Applying Regge theory

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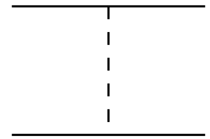
Much of the work I will describe was done in collaboration with Sandy Donnachie

History



- 1935: Yukawa predicted the existence of the pion — its exchange generates the static strong interaction
- 1960s: Nearly everybody worked on the applications of Regge theory, which sums the exchanges of many particles and generates the high-energy strong interaction
- The known particles not are enough — we need to include exchange of another object, the pomeron
- 1970s: QCD is discovered — the BFKL equation uses perturbative QCD to generate pomeron exchange as gluon exchange, but it makes total cross sections rise with energy much faster than is observed

Basic beliefs

- When two protons collide, most of the cross section results from a long-range force between them
- That force is quantum chromodynamics (QCD)
- Although QCD is weak at short range and so can be calculated by perturbation theory, this is not the case at long range
- For long range the only theory we have is Regge Theory, but it has its limitations
- Regge Theory models the exchange of families of particles ρ, ω, f_2, a_2 etc
- But to describe data it needs another exchange, called the “pomeron”
- Pomeron exchange is probably the exchange of a family of glueballs

Because of our inability to calculate, we have to inform the theory with information from the data, and build models

Pomeron Physics and QCD

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AND OTTO NACHTMANN

CAMBRIDGE MONOGRAPHS
ON PARTICLE PHYSICS, NUCLEAR PHYSICS
AND COSMOLOGY

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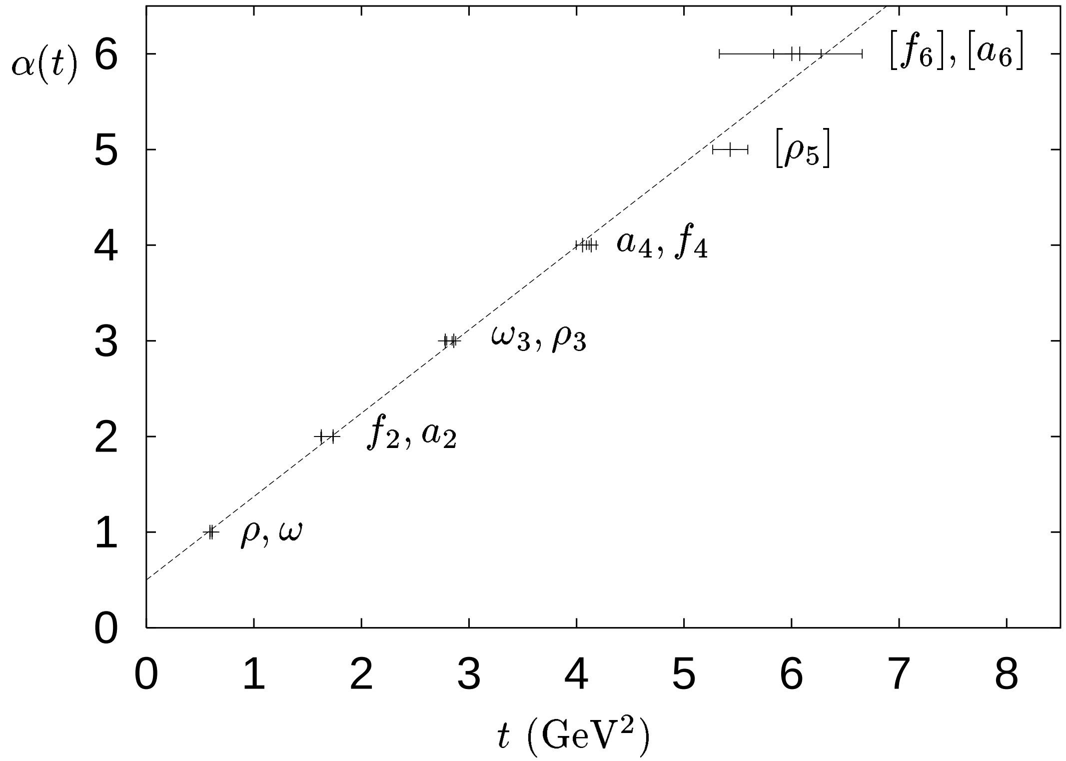
Linear particle trajectories

Plot of spins of families of particles against their squared masses:

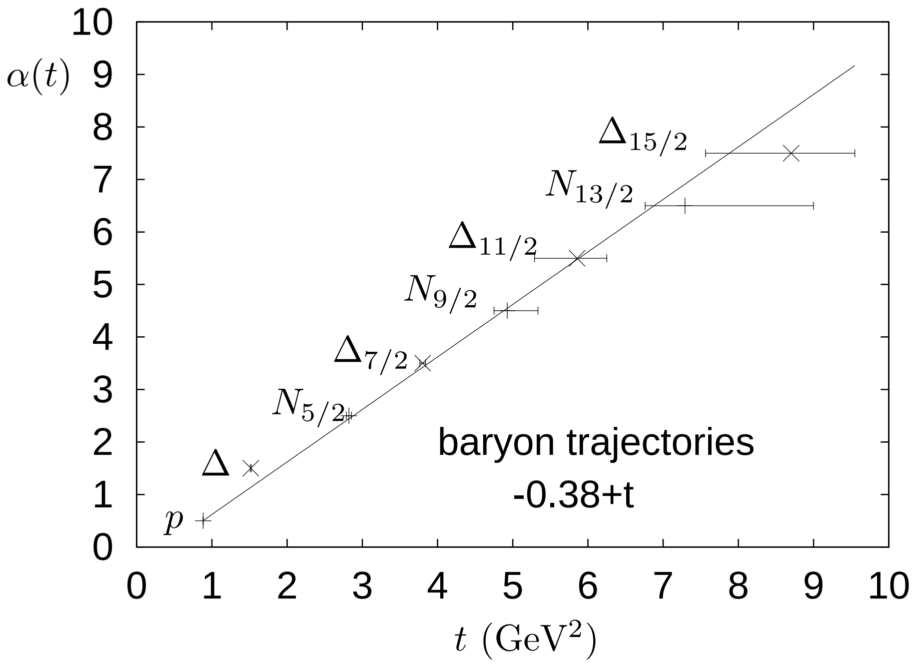


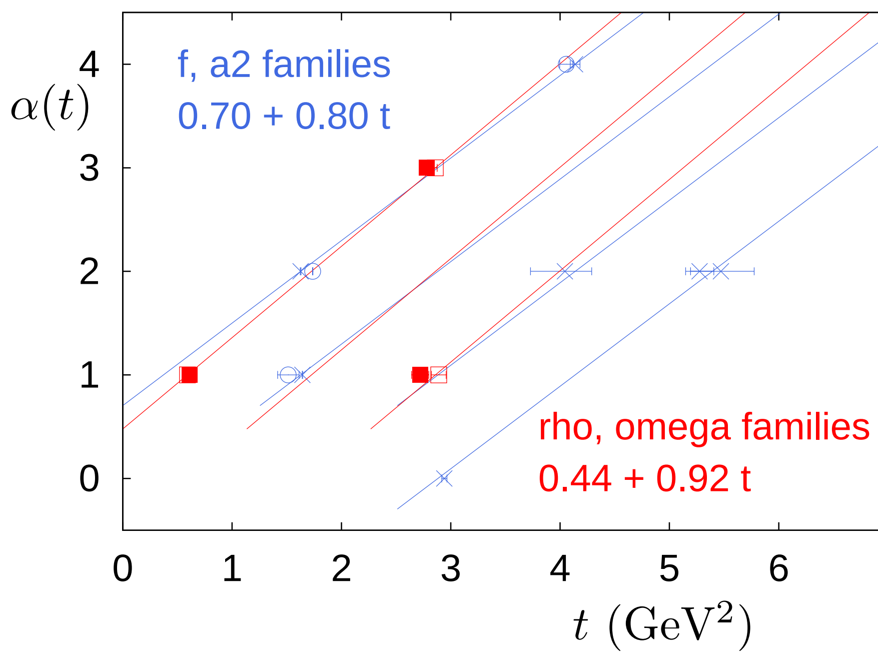
4 degenerate familes of particles: $\alpha(t) \approx \frac{1}{2} + 0.9t$

The particles in square brackets are listed in the data tables, but there is some uncertainty about whether they exist.

The function $\alpha(t)$ is called a Regge trajectory.

Regge trajectories





First daughter $\alpha_1(t) = \alpha_0(t) - 1$ Second daughter $\alpha_2(t) = \alpha_0(t) - 2$ etc

$C = +1$ and $C = -1$ trajectories not exactly degenerate

Why are trajectories almost straight lines?

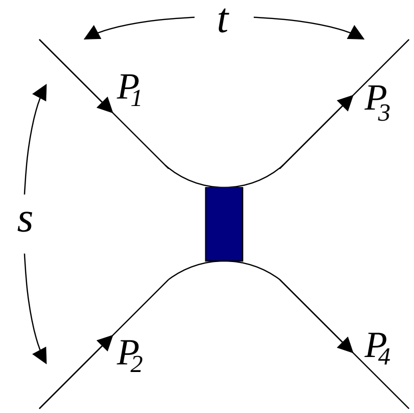
Cannot be exactly so: meson trajectories become complex for $t > 4m_\pi^2$

and baryon trajectories for $(m_N + m_\pi)^2$

Im $\alpha(t)$ at a resonance is proportional to its width

Regge theory

Regge theory sums the exchanges of many particles.



Define
$s = (P_1 + P_2)^2$ = squared CM energy
$t = (P_3 - P_1)^2$ = squared momentum transfer

At large s but $|t| << s$ each trajectory $\alpha(t)$ contributes to the amplitude a power of s:

$$A(s,t) \sim B(t)\ s^{\alpha(t)-1}$$

We know only a little about the function $B(t)$ —- later

Total cross sections

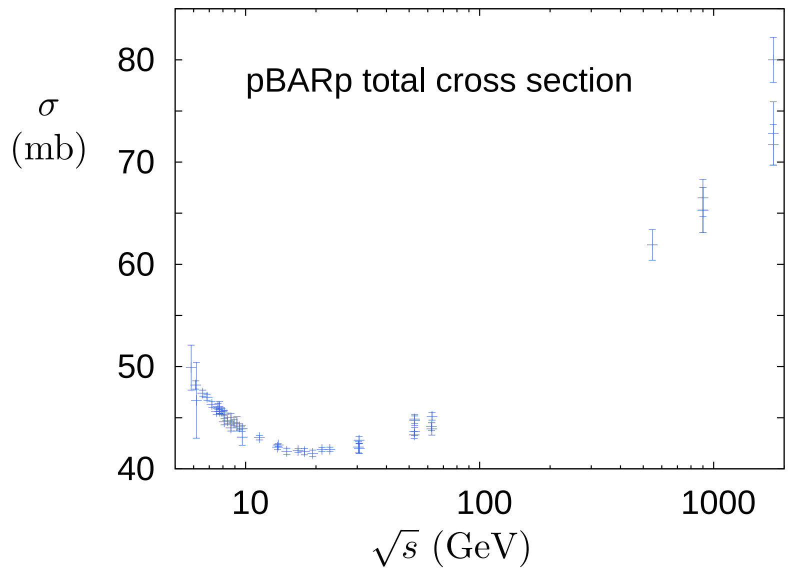
Optical theorem:

$$\sigma^{\rm TOT}(s) = \text{Im}\ A(s, t=0)$$

So each trajectory contributes a fixed power

$$s^{\alpha(0)-1} \approx s^{-\frac{1}{2}} \text{ for } \rho, \omega, f_2, a_2 \text{ trajectories}$$

The contribution from the ρ trajectory sums the exchanges of $\rho, \rho_3, \rho_5, \ldots$

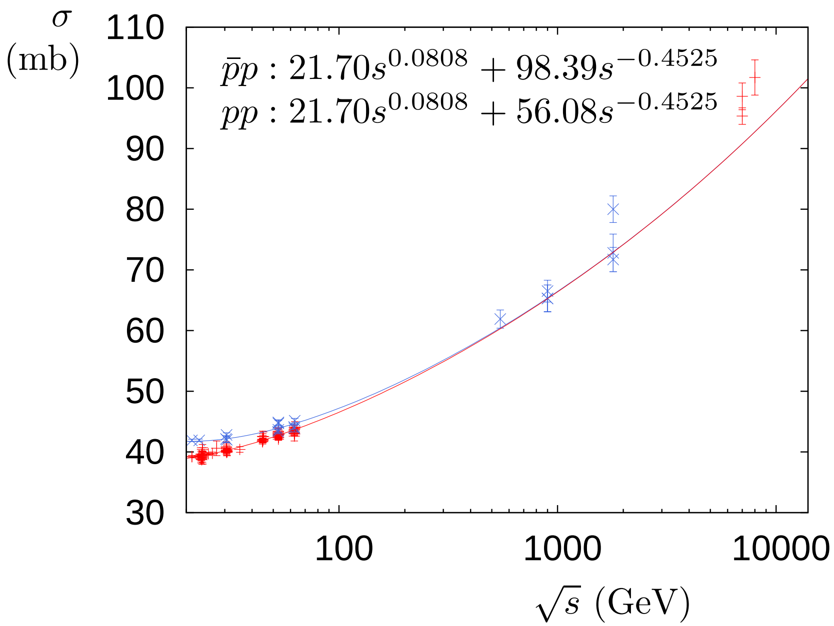


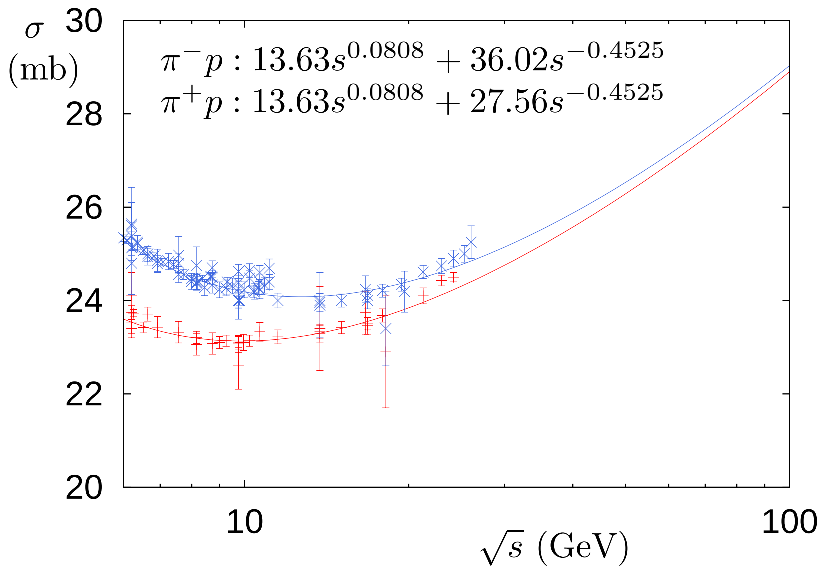
Agrees with experiment only at rather low energies

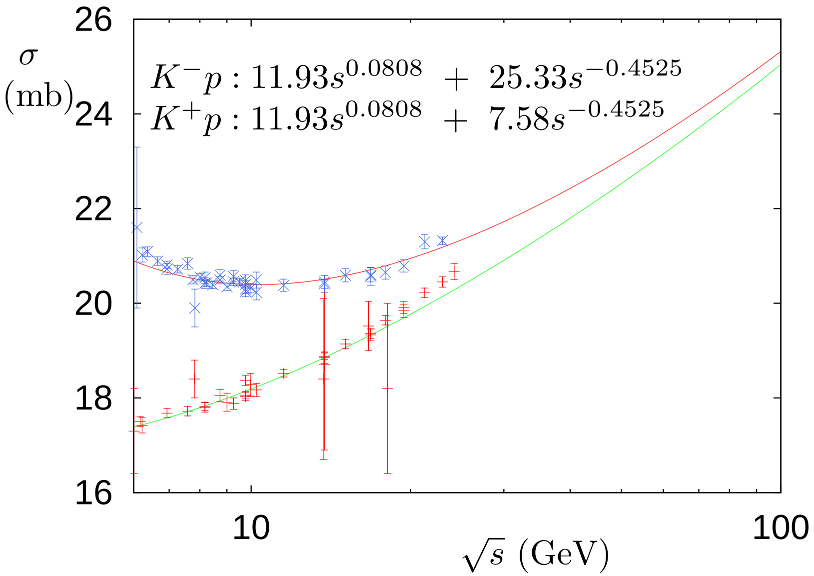
Total cross section data

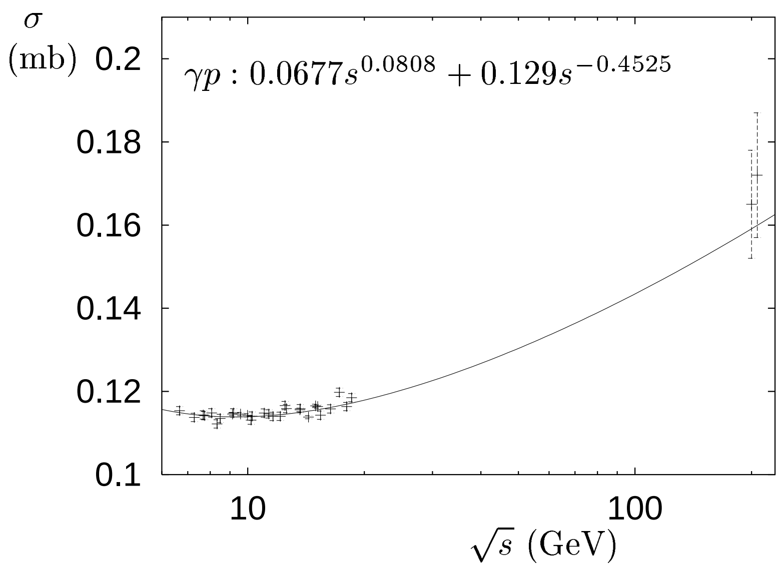
Total cross sections rise gently at large s

So we need another trajectory $\alpha_{\mathbb{P}}(t)$ with $\alpha_{\mathbb{P}}(0)$ a little > 1







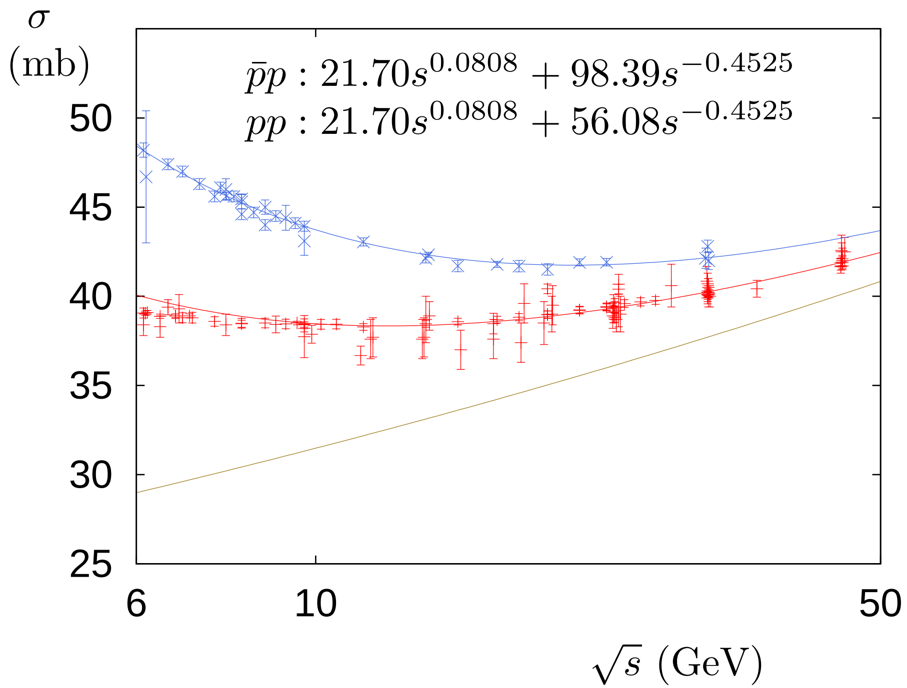


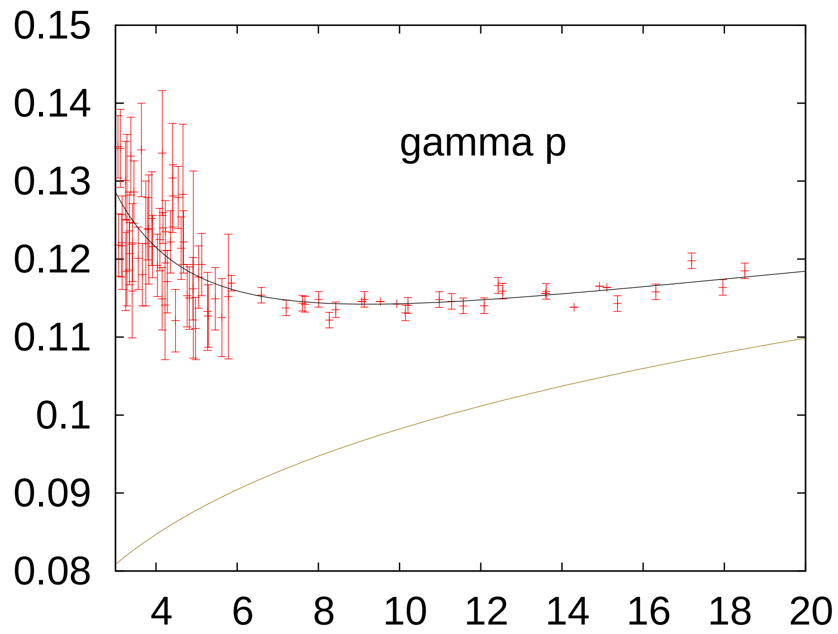
- We call this the pomeron trajectory (after the Russian physicist Pomeranchuk)

$$\frac{\text{Coupling to nucleon}}{\text{coupling to pion}} = \frac{21.7}{13.63} \approx \frac{3}{2}$$

- The pomeron seems to couple to the separate quarks in a hadron – quark counting rule
- These were fits made in 1992 – best value for $\alpha_{I\!P}(0)$ now is 1.096 rather than 1.0808
- The pomeron couples a bit more weakly to s quarks – much more so for c quarks
- Pomeranchuk theorem: $\sigma^{pp}/\sigma^{\bar{p}p} \sim 1$ at high energy. So the pomeron contributions to AB and $\bar{A}B$ scattering are equal: pomeron exchange has C-parity +1
- f_2, a_2 also have $C = +1$ and so contribute equally to AB and $\bar{A}B$ scattering. But ρ, ω have $C = -1$ and so contribute to AB and $\bar{A}B$ scattering with opposite signs.
- Probably pomeron exchange corresponds to the exchange of glueballs

Miracle



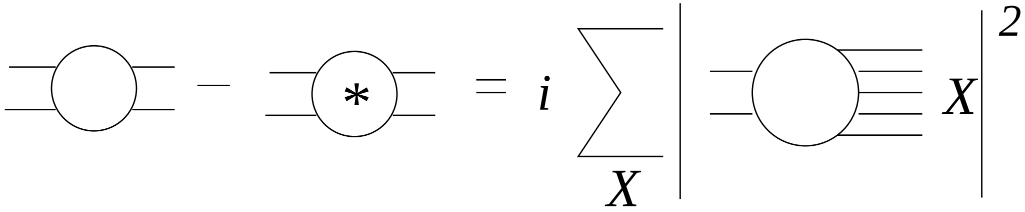


The cross sections rise smoothly even from very low energies where only pions can be produced. Thresholds for heavier particles, jets etc make no difference.

