Geometrical Methods for Data Analysis I: Dalitz Plots and Their Uses

- History of the Dalitz Plot
- Dalitz's original "plot"

  non-relativistic; in terms of kinetic energies

  applied to the "τ-θ puzzle"
- Modern-day Dalitz Plot
- Identification of resonances

  spectroscopy -- masses, spins
- Interference Effects in Dalitz Plots

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Indiana University June 8, 2015

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Thanks to Vincent Mathieu

The Dalitz Plot: Origin and Uses

Named after Richard Dalitz (1925-2006), professor at Chicago and Oxford

(and my thesis advisor)

Phil. Mag. **44**, 1058 (1953)

Visual representation of phase space decay of a particle to various final particles -- initially only 3-body decays of spin-0 particles, but often now refers to more general decay modes

- Dalitz used it to study the "τ-θ puzzle"
- strange particles that decay to 2 or 3 pions; now understood as different decay modes of kaons
- Dalitz: "I visualize geometry better than numbers"

Richard Dalitz

Contributed to a wide range of scattering phenomena, spectroscopy, elementary particle physics

Scientific biography: Aitchison, Close, Gal, Millener, Nucl. Phys. **A771**, 8 (2006).

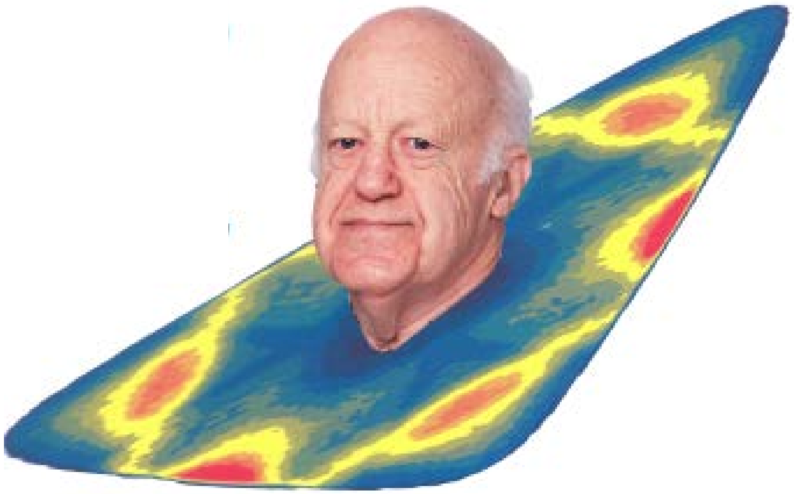


Image credit: Mike Pennington

- "Dalitz pairs" – [electron-positron pair from neutral pion decay]
- The Dalitz Plot
- physics of Hypernuclei
- Constituent quark model

  baryon & meson spectra
- "CDD poles" [Castillejo-Dalitz-Dyson]

Types of Reactions/Facilities

Nuclear Reactions involving exclusive final states;
Few-body final states (3,4, ... particles)

Electron machines (Jlab, BESIII, ...); virtual or real photons interacting with nucleons/nuclei

Future electron-ion collider (eRHIC; eLIC)

Electron-positron colliders; particularly those that look at exclusive processes (e.g., BELLE, BaBar in B-quark sector)

Medium-energy antiproton accelerators (LEAR, FAIR, ...)

Useful tools for spectroscopy, reaction mechanisms

Original Dalitz Plot

For three-body decay M → 1 + 2 + 3, have three 4-vectors for momenta of final-state particles, and 10 constraints

Constraints	Degree of freedom
3 four-vectors	12
4-momentum conservation	-4
3 masses	-3
3 Euler angles	-3
TOT	2

2 independent quantities! Choose kinetic energies of 2 different final particles.

Constraint: $T_1 + T_2 + T_3 = Q$

Q = energy release in decay,

$Q = M - m_1 - m_2 - m_3$

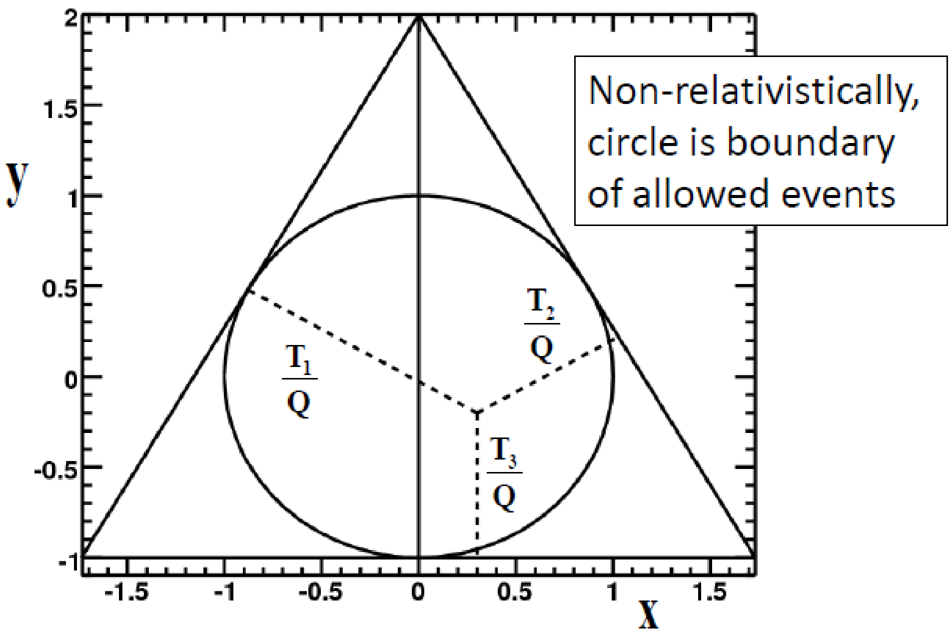
Choose an independent set of T_i/Q

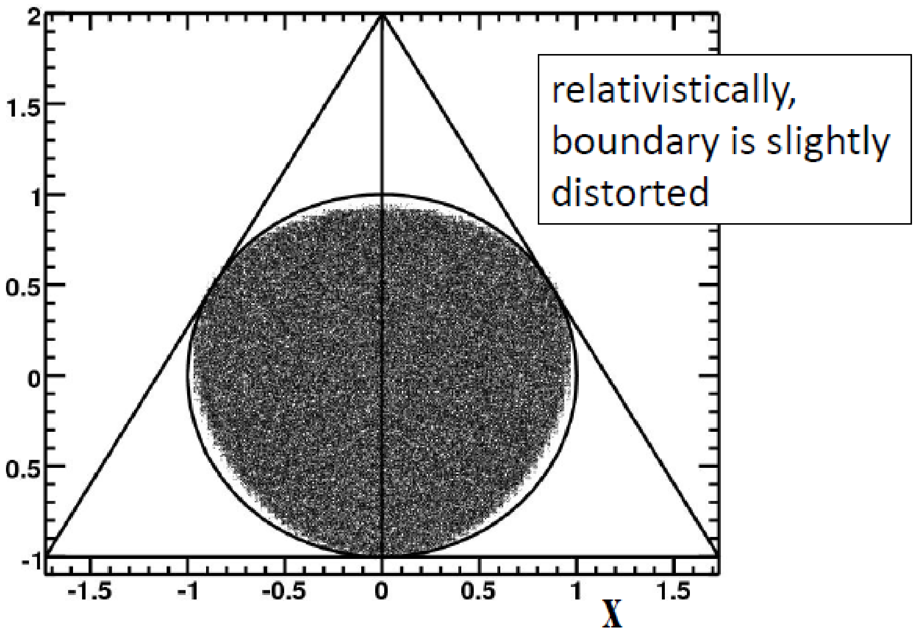
(initially, in non-relativistic regime)

Original Dalitz Plot

For three-body decay M → 1 + 2 + 3, Dalitz plot used non-rel KE to plot events.

NR, use T_i/Q : points lie within boundary of circle of unit radius:
rel: boundary slightly distorted



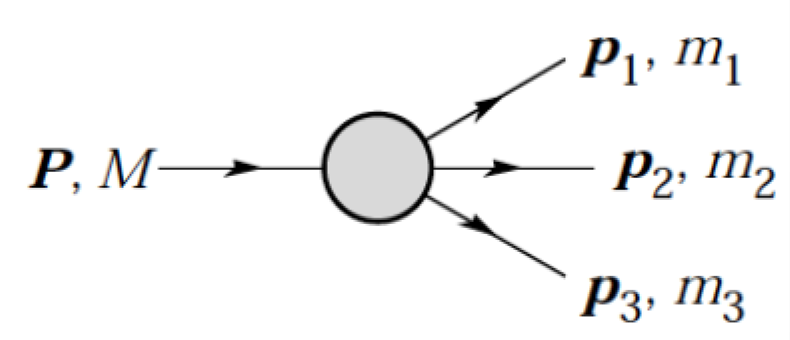


$$x = \frac{\sqrt{3}(T_1 - T_2)}{Q}; \qquad y = \frac{2T_3 - T_1 - T_2}{Q}$$

$$Q = T_1 + T_2 + T_3 = M - m_1 - m_2 - m_3$$

Interpreting the Dalitz Plot

Three-body decay M → 1 + 2 + 3, given by square of invariant amplitude
If invariant amplitude is constant, then Dalitz plot will be uniformly populated.
Non-uniformity in population of Dalitz plot gives information on final-state interactions in decay.



$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}\overline{|\mathcal{M}|^2}dm_{12}^2 dm_{23}^2$$

In particular, **2-body resonances** show up dramatically in the Dalitz plot.

