Measuring transverse size with virtual photons

Paul Hoyer
University of Helsinki

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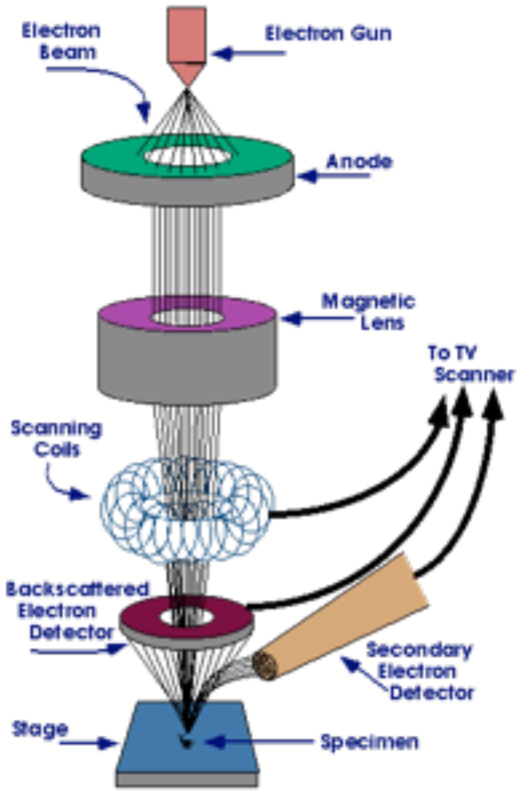


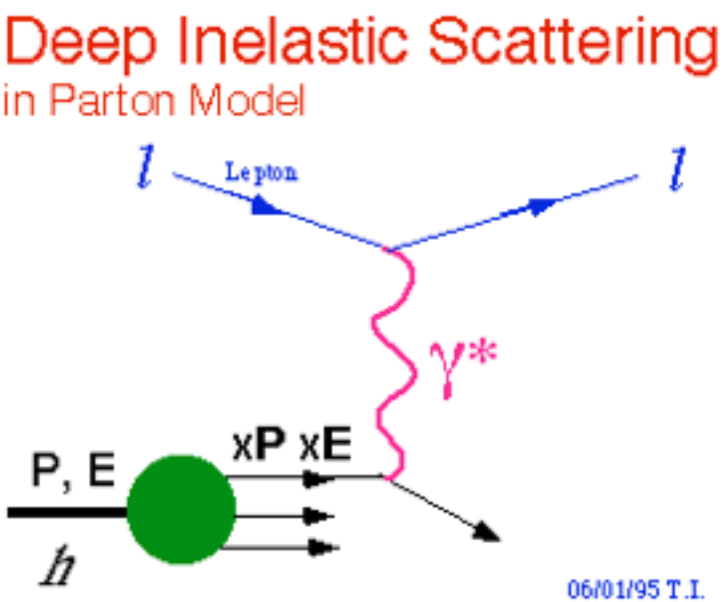
Relativistic

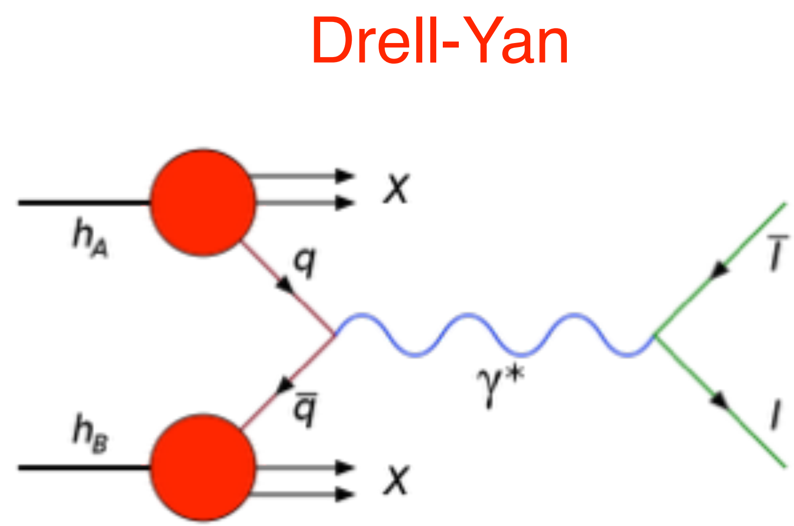
electron microscopy of

hadrons and nuclei

Photons are useful probes of strong dynamics

Photons scatter from single quarks as their virtuality $Q^2 \to \infty$



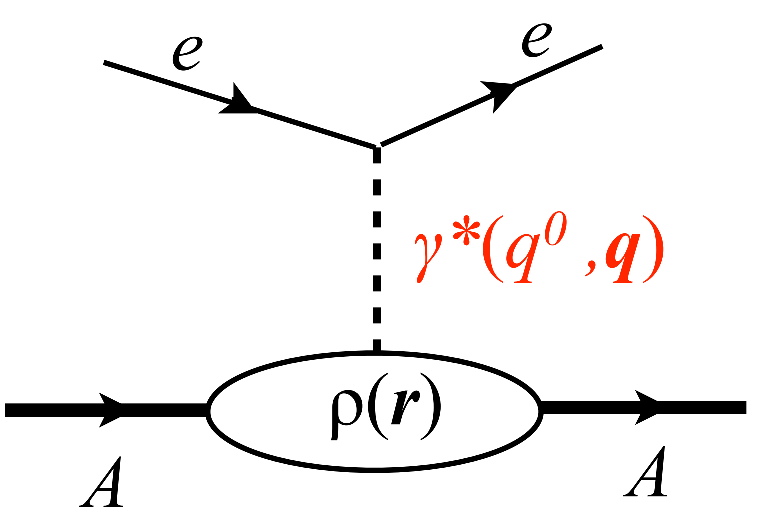


The DIS and DY processes are important
in the determination of parton distributions.

Photons are useful probes of strong dynamics at any Q^2

Unlike gluons, photons are perturbative at all Q^2 .

As in electron microscopes, the $Q^2 \equiv -q^2$ dependence of $eA \to eA$ translates into a measurement of charge distribution in position space.



$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{4E}\right)\frac{1}{\sin^4(\frac{1}{2}\theta)}\left|F(Q^2)\right|^2$$

In the static target approximation: $$F(Q^2) = \int d^3\boldsymbol{r}\,\rho(r)\exp(-i\boldsymbol{q}\cdot\boldsymbol{r})$$