The pomeron term is there down to $\sqrt{s} = 6$ GeV or lower

Unitarity

Unit probability that a given initial state results in some final state, and that any final state came from some initial state.



- Optical theorem $\sigma^{\rm TOT}(s) =$ Im $A(s, t = 0)$
- Froissart-Lukaszuk-Martin bound

$$\sigma^{\rm TOT}(s) < \frac{\pi}{m_\pi^2} \log^2(s/s_0)$$

for some unknown s_0 — probably of the order of 1 GeV^2.
At LHC energy, this gives $\sigma^{\rm TOT} <$ 4.3 barns
So the bound has little to do with physics!

- More restrictive is the bound on the elastic partial-wave amplitude

$$\text{Im } A_\ell(s) = |A_\ell(s)|^2 + \text{ inelastic terms} \qquad \text{so that } |A_\ell(s)| < 1$$

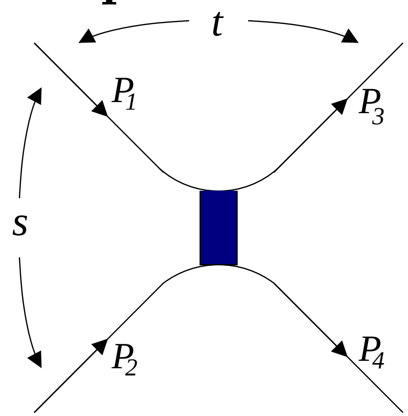
- Note that these bounds do not apply to photon or lepton beams.

Impact parameter

In the CM frame

$$P_1 = (E, \mathbf{p} + \tfrac{1}{2}\mathbf{q}) \qquad P_3 = (E, \mathbf{p} - \tfrac{1}{2}\mathbf{q})$$
$$P_2 = (E, -\mathbf{p} - \tfrac{1}{2}\mathbf{q}) \quad P_4 = (E, -\mathbf{p} + \tfrac{1}{2}\mathbf{q})$$

with $(\mathbf{p} + \frac{1}{2}\mathbf{q})^2 = (\mathbf{p} - \frac{1}{2}\mathbf{q})^2$ so that $\mathbf{p}.\mathbf{q} = 0$ and therefore $\mathbf{q}$ is in the two-dimensional space perpendicular to $\mathbf{p}$. Also $t = -\mathbf{q}^2$.



Write the amplitude as a 2-dimensional Fourier integral

$$A(s, -\mathbf{q}^2) = 4 \int d^2b \, e^{-i\mathbf{q}.\mathbf{b}} \tilde{A}(s, \mathbf{b}^2)$$
$$\tilde{A}(s, \mathbf{b}^2) = \frac{1}{16\pi^2} \int d^2q \, e^{i\mathbf{q}.\mathbf{b}} A(s, -\mathbf{q}^2)$$

b is called the “impact parameter”. Roughly speaking, it is the transverse distance between the two scattering particles.

Eikonal representation

Define

$$\chi(s,b) = -\log(1 + 2i\tilde{A}/s)$$

so that

$$\tilde{A}(s,\mathbf{b}^2) = \tfrac{1}{2} is(1 - e^{-\chi(s,b)})$$

and

$$A(s,-\mathbf{q}^2) = 2is \int d^2b\, e^{-i\mathbf{q}.\mathbf{b}}(1 - e^{-\chi(s,b)}).$$

It can be shown that the unitarity condition $|A_\ell(s)| < 1$ is just

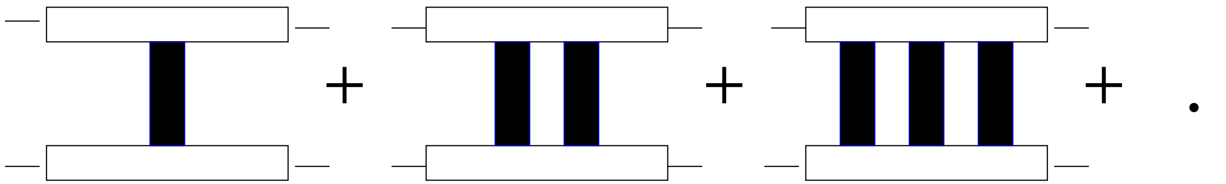
$$\text{Re}\ \chi(s,b) \geq 0$$

so this is an easy way to satisfy unitarity when one makes models

Multiple exchanges

$$A(s,-\mathbf{q}^2) = 2is\int d^2b\, e^{-i\mathbf{q}.\mathbf{b}}\left(\chi - \frac{\chi^2}{2!} + \frac{\chi^3}{3!}\ldots - \frac{(-\chi)^n}{n!}\ldots\right).$$

If we choose $\chi(s,b)$ so that the first term represents the exchange of a single pomeron, then the next term will be two-pomeron exchange, then three-pomeron etc.



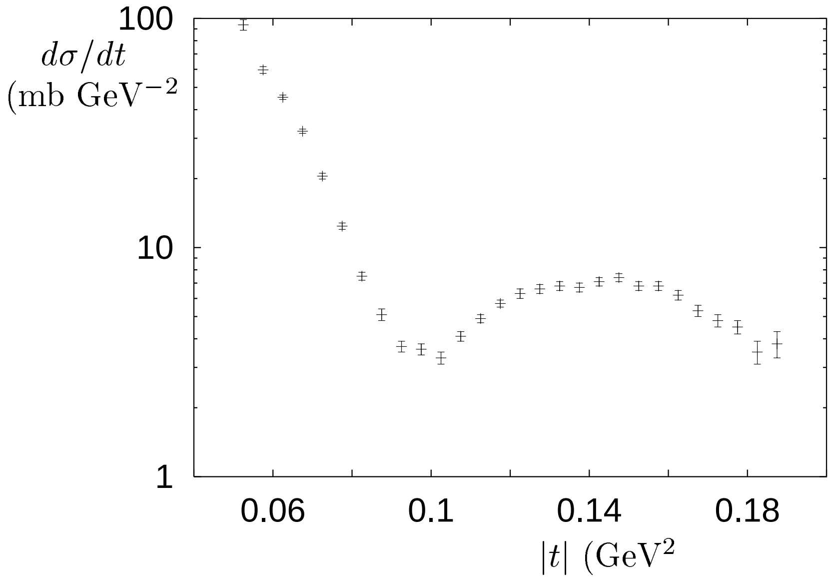
But this is only a model: we do not know how to calculate multiple exchanges properly.

Only single exchange is a simple power of s. So in our fits with $s^{0.08}$ this is an effective power, representing single plus mutliple exchanges.

This is usually good enough, but not always.

Multiple exchanges

This is data for elastic $\alpha\alpha$ scattering at $\sqrt{s} = 126$ GeV from the CERN Intersecting Storage Rings:



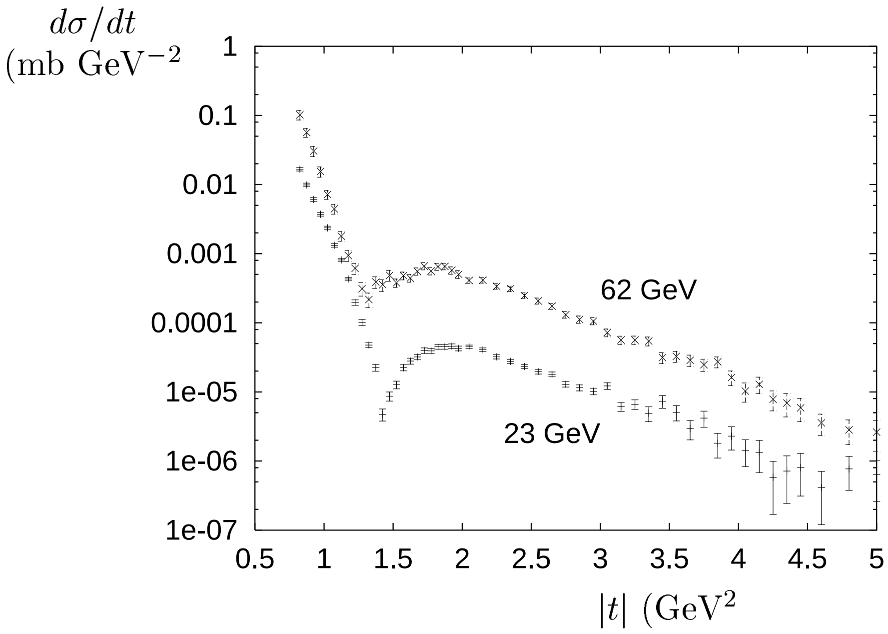
The energy is high enough for pomeron exchange to dominate

The shape of the curve is similar to the intensity pattern for optical diffraction, so pomeron exchange is often called diffractive scattering

At small t the dominant contribtion is from one pair of nucleons scattering

But the dip comes from interference with double exchange: two pairs of nucleons scattering, calculated by Glauber formula which is just the eikonal model

Proton-proton scattering



Now the scattering is mainly between pairs of constituent quarks:

For pp scattering there are $3 \times 3 = 9$ pairings and for πp there are $2 \times 3 = 6$

Ratio of coefficients of $s^{0.08}$ in fits to pp and πp total cross sections is $21.7/13.63 \approx 3/2$

Note:

- The forward peak gets steeper as the energy increases
- The dip is at larger $|t|$ than in $\alpha\alpha$ scattering, which makes the theory more complicated

The Regge formula

The theory of pomeron exchange uses the same mathematics as for optical diffraction: promote the orbital angular momentum ℓ into a complex variable and transform the partial-wave series into an integral

After some quite subtle mathematics find that the exchange of particles associated with a Regge trajectory $\alpha(t)$ contributes at high energy

$$A(s,t) = \beta(t)\,\xi(\alpha(t))\,(s/s_0)^{\alpha(t)-1}$$

for $|t| << s$ and fixed s_0.

$\xi(\alpha(t))$ is a phase factor:

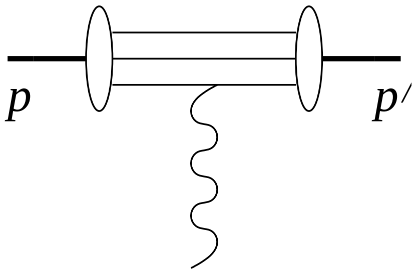
$$\xi(\alpha(t)) = \begin{cases} e^{-\frac{1}{2}i\pi\alpha(t)} & C=+1 \text{ exchange} \\ -ie^{-\frac{1}{2}i\pi\alpha(t)} & C=-1 \text{ exchange} \end{cases}$$

The theory does not tell us what is the function $\beta(t)$, except

It is real for $t < 0$

It has a pole for values of t corresponding to each particle on the trajectory (unimportant when we are interested in $t < 0$)

Photon coupling to a nucleon



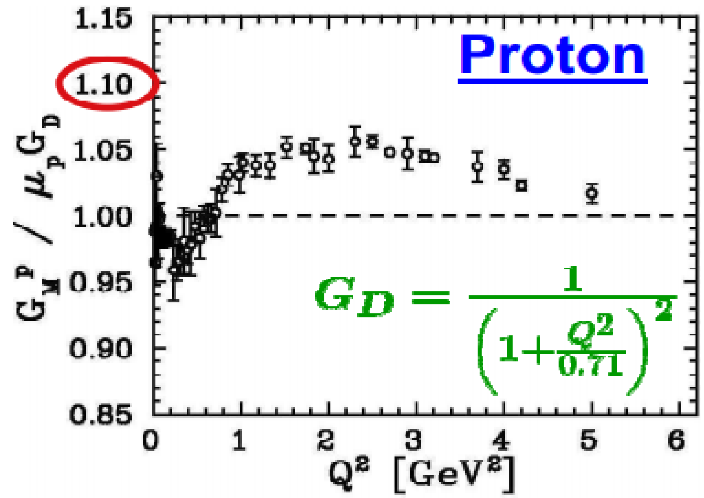
The coupling to a quark is (quark charge) $\times \gamma^\mu$

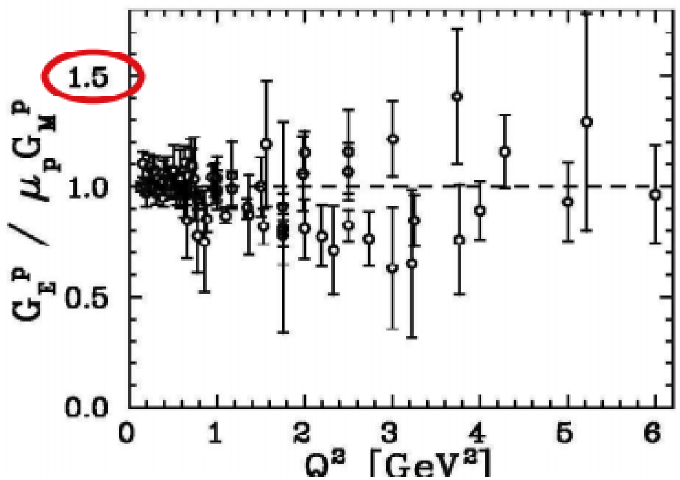
To get the coupling to the nucleon we couple to each quark in turn and take account of the nucleon wave function, giving

$$eF_1(t)\gamma^\mu + \frac{\kappa}{2m} F_2(t) i\sigma^{\mu\nu}(p'_\nu - p_\nu) \qquad\qquad F_1(0) = F_2(0) = 1$$

e is the sum of the quark charges, ie the charge of the nucleon
κ is its anomalous magnetic moment, ie the amount it differs from the Dirac-equation value
If this is used to calculate scattering from photon exchange, the last term flips the helicity of the nucleon
The photon is a mixture of isospin 0 and 1. The last term comes almost entirely from the $I = 1$ part, because $\kappa_p = 1.79 \quad \kappa_n = -1.91$ so that $\frac{1}{2}(\kappa_p + \kappa_n) \approx 0$
The F_2 term flips the helicity of the nucleon

The proton form factors





$\mu_p = 1 + \kappa_p$

http://www.phy.anl.gov/theory/PHYTI09/
PHYTI09_fichiers/ArringtonQCD09.pdf

Define

$$G_E(t) = F_1(t) + \frac{t}{4m_p^2} F_2(t) \quad G_M(t) = F_1(t) + F_2(t)$$

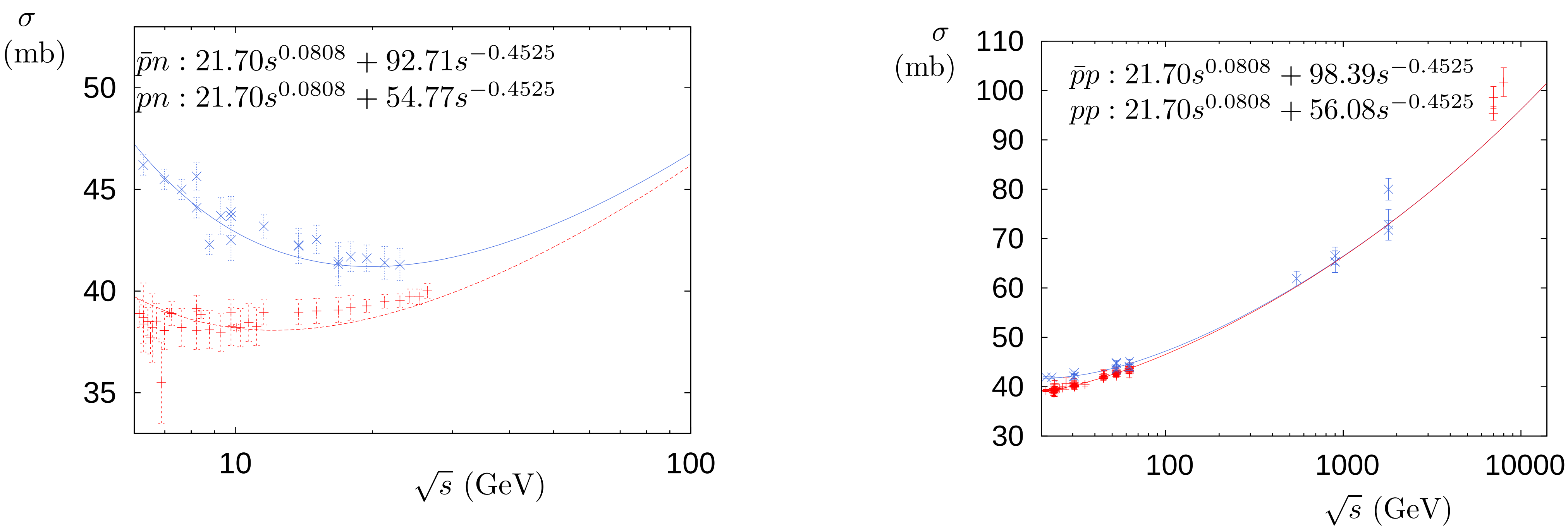
Data:

$$G_M(t) \approx \mu G_E(t) \quad G_E(t) \approx \frac{1}{1 - t/0.71)^2}$$

This gives

$$F_1(t) \approx \frac{4m_p^2 - 2.79t}{4m_p^2 - t} \frac{1}{(1 - t/0.71)^2}$$

Pomeron coupling to nucleon (1974!)

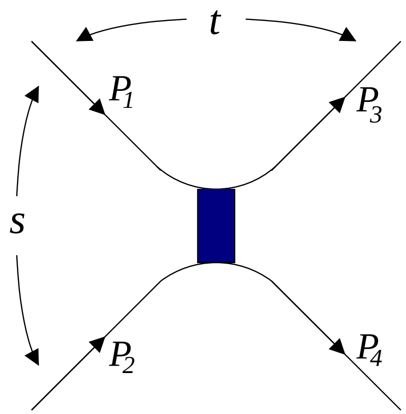


Coefficient of $s^{0.0808}$ same as for pp and $\bar{p}p$: the pomeron is isosinglet
Also $C = +1$: it couples equally to quarks and antiquarks

Big assumption: its coupling to quarks is just like an isoscalar photon, so to a hadron it is $n\beta F_1(t)$ where $F_1(t)$ is its electromagnetic form factor and β is a constant.
n is the number of constituent (quarks + antiquarks) in the hadron: $n = 3$ for a nucleon and 2 for a pion.
The absence of an F_2 term means no helicity flip – fits data

[Note that the coefficients of the $s^{-0.4525}$ term in pn are also not very different from pp, so the ω couples to the nucleon more strongly than the ρ]

Pomeron exchange



$$A(s,t) = \bar{u}(p_3)\, 3\gamma^\mu \beta_{I\!P} F_1(t)\, u(p_1) \cdot \bar{u}(p_4)\, 3\gamma^\mu \beta_{I\!P} F_1(t)\, u(p_2)\, e^{-\frac{1}{2}\pi\alpha(t)}\, (s/s_0)^{\alpha_{I\!P}(t)-1}$$

Assume the pomeron trajectory is linear (like ρ, ω, f_2, a_2):

$\alpha_{I\!P}(t) = \epsilon_{I\!P} + \alpha'_{I\!P} t$

The choice of s_0 is not critical, but $s_0 = 1/\alpha'_{I\!P}$ works well

$\beta_{I\!P}$ and $\epsilon_{I\!P}$ are known from σ^{TOT}

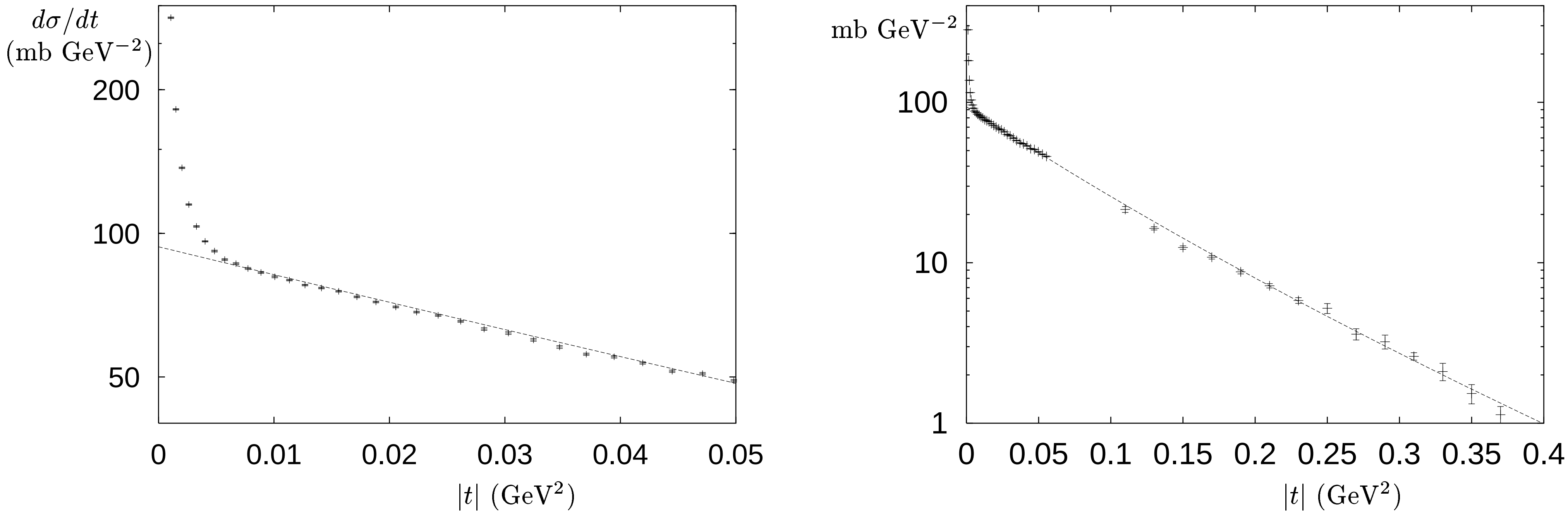
The only free parameter is $\alpha'_{I\!P}$

$$\frac{d\sigma}{dt} = \frac{[3\beta_{I\!P} F_1(t)]^4}{4\pi} (\alpha'_{I\!P} s)^{2(\epsilon_{I\!P} + \alpha'_{I\!P} t)}$$

Include also the ρ, ω, f_2, a_2 trajectory $\quad \alpha_R(t) = 1 - 0.4525 + 0.9t$

Proton-proton scattering

Fix α' from the very-low-t data at some energy, say $\sqrt{s}$ =53 Gev:



Determines $\alpha' = 0.25$ GeV^{-2}

Then the formula works well out to larger t at the same energy
It also fits well to pp and $p\bar{p}$ elastic scattering data at all other available energies.
Because $F_1(t)$ is raised to the power 4 in the formula, this gives a good test that it is the correct form factor, but why this should be so is not understood.

