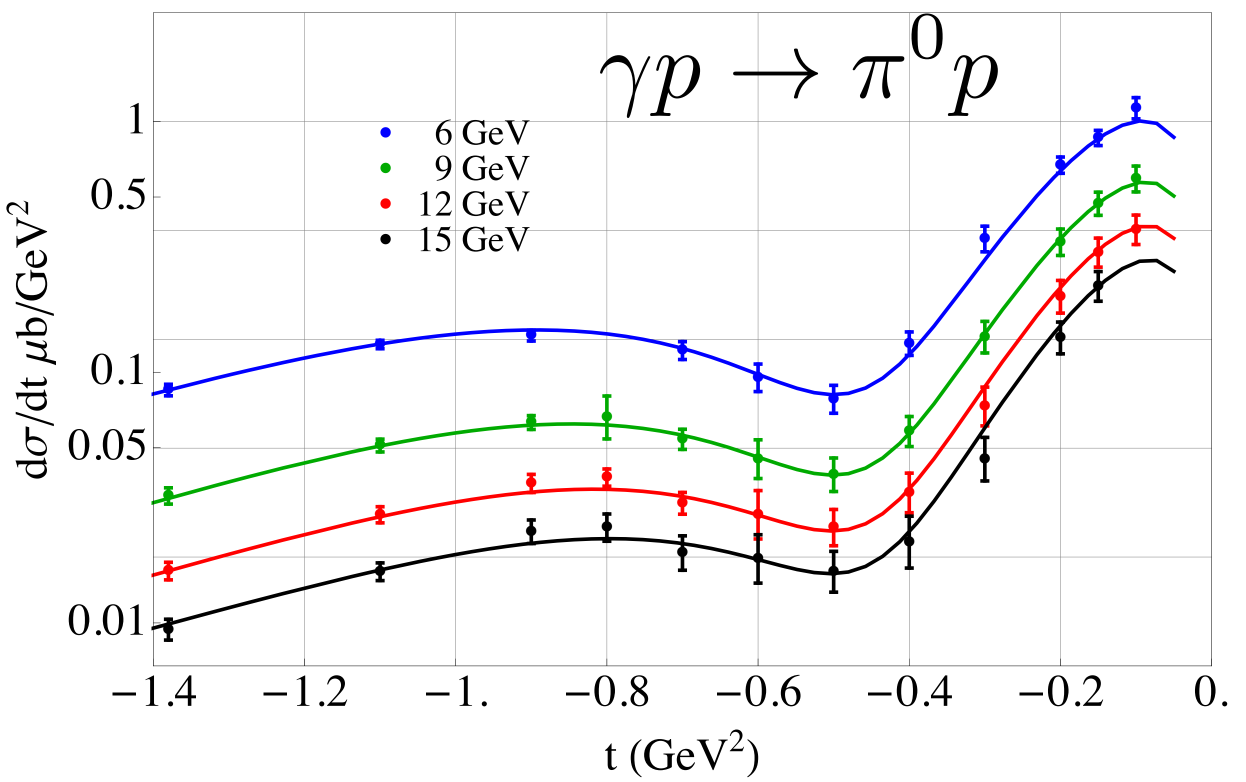


no dip at alpha=0
different residues



Summary

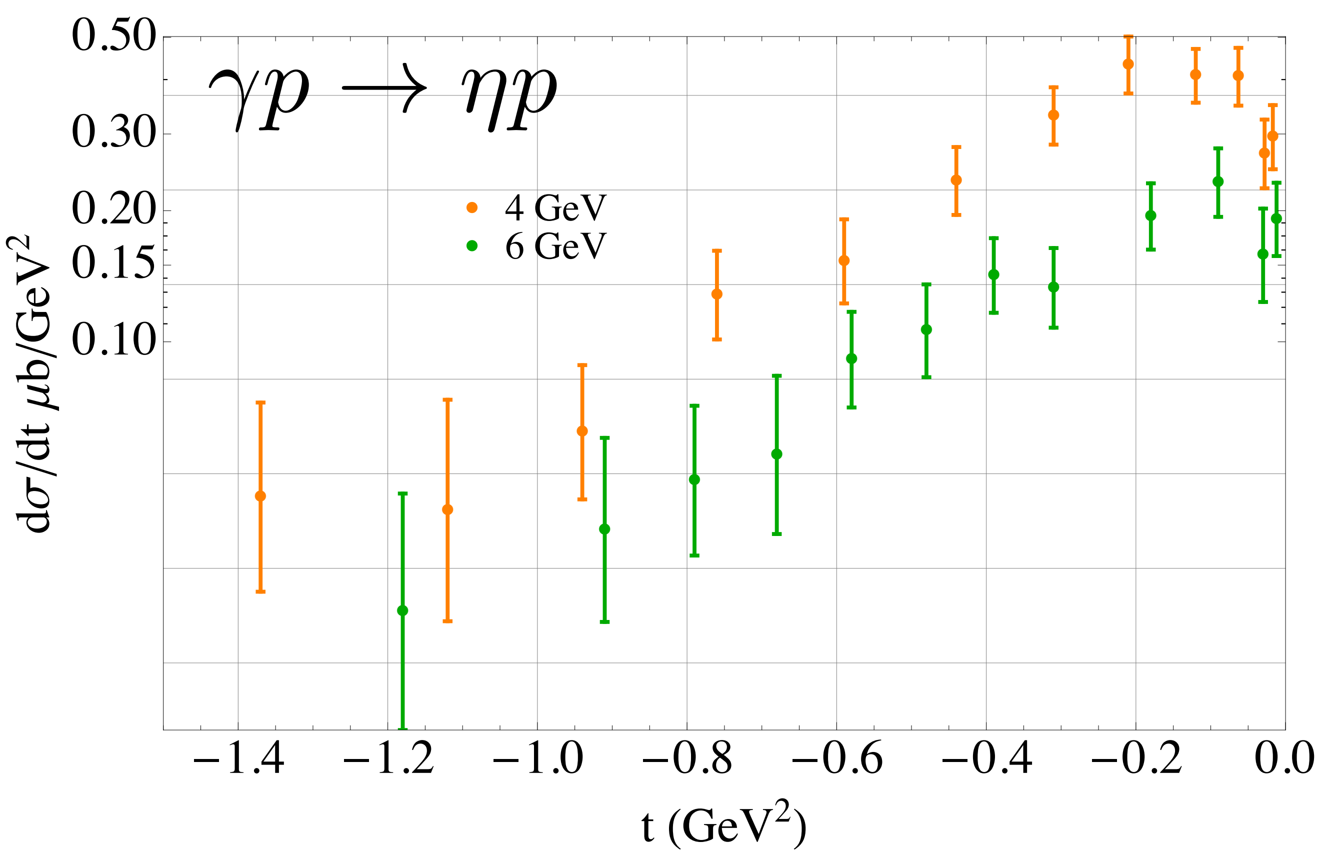
Choose appropriate variable(s)



Simple energy dependence at high energy $A \sim \beta(t) s^{\alpha(t)}$

Separate kinematics (Mi) from model (Ai)