Note: we assume all final particles are spinless here (e.g, π, K, η)

The Dalitz Plot and the "τ-θ Puzzle"

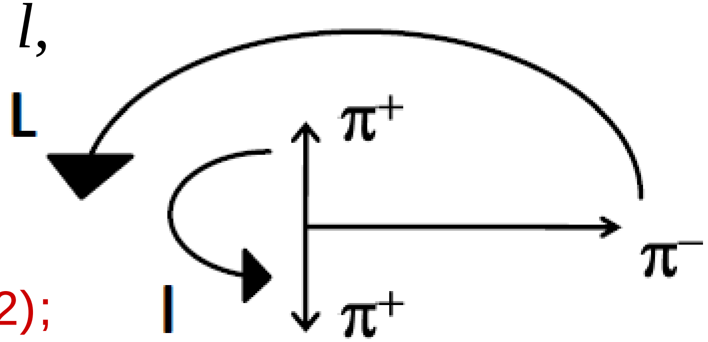
Observed 2 mesons, τ^+ and θ^+, with identical masses

τ^+ had 3-body decays, $\tau^+ \longrightarrow \pi^+ + \pi^+ + \pi^-$

θ^+ had 2-body decays, $\theta^+ \longrightarrow \pi^+ + \pi^0$

Assuming parity conservation, spin-parity of θ would be 0^+, 1^-, 2^+, 3^-, ...
Dalitz: utilize the 3-pion distributions on the Dalitz plot to determine the spin-parity of the τ

Decay of τ: two π^+ with angular momentum l, π^- with L, coupled to overall J.



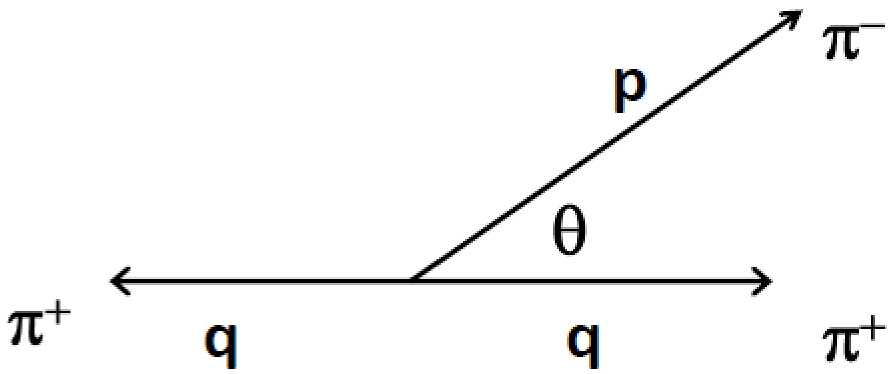
Calculate in Gottfried-Jackson frame (cm of particles 1,2); 1 and 2 have (equal + opposite) momentum **q**, particle 3 has mom. **p**

3-Pion Decays of the τ Particle

Three-pion decay of τ^+ given by square of matrix element

$$\rho(x,y) = \frac{1}{2J+1} \sum_{m_J} \left| \mathcal{A}(m_J) \right|^2$$

$$\mathcal{A}(m_J) = \sum_{L,\ell} [f_{L,\ell}(p,q) \otimes Y_L]^J_{m_J}$$



Assume that $f_{L,l}$ is slowly varying except for centrifugal barrier,

$$f_{L,\ell} \sim (pr)^L \, (qr)^\ell$$

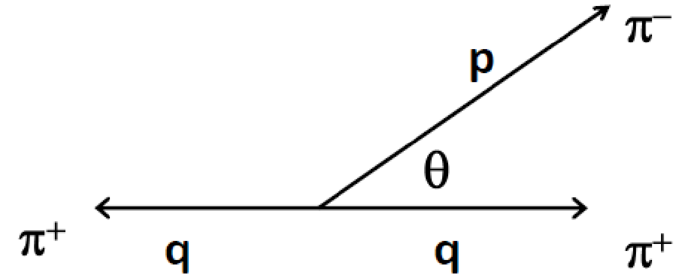
And that for low energies only the lowest value of $L+l$ contributes

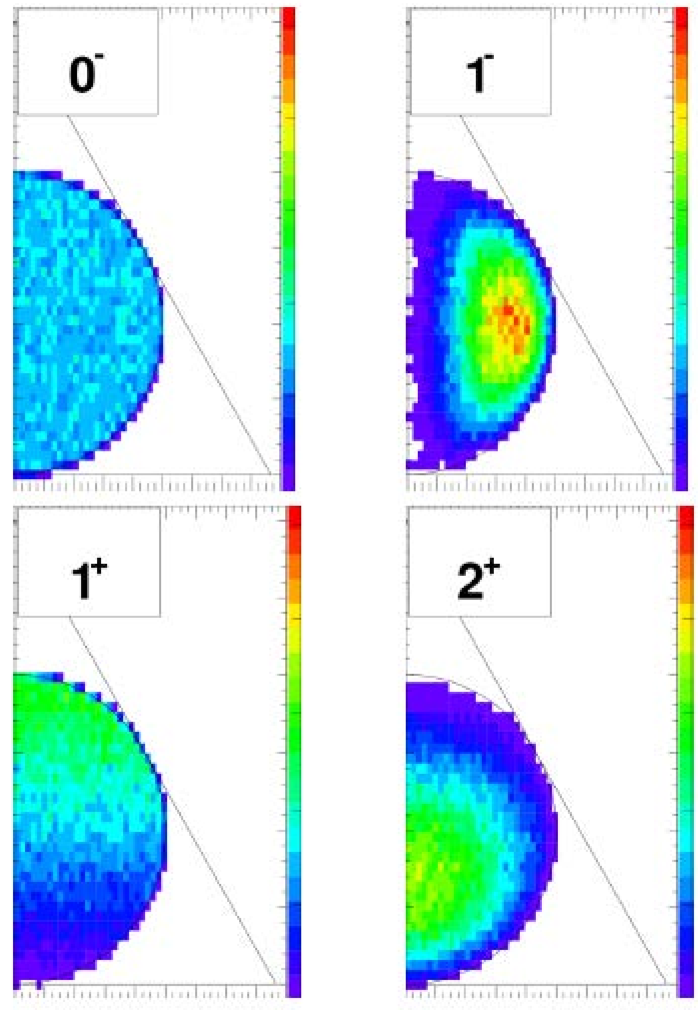
Then we can determine the spin-parity of the τ from the distribution of points in the Dalitz plot.

Spin-Parity of the τ Particle

Form of the distribution vs. spin-parity of the τ

J^P	dist'n
0^{--}	1
1^+	p^2
1^{--}	$(p^2\, q^2 \sin\theta \cos\theta)^2$
2^+	$(p\, q^2 \sin\theta)^2$





Dalitz plot of decays showed that τ was a 0^- particle.

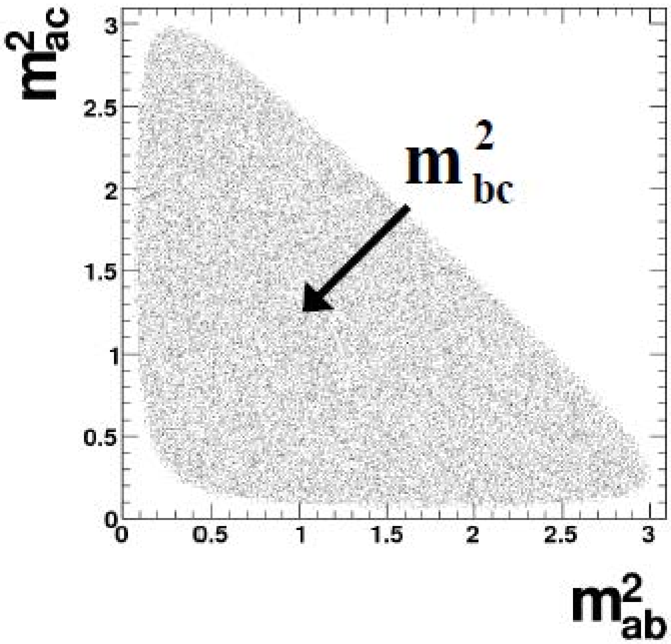
"τ-θ Puzzle": "Why do 2 particles with identical masses appear to have different spin-parities?" (Dalitz, PR **94**, 1046 (1954))

Answer: analysis assumed P conservation; once P non-conservation observed (1957), these were different decays of the same particle (the K^+, $J^P = 0^{--}$)

Modern Dalitz Plot (Relativistic Kin.)