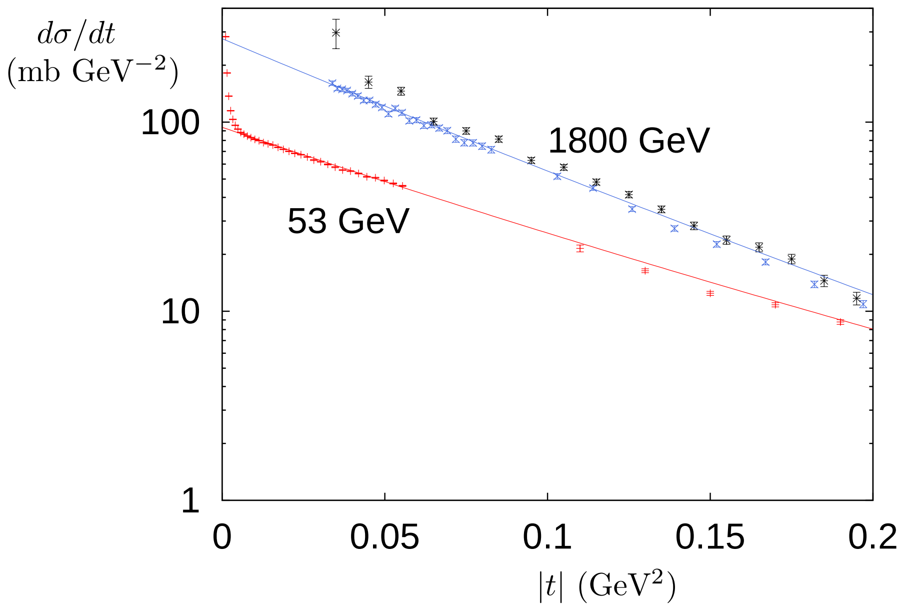
Note that the curves do not include photon exchange, which adds e^2/t to the amplitude and contributes significantly at very small t.

Shrinkage of the forward peak

Because the formula contains the factor

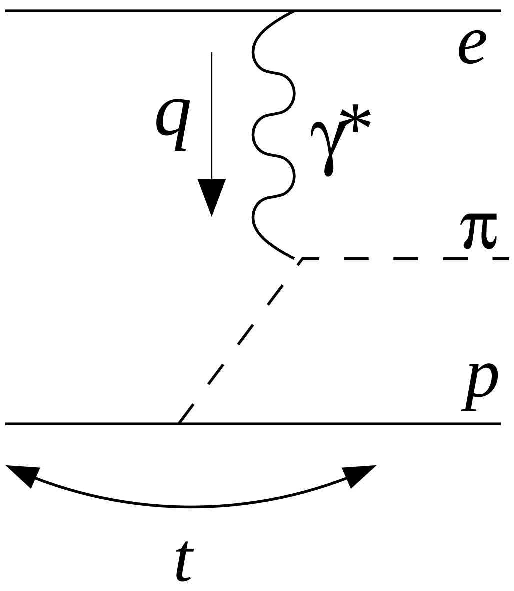
$$(\alpha'_{I\!P} s)^{2\alpha'_{I\!P} t} = \exp(-2\alpha'_{I\!P} \log(\alpha'_{I\!P} s)|t|)$$

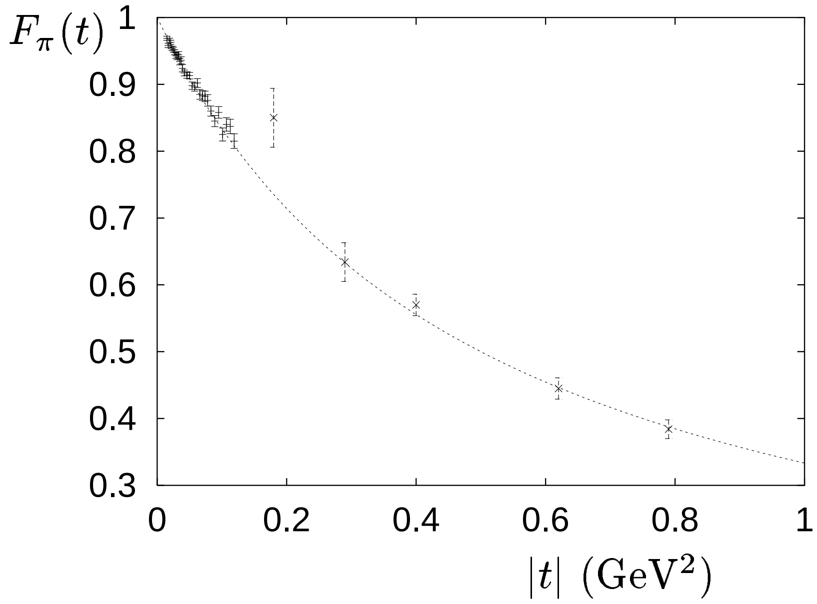
the contribution of pomeron exchange to the forward peak in $d\sigma/dt$ becomes steeper as the energy increases:



Note the discrepancy between the data from the two Tevatron experiments.

Pion form factor





Measure $ep \to ep\pi$ at $t = 0$

The exchanged π is almost on shell
So gives pion form factor $F_\pi(q^2)$ (Chew-Low theory)

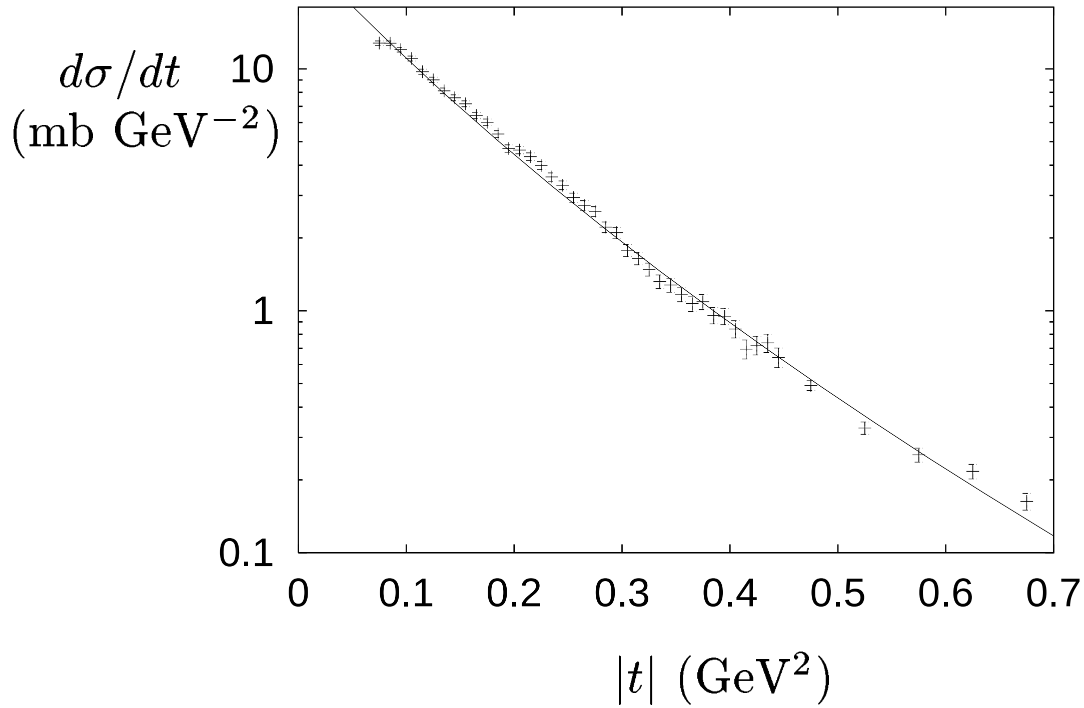
The curve is

$$F_\pi(t) = \frac{1}{1 - t/m_0^2} \qquad m_0^2 = 0.5\ \text{GeV}^2 \approx m_\rho^2$$

Pion-proton elastic scattering

$$\frac{d\sigma}{dt} = \frac{[3\beta_{I\!P} F_1(t)]^4}{4\pi} (\alpha'_{I\!P} s)^{2(\epsilon_{I\!P} + \alpha'_{I\!P} t)} \qquad pp$$

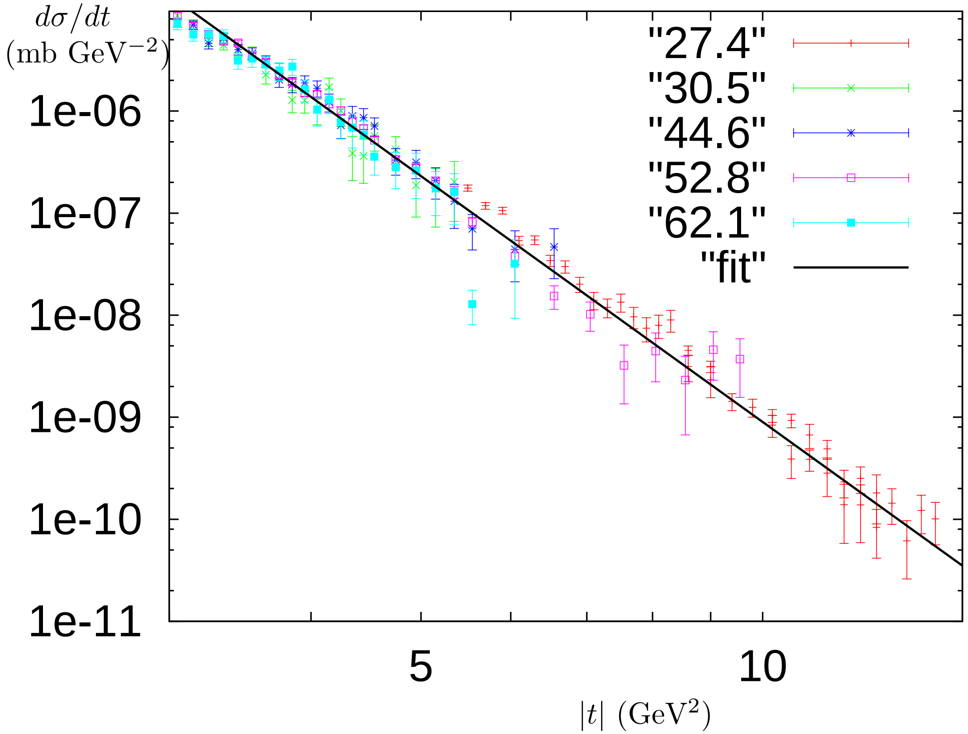
$$\frac{d\sigma}{dt} = \frac{[2\beta_{I\!P} F_\pi(t)]^2 [3\beta_{I\!P} F_1(t)]^2}{4\pi} (\alpha'_{I\!P} s)^{2(\epsilon_{I\!P} + \alpha'_{I\!P} t)} \qquad \pi p$$

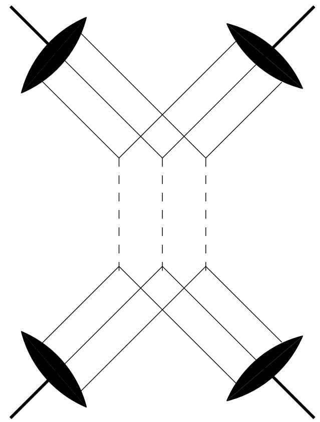


Pomeron exchange dominates already at $\sqrt{s} = 19.4$ GeV, because ω couples strongly to the nucleon but not to the pion

Proton-proton elastic scattering at large t

For $|t|$ greater than about 3 Gev^2, the data are consistent with being energy-independent and fit well to a simple power of t: $d\sigma/dt = 0.09\,t^{-8}$

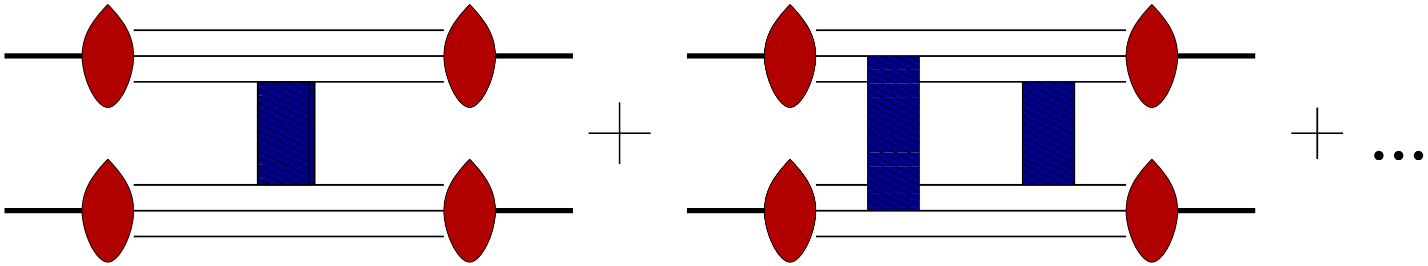




This behaviour is what is calculated from triple-gluon exchange
Why this simple mechanism, with no higher-order perturbative QCD corrections?

Note that triple-gluon exchange is $C = -1$ — its contributions to the pp and $\bar{p}p$ amplitudes are opposite in sign

Double pomeron exchange



What we know about double exchange:

$$\alpha_{I\!P}(t) = 1 + \epsilon_{I\!P} + \alpha'_{I\!P}(t) \qquad \alpha_{I\!P I\!P}(t) = 1 + 2\epsilon_{I\!P} + \tfrac{1}{2}\alpha'_{I\!P}(t)$$

Phase $\quad e^{-\frac{1}{2}i\pi\alpha_{I\!P}(t)} \qquad -\, e^{-\frac{1}{2}i\pi\alpha_{I\!P I\!P}(t)}$

So $I\!P I\!P$ has less steep t-dependence than $I\!P$ and is opposite in sign at small t

But:

Not simple power $s^{\alpha_{I\!P I\!P}(t)}$ – there are also unknown log factors

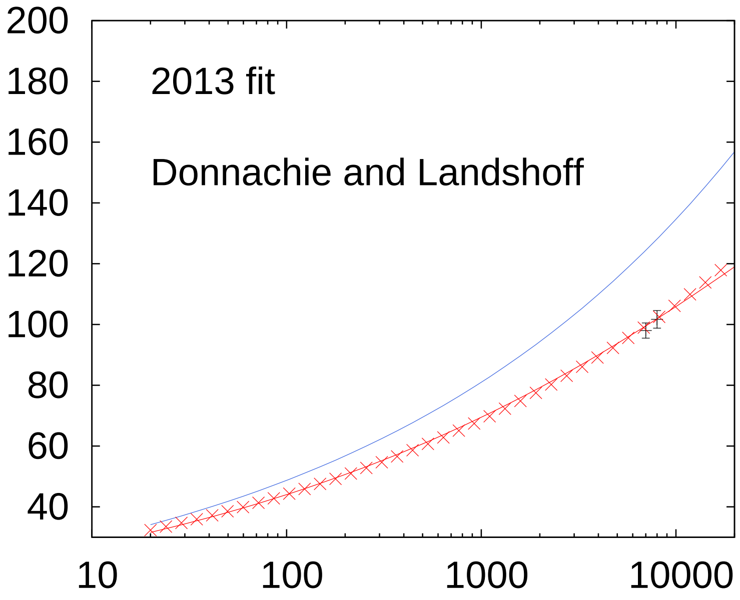
Unknown factor $F_{I\!P I\!P}(t)$, both t-dependence and magnitude

$$A(s, -\mathbf{q}^2) = 2is \int d^2b\, e^{-i\mathbf{q}.\mathbf{b}} \left(\chi - \frac{\chi^2}{2!} + \ldots\right)$$

If choose first term to be single exchange $I\!P$, second term has the right structure to be $I\!P I\!P$, but is wrong in detail

Effective power

The $I\!\!P I\!\!P$ term gives a negative contribution to the total cross sections

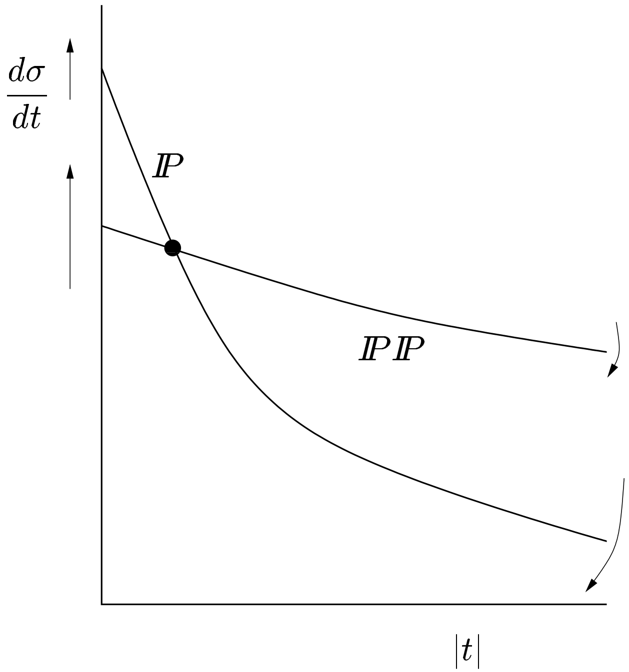


Blue line $17.55\ s^{0.110}$ $I\!\!P$

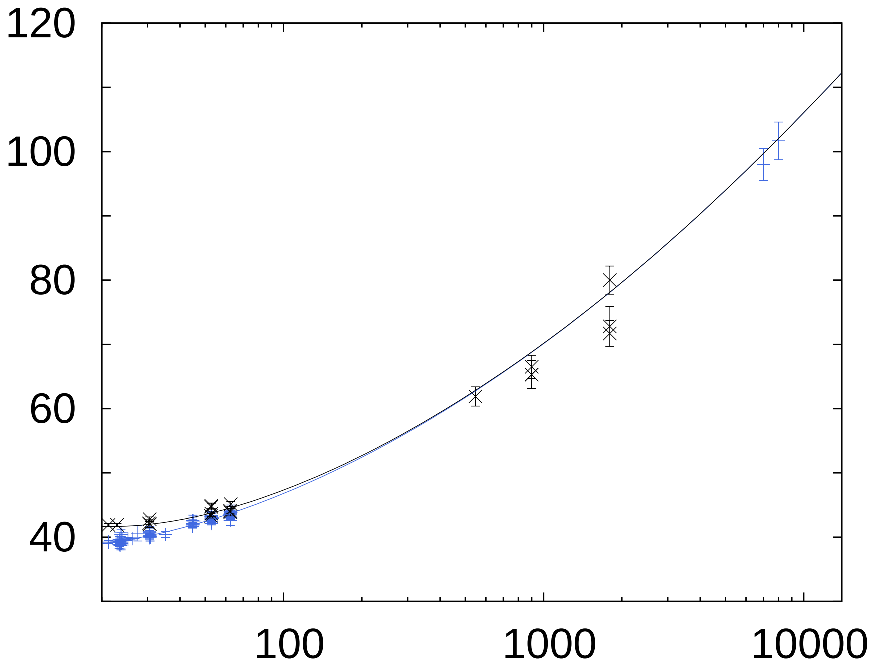
Red crosses $I\!\!P + I\!\!P I\!\!P$

Red line $18.23\ s^{0.096}$ effective power

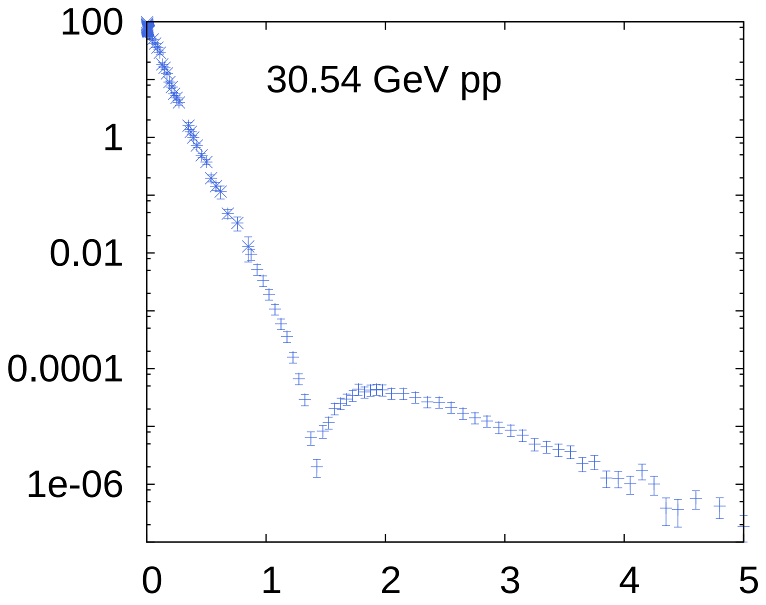
Black points TOTEM data



Current fit to pp and $\bar{p}p$ total cross sections with $I\!\!P, I\!\!P I\!\!P, \rho, \omega, f_2, a_2$