$$A^t_{\lambda_4\lambda_2;\lambda_1} = \bar{u}(p_4,\lambda_4)\left[\sum_{i=1}^{4} A_i M_i\right] v(-p_2,\lambda_2)$$



Physics contraint on residues

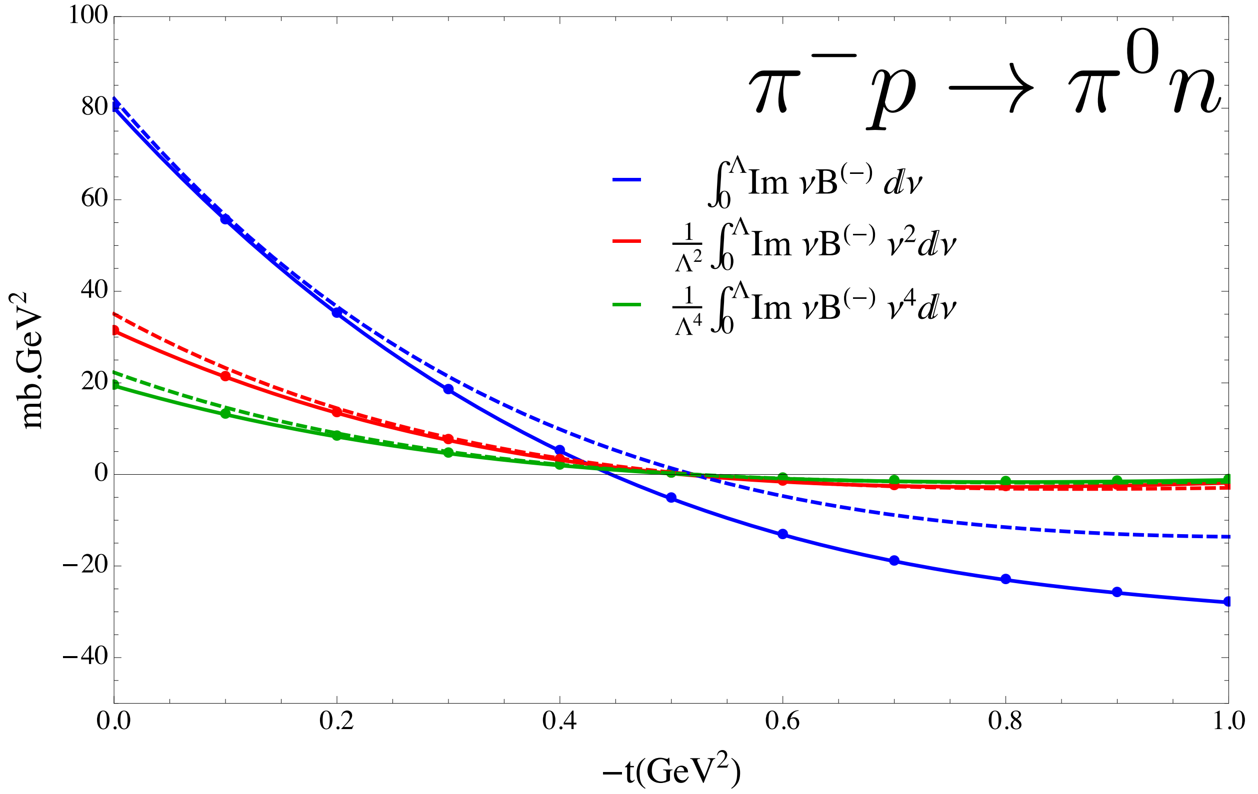
$$\beta(t) \propto \alpha(t)\left[\alpha(t)+1\right]$$