For three-body decay M → a + b + c, have three 4-vectors for momenta of final-state particles, and 10 constraints

Constraints	Degree of freedom
3 four-vectors	12
4-momentum conservation	-4
3 masses	-3
3 Euler angles	-3
TOT	2



2 independent quantities: Choose "invariant mass squared" of two different pairs.

$$x = m^2_{ab} = s_{ab} = (p^\mu_a + p^\mu_b)^2$$

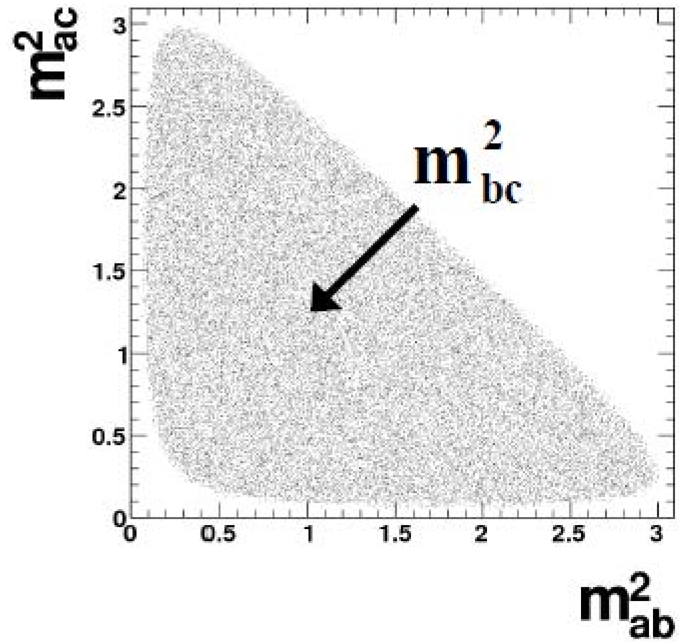
$$y = m^2_{ac} = s_{ac} = (p^\mu_a + p^\mu_c)^2$$

Modern Dalitz Plot Kinematics

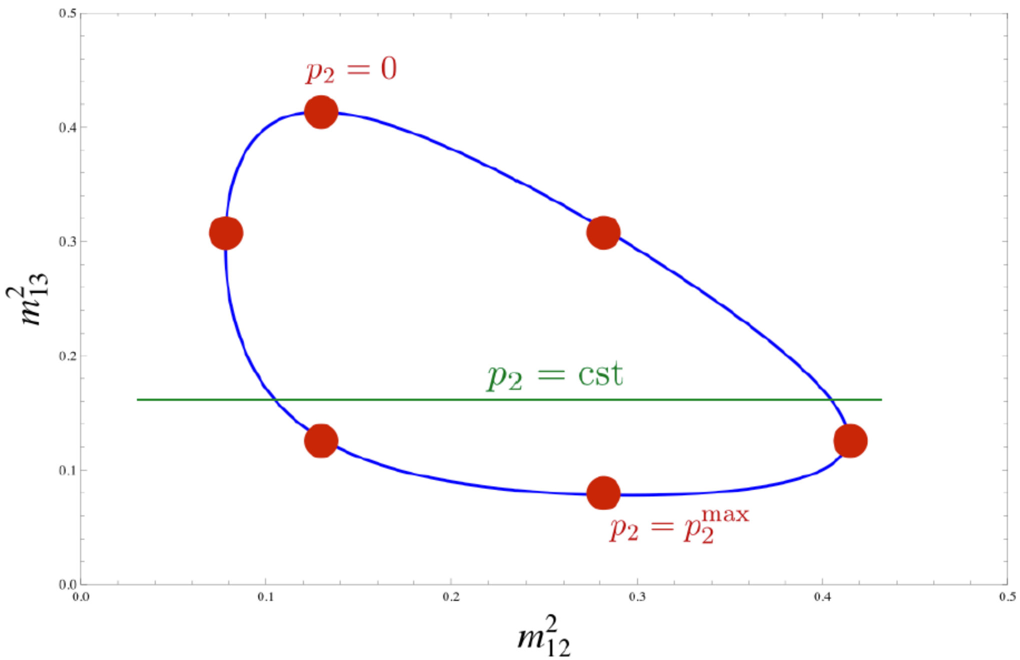
For three-body decay M → a + b + c, use "invariant mass squared"

$$x = m_{ab}^2 = s_{ab} = (p_a^\mu + p_b^\mu)^2$$

$$y = m_{ac}^2 = s_{ac} = (p_a^\mu + p_c^\mu)^2$$



Boundaries of Dalitz Plot: M → 1+ 2 + 3, look at kinematic conditions that determine boundaries: consider selected points.



Work in cm of 3-body system
Max value of m_{13} corresponds to $p_2 = 0$
Moving along line of constant m_{13} corresponds to constant p_2

Dalitz Plot, Relativistic Kinematics

$$s_{12} = (p_1^\mu + p_2^\mu)^2 = p_1^2 + p_2^2 + p_1 \cdot p_2$$
$$= m_1^2 + m_2^2 + E_1 E_2 - \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2$$

Example: kinematics in CM of 3-particle system

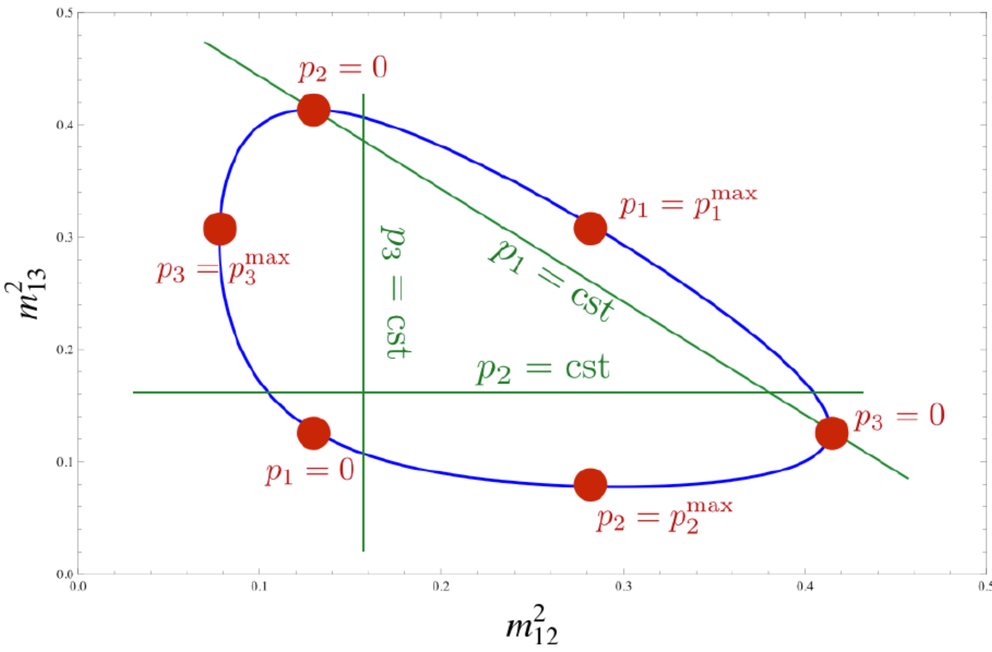
For given $\mathbf{p}_1$ and $\mathbf{p}_2$, $(s_{12})_{max}$ occurs when $\mathbf{p}_1$ and $\mathbf{p}_2$ are in opposite directions

Max value of s_{13} (top of DP) corresponds to $\mathbf{p}_2 = 0$

Max value of $\mathbf{p}_2$ occurs at bottom of DP

If you move along a line of constant m_{13}, then p_2 is constant.

Similarly along lines of constant m_{12}, m_{23}



Dalitz Plot, Relativistic Kinematics

$$s_{12} = (p_1^\mu + p_2^\mu)^2 = p_1^2 + p_2^2 + p_1 \cdot p_2$$
$$= m_1^2 + m_2^2 + E_1 E_2 - \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2$$

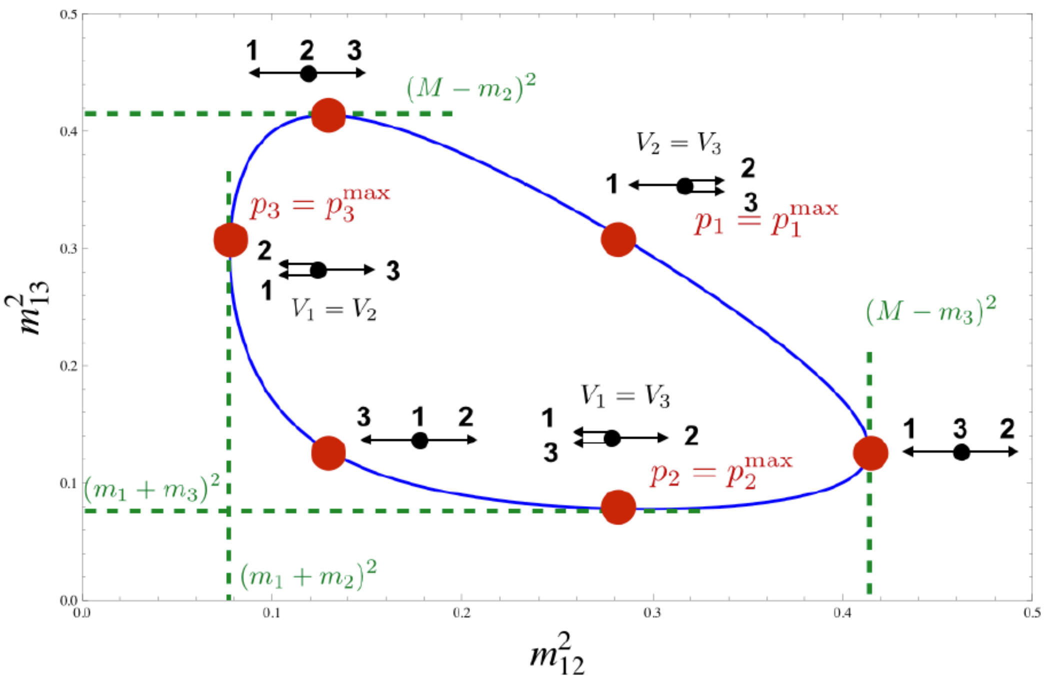
Example: kinematics in CM of 3-particle system

For given $\mathbf{p}_1$ and $\mathbf{p}_2$, $(s_{12})_{max}$ occurs when $\mathbf{p}_1$ and $\mathbf{p}_2$ are in opposite directions

Max value of p_3 occurs on LH boundary of DP; here all 3 momenta are collinear, with 1 and 2 moving in same direction.

If you move along a line of constant m_{13}, then p_2 is constant.