Dips in proton-proton scattering



Really deep dip at $\sqrt{s}$ =31 GeV

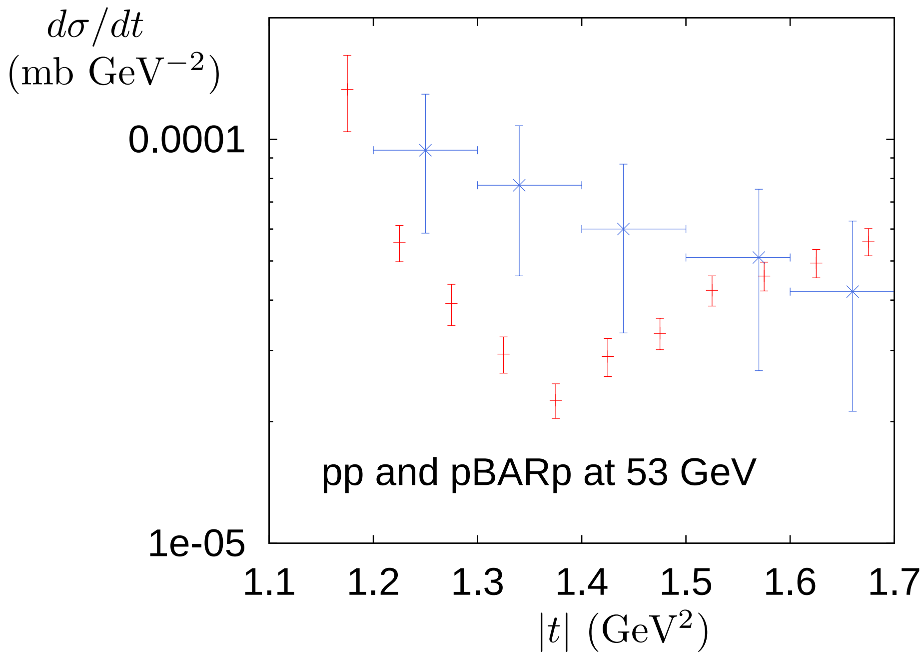
Both real and imaginary parts of amplitude almost vanish at same t value

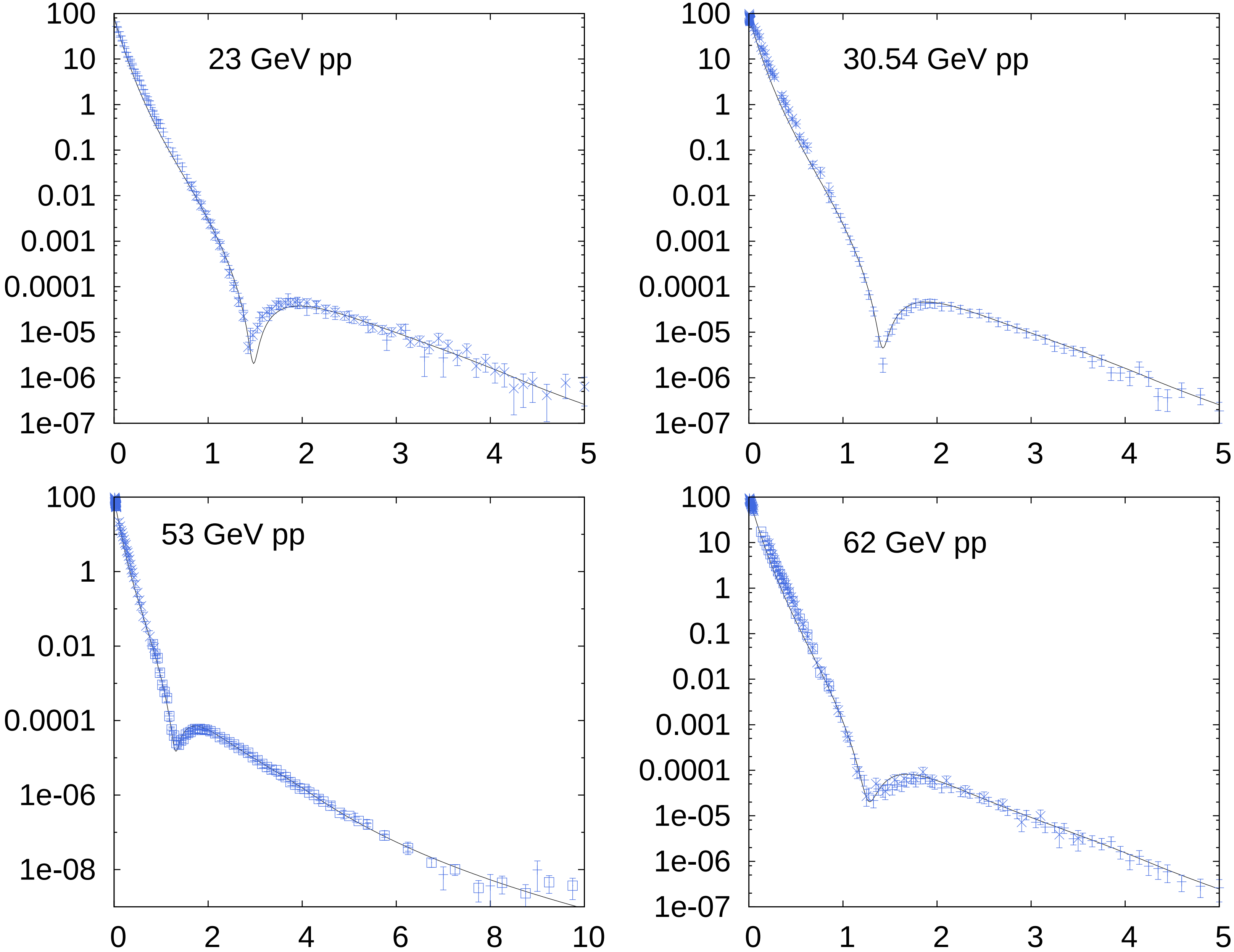
$I\!P$ and $I\!P I\!P$ have different phases $e^{-\frac{1}{2}i\pi\alpha_{I\!P}(t)}$ and $-e^{-\frac{1}{2}i\pi\alpha_{I\!P I\!P}(t)}$

Adjust $I\!P I\!P$ to cancel imaginary parts

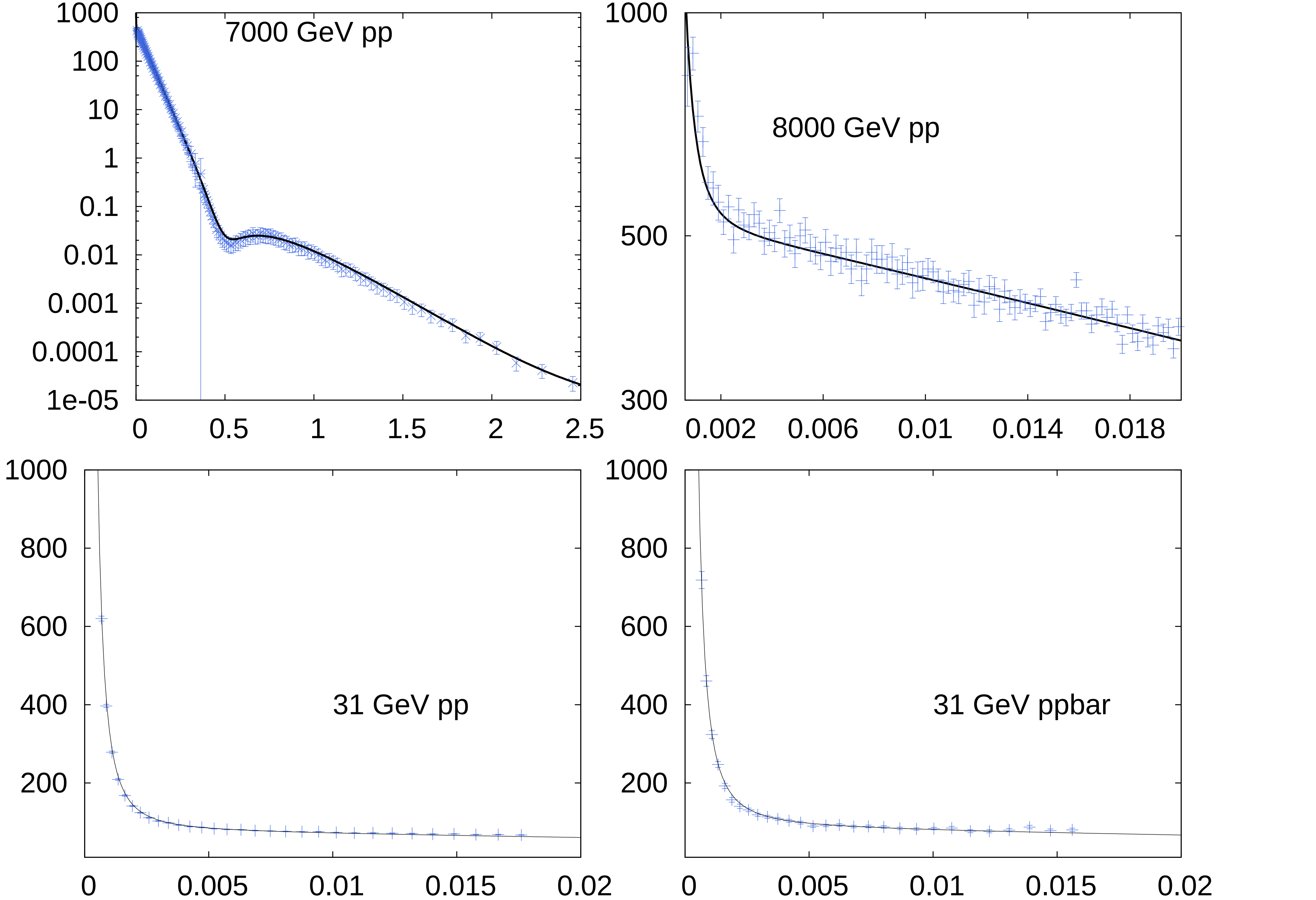
Need another term to cancel real parts: perhaps ggg

But ggg is $C = -1$ so then no dip in $\bar{p}p$ scattering

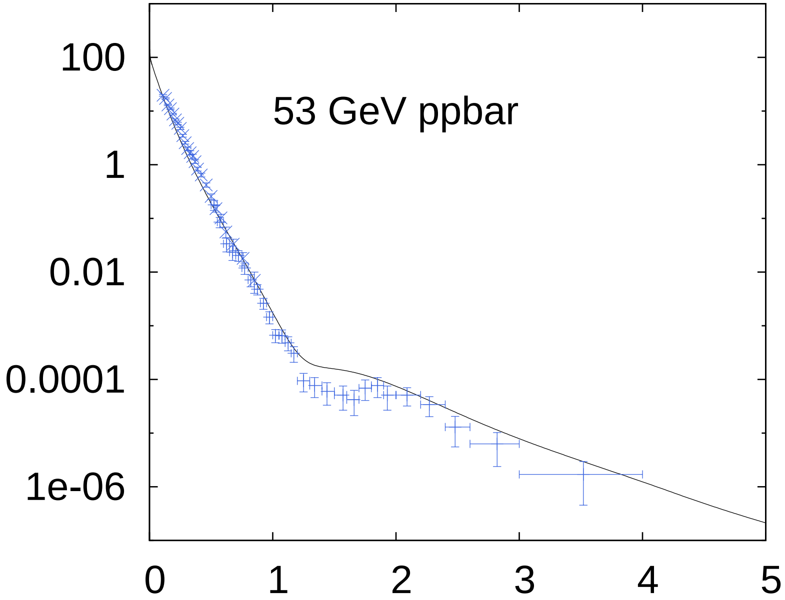


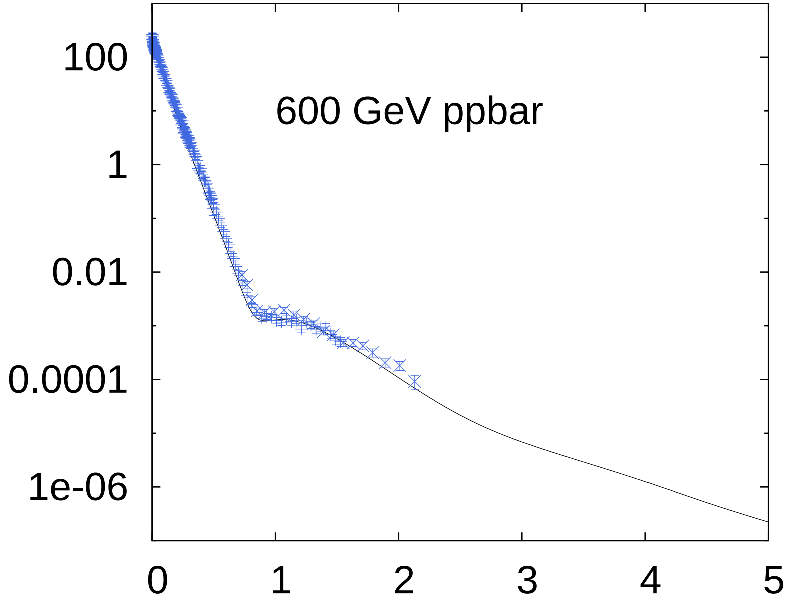


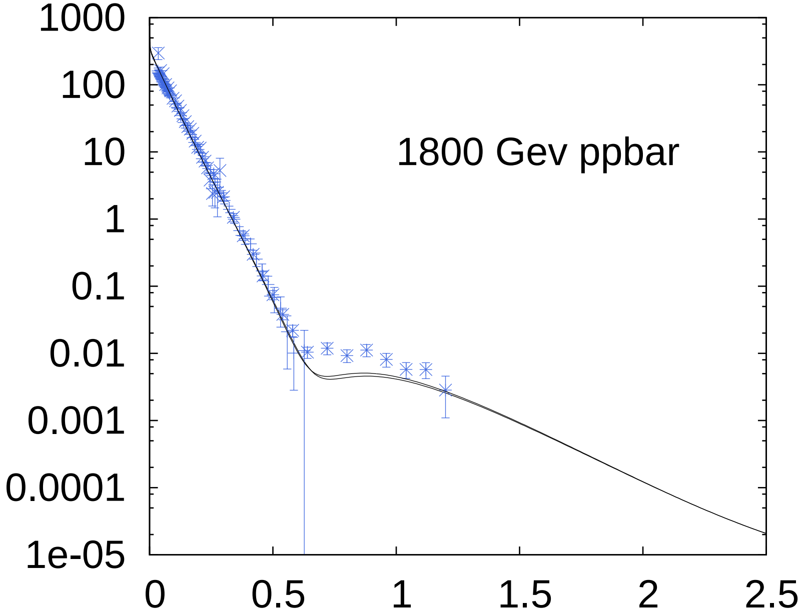
23 GeV pp
30.54 GeV pp
53 GeV pp
62 GeV pp
100
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
0
1
2
3
4
5
6
8
10



7000 GeV pp
1000
100
10
1
0.1
0.01
0.001
0.0001
1e-05
0
0.5
1
1.5
2
2.5
8000 GeV pp
1000
500
300
0.002
0.006
0.01
0.014
0.018
31 GeV pp
1000
800
600
400
200
0
0.005
0.01
0.015
0.02
31 GeV ppbar
1000
800
600
400
200
0
0.005
0.01
0.015
0.02

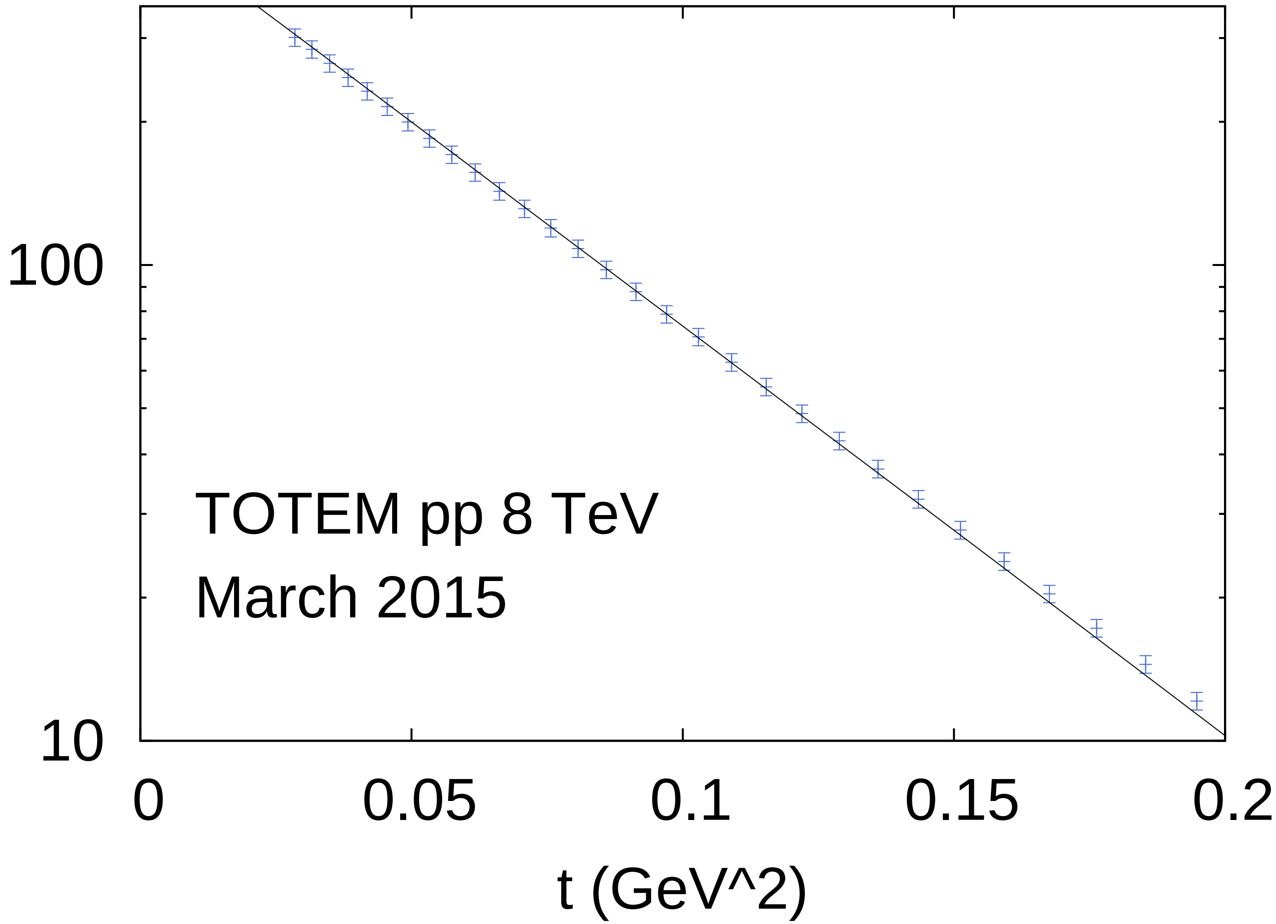






Details: A Donnachie and P V Landshoff
Physics Letters B727 (2013) 500
http://arxiv.org/abs/1309.1292

Challenge: get a better fit to <u>all</u> the data!