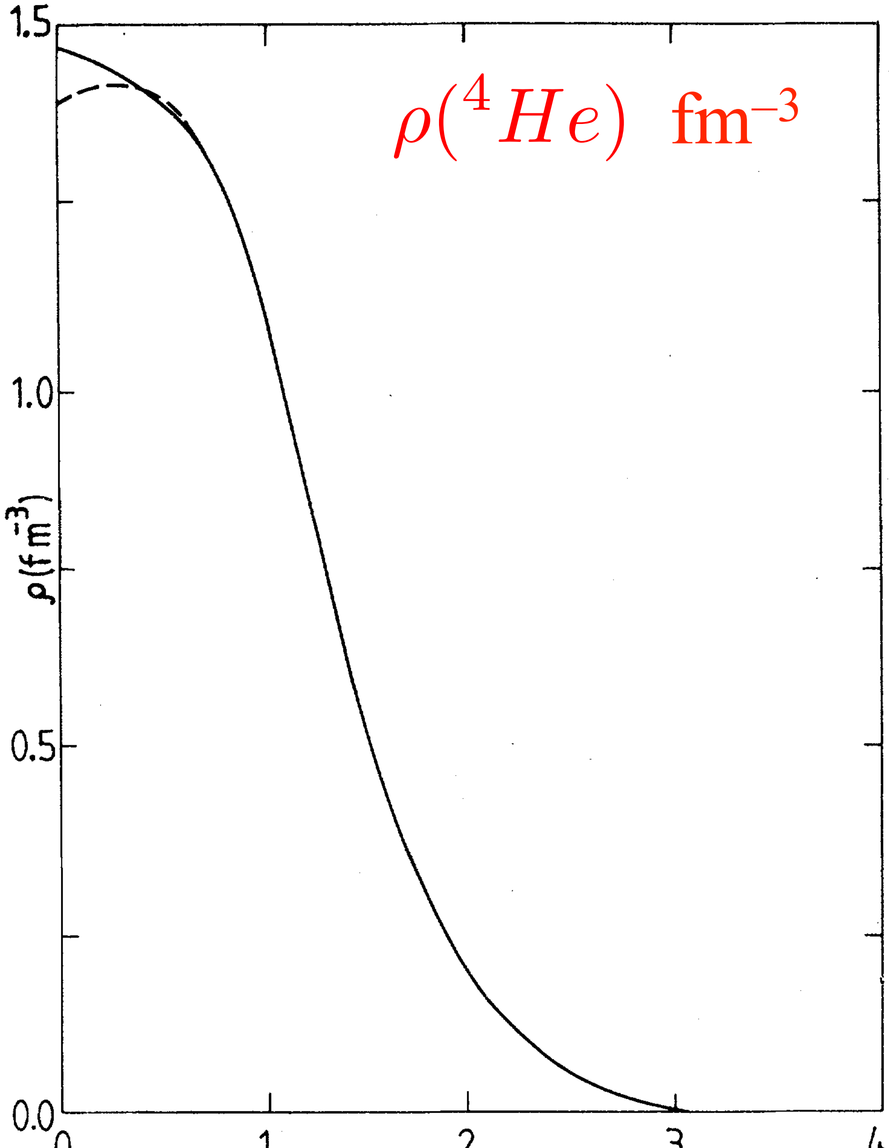
$$q^0 \simeq \frac{\boldsymbol{q}^2}{2M_A} \to 0$$

$$\langle \boldsymbol{r}^2 \rangle = -6\,\frac{dF}{dQ^2}\bigg|_{Q^2=0}$$

Exercise 4.1

Show that $\langle \boldsymbol{r}^2 \rangle = -6 \left. \frac{dF}{dQ^2} \right|_{Q^2=0}$ when $\rho(r) = \rho(|\boldsymbol{r}|)$.

Charge Profile of a ^{4}He nucleus



Atkin & Dumbrajs, PRC 26 (1982) 680

Note model dependence for $r \to 0$: —— *vs.* - - - - -

At large $\boldsymbol{q}$, the recoil of the ^{4}He target cannot be neglected.

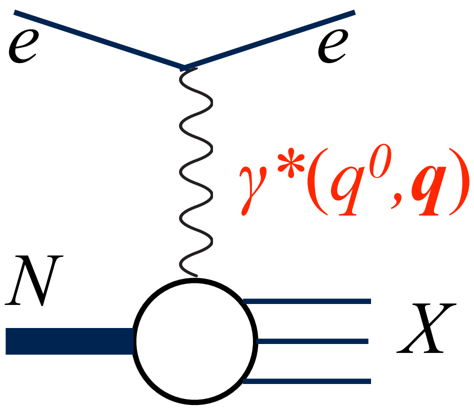
- Which frame to choose then? The Fourier transfrom is not covariant under boosts:

$$F(Q^2) = \int d^3\boldsymbol{r}\,\rho(r)\exp(-i\boldsymbol{q}\cdot\boldsymbol{r})$$

- The quarks move with the speed of light: $v \approx c$. How can the γ^* pinpoint their positions?

Relativistic effects on photon resolution

From DIS, recall that resolution of photon with $q^2 = -Q^2$ is different in the longitudinal and transverse directions due to Lorentz dilation



In the target rest frame: $q^0 = \nu$

Photon mass^2 : $Q^2 = -q^2$

Lorentz γ-factor: $E/M \sim \nu/Q$

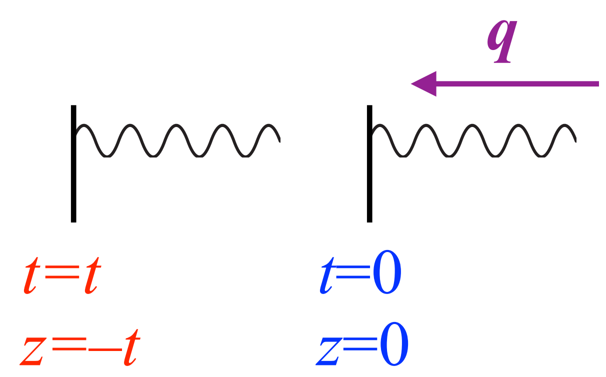
Virtual photon resolution in the target rest frame:

$$\Delta r_{\parallel} \sim \frac{1}{Q}\,\frac{\nu}{Q} = \frac{1}{2m\,x_{Bj}} \geq \frac{1}{2m} \qquad\qquad \Delta r_{\perp} \sim \frac{1}{Q} \geq 0$$

Only the transverse resolution grows with Q!

Boosting to the Infinite Momentum Frame (IMF)

A photon moving along the negative z-axis with speed v = c = 1 scatters from target quarks at an instant of Light-Front (LF) time $x^+ = t + z$, not at an instant of ordinary time t



Since DIS probes the target at equal LF time it is convenient to use LF quantization: Fock states are defined at $x^+ = 0$ (rather than $t = 0$).

In the IMF ≈ LF, quarks with a momentum fraction $x > 0$ of the target h have energy $E_q = x\, E_h \to \infty$

Hence their speed in the transverse direction is small: $v_{q\perp} = \dfrac{p_{q\perp}}{x E_h} \to 0$

This makes it easier to understand why the photon can resolve the transverse positions of the quarks with infinite accuracy as $Q \to \infty$

Exercise 4.2

Consider the resolution in DIS, $e+p \to e+X$. Take the virtual photon momentum in the proton rest frame to be along the negative z-axis:

$$q = (q^0, q^1, q^2, q^3) = (\nu, 0, 0, -\sqrt{\nu^2 + Q^2})$$

Determine $q^\pm = q^0 \pm q^3$ in the Bjorken limit. Express the Fourier factor $\exp(-iq \cdot x)$ in terms of $q^\pm$ and $x^\pm$. From this find the resolution of the virtual photon in x^+ and x^-.