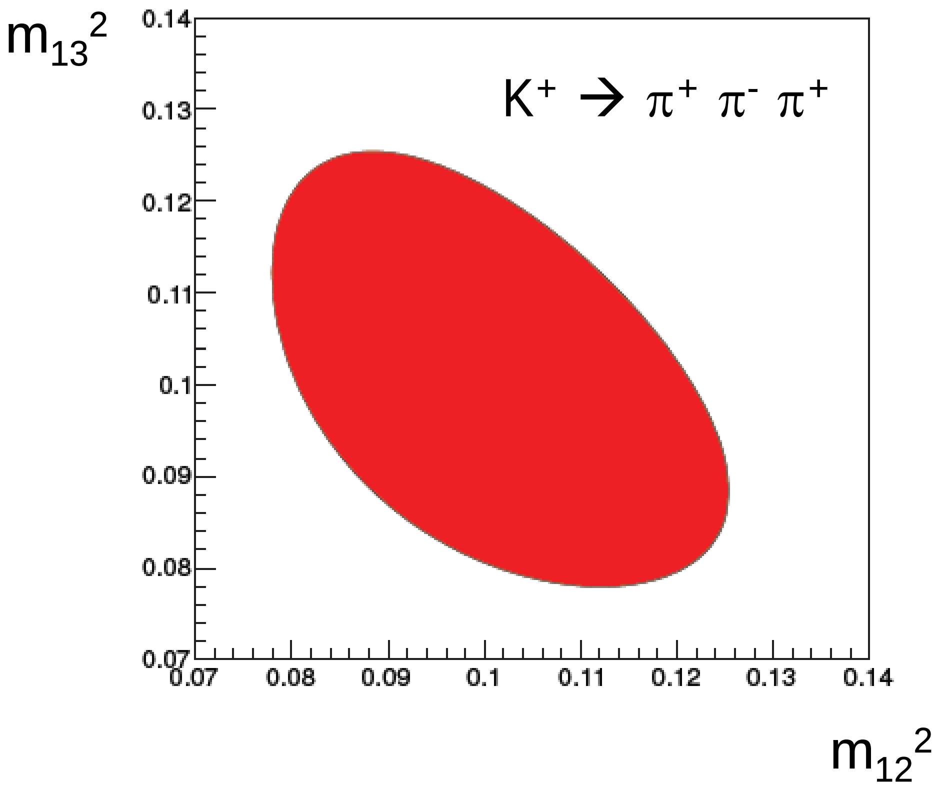
Q: what is kinematics at center of Dalitz Plot?



Shape of Dalitz Plot Boundary

- $Q \rightarrow 0$ (Non-relativistic regime) ⟷ DP shape → "Egg"
- $Q \rightarrow \infty$ (Relativistic regime) ⟷ DP shape → Triangle

Q = energy release in 3-body decay = $M - m_1 - m_2 - m_3$



Shape of Dalitz Plot Boundary

- $Q \to 0$ (Non-relativistic regime) ⟷ DP shape → "Egg"
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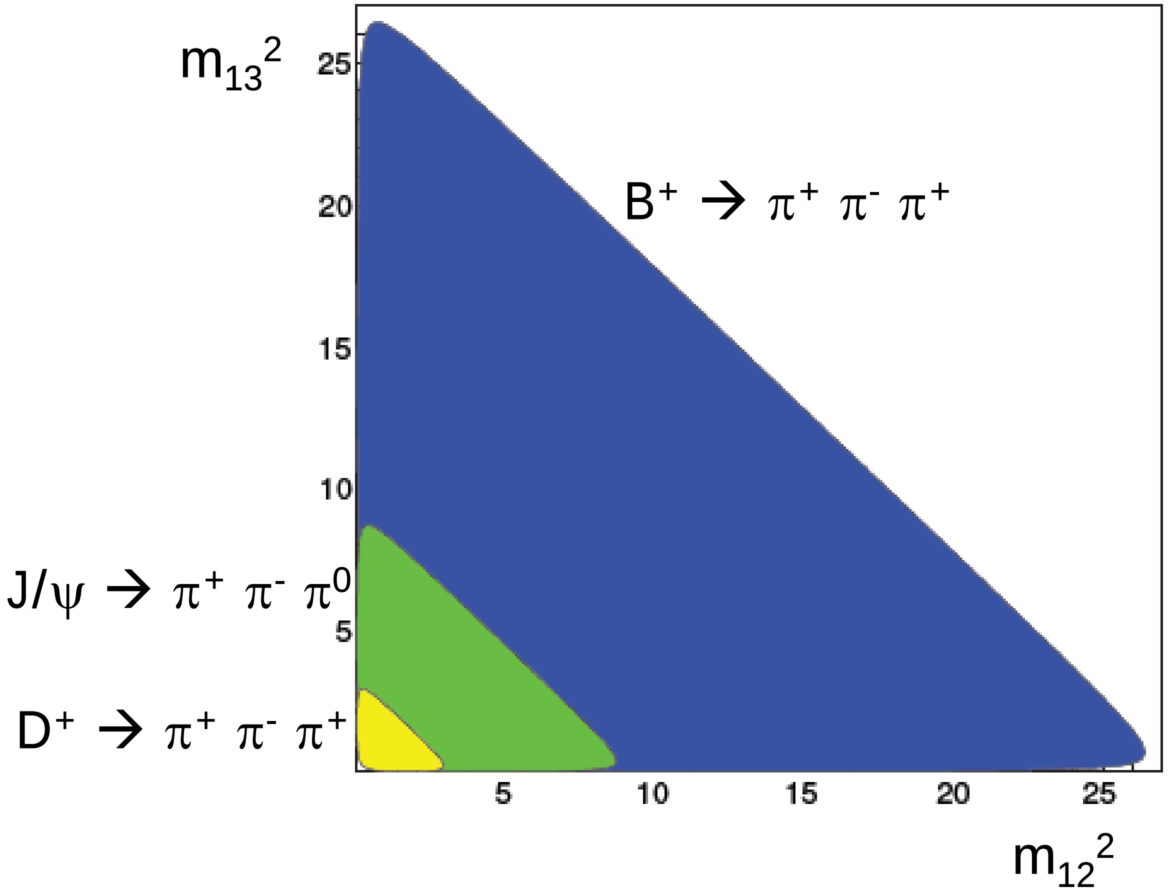


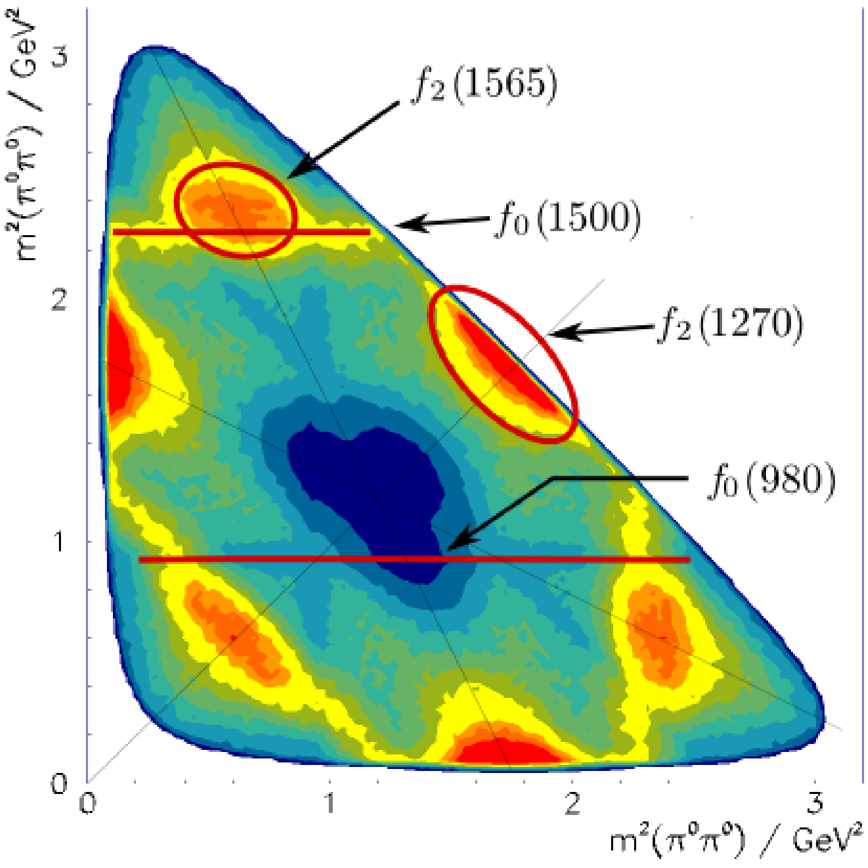
Symmetries of Dalitz Plots

Often final-state particles are identical, in which case Dalitz plots will respect exchange symmetries

$$\bar{p} + p \rightarrow 3\,\pi^0$$

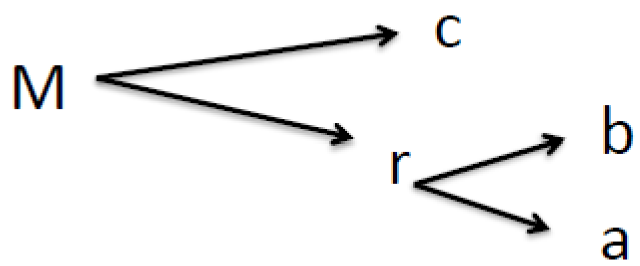
In this reaction, since all final particles are identical bosons, DP is symmetric with respect to reflection about any of 3 axes

C. Amsler etal (LEAR expt), Eur Phys J **C23**, 29 (2002)



Three-Body Decay Through Intermediate Resonance

Three-body decays will often take place through an intermediate resonant state “r” that subsequently undergoes two-body decay,