100
10
0
0.05
0.1
0.15
0.2
t (GeV^2)
TOTEM pp 8 TeV
March 2015

HEP 205 records found 1 - 25 jump to record: 1

1. **Photoproduction of pi+ pi- pairs in a model with tensor-pomeron and vector-odderon exchange**
Arthur Bolz (Heidelberg U.), Carlo Ewerz (U. Heidelberg, ITP & Darmstadt, EMMI & Darmstadt, GSI), Markos Maniatis (Biobio U
e-Print: **arXiv:1409.8483 [hep-ph]** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record

2. **Strong Coupling Expansion for the Conformal Pomeron/Odderon Trajectories**
Richard C. Brower (Boston U.), Miguel S. Costa, Marko Djurić (Porto U. & Porto U., Astron. Dept.), Timothy Raben, Chung-I Ta
e-Print: **arXiv:1409.2730 [hep-th]** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 1 record

3. **QCD at small x: From Color Glass Condensate to Pomerons and Odderons and more**
Jamal Jalilian-Marian (Baruch Coll. & CUNY, Graduate School - U. Ctr.). 2014. 4 pp.
Published in **EPJ Web Conf. 66 (2014) 04012**
DOI: 10.1051/epjconf/20146604012
Conference: C13-06-02.3 Proceedings
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record

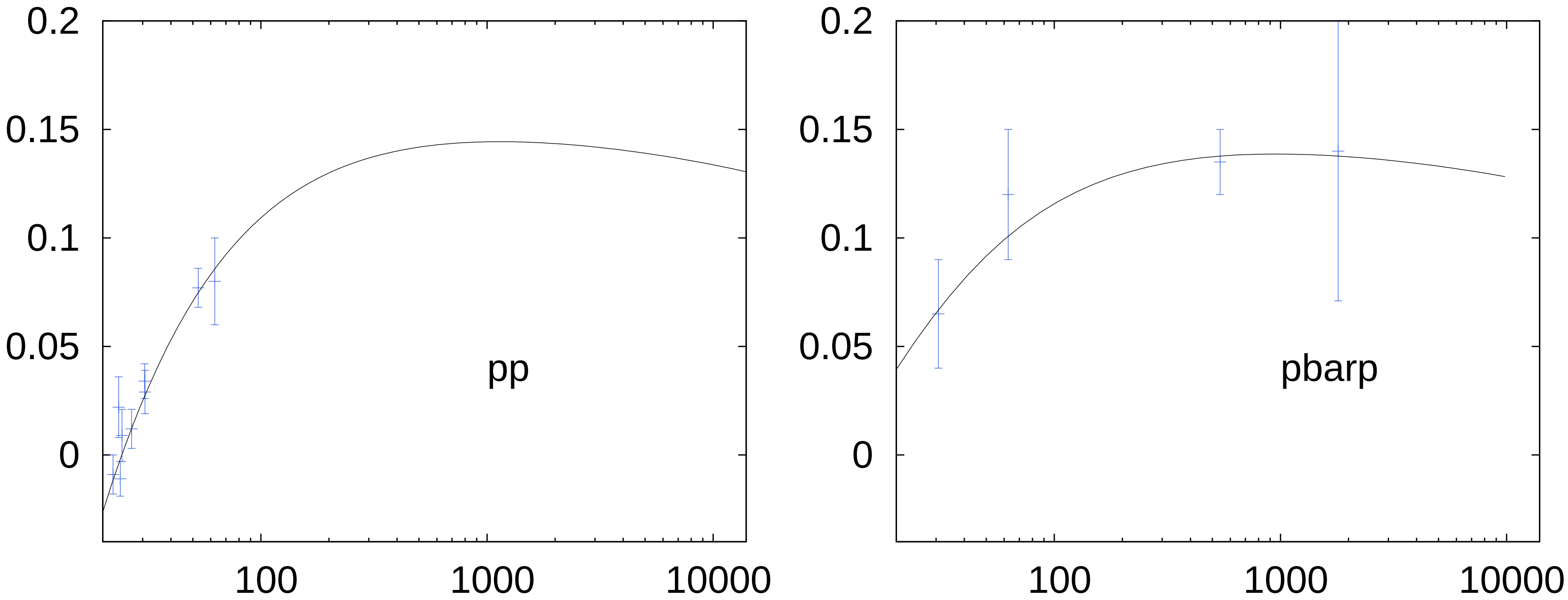
4. **Conformal Pomeron and Odderon in Strong Coupling**
Richard C. Brower (Boston U.), Miguel Costa (Porto U.), Marko Djuric (Porto U. & Porto U., Astron. Dept. & Porto U.), Timothy
Conference: C13-05-30
e-Print: **arXiv:1312.1419 [hep-ph]** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 4 records

5. **A Model for Soft High-Energy Scattering: Tensor Pomeron and Vector Odderon**
Carlo Ewerz (Heidelberg U. & Darmstadt, EMMI), Markos Maniatis (Biobio U.), Otto Nachtmann (Heidelberg U.). Sep 13, 2013.
Published in **Annals Phys. 342 (2014) 31-77**
DOI: 10.1016/j.aop.2013.12.001
e-Print: **arXiv:1309.3478 [hep-ph]** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 6 records

6. **Generalized Bootstrap Equations and possible implications for the NLO Odderon**

A $C = -1$ term like ggg which survives at high energy is called an odderon.
There have been many attempts to see signs of an odderon at $t = 0$.
A sensitive test is thought to be in

$$\rho = \frac{\text{Re}A(s,t)}{\text{Im}A(s,t)}\Big|_{t=0}$$



Not there?

Glueball trajectory??

Our fit has

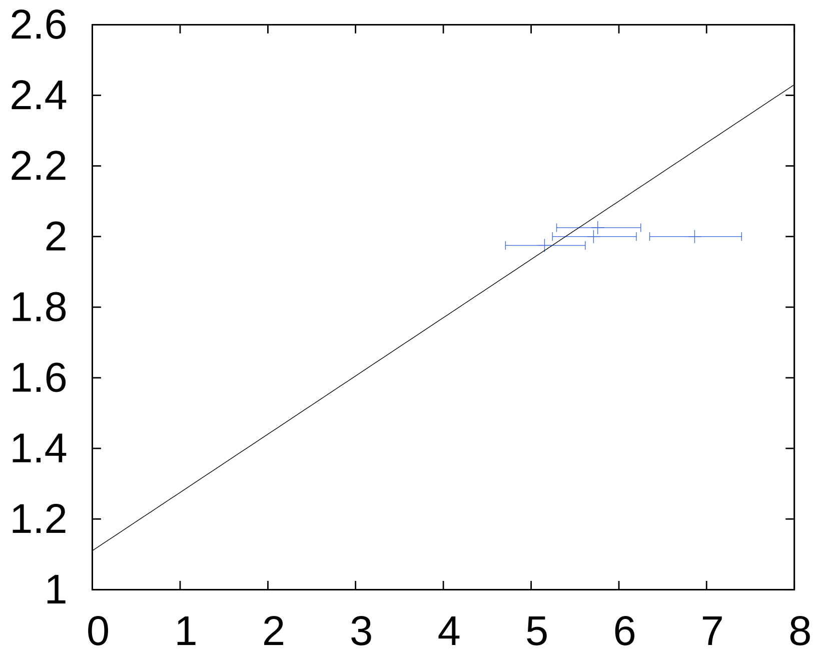
$$\alpha_{I\!P}(t) = 1.1 + 0.165t$$

If this linear form extends to positive t it should pass through the mass of a $J^{PC} = 2^{++}$ glueball

Experimental situation with glueballs obscure

Lattice calculations:

- Bali et al (1993) 2.27 ± 0.1 GeV
- Morningstar et al (1999) 2.4 ± 0.12 GeV
- Chen et al (2006) 2.39 ± 0.12 GeV
- Gregory et al (2012) 2620 ± 0.05 GeV



Summary so far

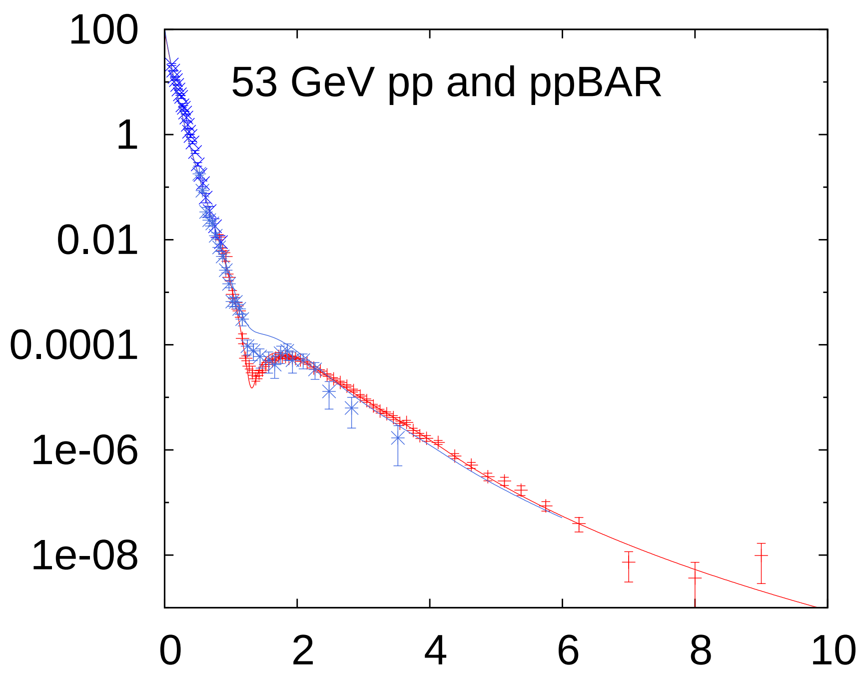
Effective-power approach describes small-t elastic scattering and total cross sections:

$$pp \quad \bar{p}p \quad \pi p \quad Kp \quad \gamma p$$

But for larger t need to consider

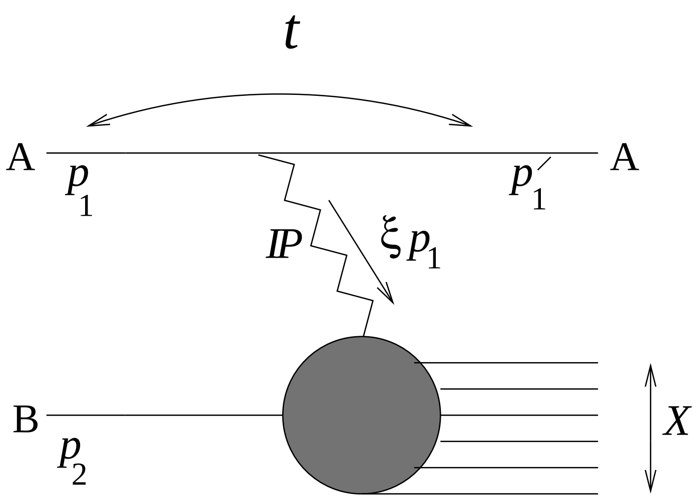
$$I\!\!P + I\!\!P I\!\!P + ggg + \ldots ?$$

Easy to fit subsets of data, but universal fit more challenging



Diffraction dissociation

$AB \to AX$ with particle A losing a very small fraction ξ of its momentum



Can think of the lower part of the diagram as the pomeron $I\!P$ scattering on particle B

$$M_X^2 = (p_1 + p_2 - p_1')^2 \sim \xi s$$

If p is the momentum of one of the particles of system X, define its rapidity

$$Y = \tfrac{1}{2} \log \frac{E + p_L}{E - p_L}$$

(which is invariant under longitudinal Lorentz-frame boosts)

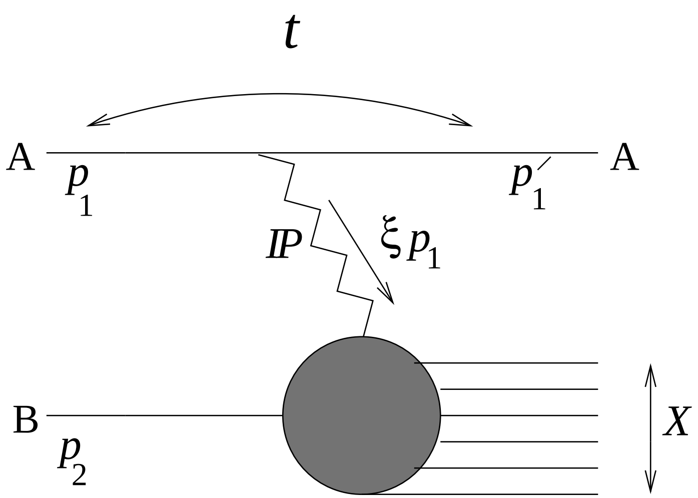
If ξ is small, the rapidity of particle p_1' is approximately

$\log(\sqrt{s}/m'_{1T}) \qquad m'^2_{1T} = p'^2_{1T} + m^2$

while the rapidity of the fastest particle in the system X is approximately 0

So there is a large rapidity gap

Mueller's generalised optical theorem



Sum over systems of hadrons X

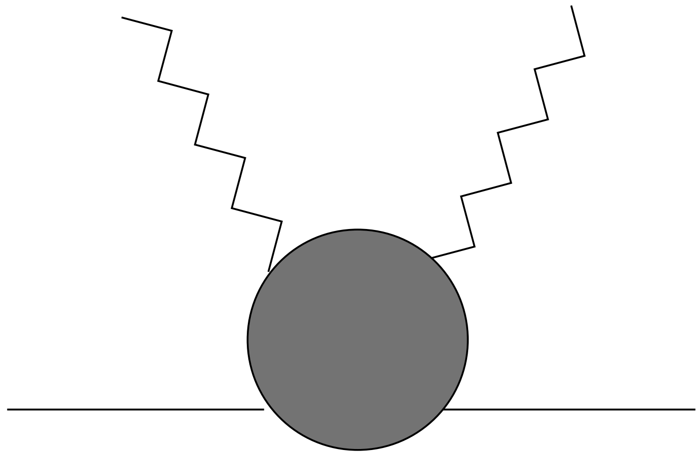
$$\frac{d^2\sigma}{dt\,d\xi} = D^{I\!\!P/a}(t,\xi)\;\sigma^{I\!\!Pb}(M_X^2,t)$$

$D^{I\!\!P/a}(t,\xi)$ is the "flux" of pomerons emitted by A:

$$D^{I\!\!P/a}(t,\xi) = \frac{9\beta_{I\!\!P}^2}{4\pi^2}\,(F_1(t))^2\,\xi^{1-2\alpha_{I\!\!P}(t)}$$

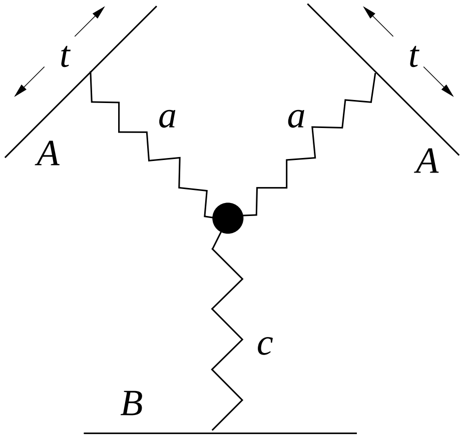
$\sigma^{I\!\!Pb}(M_X^2,t)$ is the cross section for a pomeron of squared mass t scattering on particle B

Generalised optical theorem: it is calculated from the imaginary part of the forward $I\!\!PB$ elastic scattering amplitude



Triple-reggeon vertex

At large M_X the amplitude $I\!\!P B \to I\!\!P B$ should be dominated by pomeron exchange:

$\frac{d^2\sigma}{dt\, d\xi} =$ 

Upper two pomerons: squared 4-momentum t
Lower pomeron: zero 4-momentum

We know all the factors, except the triple-pomeron vertex $V_{I\!\!P I\!\!P I\!\!P}(t)$ in the middle

Complications: unless ξ is very small, also need ρ, ω, f_2, a_2 instead of the top two pomerons, and unless $M_X^2 = \xi s$ is very large the same for the lower reggeon

So need

$$\begin{matrix} I\!\!P I\!\!P & I\!\!P I\!\!P & f_2 I\!\!P & I\!\!P f_2 & f_2 I\!\!P & \omega I\!\!P & \\ I\!\!P & f_2 & I\!\!P & I\!\!P & f_2 & \omega & \dots \end{matrix}$$

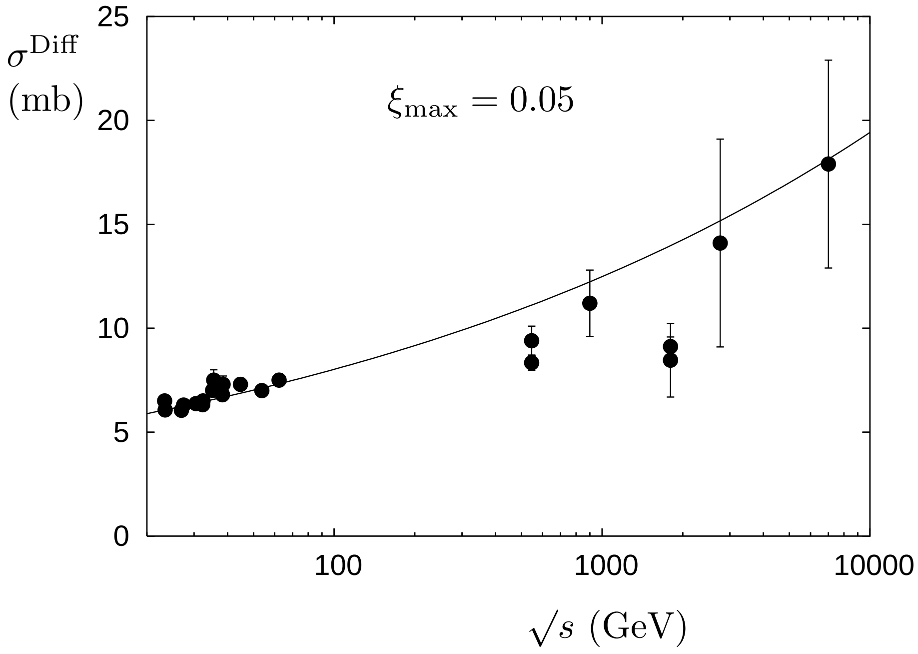
each with its unknown triple vertex in the middle

Usually, for simplicity, only terms of the form aac are considered:

$$F_a^A(t) F_a^A(t) F_c^B(0) V_c^{aa}(t)\; \xi^{\alpha_c(0) - 2\alpha_a(t)}\; (\alpha_c' s)^{\alpha_c(0)-1}$$

Non-pomeron terms are often important!

Diffraction dissociation data problems



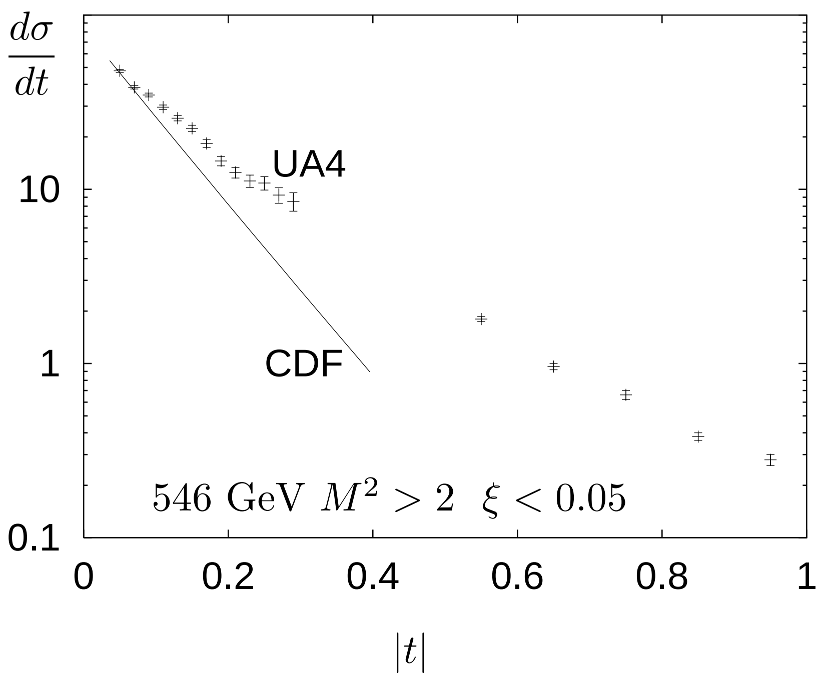
$\sigma^{\rm Diff}(s) = \int dt \int_{\xi_{\rm min}}^{\xi_{\rm max}} d\xi \; d^2\sigma/dtd\xi$