The resolution in $\boldsymbol{x}^\perp = (x^1, x^2)$ would appear to vanish. However, x represents the distance between the virtual photon vertices in the amplitude and its complex conjugate. For the handbag diagram to have a discontinuity this distance must be timelike: $x^2 = x^+ x^- - (\boldsymbol{x}^\perp)^2 \geq 0$. Conclude from this that $|\boldsymbol{x}^\perp| = \mathrm{O}(1/Q)$.

The LF Fock state expansion

A hadron state of momentum $P^+ = P^0 + P^3$ defined at given $x^+ = x^0 + x^3$ can be expanded in terms its quark and gluon Fock states as

$$|P^+, \boldsymbol{P}_\perp, \lambda\rangle_{x^+=0} = \sum_{n,\lambda_i} \prod_{i=1}^{n} \Big[\int_0^1 \frac{dx_i}{\sqrt{x_i}} \int \frac{d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1 - \sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

$$\times \psi_n(x_i, \boldsymbol{k}_i, \lambda_i)\, |n;\, x_i P^+, x_i \boldsymbol{P}_\perp + \boldsymbol{k}_i, \lambda_i\rangle_{x^+=0}$$

where the LF wave functions $\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)$ are independent of the hadron momentum P^+, $P_\perp$.

The boost invariance is a consequence of the quantization surface being $x^+=0$

$x^+ \to e^\zeta x^+$ under a longitudinal Lorentz boost ζ

Note: The partons carry fractions $x_i \geq 0$ of the hadron momentum P^+ and $P_\perp$.

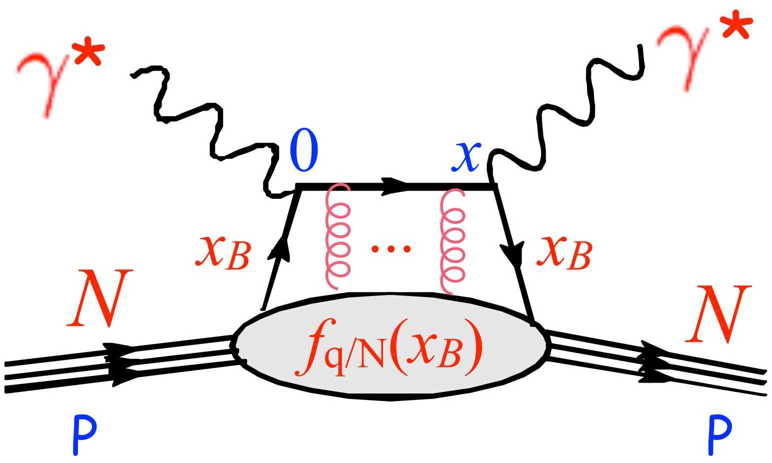
Since $P^+ > 0$, the quark momenta $x_i P^+ \geq 0$.

Exercise 4.3

Show that $x^+ \to e^\zeta x^+$ under a longitudinal Lorentz boost ζ

Inclusive Deep Inelastic Scattering (DIS)

In the Bj limit ($Q^2 \to \infty$ at fixed $x_B = Q^2/2m\nu$) the DIS cross section factorizes,



$$\frac{d\sigma}{dx_B dQ^2}(\ell p \to \ell X)$$

$$= \sum_q f_{q/N}(x_B, Q^2) \frac{d\sigma}{dQ^2}(\ell \mathrm{q} \to \ell \mathrm{q})$$

The quark distribution in $A^+ = 0$ gauge is, with $x^- = t - z$

$$f_{q/N}(x_B, Q^2) = \frac{1}{8\pi} \int dx^- \exp(-ix_B p^+ x^- /2) \langle N(p)|\bar{\psi}(x^-)\gamma^+\psi(0)\,|N(p)\rangle$$

where x^+ , $x_\perp = 0$. The longitudinal resolution is $x^- \sim 1/(x_B p+) \sim 1/(2mx_{Bj})$

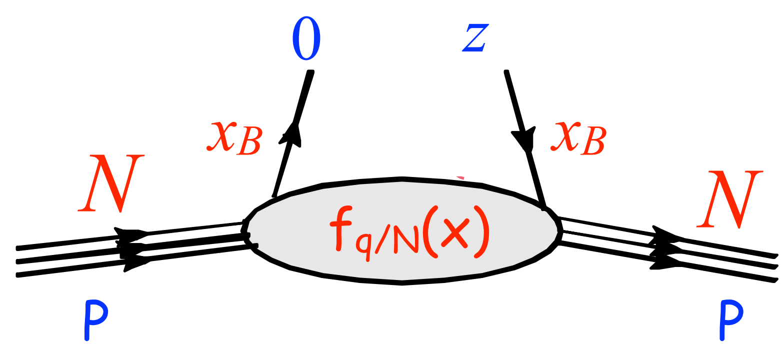
If the nucleon state $|N(p)\rangle$ is expressed in terms of the LF Fock expansion we find the quark distribution in terms of the LF wave functions

Quark distribution in terms of LF wave functions

$$f_{q/N}(x) = \sum_{n,\lambda_i,k} \prod_{i=1}^{n} \Big[\int \frac{dx_i\, d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1 - \sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

$$\times \delta(x - x_k) |\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)|^2$$

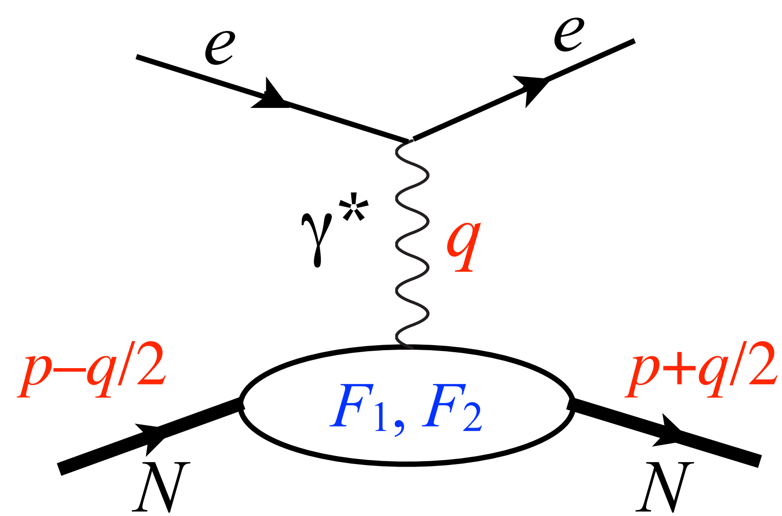
The quark distribution is given by the probability to find a quark with momentum fraction *x*, summed over all Fock states *n*.



Note: – The parton distribution is obtained from data in the $Q^2 \to \infty$ limit.

Nucleon Form Factors

Using Lorentz and gauge invariance, the scattering amplitude is expressed in terms of the Dirac F_1 and Pauli F_2 form factors, which only depend on $Q^2 = -q^2$

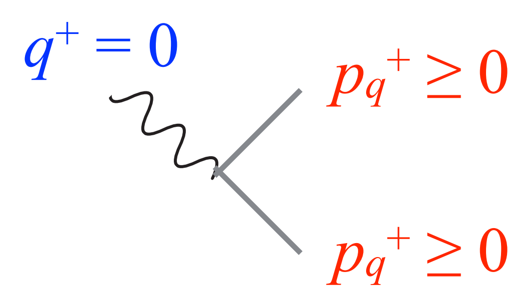


$$A^{\mu}_{\lambda\lambda'} = \langle p + \tfrac{1}{2}q, \lambda' | J^{\mu}(0) | p - \tfrac{1}{2}q, \lambda \rangle$$

$$= \bar{u}(p + \tfrac{1}{2}q, \lambda') \left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2)\frac{i}{2m}\sigma^{\mu\nu}q_{\nu} \right] u(p - \tfrac{1}{2}q, \lambda)$$

The frame is conventionally chosen such that $\boldsymbol{p}_{\perp} = q^+ = q^- = 0$.