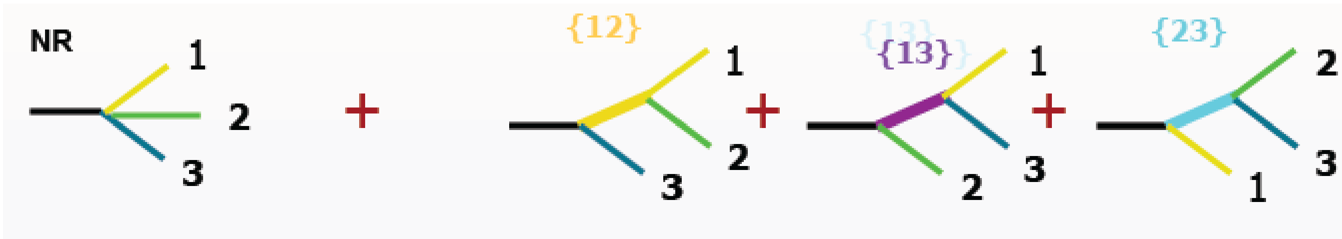
We can describe the behavior of the resonance by using a relativistic Breit-Wigner amplitude

$$\mathcal{A}_{BW} \sim \frac{1}{M_r^2 - s_{ab} - i\Gamma M_r} \;;\quad \Gamma = \frac{\hbar}{\tau}$$

Γ is inverse of lifetime τ of resonant state

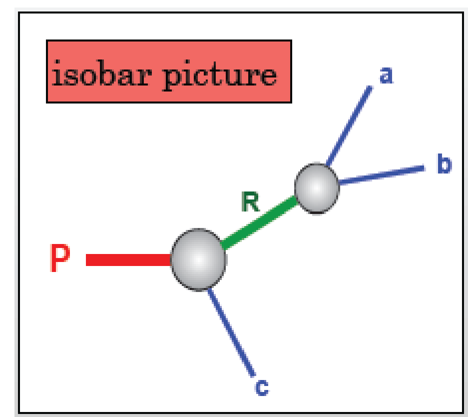
Isobar Model for 3-Body Decays

Approximate the total 3-body decay amplitude as a coherent sum of processes where one particle is a spectator (plus background?)

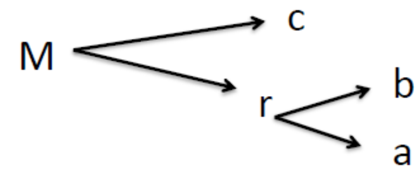


$$\mathcal{A}_{\mathcal{D}}(s_{12},s_{13}) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} \mathcal{A}_r(s_{12},s_{13})$$

NR term (direct 3 body decay)

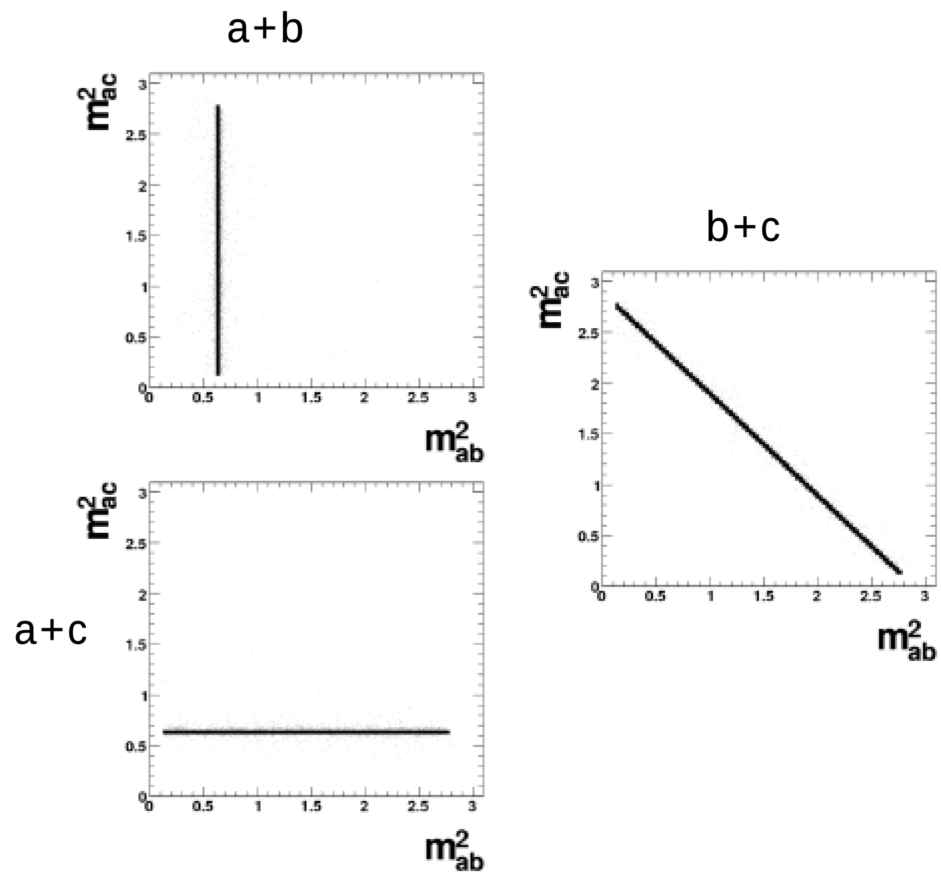


2-Body Resonances in Dalitz Plots



A 2-body resonance will appear on a Dalitz plot as a sharp enhancement, corresponding to the pair of particles that forms the resonance

$$\mathcal{A}_{BW}(ab) \sim \frac{1}{M_r^2 - s_{ab} - i\Gamma M_r}$$



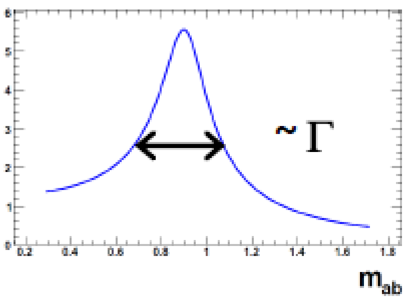
Identification of Resonances in Dalitz Plot

Use relativistic Breit-Wigner form for resonance,

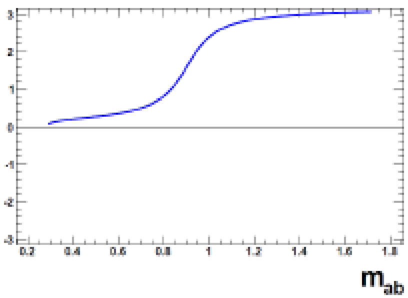
$$\mathcal{A}_{BW}(ab) \sim \frac{1}{M_r^2 - s_{ab} - i\Gamma M_r}$$

Γ is inverse of lifetime of resonant state

Magnitude

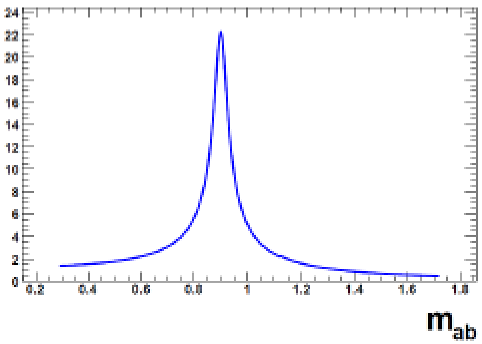


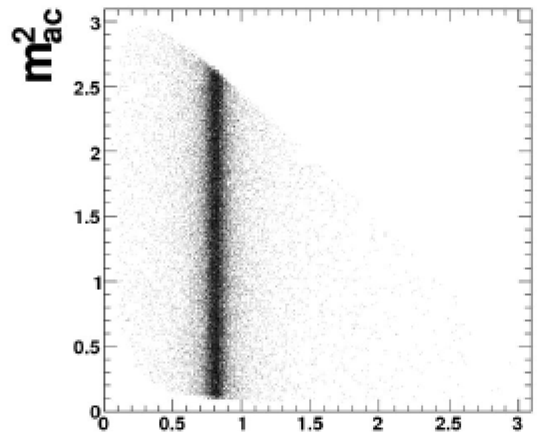
Phase



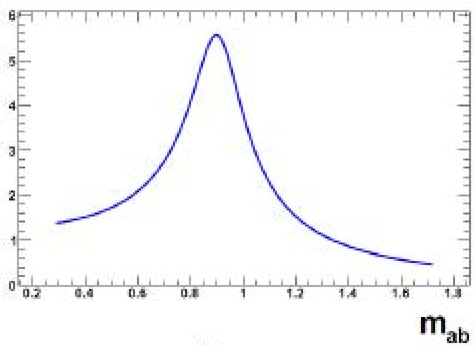
Narrow and Broad Resonances in Dalitz Plot

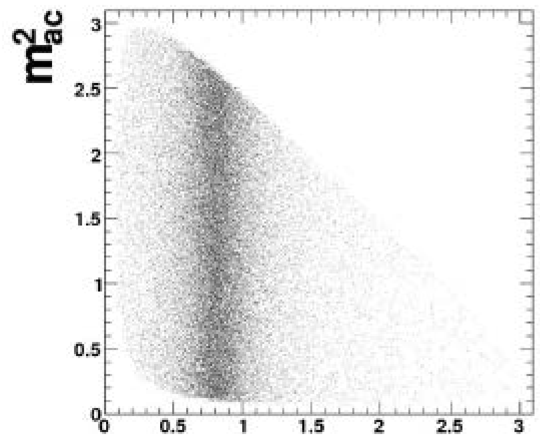
Magnitude





Magnitude



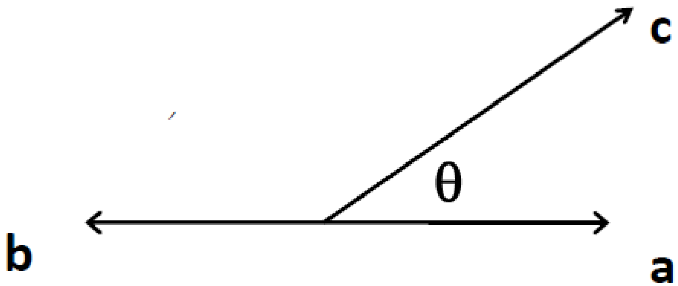


Width of resonance can be inferred from sharpness of resonant line; mass M_r from location on axis.