Ok for neutral pion but not for eta

Improvements

FESR constrain residues

$$\beta(t) \propto \alpha(t)\,[\alpha(t) + 1]$$

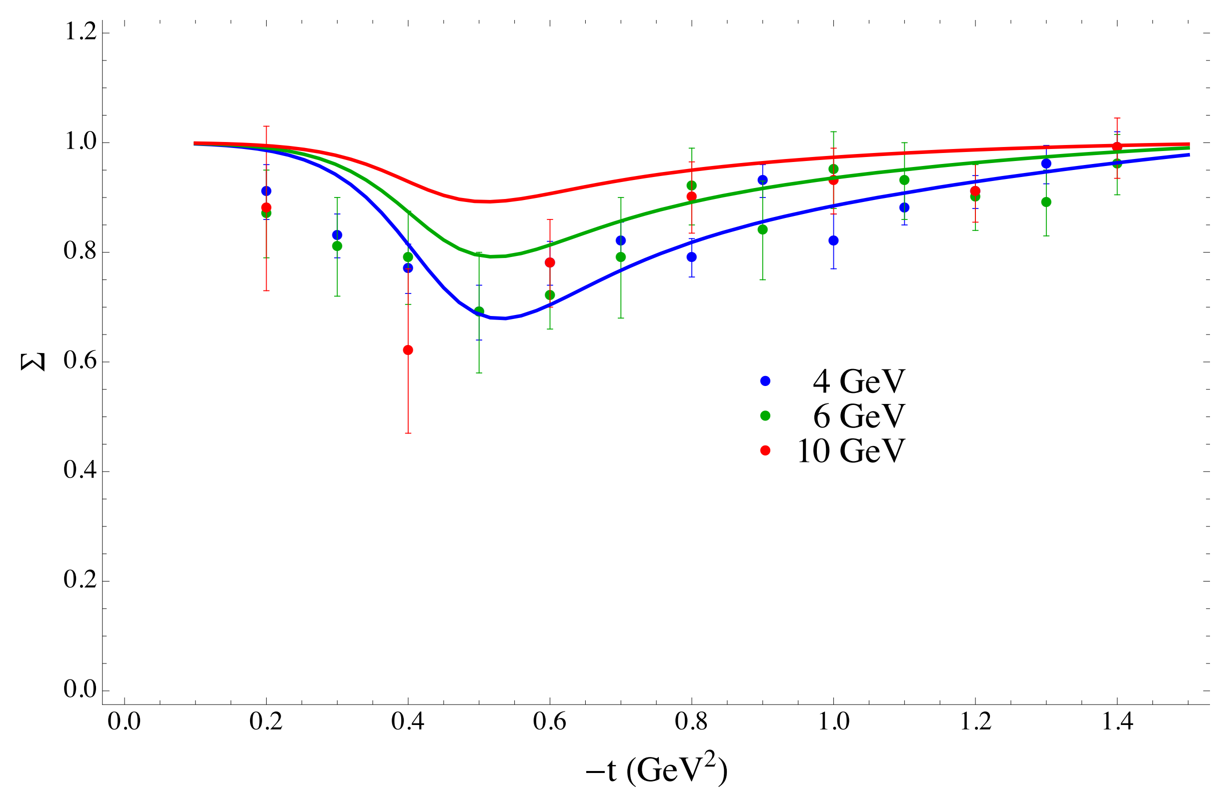


Polarization observables sub-leading contributions

Not intuitive in photoproduction

$$\Sigma = \frac{\sigma_\parallel - \sigma_\perp}{\sigma_\parallel + \sigma_\perp} = \frac{(\rho+\omega) - (b+h)}{(\rho+\omega) + (b+h)}$$

$$\sim \frac{s^{0.5+0.9t} - s^{0.7t}}{s^{0.5+0.9t} + s^{0.7t}}$$



Sources and References

Paper: **VM, G. Fox and A. Szczepaniak** http://arxiv.org/abs/1505.02321

Interactive webpage: http://www.indiana.edu/~jpac/index.html

Data and Mathematica code: http://www.indiana.edu/~jpac/Resources.html