Sensitive to $\xi_{\rm max}$, and more so to $\xi_{\rm min}$

Uppermost three points: ALICE at LHC

The curve rises as $s^{0.08}$

Issues with the data – often unclear experiments are measuring the same thing

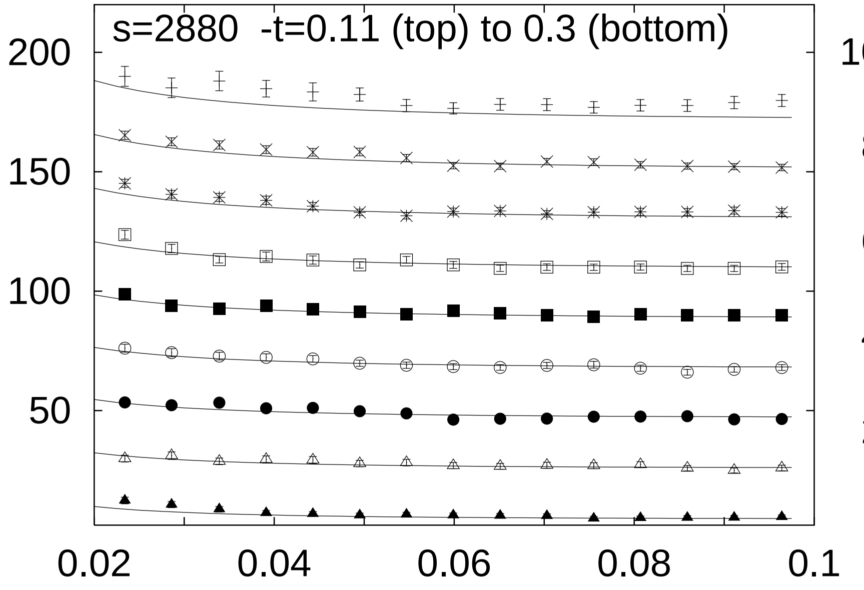


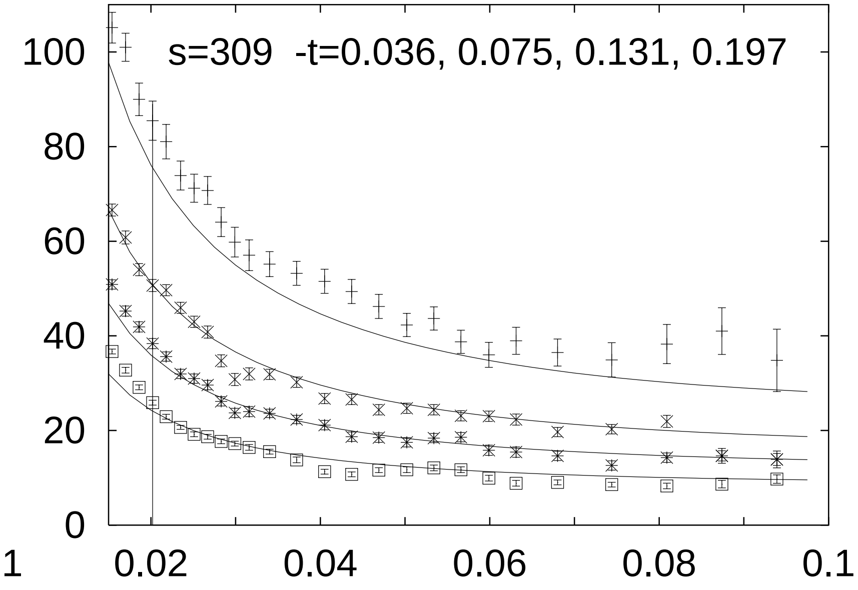
Data at $\sqrt{s}$ = 546 GeV

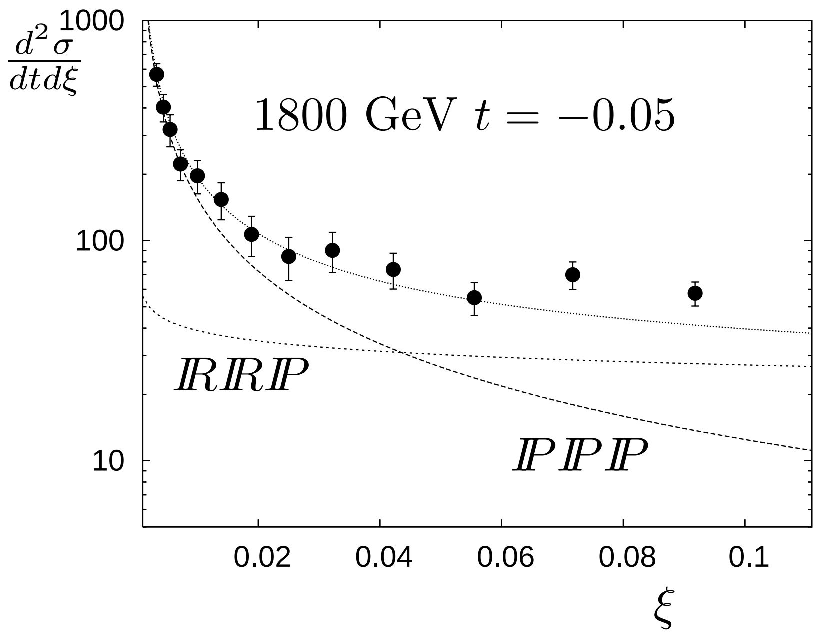
The line is CDF's parametrisation of its data for $d^2\sigma/dtd\xi$ integrated over ξ

Diffraction dissociation data fits

Data for $d^2\sigma/dtd\xi$



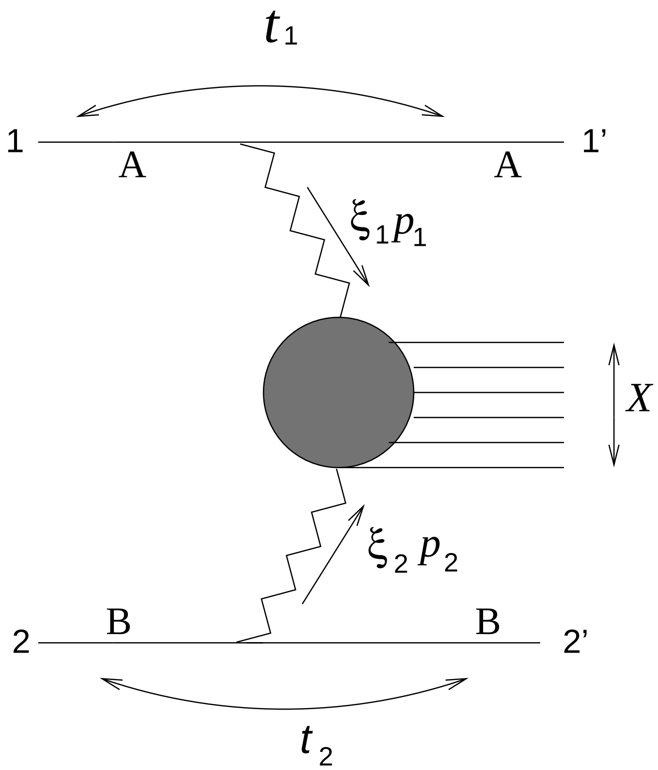


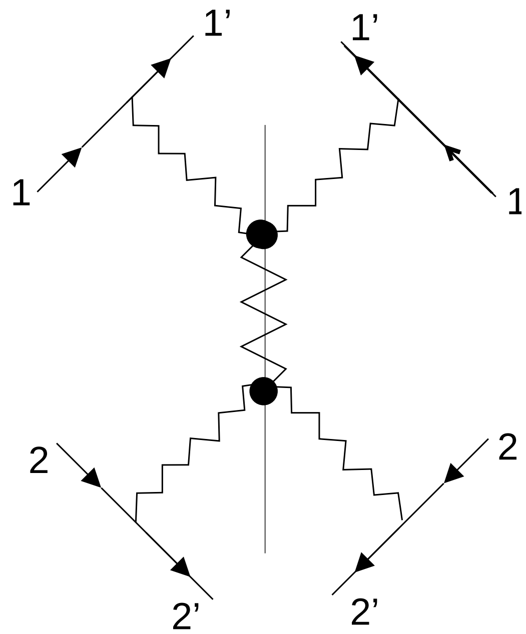


$I\!\!P I\!\!P I\!\!P$ does not dominate until ξ is extremely small

Double diffraction dissociation

Both initial particles lose a very small fraction of their momentum



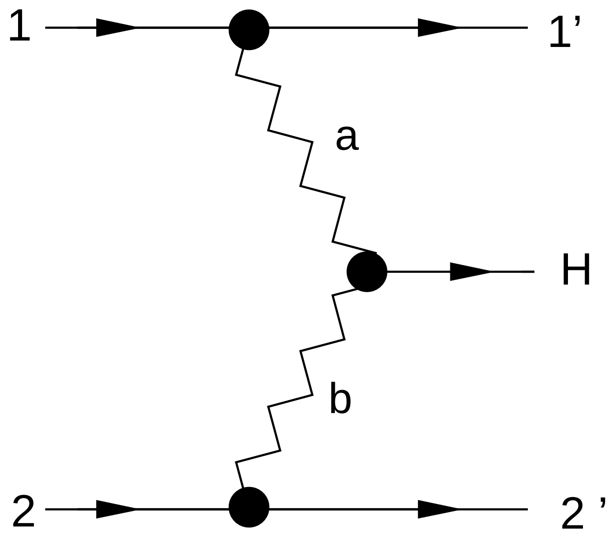


$$\frac{d^4\sigma}{dt_1 d\xi_1 dt_2 d\xi_2} = D^{I\!P/A}(t_1,\xi_1) D^{I\!P/B}(t_2,\xi_2) \sigma^{I\!P I\!P}(M_X^2, t_1, t_2)$$

If M_X is large enough for $\sigma_{I\!P I\!P}(M_X^2)$ to be dominated by $I\!P$ exchange

$$\frac{d^4\sigma(s)}{dt_1 d\xi_1 dt_2 d\xi_2} = \frac{d^2\sigma(s)}{dt_1 d\xi_1} \frac{d^2\sigma(s)}{dt_2 d\xi_2} \sigma_{\rm pp}^{\rm Tot}(s)$$

Exclusive central production



5 independent variables, eg

$s=(p_1+p_2)^2 \quad s_1=(p_1'+p_H)^2 \quad s_2=(p_H+p_2')^2$

$t_1=(p_1'-p_1)^2 \quad t_2=(p_2'-p_2)^2$

In most events s_1, s_2 are large and t_1, t_2 small

$s_1 \sim s\,\xi_2 \quad s_2 \sim s\,\xi_1 \qquad \xi_1\xi_2 s \sim M_H^2$

Energy dependence of amplitude: $(\alpha_a' s\xi_2)^{\alpha_a(t_1)}(\alpha_b' s\xi_1)^{\alpha_b(t_2)}$

Square and apply

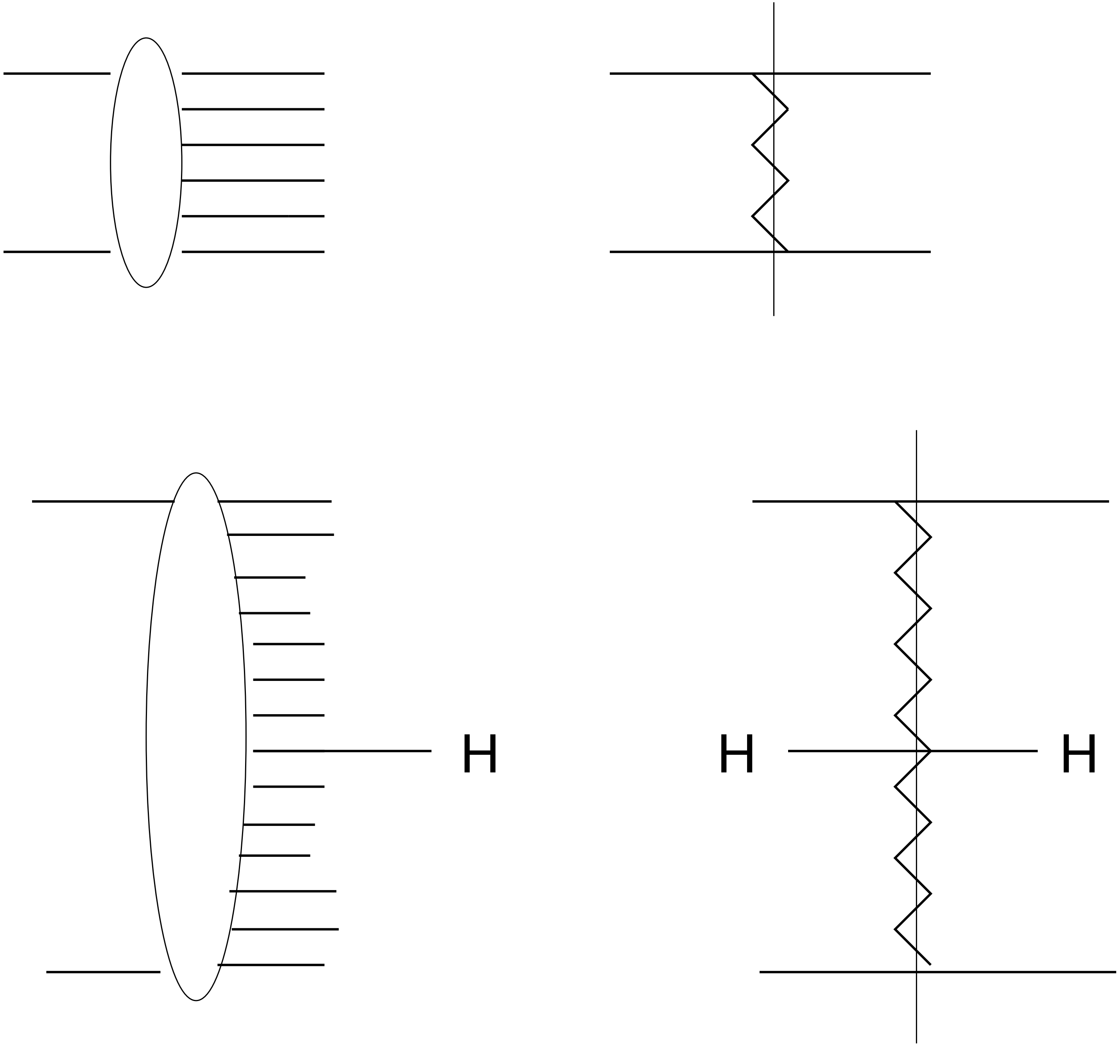
$$\int_{M_H^2/s}^{1} d\xi_1 d\xi_2\; \delta(\xi_1\xi_2 - m_H^2/s)$$

$\sigma(I\!R I\!P)$ and $\sigma(I\!P I\!P)$ increase with energy, $\sigma(I\!R I\!R) \sim 1/s$

Hope that this is a good mechanism to produce glueballs

ALICE has detected $f_0(980)$ and $f_2(1270)$ production in $pp \to pp\pi\pi$ double-gap events

Inclusive central production

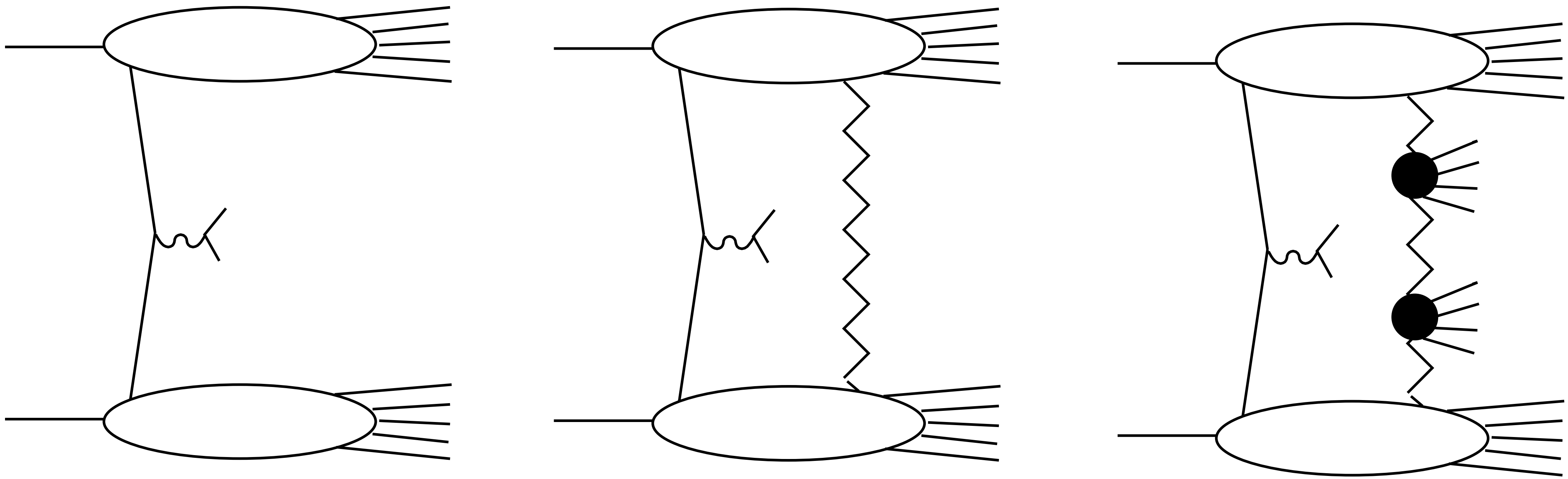


$p\,p \;\rightarrow H\;\; X$

AGK cancellation

(Abramovskii, Gribov, Kancheli)

Simplest example: Drell-Yan $pp \to \ell^+ \ell^- X$

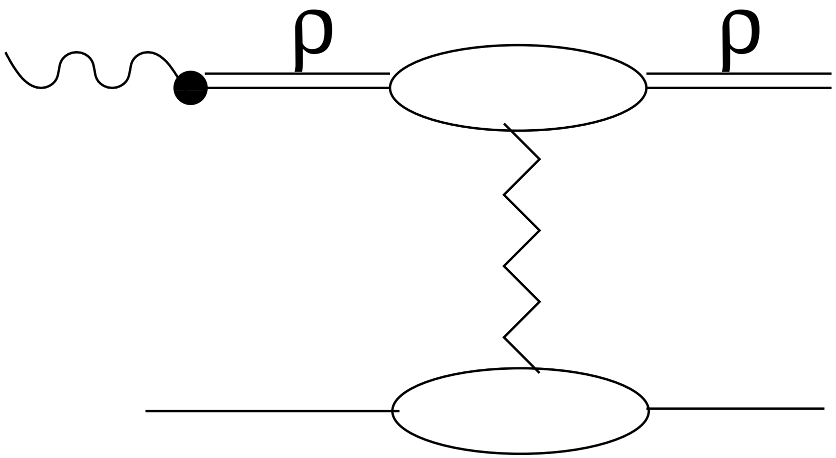


Add initial-state interactions, and cross interactions between initial and final states

They all cancel in the inclusive cross section $d\sigma/dq^2$

But they do change the final state

Real photons

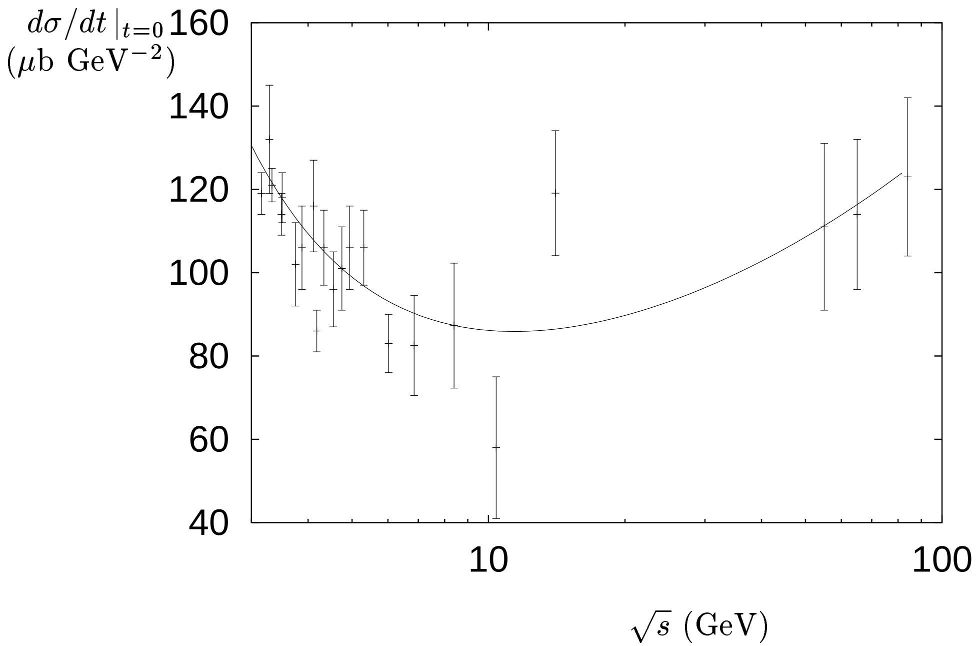


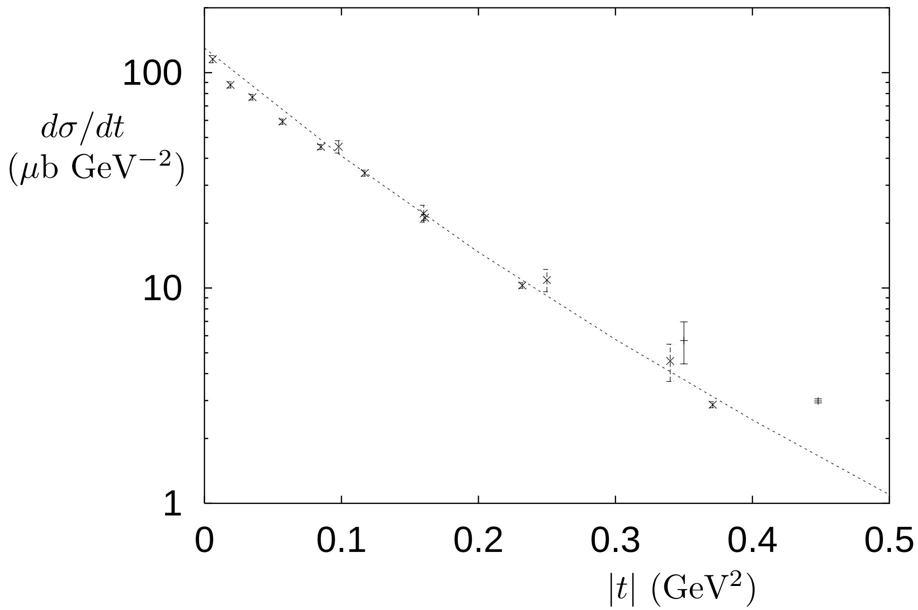
Simplest form of vector dominance:

$$\frac{d\sigma}{dt}(t) = \alpha_{EM}\ \frac{4\pi}{\gamma_\rho^2}\ \frac{d\sigma}{dt}(\rho^0 p \to \rho^0 p : t)$$

γ_ρ is ρ-photon coupling got from $\rho \to e^+e^-$

Assume $I\!\!P$ coupling to ρ same as to π and include f_2, a_2 exchange – need a fudge factor 0.84 (ρ width? higher resonances?)