$$\mathcal{A}_{BW}(ab) \sim \frac{1}{M_r^2 - s_{ab} - i\Gamma M_r}$$

Resonance Spin in Dalitz Plot

If resonant state has spin S and a, b and c are all spinless particles, then decay amplitude is proportional to Legendre Polynomial;



In Gottfried-Jackson frame (rest frame of [ab] pair),

$$\mathcal{A} \sim \mathcal{A}_{BW}\, P_S(\cos\theta)\ ;$$

$$P_0(\cos\theta) = 1$$

$$P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

Resonance Spin in Dalitz Plot

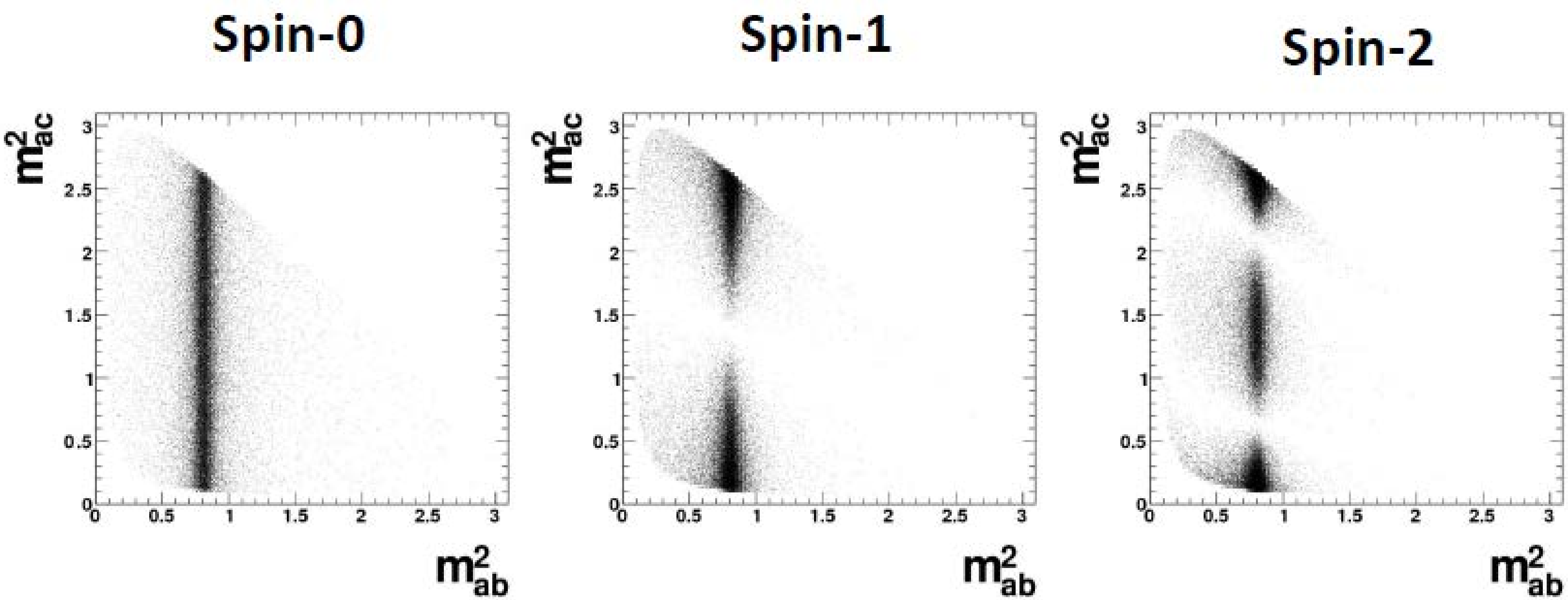
If resonance has spin S and a, b and c are all spinless particles, then decay amplitude will have zeroes corresponding to Legendre Polynomial;

$$\mathcal{A} \sim \mathcal{A}_{BW} \, P_S(\cos\theta) \; ;$$

$$P_0(\cos\theta) = 1$$

$$P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

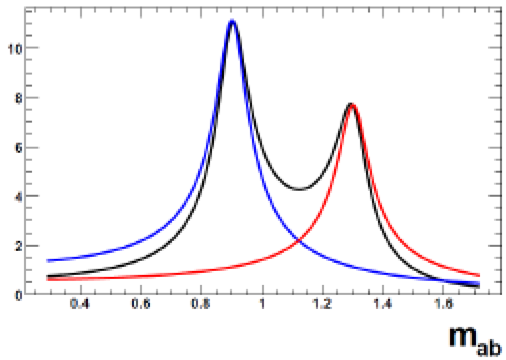


Spin-S resonance will have S zeroes in Dalitz plot

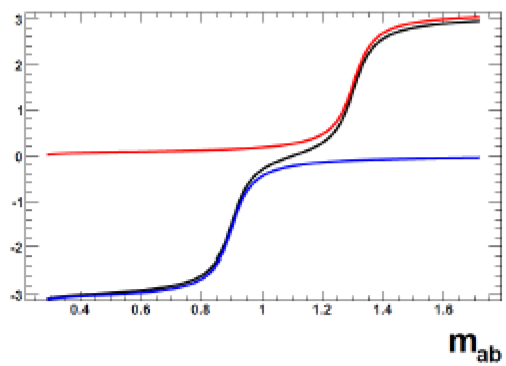
Interference of Overlapping Resonances in Dalitz Plot

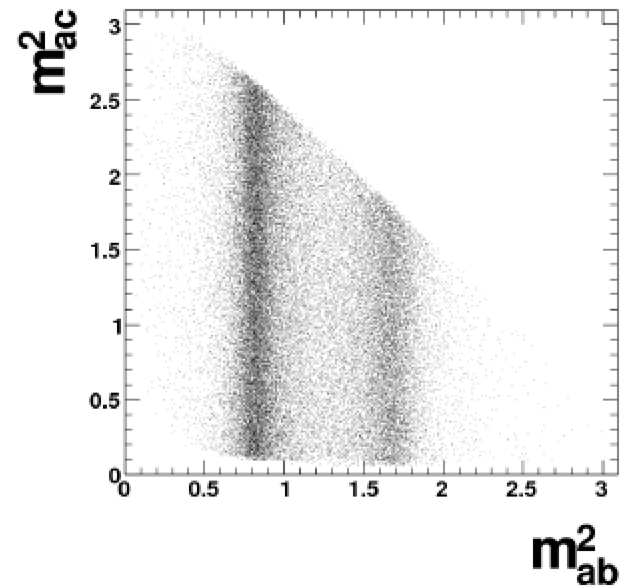
Constructive interference between a pair of resonances in [ab]

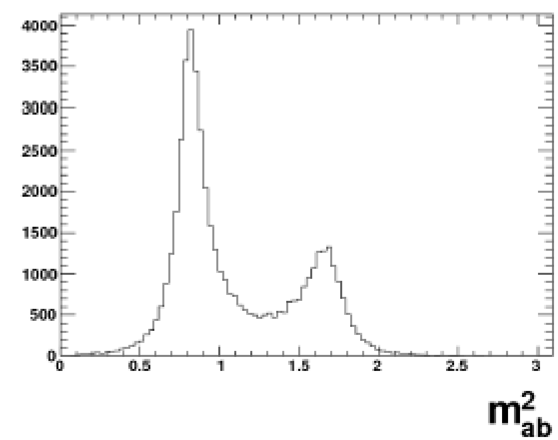
Magnitude



Phase



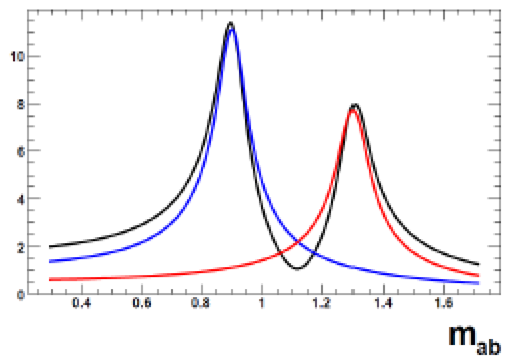




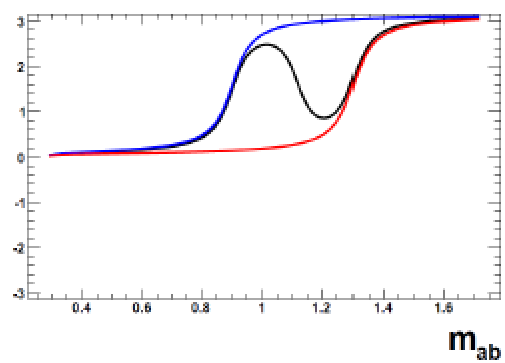
Interference of Resonances in Dalitz Plot (2)

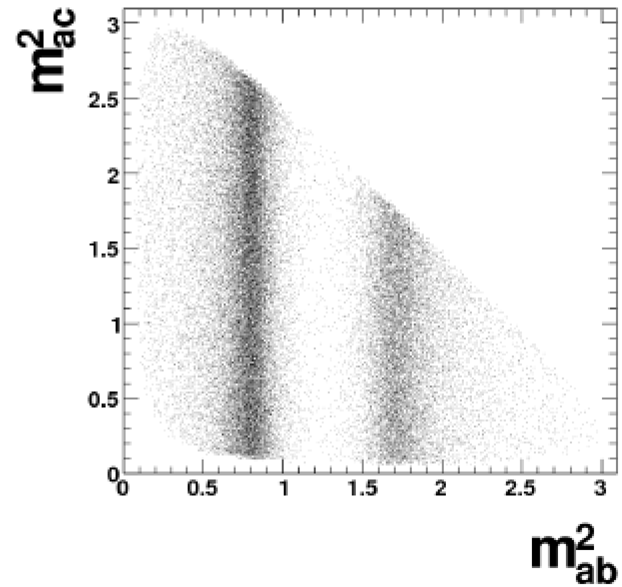
Destructive interference between a pair of resonances in [ab]