For $q^+ = 0$ the virtual photon cannot create/destroy a pair of quarks on the LF:



Nucleon Charge Distribution

The charge density for an unpolarized nucleon is defined as a (2-dimensional!) Fourier transform over $\boldsymbol{q}_\perp$:

$$\rho_0(\boldsymbol{b}) = \frac{1}{2p^+}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \left\langle p^+, \tfrac{1}{2}\boldsymbol{q}, \lambda \right| J^+(0) \left| p^+, -\tfrac{1}{2}\boldsymbol{q}, \lambda \right\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi}\, Q\, J_0(b\,Q) F_1(Q^2)$$

Inserting the LF Fock expansion for the nucleon states gives

$$\rho_0(\boldsymbol{b}) \;= \sum_{n,\lambda_i,k} e_k \Big[\prod_{i=1}^{n}\int dx_i \int 4\pi d^2\boldsymbol{b}_i\Big]\delta(1-\sum_i x_i)\frac{1}{4\pi}\delta^{(2)}(\sum_i x_i \boldsymbol{b}_i)$$

$$\times\; \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_k)\,|\psi_n^\lambda(x_i, \boldsymbol{b}_i, \lambda_i)|^2$$

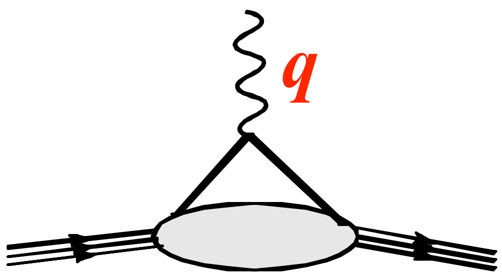
where the wave functions $\psi(x_i,\boldsymbol{b}_i)$ are Fourier transforms of the $\psi(x_i,\boldsymbol{k}_i)$.

Impact parameter picture of Form Factors

$$\rho_0(\boldsymbol{b}) = \sum_{n,\lambda_i,k} e_k \Big[\prod_{i=1}^{n} \int dx_i \int 4\pi d^2\boldsymbol{b}_i\Big] \delta(1-\sum_i x_i)\frac{1}{4\pi}\delta^{(2)}(\sum_i x_i \boldsymbol{b}_i)$$
$$\times\ \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_k)\,|\psi_n^{\lambda}(x_i,\boldsymbol{b}_i,\lambda_i)|^2$$

This is the probability to find a quark at impact parameter ***b***, from which the the photon scatters.

$$\rho_0(\boldsymbol{b}) = \int_0^\infty \frac{dQ}{2\pi}\, Q\, J_0(b\,Q) F_1(Q^2)$$



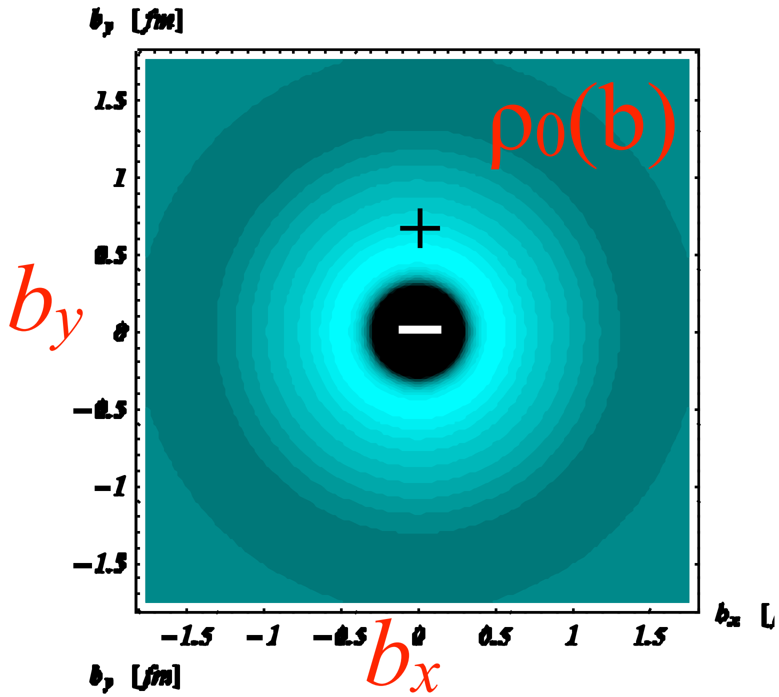
No QCD factorization (Bj limit) required!

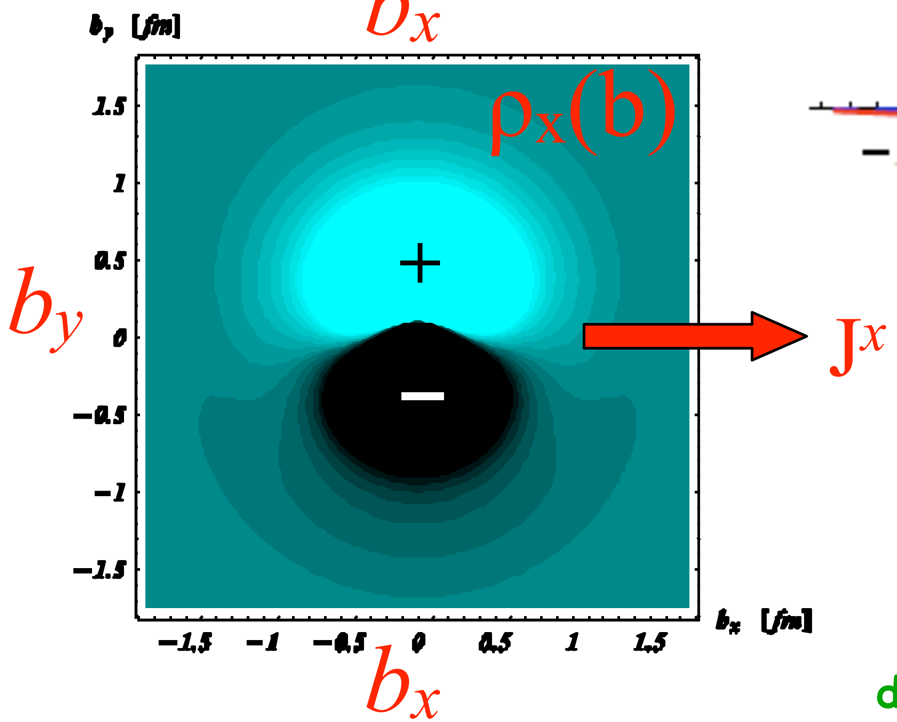
All Q^2 contribute: Resolution in $b \sim 1/Q_{max}$