$d\sigma/dt$ at $t = 0$ and at $\sqrt{s} = 94$ GeV

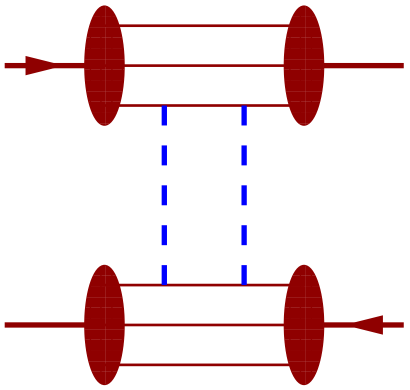
Summary on inelastic diffractive processes

- There are severe problems with the lack of good data
- Single effective-power exchanges should be tried first
- But there are indications that for exclusive production of heavy systems, such as Higgs or a pair of high-p_T jets, initial and final state interactions are important – people talk about rapidity gap survival probabilities

Models of the pomeron

There are many:

dipole, geometrical, dual, string, color glass, etc



Simplest QCD model: two-gluon exchange

At $t = 0$ the amplitude is proportional to

$\int_{-\infty}^{0} dk^2 [\alpha_S D(k^2)]^2$

where $D(k^2)$ is the gluon propagator

The perturbative propagator $D(k^2) = 1/k^2$ would make the integral diverge

Introduce a length scale a: the largest distance the gluon can propagate in the vacuum before confinement pulls it back

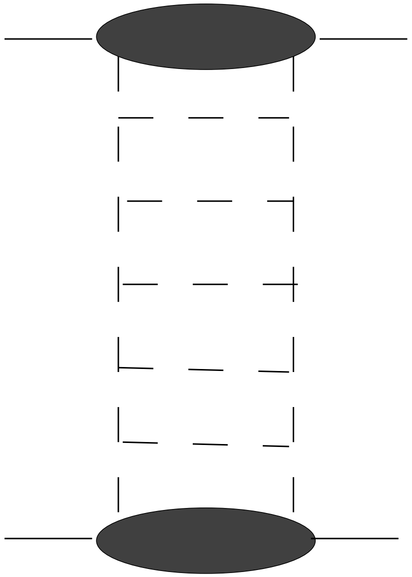
Simplest: $D(k^2) = a^2/(1 + a^2 k^2)$

Then it turns out that if a is much less than the nucleon radius both gluons want to couple to the same quark in each nucleon – in line with the quark-counting rule for total cross sections

(This is the Landshoff-Nachtmann model)

BFKL pomeron

(Balitsky, Fadin, Kuraev, Lipatov)



Sums generalised perturbative gluon ladder graphs

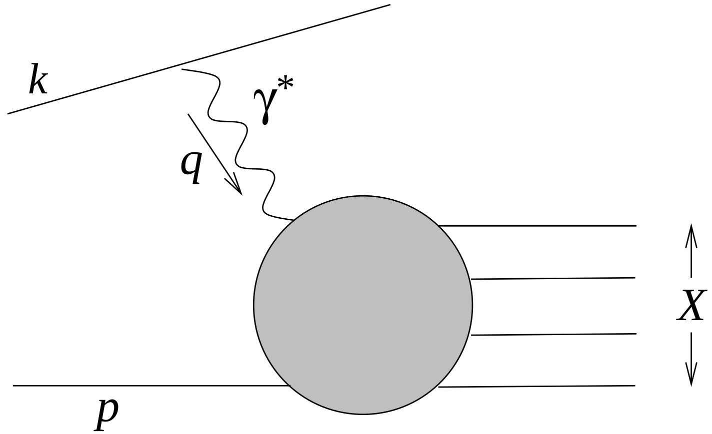
Leading-order calculation: effective power s^{ϵ}

$\epsilon = 12(\alpha_s/\pi)\log 2 \approx 0.4$

But DISASTER: next-order calculation makes ϵ negative

0.4 would have been good!

Inelastic lepton scattering

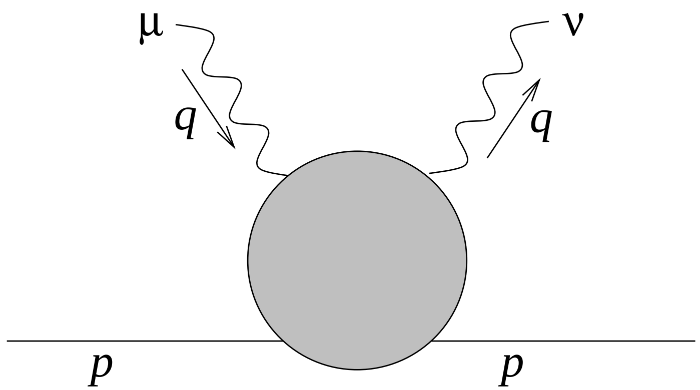


$$Q^2 = -q^2 \qquad \nu = p.q \qquad x = \frac{Q^2}{2\nu} \qquad y = \frac{p.q}{p.k}$$

$$W^2 = (p+q)^2 = Q^2\left(\frac{1}{x} - 1\right) + m^2$$

Square and sum over X

Need imaginary part of virtual-Compton amplitude



$$\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) F_1(x, Q^2)$$
$$+ \frac{1}{\nu}\left(p^\mu + \frac{\nu}{Q^2} q^\mu\right)\left(p^\nu + \frac{\nu}{Q^2} q^\nu\right) F_2(x, Q^2)$$

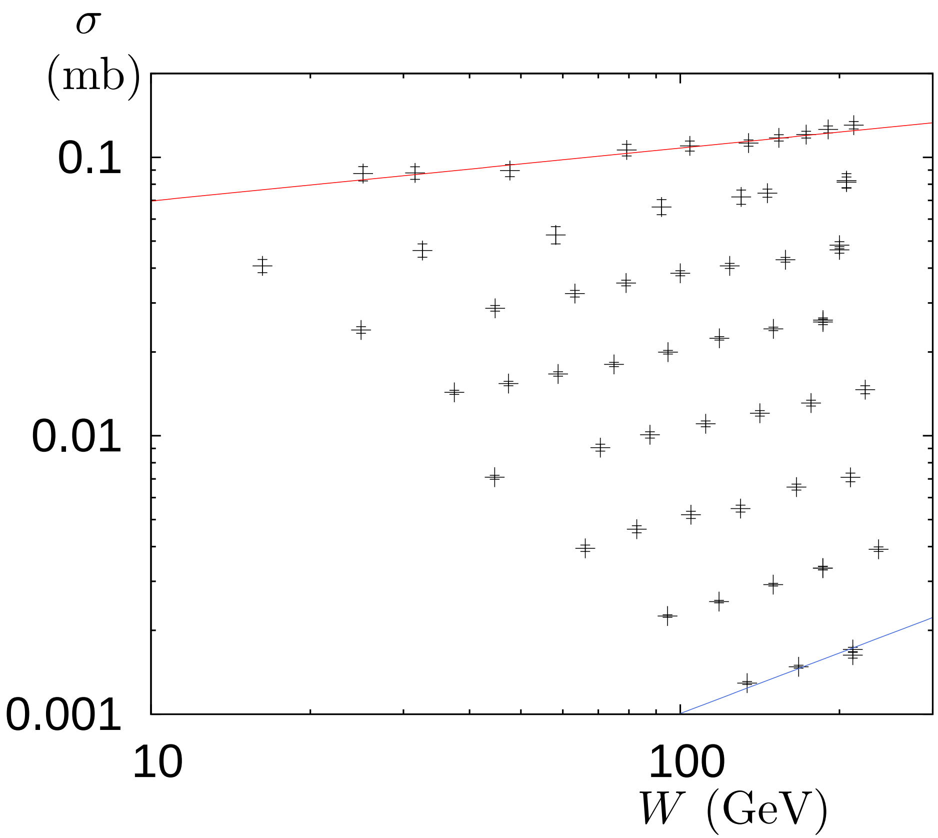
Does Regge theory apply when $W^2 >> Q^2$, ie at small x ?

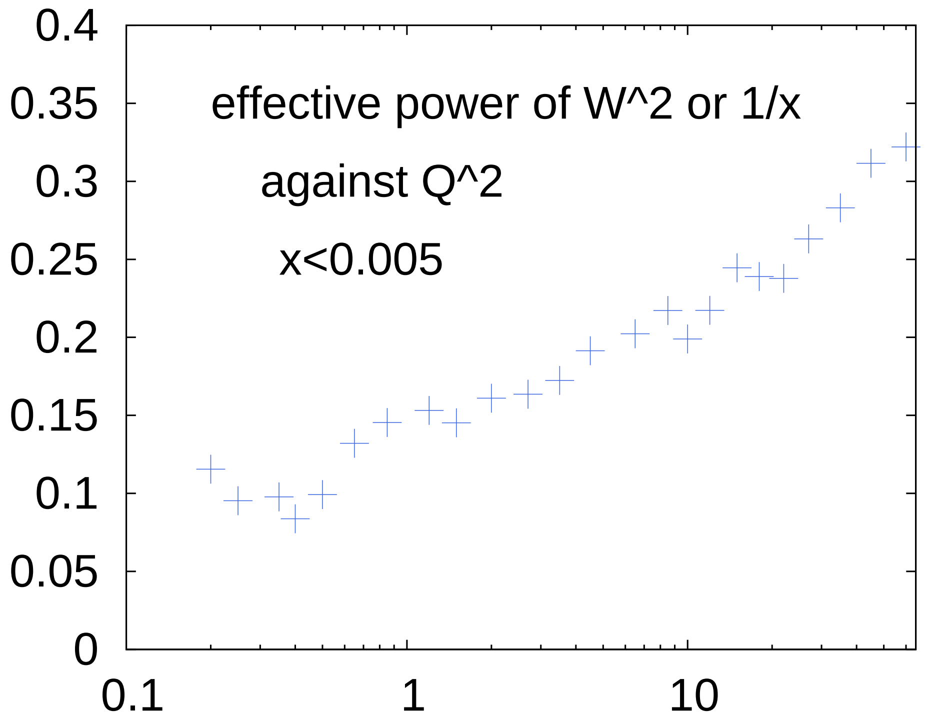
Deep inelastic total cross sections

We already have seen that it applies for the real γp total cross section

$$\sigma^{\gamma p}(W) = \frac{4\pi^2 \alpha_{EM}}{Q^2} F_2(x, Q^2)\Big|_{Q^2=0}$$

Adopt same definition for nonzero Q^2





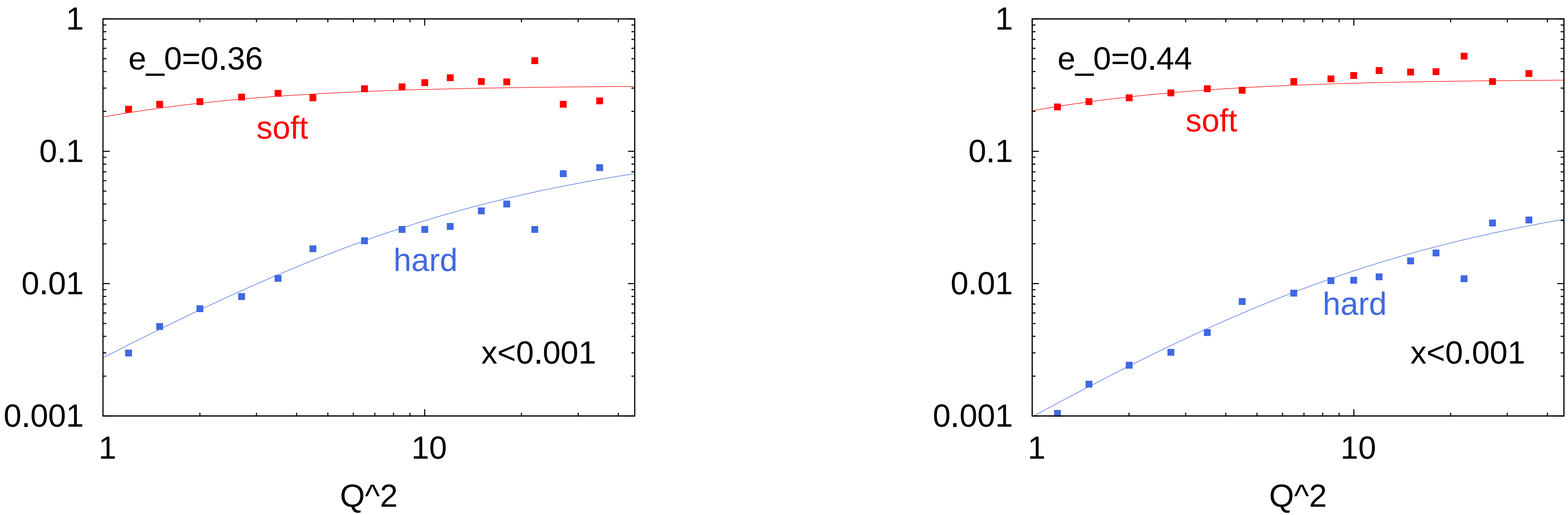
Q^2 from 0.25 to 90 GeV^2 —- power increases from a bit less than 0.1 to about 0.35

Regge approach

$F_2(x,Q^2) = F_0(Q^2)x^{-e_0} + F_1(Q^2)x^{-e_1}$ at small x ???

$e_1 = 0.096$ "soft pomeron"

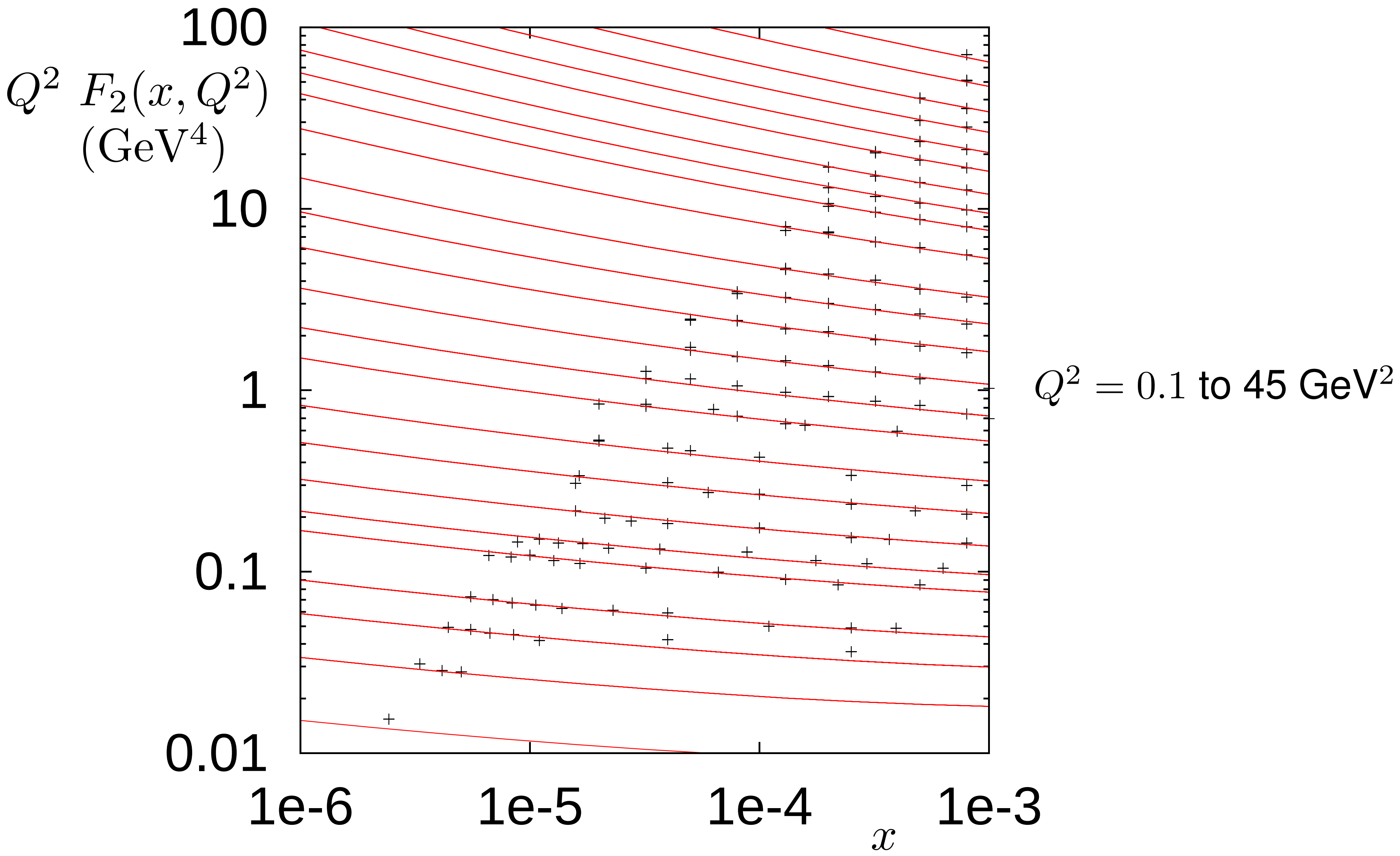
e_0 between 0.35 and 0.45 "hard pomeron"



Fit all $x < 0.001$ data with

$$F_0(Q^2) = A_0\left(\frac{Q^2}{a_0+Q^2}\right)^{1+e_0}\left(1+\frac{Q^2}{a_0}\right)^{\frac{1}{2}e_0} \qquad F_1(Q^2) = A_1\left(\frac{Q^2}{a_1+Q^2}\right)^{1+e_1}$$

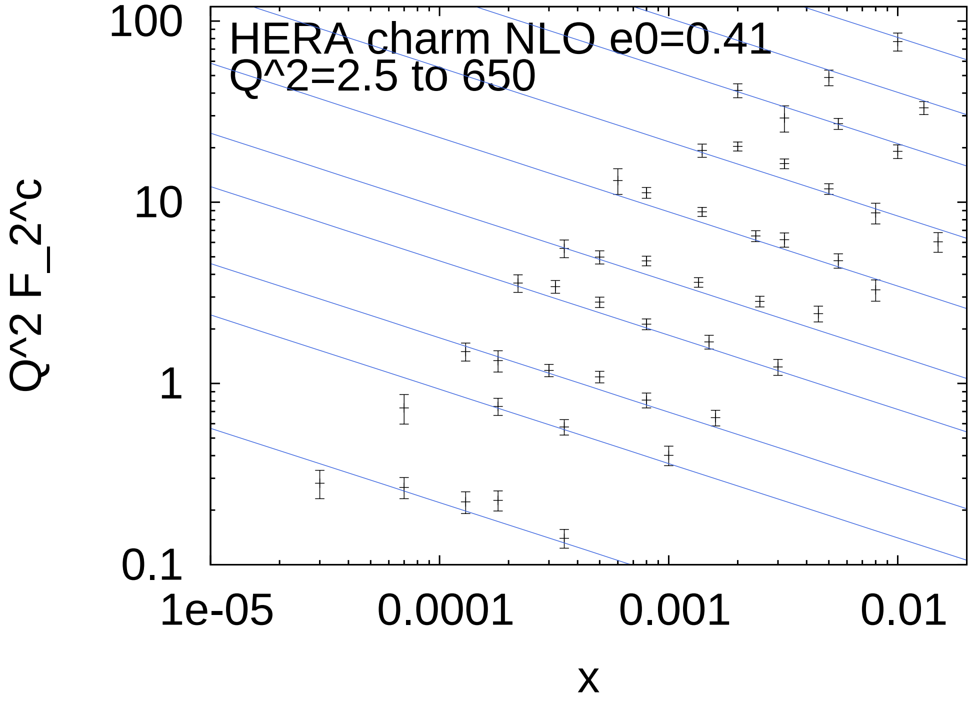
Include also f_2, a_2 exchange term $e_2 = -0.34$

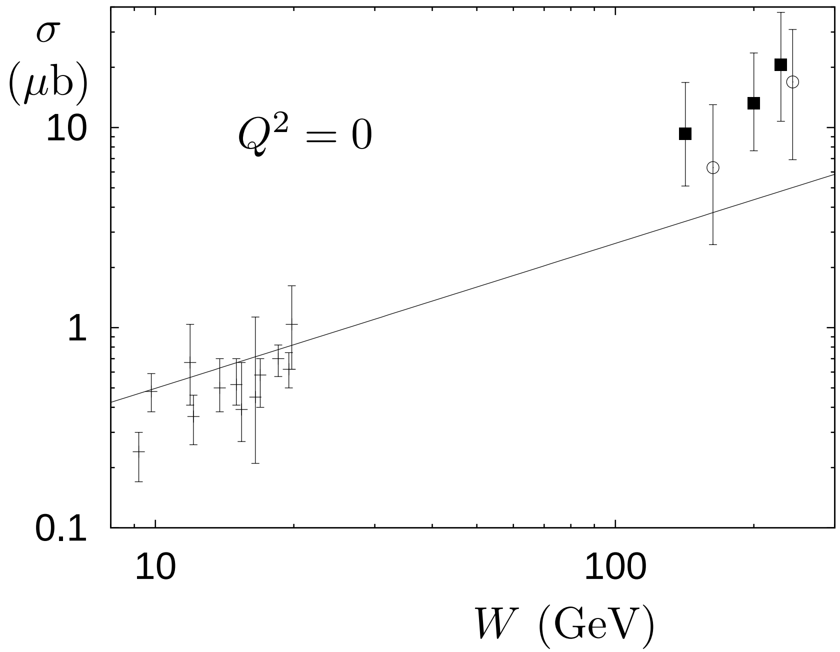


Fit data with $x < 0.001$ $\quad e0 = 0.41 \quad e1 = 0.08$

Very accurate data, though with some irregularities

Charm cross section





The lines are 2/5 of the hard pomeron contribution to the complete F_2

$$\frac{2}{5} = \frac{\frac{4}{9}}{\frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9}} = \frac{e_c^2}{e_u^2 + e_d^2 + e_s^2 + e_c^2}$$