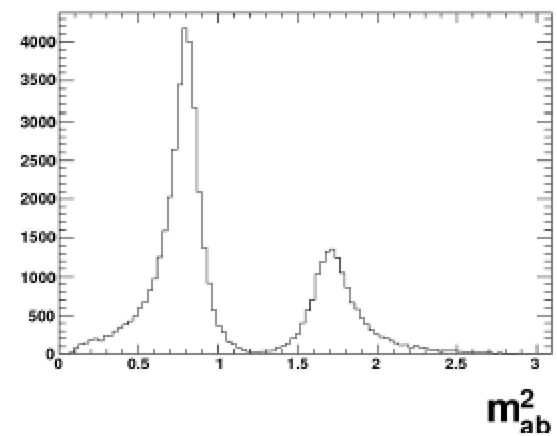
Magnitude



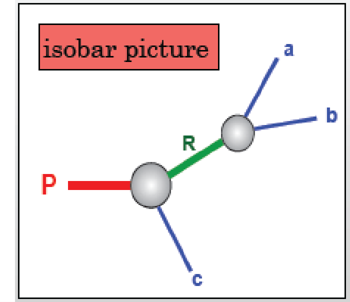
Phase



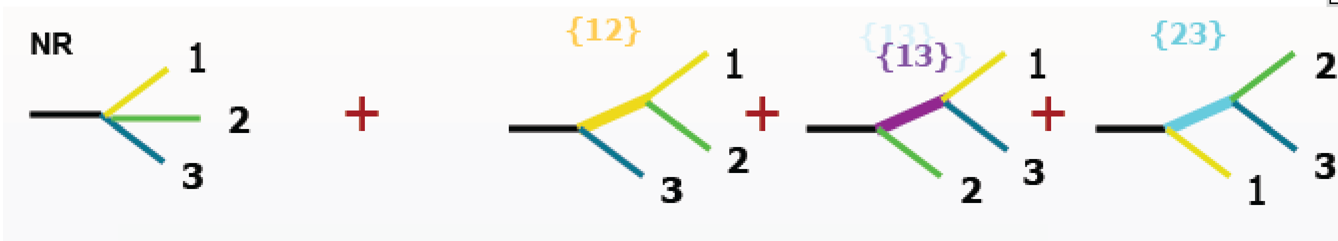




Applications of Isobar Model



Total 3-body decay amplitude = coherent sum of 2-body resonant processes plus background?



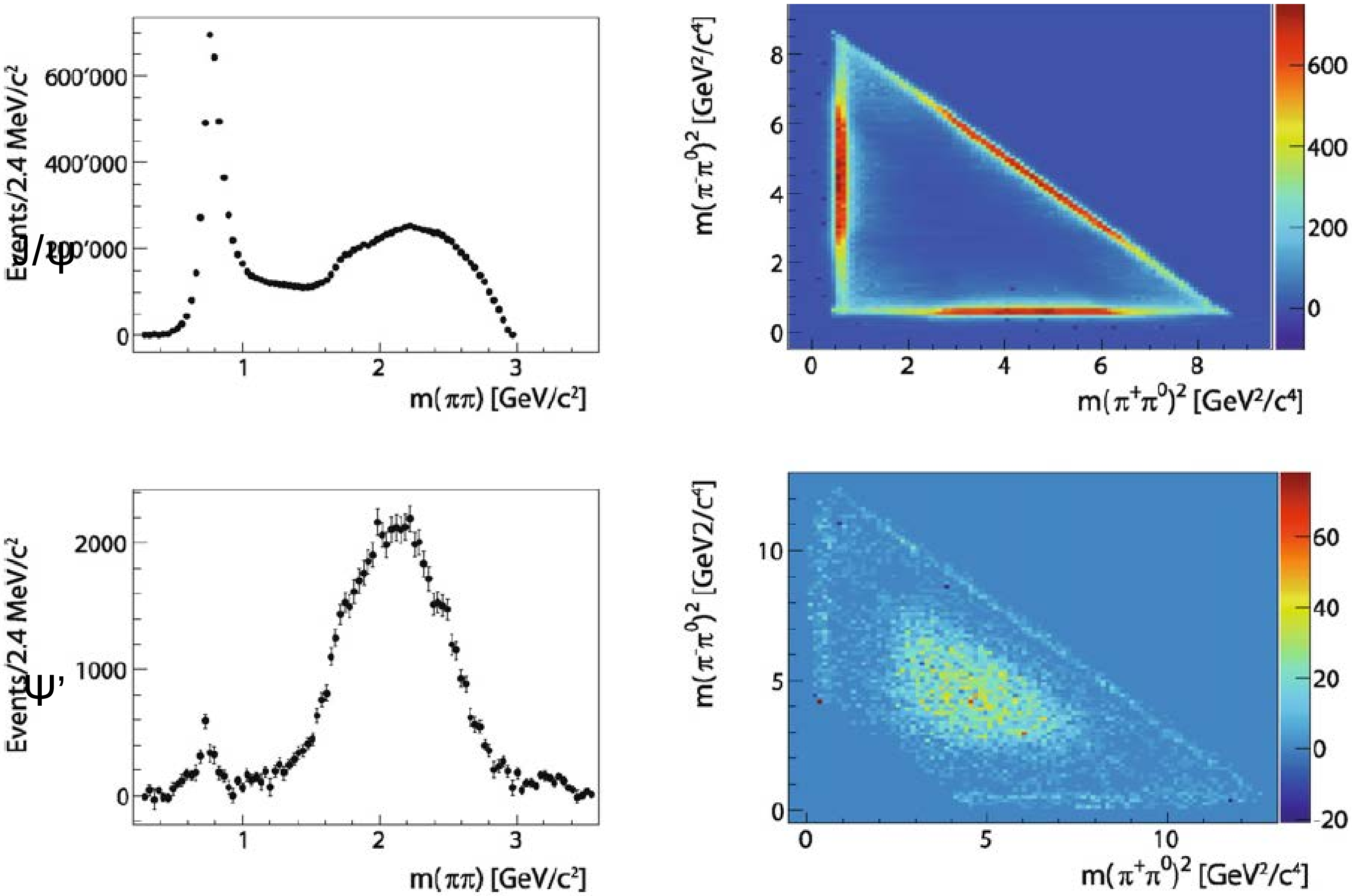
$$\mathcal{A}_{\mathcal{D}}(s_{12}, s_{13}) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta r} \mathcal{A}_r(s_{12}, s_{13})$$

↳ NR term(direct 3 body decay)

Relativistic Breit-Wigner parameters extracted from resonance properties (or from particle data tables);
a_i, δ_i are constants that are determined through a maximum-likelihood fit
Can measure fractions and relative phases of different isobars

A Current Puzzle

Compare decays $J/\psi \rightarrow \pi^+ + \pi^- + \pi^0$ to $\psi' \rightarrow \pi^+ + \pi^- + \pi^0$



The "ρ-π puzzle"

$$J/\psi, \psi' \text{ both } c\bar{c} \text{ bound states, but } 3\pi \text{ decays very different}$$

J/ψ: almost excusively through ρ (770)

ψ' decays through cluster of states ~ 2.2 GeV

BES-III Collab, P. Lett. **B710**, 594 (2012)

Higher-Order Dalitz Plots

Three-body decay M → 1 + 2 + 3, with 2 scalar and 1 vector final particles.

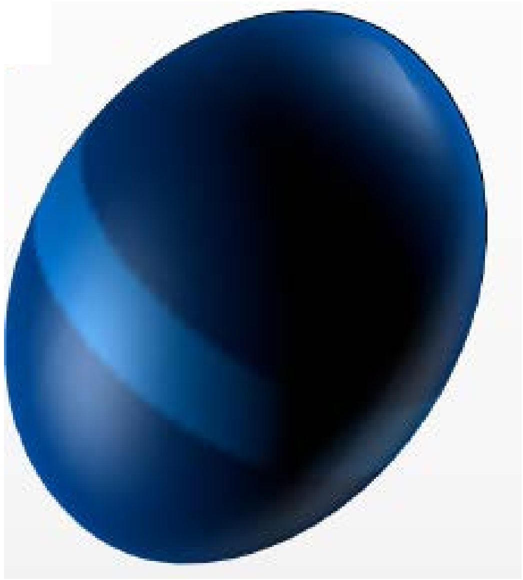
Constraints	Degree of freedom
3 four-vectors	12
4-momentum conservation	-4
3 masses	-3
3 Euler angles	-3
Vector helicity	2
TOT	4

Now 4 independent quantities! 4-D phase space, "4-D Dalitz Plot"

Recently used because decays to vector + scalars showed enhancements.

