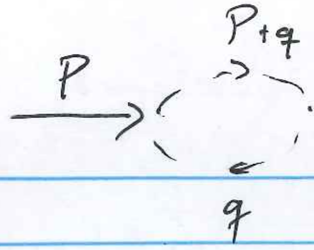


1-loop

$$\int \frac{d^4q}{(2\pi)^4}\,\underbrace{\frac{i}{(P+q)^2-m_1^2+i\varepsilon}}\;\frac{i}{q^2-m_2^2+i\varepsilon}$$

$$P=\begin{pmatrix}\sqrt{s}\\0\\0\\0\end{pmatrix}$$

$$\omega_1^2=(\overset{0}{\vec{P}}+\vec{q})^2+m_1^2$$

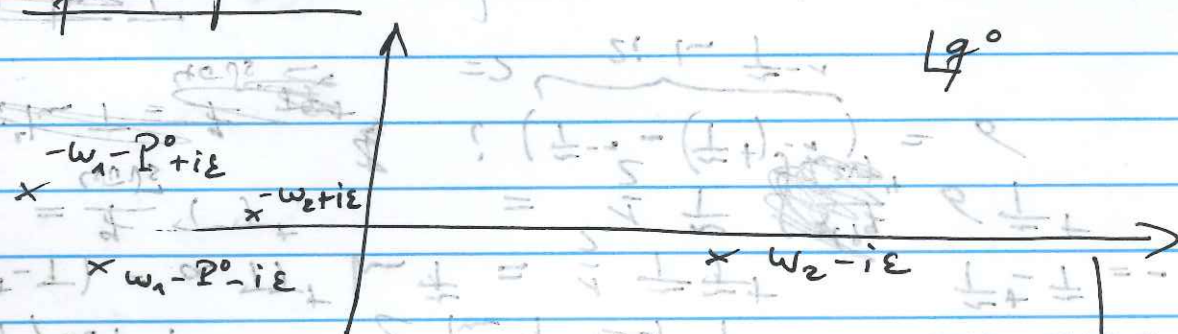
$$\omega_2^2=\vec{q}^{\,2}+m_2^2$$

$$(P^0+q^0)^2-\omega_1^2+i\varepsilon$$

$$q^{0\,2}-\omega_2^2+i\varepsilon$$

$$=(P^0+q^0-\omega_1+i\varepsilon)(P^0+q^0+\omega_1-i\varepsilon)(q^0-\omega_2+i\varepsilon)(q^0+\omega_2-i\varepsilon)\qquad(*)$$

q^0-integration



close integration in lower q^0 half plane $\Rightarrow$ factor (-1) from residue theorem

$$\Rightarrow \frac{1}{2\pi i}\int dq^0 \frac{1}{(\;)(\;)(\;)(\;)} = (-1)\sum_{\text{Residues}}$$

$(*)\rightarrow$

$$=(-1)\,\frac{q^0-\omega_2+i\varepsilon}{(\;)(\;)(\;)(q^0-\omega_2+i\varepsilon)} + \text{other poles}$$

$$\underset{\text{exercise}}{=} -\frac{\omega_1+\omega_2}{2\omega_1\omega_2}\,\frac{1}{P^{0\,2}-(\omega_1+\omega_2)^2+i\varepsilon}$$

derive!

$$\Rightarrow \int \frac{d^4q}{(2\pi)^4} \frac{i}{(P+q)^2 - m_1^2 + i\varepsilon} \frac{i}{q^2 - m_2^2 + i\varepsilon}$$

$$= i \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{1}{\underset{\overset{\parallel}{s}}{(P^0)^2} - (\omega_1+\omega_2)^2 + i\varepsilon}$$

$$\equiv (+i)\, G$$

$$i \,\mathrm{Im}\, G = (+i\pi) \underbrace{\int \frac{d^3q}{(2\pi)^3}}_{\frac{4\pi}{(2\pi)^3}\int dq\, q^2} \frac{\omega_1+\omega_2}{2\omega_1\omega_2}\, \delta\Big(\underbrace{P^{0\,2} - (\omega_1+\omega_2)^2}_{= f(q)}\Big)$$

$$f(q) = 0 \iff q = q_{cm} = \frac{1}{2\sqrt{s}} \Big[(s - (m_1+m_2)^2) \times (s - (m_1 - m_2)^2) \Big]^{1/2}$$

$$\delta(f(q)) = \frac{1}{\left| \frac{\partial}{\partial q} f(q) \right|_{q = q_{cm}}} \delta(q - q_{cm})$$

$$\frac{\partial}{\partial q} f(q) \Big|_{q = q_{cm}} = -2\, q_{cm} \underbrace{\frac{\omega_1 + \omega_2}{\omega_1\omega_2}}_{\omega_1+\omega_2 = \sqrt{s}} \underbrace{(\omega_1 + \omega_2)}_{= \sqrt{s}}$$

$$\Rightarrow i \,\mathrm{Im}\, G = \frac{-i\pi}{8\pi^3}\, 4\pi\, q_{cm}^2\, \frac{\sqrt{s}}{2\omega_1\omega_2} \frac{1}{2 q_{cm} \frac{s}{\omega_1 \omega_2}}$$

$$= -\frac{i}{8\pi} \frac{q_{cm}}{\sqrt{s}}\, \theta\left(\sqrt{s} - (m_1 + m_2)\right)$$

$$\equiv -i\sigma$$

$$(\mathrm{Im}\, G = -\sigma)$$

$$\sigma = \frac{q}{8\pi\sqrt{s}}$$

(1)

(unitary normalization!)

partial wave!