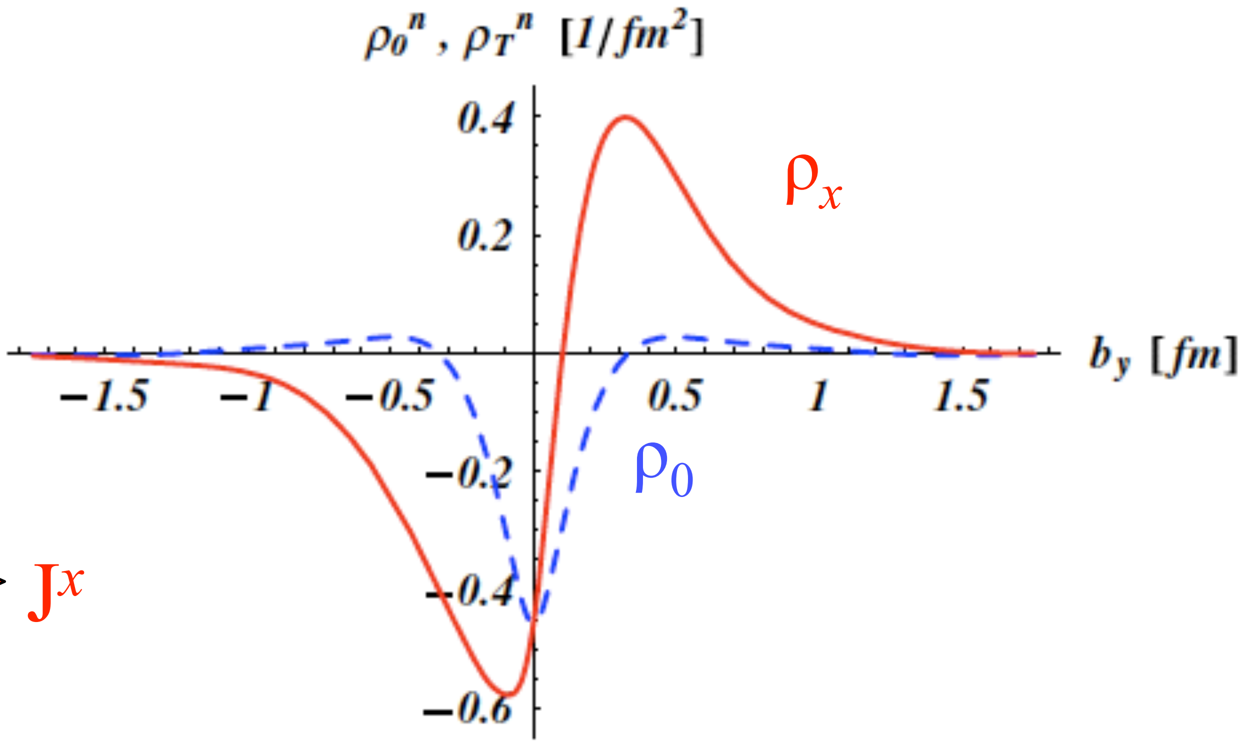
The charge density of a nucleon polarized in the transverse (x-) direction is similarly given by a Fourier transform of the F_2 form factor.

Using measured form factors, find the

empirical quark transverse densities in neutron



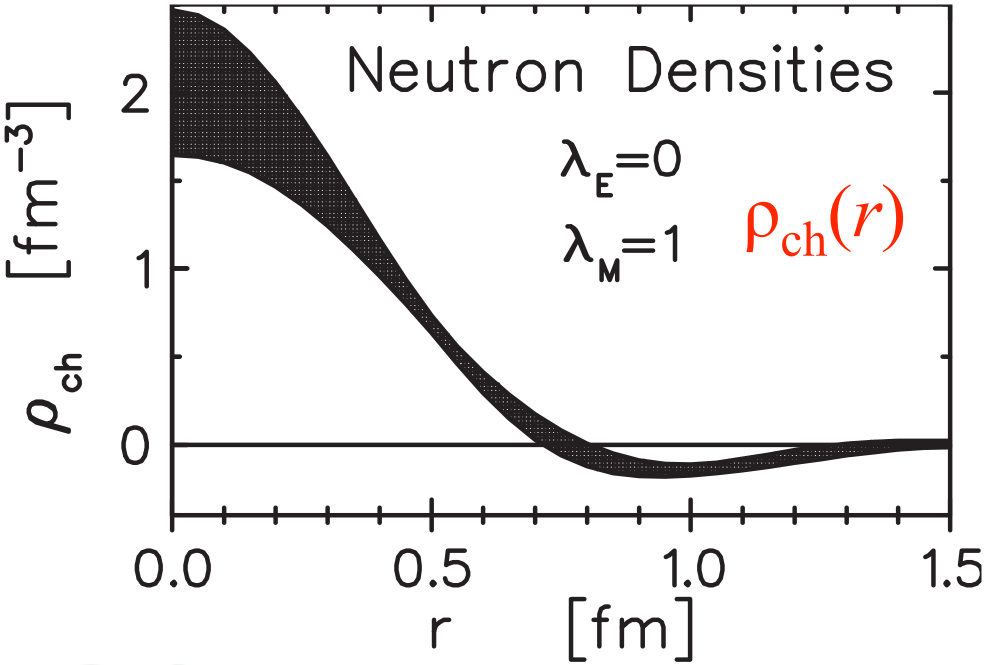




Miller (2007)
Carlson and Vanderhaeghen (2008)

data : Bradford, Bodek, Budd, Arrington (2006)

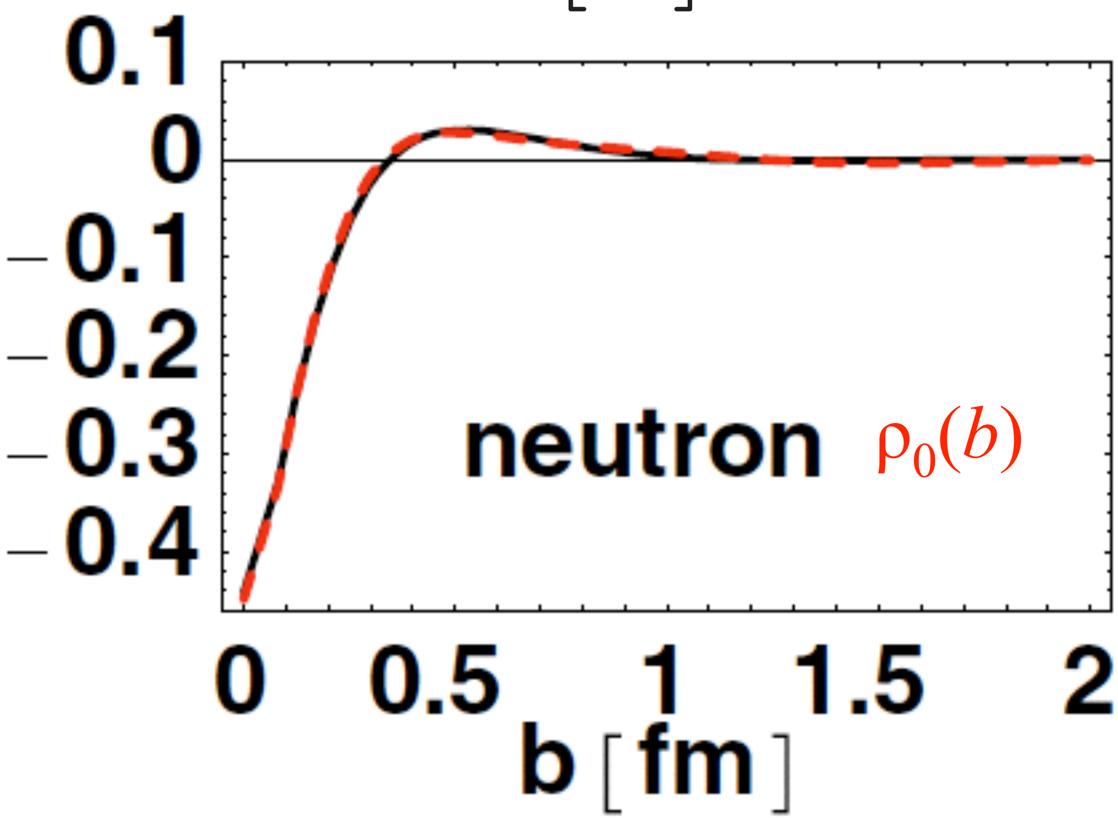
Qualitative change in central neutron charge density