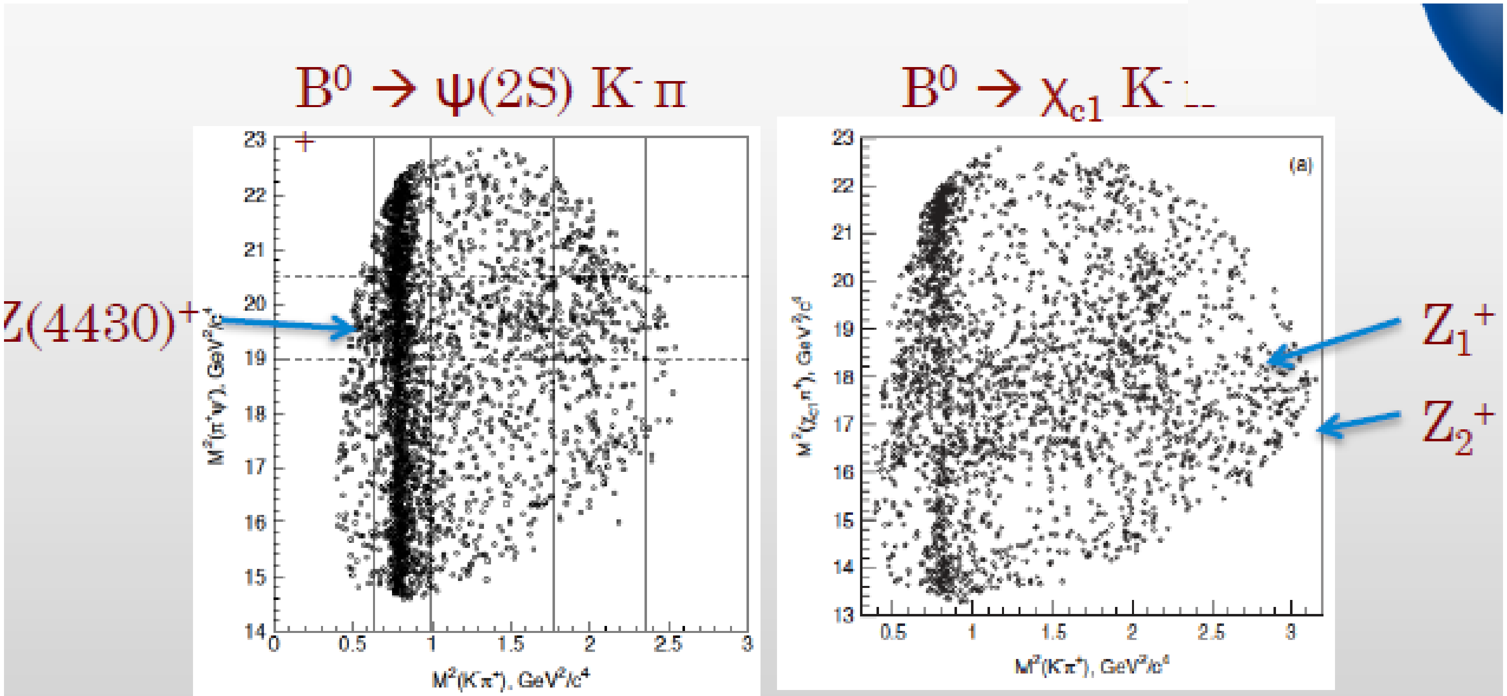
Must use helicity formalism

$B^0 \to \psi(2S) K^- \pi$ $B^0 \to \chi_{c1} K^- \pi^+$



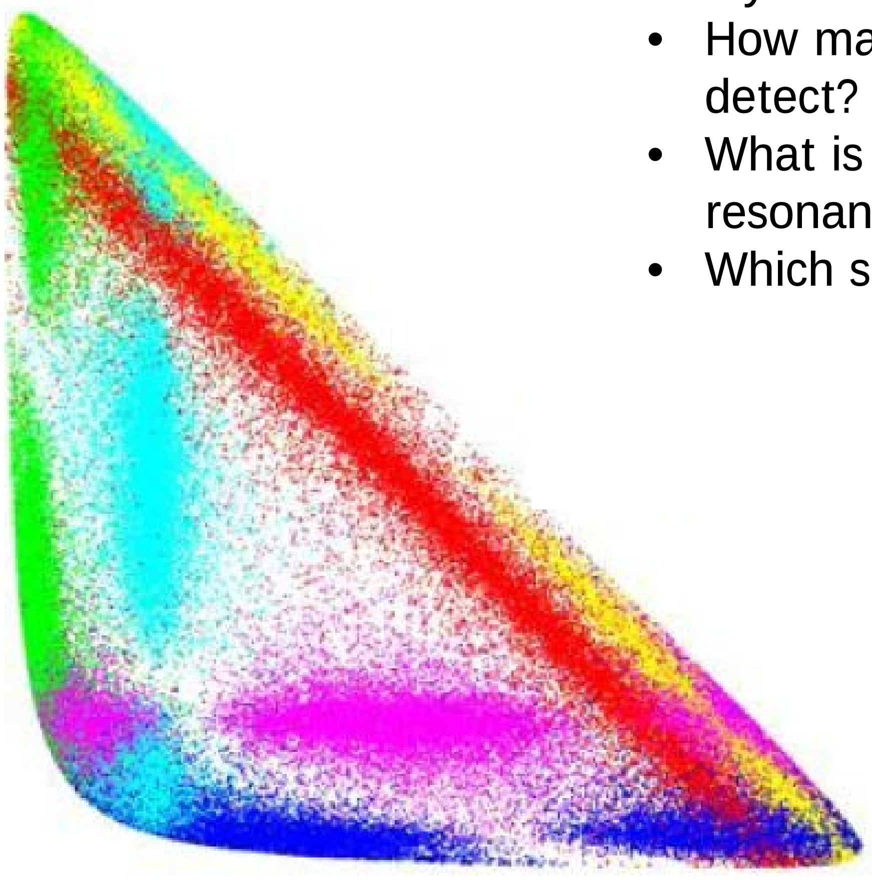
4-D Dalitz Plots

Three-body decay M → 1 + 2 + 3, with 2 scalar and 1 vector final particles.



Shadows? Interference? Or, new states?

Detection and Spin of Resonances in Dalitz Plot

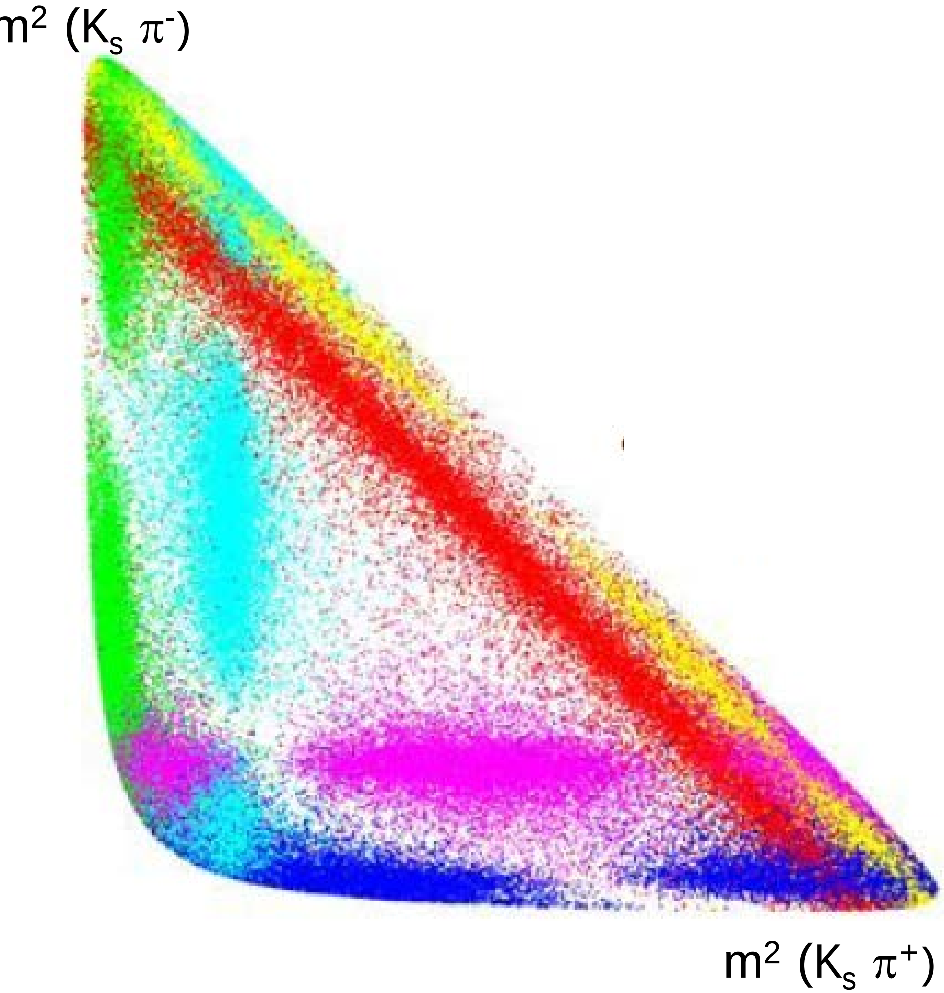


Here is a Dalitz plot of a 3-body decay

- How many resonances can you detect?
- What is the spin of each resonance?
- Which states have higher masses?

Detection and Spin of Resonances in Dalitz Plot

Decay $D^0 \rightarrow K_s \pi^+ \pi^-$



Because both x and y axes have $K_s + \pi$, symmetric about reflection (diagonal = $\pi + \pi$)

green + blue: **K* (892), S=1**

cyan + magenta: **K_2* (1430), S=2**

yellow: **ρ (770), S=1**

red: **f_0 (980), S=0**

Conclusions:

Dalitz Plots convert 3-body decays into plots that allow one to intuit important processes driving decays:

- Any non-random processes are highlighted in DP
- Resonances occur as **sharp bands in DP**

  can read off **position, width** of resonance

  spin of resonance determined by zeroes in DP
- Can also determine branching fractions, phases
- High-energy physics: now measuring CP-violating phases, etc. using Dalitz plots

Dalitz Plot: one of the most useful tools in particle spectroscopy