$$e^{2i\delta_\ell} = S_\ell = 1 - \frac{i\, q_{cm}}{4\pi\sqrt{s}}\, T_\ell$$

$$S_\ell S_\ell^\dagger = 1$$

$$\left(1 - \frac{iq}{4\pi\sqrt{s}}\, T_\ell\right)\left(1 + \frac{iq}{4\pi\sqrt{s}}\, T_\ell^\dagger\right) = 1$$

$$\Rightarrow \frac{iq}{4\pi\sqrt{s}}\left(T_\ell^\dagger - T_\ell\right) + \left(\frac{q}{4\pi\sqrt{s}}\right)^2 T_\ell T_\ell^\dagger = 0$$

$$\Rightarrow i\left(T_\ell - T_\ell^\dagger\right) = \frac{q}{4\pi\sqrt{s}}\, T_\ell T_\ell^\dagger$$

$$i\underbrace{\left(\frac{1}{T_\ell^\dagger} - \frac{1}{T_\ell}\right)}_{-2i\,\mathrm{Im}\, T_\ell^{-1}} = \frac{q}{4\pi\sqrt{s}}$$

$$\Rightarrow \mathrm{Im}\, T_\ell^{-1} = \frac{q}{8\pi\sqrt{s}} = +\sigma = -\mathrm{Im}\, G$$

$$T_\ell = V_\ell + V_\ell G T_\ell$$

$$\Rightarrow T_\ell = \left(V_\ell^{-1} - G\right)^{-1}$$

$$\Rightarrow \mathrm{Im}\, T_\ell^{-1} = -\mathrm{Im}\, G \quad \underline{\text{if}}\ V_\ell \in \mathbb{R}$$

✓

$\Rightarrow$ choose <u>real</u> V_ℓ

<u>Simplification:</u> <u>K-matrix</u> : $\mathrm{Re}\, G \equiv 0$

dispersive part

$$\Rightarrow T_\ell = V_\ell - iV_\ell \sigma T_\ell \Rightarrow T_\ell = \frac{1}{K^{-1} + i\sigma}$$

<u>Remark:</u> Re G can be reconstructed from Im G by dispersion relation

Coupled channels

$$\mathrm{Im}\, T^{-1} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} + \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \times \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}$$

$$\Rightarrow T = \left(V^{-1} - G\right)^{-1}$$

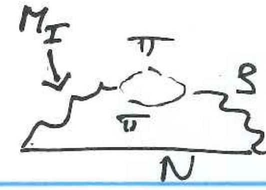
matrix equation

common parametrization: $$S_\ell = \begin{pmatrix} \eta e^{2i\delta_1} & i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} \\ i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

δ: phase shift

η: elasticity

3 - particles



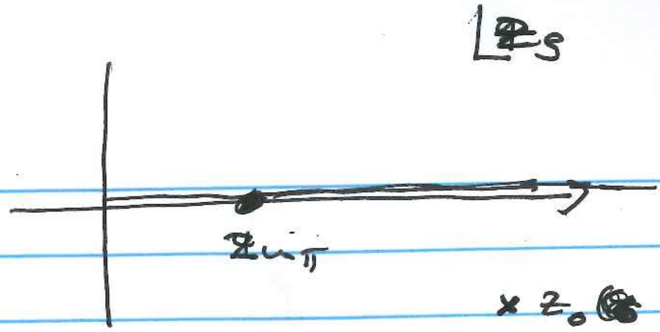
$2m_\pi < M_I < \sqrt{s} - m_N$

$$L = \int dp\, p^2\, G^{\rho N}(p)$$

$$= \int dp\, p^2 \frac{1}{\sqrt{s} - E_N(p) - E_\rho(p) - \Sigma_\rho(s_\rho)}$$

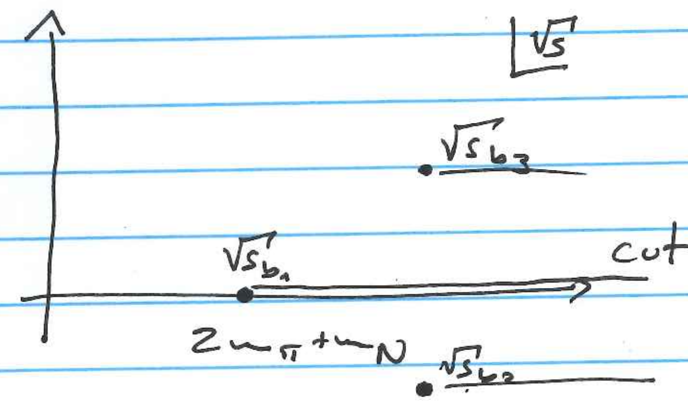
$$\Sigma_\rho(s_\rho) = \int_0^\infty dq\, q^2 \frac{V^2_{\rho\pi\pi}}{s_\rho - 4\omega_\pi^2(q) + i\varepsilon}$$

$$s_\rho = s + m_N^2 - 2\sqrt{s}\, E_N(p) \qquad E_N^2(p) = m_N^2 + p^2$$



branch point: $4m_\pi^2 = s_{b_1} + m_N^2 - 2\sqrt{s_{b_1}}\, m_N = (\sqrt{s} - m_N)^2$

$$\Rightarrow \sqrt{s_{b_1}} = 2m_\pi + m_N$$



but more: $z_g^2 = s_{b_2} + m_N^2 - 2\sqrt{s_{b_2}}\, m_N = (\sqrt{s_{b_2}} - m_N)^2$

$$\left.\begin{aligned} &\Rightarrow \sqrt{s_{b_2}} = m_N + z_g \\ &\underset{\text{Schwarz-reflection}}{\Rightarrow} \sqrt{s_{b_3}} = m_N + z_g^* \end{aligned}\right\} \text{2nd sheet w.r.t. } \sqrt{s_{b_1}}$$

$\Rightarrow$ first sheet still free of non-analyticities