3-dimensional Fourier transform with phenomenological factors (2001)

J. J. Kelly, hep-ph/0111251

$\rho_{ch}(r=0) > 0$



$\rho_0(b=0) < 0$

Transverse Fourier transform (2007)

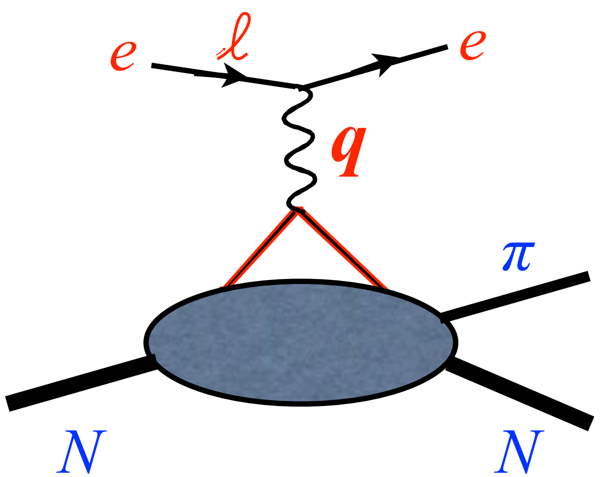
G. Miller, PRL 99 (2007) 112001

Generalization to any γ^* transition

PH and S. Kurki
arXiv:1101.4810

Measure the transverse distribution of charge contributing to a general process:

$$\mathcal{M}(\ell N \to \ell' f) = -e^2 \bar{u}(\ell')\gamma_\mu u(\ell)\frac{1}{q^2}\int d^4x e^{-iq\cdot x}\langle f|J^\mu(x)|N(p)\rangle$$



Need to select J^+ current contribution, which gives a probability density:

$$J^+(x) = e_q\,\bar{\mathrm{q}}(x)\gamma^+\mathrm{q}(x) = 2e_q\,\mathrm{q}_+^\dagger(x)\mathrm{q}_+(x)$$

$\mathrm{q}_+(x) = \frac{1}{4}\gamma^-\gamma^+\mathrm{q}(x)$

E.g.: $\ell^- \to \infty$ at fixed q selects J^+

Consider states in impact parameter: $|p^+,\boldsymbol{p}\rangle = 4\pi\int d^2\boldsymbol{b}\, e^{i\boldsymbol{p}\cdot\boldsymbol{b}}|p^+,\boldsymbol{b}\rangle$

Expand into LF Fock states:

$$|p^+,\boldsymbol{b}\rangle = \frac{1}{4\pi}\sum_n \Big[\prod_{i=1}^n \int_0^1 \frac{dx_i}{\sqrt{x_i}} \int 4\pi d^2\boldsymbol{b}_i\Big]\delta(1-\sum_i x_i)\delta^2(\boldsymbol{b}-\sum_i x_i\boldsymbol{b}_i)$$

$$\times\ \psi_n(x_i,\boldsymbol{b}_i-\boldsymbol{b})\prod^n b^\dagger(x_i p^+,\boldsymbol{b}_i)d^\dagger(\ )a^\dagger(\ )|0\rangle$$

$\Rightarrow$

$$\frac{1}{2p^+}\langle f(p^+,\boldsymbol{b}_f)|J^+(0)|N(p^+,\boldsymbol{b}_N)\rangle \equiv \frac{1}{(4\pi)^2}\delta^2(\boldsymbol{b}_f-\boldsymbol{b}_N)\mathcal{A}_{fN}(-\boldsymbol{b}_N)$$

where

$$\mathcal{A}_{fN}(\boldsymbol{b}) = \frac{1}{4\pi}\sum_n \Big[\prod_{i=1}^n \int_0^1 dx_i \int 4\pi d^2\boldsymbol{b}_i\Big]\delta(1-\sum_i x_i)\delta^2(\sum_i x_i\boldsymbol{b}_i)$$

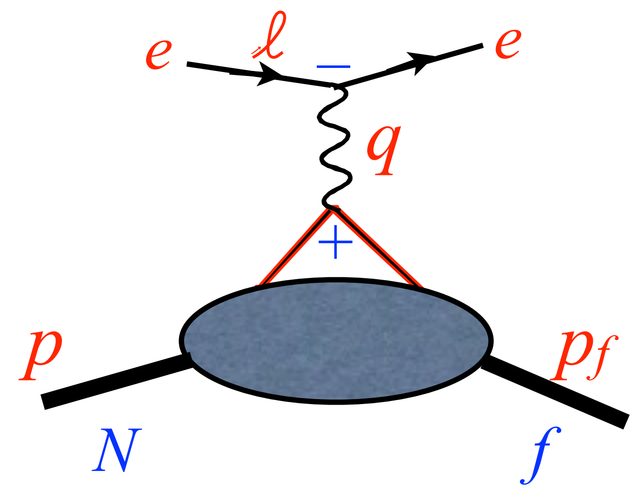
$$\times\ \psi_n^{f\,*}(x_i,\boldsymbol{b}_i)\psi_n^N(x_i,\boldsymbol{b}_i)\sum_k e_k\delta^2(\boldsymbol{b}_k-\boldsymbol{b})$$

is diagonal in Fock states n provided $q^+ = 0$

FT of γ* matrix element in momentum space

In the frame:

$$\begin{aligned} p &= (p^+, p^-, -\tfrac{1}{2}\boldsymbol{q}) \\ q &= (0^+, q^-, \boldsymbol{q}) \\ p_f &= (p^+, p^- + q^-, \tfrac{1}{2}\boldsymbol{q}) \end{aligned}$$



we have

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle = \mathcal{A}_{fN}(\boldsymbol{b})$$

where $\mathcal{A}_{fN}(\boldsymbol{b})$ is given by the previous overlap of Fock amplitudes, which are universal features of N and f.

The ***b***-distribution may be studied as a function of the final state *f*, providing information about the transverse size of the contributing Fock states.