The contribution from soft pomeron exchange is negligible

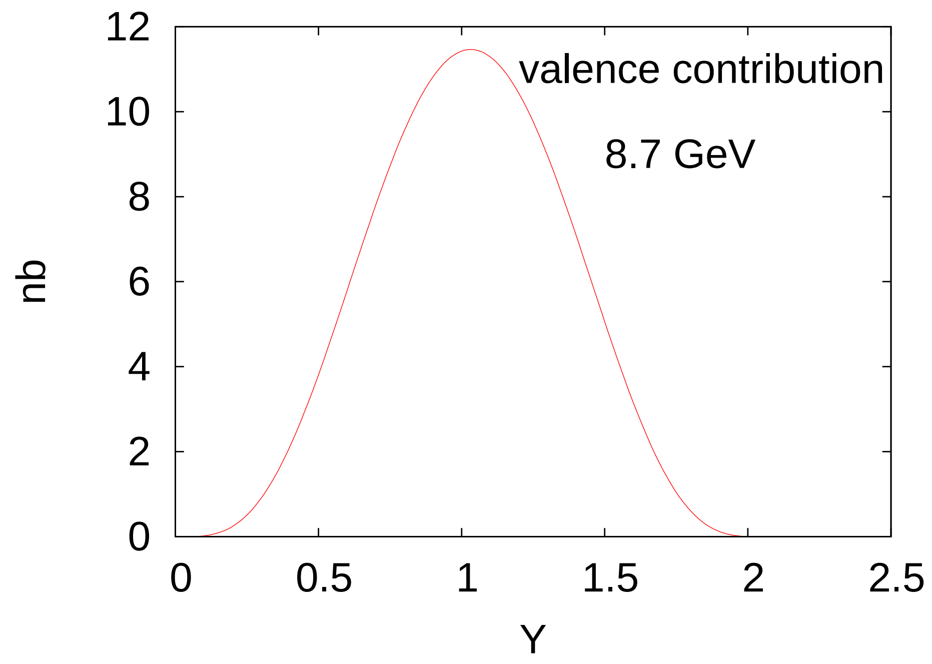
J/ψ production

Omega experiment (1977):

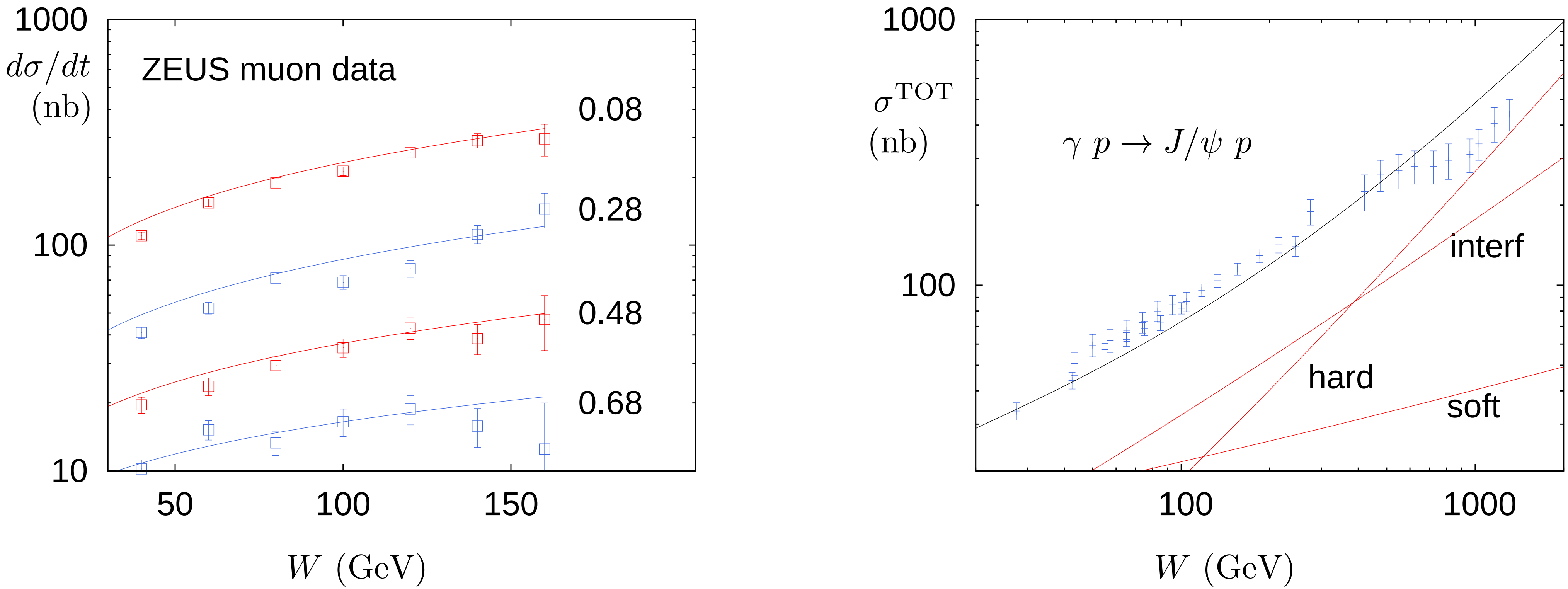
$$\frac{p + \text{Cu} \to J/\psi + X}{\bar{p} + \text{Cu} \to J/\psi + X} = 0.15 \pm 0.08$$

$\sigma(\bar{p}\, p \to J/\psi\ X \ = \ 12 \pm 5$ nb

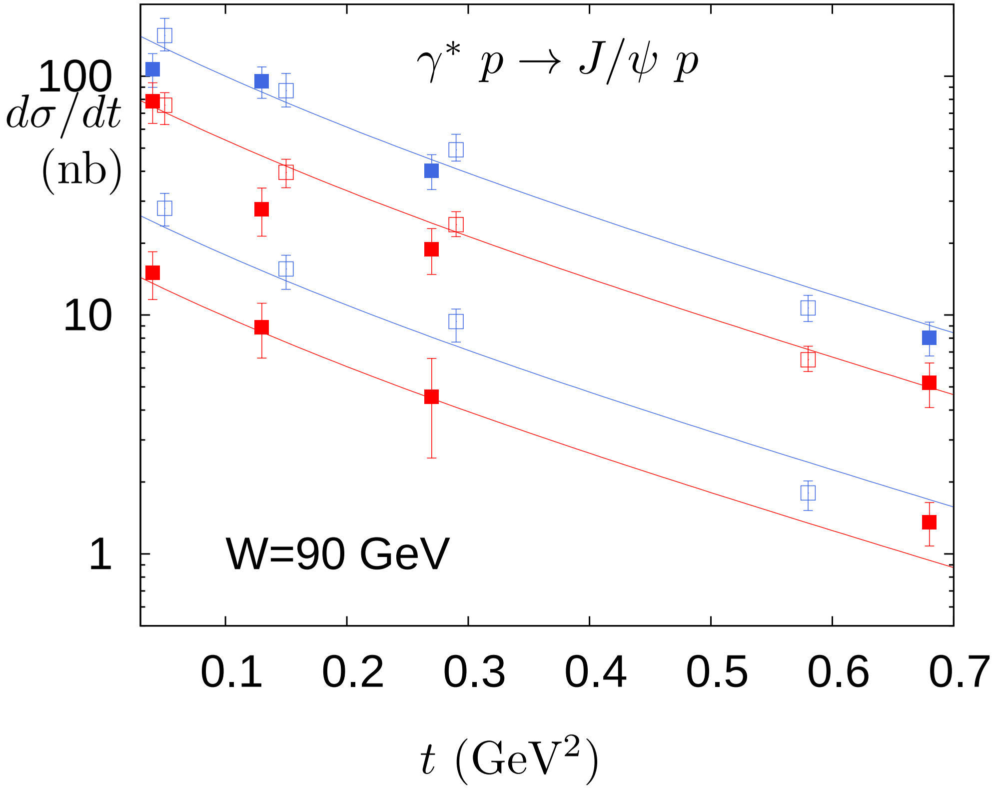
So the valence quarks of the beam couple to J/ψ: $|c\bar{c}\rangle$ mixes with $|q\bar{q} >$ at 10^{-4} level



J/ψ production



A mixture of hard and soft pomeron contributions

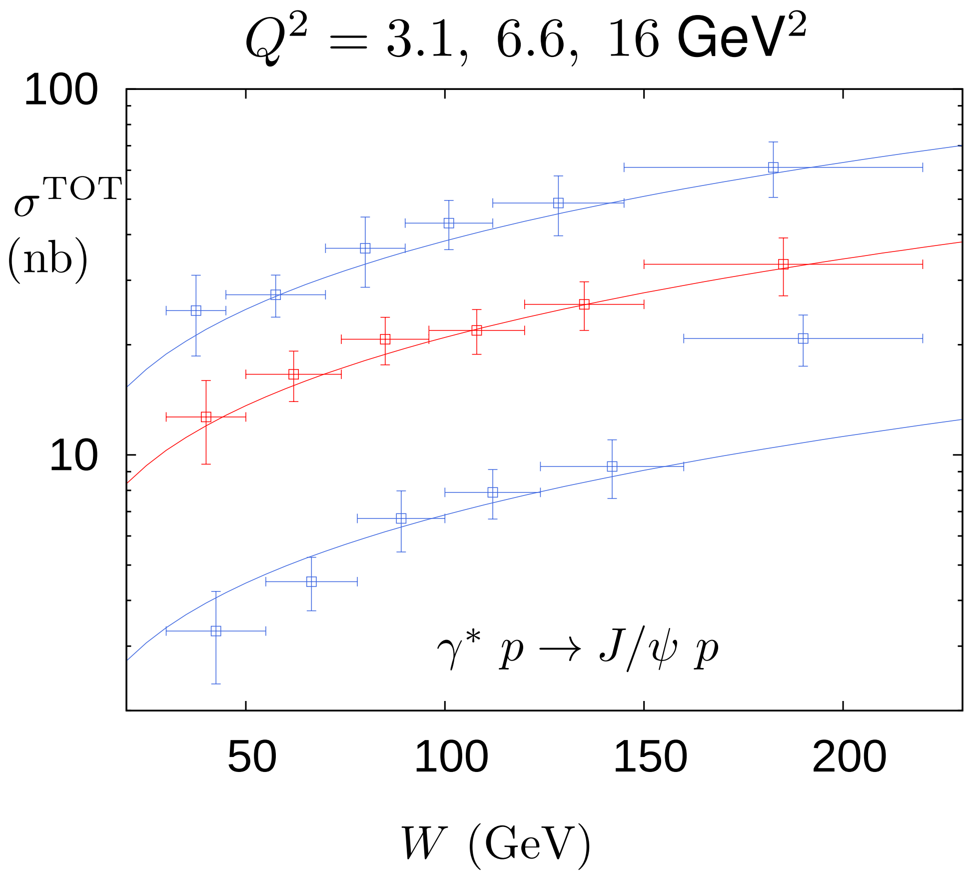


Open points:

ZEUS, $Q^2 = 3.1, 6.8, 16.0$ GeV^2

Solid points:

H1, $Q^2 = 3.2, 7.0, 22.4$ GeV^2



Proton vertex: form factor $F_1(t)$

J/ψ vertex:

$$\frac{1}{1.0 + t/(M^2_{J/\psi} + Q^2)} \; \frac{1}{1 + Q^2/B}$$

Additional $(1 + Kt)$ for hardpom

$B = 66.6$ GeV^2 $\qquad$ $K = 2.14$ GeV^{-2}

Perturbative QCD

Perturbative QCD and Regge theory have to live together

Singlet DGLAP evolution eqation:

$$Q^2 \frac{\partial}{\partial Q^2} \mathbf{u}(x, Q^2) = \int_x^1 dz\, \mathbf{P}(z, \alpha_s(Q^2))\, \mathbf{u}(x/z, Q^2)$$

$$\mathbf{u}(x, Q^2) = \begin{pmatrix} \sum_i (q_i(x, Q^2) + \bar{q}_i(x, Q^2)) \\ g(x, Q^2) \end{pmatrix}$$

$$xF_2(x, Q^2) = \tfrac{4}{9} u(x, Q^2) + \tfrac{1}{9} d(x, Q^2) + \tfrac{1}{9} s(x, Q^2) + \tfrac{4}{9} c(x, Q^2)$$

Mellin transform with respect to x:

$$\mathbf{u}(N, Q^2) = \int_0^1 dx\, x^{N-1} \mathbf{u}(x, Q^2) \qquad \mathbf{P}(N, \alpha_s(Q^2)) = \int_0^1 dz\, z^N \mathbf{P}(z, \alpha_s(Q^2))$$

Then

$$Q^2 \frac{\partial}{\partial Q^2} \mathbf{u}(N, Q^2) = \mathbf{P}(N, \alpha_s(Q^2))\, \mathbf{u}(N, Q^2).$$

Problem

If $\mathbf{u}(x,Q^2) \sim \mathbf{f}(Q^2)x^{-\epsilon}$ at small x, then

$$\mathbf{u}(N,Q^2) \sim \frac{\mathbf{f}(Q^2)}{N-\epsilon}$$

and

$$Q^2 \frac{\partial}{\partial Q^2}\mathbf{f}(Q^2) = \mathbf{P}(N=\epsilon,\alpha_s(Q^2))\,\mathbf{f}(Q^2).$$

For small z

$$P_{gg}(z,\alpha_s) \sim (6\alpha_s/2\pi)(1/z) \qquad\qquad P_{gg}(N,\alpha_s) \sim (6\alpha_s/2\pi)(1/N)$$

$P_{gg}(N,\alpha_s)$ is known to be finite at N=0
At small N expansion in powers of α_s is illegal
Compare

$$\sqrt{N^2+\alpha_s}-N = \frac{\alpha_s}{2N} - \frac{\alpha_s^2}{8N^3}\ldots$$

The expansion is certainly not OK for the soft pomeron $N = 0.096$,
but may be OK for the hard pomeron $N \approx 0.4$

Procedure

1 Choose some Q^2. Calculate hardpom coefficent function $f_q(Q^2)$ from the fit.

2 Fix the gluon coefficient function $f_g(x,Q^2)$ by requiring that the perturbative calculation of

$$\gamma^* \ + \ g \to c\,\bar{c}$$

at is equal to 0.4 $f_q(Q^2)$

Use running mass

$$m(Q^2) = m_{0c}(\alpha_s(Q^2)/\alpha_s(m_{0c}^2))^C \qquad C = 12/(33-2n_f)$$

and $\alpha_s(Q^2+4m^2(Q^2)),\ g(x,Q^2+4m^2(Q^2))$.

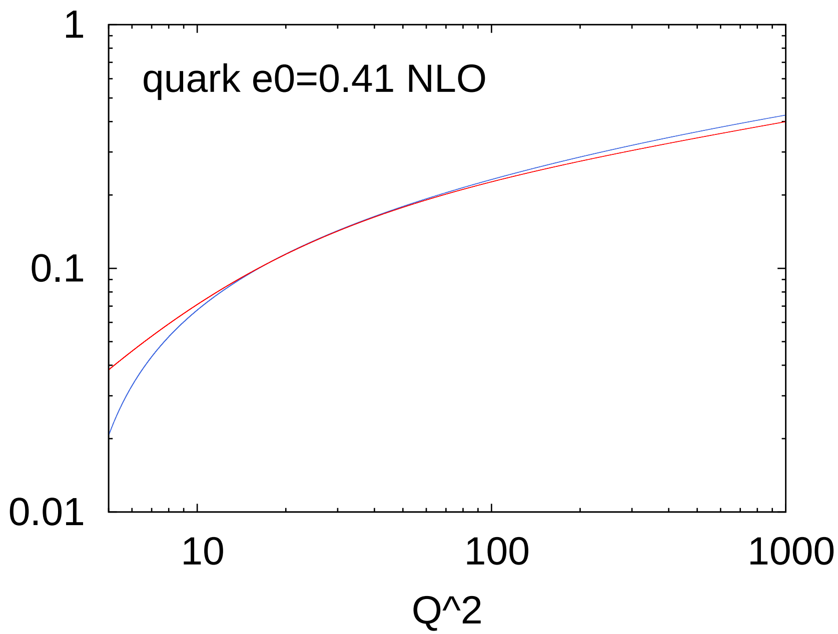
According to PDG $\quad m_{0c} = 1.275 \pm 0.025$GeV $\quad m_{0b} = 4.18 \pm 0.02$GeV

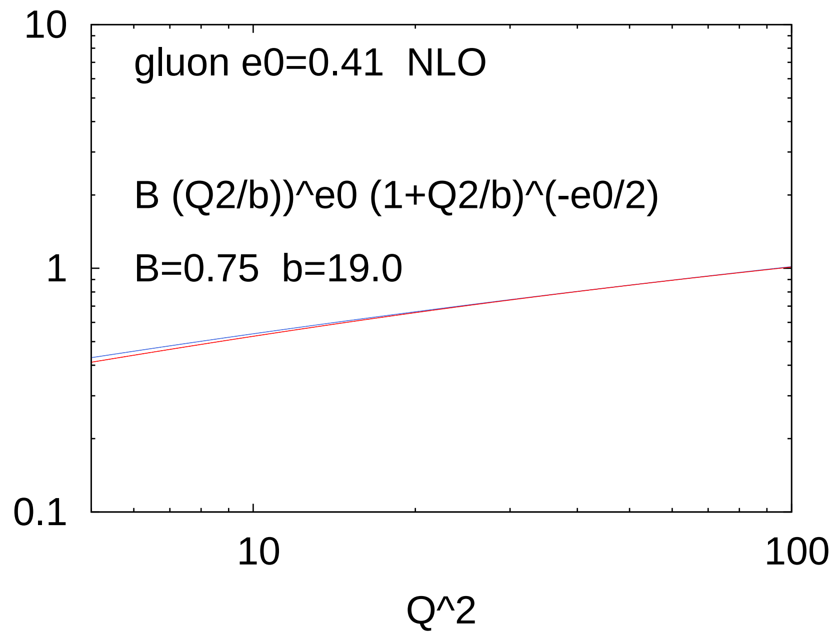
Require $\alpha_s(M_Z^2) \approx 0.1184$ giving $\Lambda_5 = 230$ MeV at NLO. Continuity at $Q^2 = m_{0b}^2$ gives $\Lambda_4 = 330$ MeV.

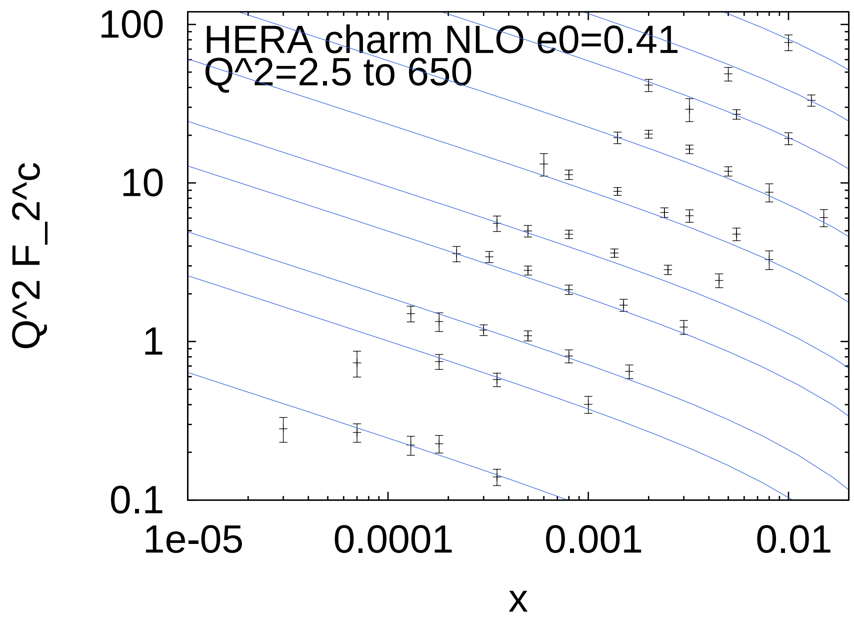
3 Apply DGLAP evolution to calculate $f_q(Q^2)$ and $f_g(x,Q^2)$ at all Q^2.

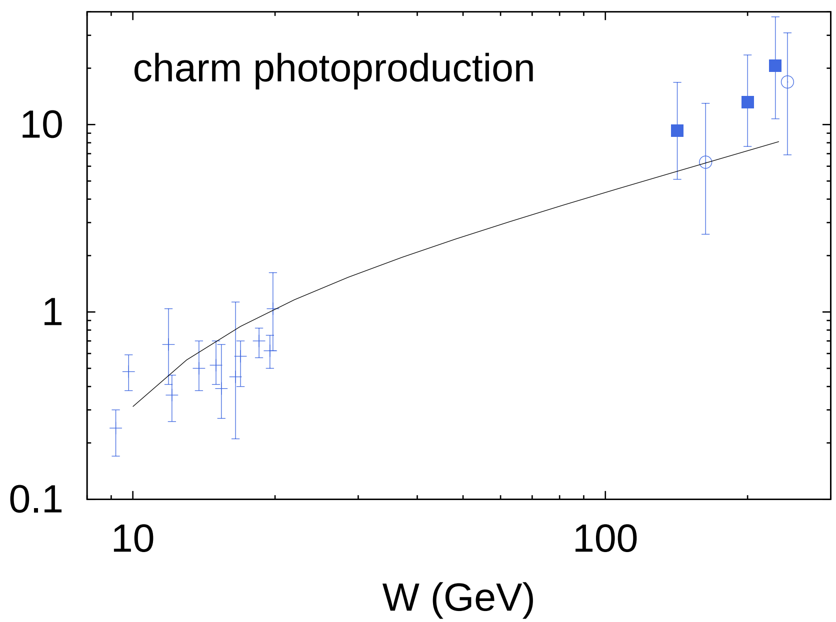
4 Fit $f_g(x,Q^2)$ to some simple function of Q^2

5 Calculate $F_2^c(x,Q^2)$ and compare with data.









(use mass m_{0c} and $\alpha_s(4m_{0c}^2)$)

Summary on electroproduction

- Small-x data show need for hard pomeron $0.35 < \epsilon_0 < 0.45$
- Charm couples only to the hard pomeron. Why??
- J/ψ contains a small $\bar{u}u.\bar{d}d$ component: both pomerons couple to it, but problems fitting LHC data
- DGLAP evolution can be applied only to the hard pomeron component of $F_2(x, Q^2)$

Overall summary

- Regge theory successfully correlates a large amount of data
- But we do not know how to calculate multiple exchanges