When f consists of several hadrons their relative momenta must be consistent with the LF Fock expansion at all $p_f = q + p$

Example: f = $\pi\,(p_1)\,N(p_2)$

In order to conform with the Lorentz covariance of LF states, at any p_f :

$$|\pi N(p_f^+, \boldsymbol{p}_f; \Psi^f)\rangle \equiv \int_0^1 \frac{dx}{\sqrt{x(1-x)}} \int \frac{d^2\boldsymbol{k}}{16\pi^3}\, \Psi^f(x,\boldsymbol{k}) |\pi(p_1)N(p_2)\rangle$$

where $\Psi^f(x,\boldsymbol{k})$ defines the final state in terms of the relative variables x, $\boldsymbol{k}$:

$$p_1^+ = x p_f^+ \qquad\qquad \boldsymbol{p}_1 = x\boldsymbol{p}_f + \boldsymbol{k}$$

$$p_2^+ = (1-x)p_f^+ \qquad\qquad \boldsymbol{p}_2 = (1-x)\boldsymbol{p}_f - \boldsymbol{k}$$

With x, $\boldsymbol{k}$ being independent of p_f, this defines the pion and nucleon momenta p_1, p_2 at all photon momenta q.

The $|\pi N(p_f^+, \boldsymbol{p}_f; \Psi^f)\rangle$ state-to-be is created by the photon at an instant of x^+.

The pion and nucleon are formed later, via final state interactions.

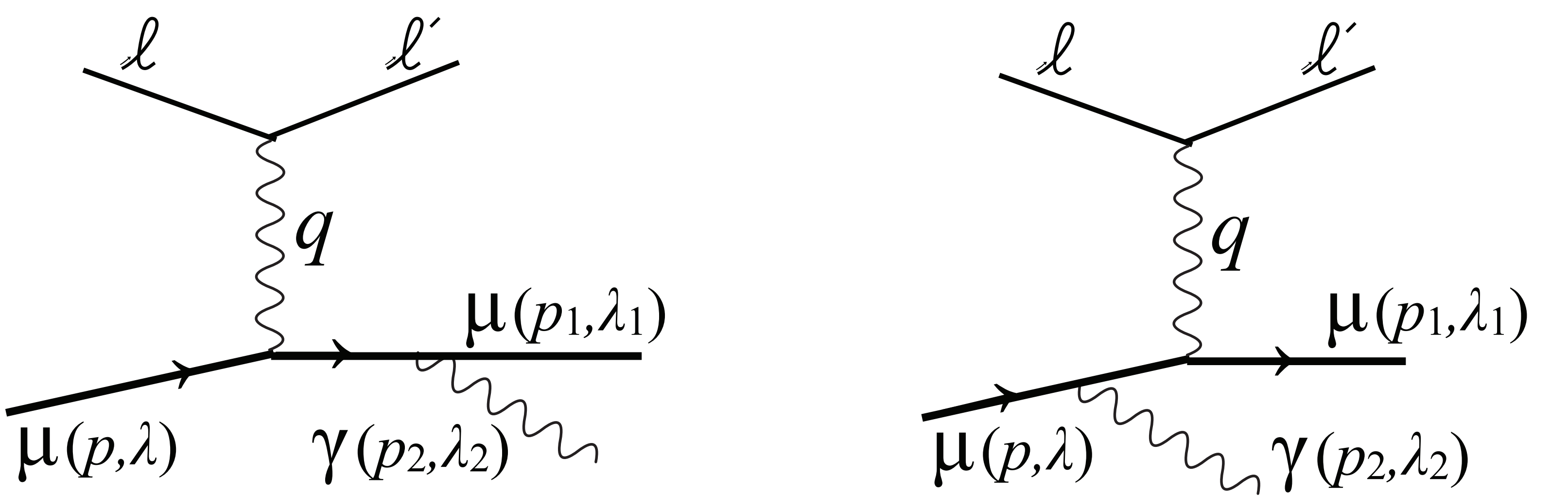
Illustration (1): $\gamma^* + \mu \to \mu + \gamma$

The QED matrix element $\mathcal{A}^{\mu\gamma}_{\lambda_1,\lambda_2} = \dfrac{1}{2p^+}\langle \mu(p_1,\lambda_1)\gamma(p_2,\lambda_2)|J^+(0)|\mu(p,\lambda=\tfrac{1}{2})\rangle$

expressed in terms of the relative variables x, $\boldsymbol{k}$ is:

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{q};x,\boldsymbol{k}) = 2e\sqrt{x}\left[\frac{\boldsymbol{e}_-\cdot\boldsymbol{k}}{(1-x)^2m^2+\boldsymbol{k}^2} - \frac{\boldsymbol{e}_-\cdot(\boldsymbol{k}-(1-x)\boldsymbol{q})}{(1-x)^2m^2+(\boldsymbol{k}-(1-x)\boldsymbol{q})^2}\right]$$

where $\boldsymbol{e}_\lambda \cdot \boldsymbol{k} = -\lambda e^{i\lambda\phi_k}|\boldsymbol{k}|/\sqrt{2}$.



(a) (b)

Illustration (1): $\gamma^* + \mu \to \mu + \gamma$ (cont.)

The Fourier transform gives:

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{b};x,\boldsymbol{k}) = 2e\sqrt{x}\left[\frac{\boldsymbol{e}_-\cdot\boldsymbol{k}}{(1-x)^2m^2+\boldsymbol{k}^2}\delta^2(\boldsymbol{b}) - \frac{i}{2\sqrt{2}\pi}\frac{m\,e^{-i\phi_b}}{1-x}K_1(mb)\exp\left(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\right)\right]$$

In the first term the γ^* interacts with the initial muon, which by definition is at $\boldsymbol{b} = 0$. The second term reflects the $\boldsymbol{b}$-distribution of the final muon.

This expression agrees exactly with the wave function overlap formula:

$$\mathcal{A}_{fN}(\boldsymbol{b}) = \frac{1}{4\pi}\sum_n\left[\prod_{i=1}^n\int_0^1 dx_i\int 4\pi d^2\boldsymbol{b}_i\right]\delta(1-\sum_i x_i)\delta^2(\sum_i x_i\boldsymbol{b}_i)\,\psi_n^{f\,*}(x_i,\boldsymbol{b}_i)\psi_n^N(x_i,\boldsymbol{b}_i)\sum_k e_k\delta^2(\boldsymbol{b}_k-\boldsymbol{b})$$

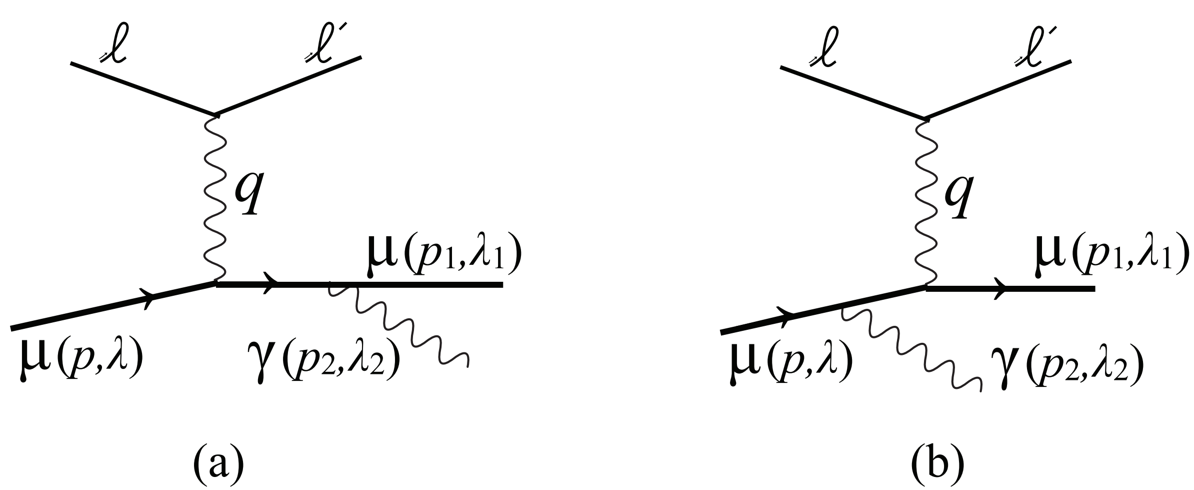


Illustration (2): $\gamma^* + \mu \to \mu + \gamma$

Choosing $$\Psi(x', \boldsymbol{k}) = \delta(x'-x)\sqrt{x(1-x)}\,\exp\Big(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}'_\mu}{1-x}\Big)$$

corresponds to fixing the impact parameter $\boldsymbol{b}_\mu{}'$ of the final muon. Then

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{b}; x, \boldsymbol{b}'_\mu) = \sqrt{x(1-x)}\,\psi^{\uparrow}_{+\frac{1}{2}+1}(x, \boldsymbol{b}'_\mu)\Big[-\delta^{(2)}(\boldsymbol{b}) + \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}'_\mu)\Big]$$

which again conforms with the general overlap expression of LF Fock state wave functions.

Fourier transform of the cross section

The $\gamma^*+N \to f$ amplitudes have dynamical phases (resonances,...).

$\Rightarrow$ Calculating their Fourier transforms requires an amplitude analysis.

However, one can Fourier transform the measured cross section itself.

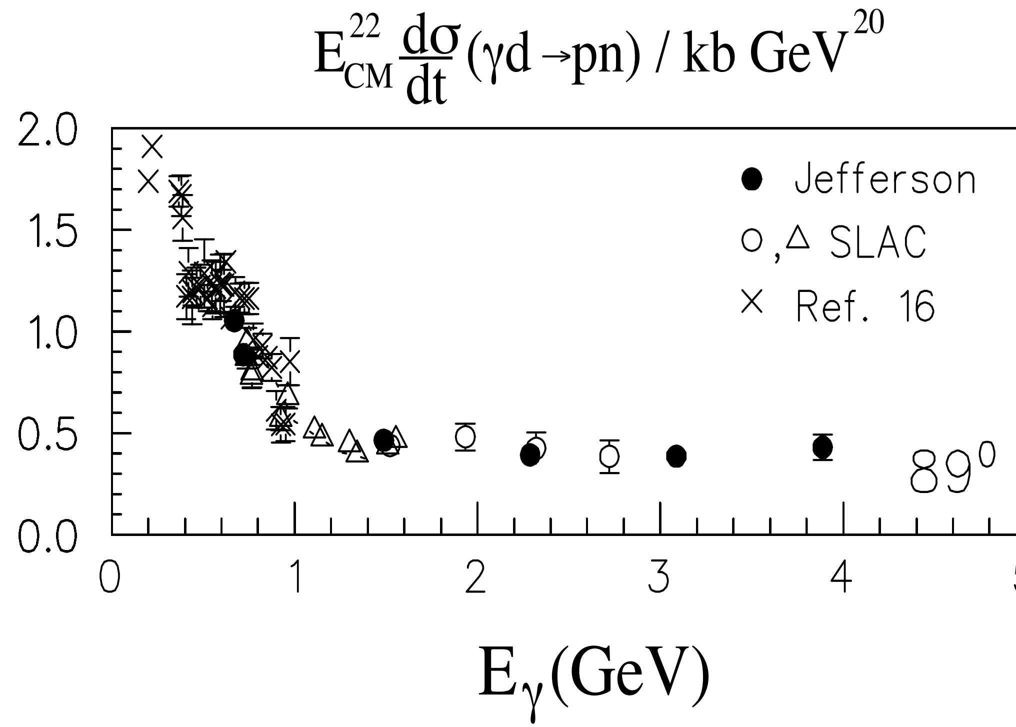
Then the $\boldsymbol{b}$-distribution reflects the difference between the impact parameters of the photon vertex in the amplitude and its complex conjugate:

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \left| \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle \right|^2 = \int d^2\boldsymbol{b}_q \, \mathcal{A}_{fN}(\boldsymbol{b}_q) \, \mathcal{A}^*_{fN}(\boldsymbol{b}_q - \boldsymbol{b})$$

Remarks

In $\gamma^* N \to \pi N$, expect the b-distribution to narrow with the relative transverse momentum k between the π and the N.

$\sigma(\gamma D \to pn) \propto E^{-22}$ at large angles, suggesting compact states. A measurement of the q^2-dependence would allow a direct measurement of the transverse size.



In heavy quark production:
$$\gamma^* N \to K\Lambda$$
$$\gamma^* N \to D\Lambda_c$$
the b-distribution should narrow with the quark mass if the photon couples directly to the heavy quarks.

Summary (1)

Intuitively, the ***q*** -dependence of a virtual photon interaction gives information about the charge distribution in space.

The target is illuminated "instantaneously" only when the charge carriers are non-relativistic. This is the case in standard electron microscopy.

Quarks move inside hadrons with ≈ velocity of light.

The photon phase is constant at fixed Light-Front time $x^+ = t + z$

In the IMF ≈ LF formulation, transverse quark velocities are non-relativistic

2-dim. FT's of form factors describe charge densities in transverse space

Unlike pdf's, no "leading twist" limit is implied.

The resolution in impact parameter is expected to be $\Delta b \sim 1/Q_{max}$

Summary (2)

The formulation can be generalised to transition form factors $\gamma^* N \to N^*$ and to any (multi-hadron) final (and initial) state: $\gamma^* \mathrm{A} \to f$

FT of the cross section $\sigma(\gamma^* N \to f)$ gives the distribution in the transverse distance $\boldsymbol{b}$ between the photon vertex in $\mathrm{T}(\gamma^* N \to f)$ and $[\mathrm{T}(\gamma^* N \to f)]^*$

Comparisons of $\boldsymbol{b}$-distributions in different processes can give insights into the scattering dynamics in transverse space.

The impact parameter distribution of the quarks with which the photon interacts is measured at an instant of x^+.

In color transparency measurements of $eN \to eN$ the nucleon size is measured later, through rescattering in the surrounding nucleus.

Furthermore, CT measures the transverse size of the entire state, including spectator quarks and gluons.

Scoop?!

You could be the first one
to apply the impact parameter analysis
to data on inelastic electroproduction*!

*Try also hadroproduction: $\gamma^* \rightarrow$ Reggeon/Pomeron