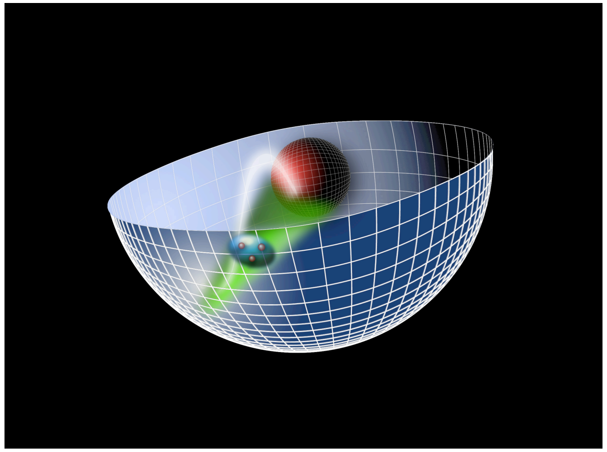
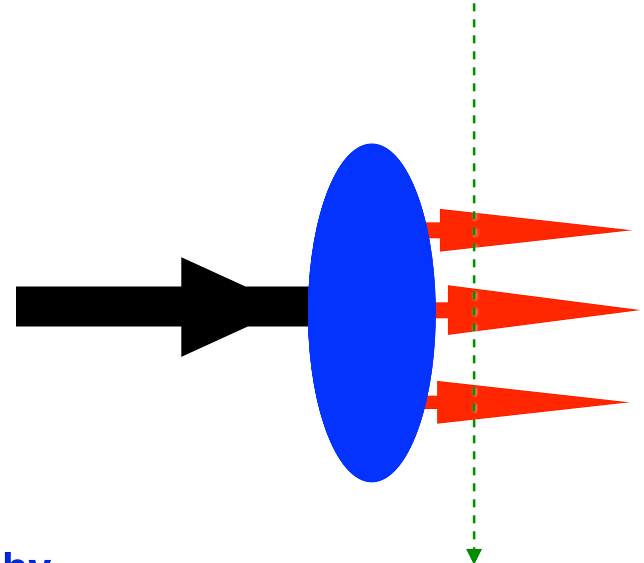
Scattering Theory and Light-Front QCD

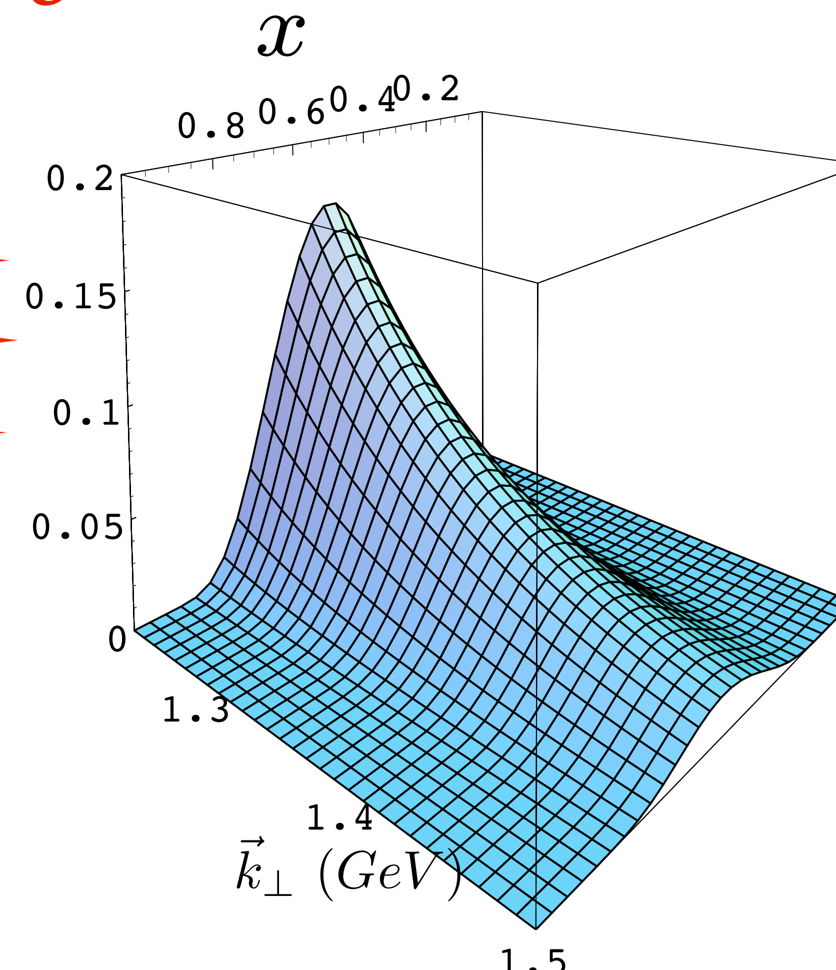


AdS/QCD : Light-Front Holography



Fixed $\tau = t + z/c$

$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$



2015 International Summer Workshop on Reaction Theory

June 12, 2015

INDIANA UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

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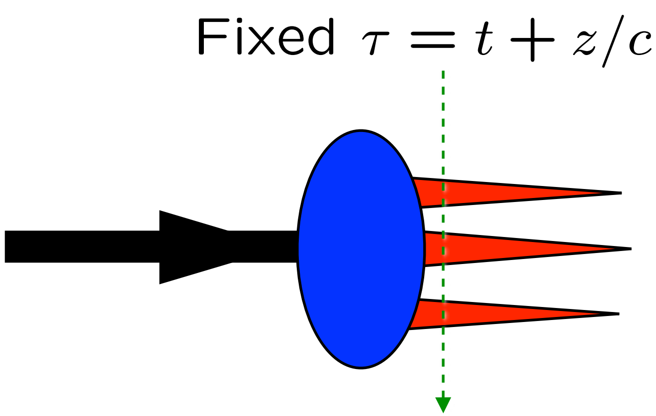
Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

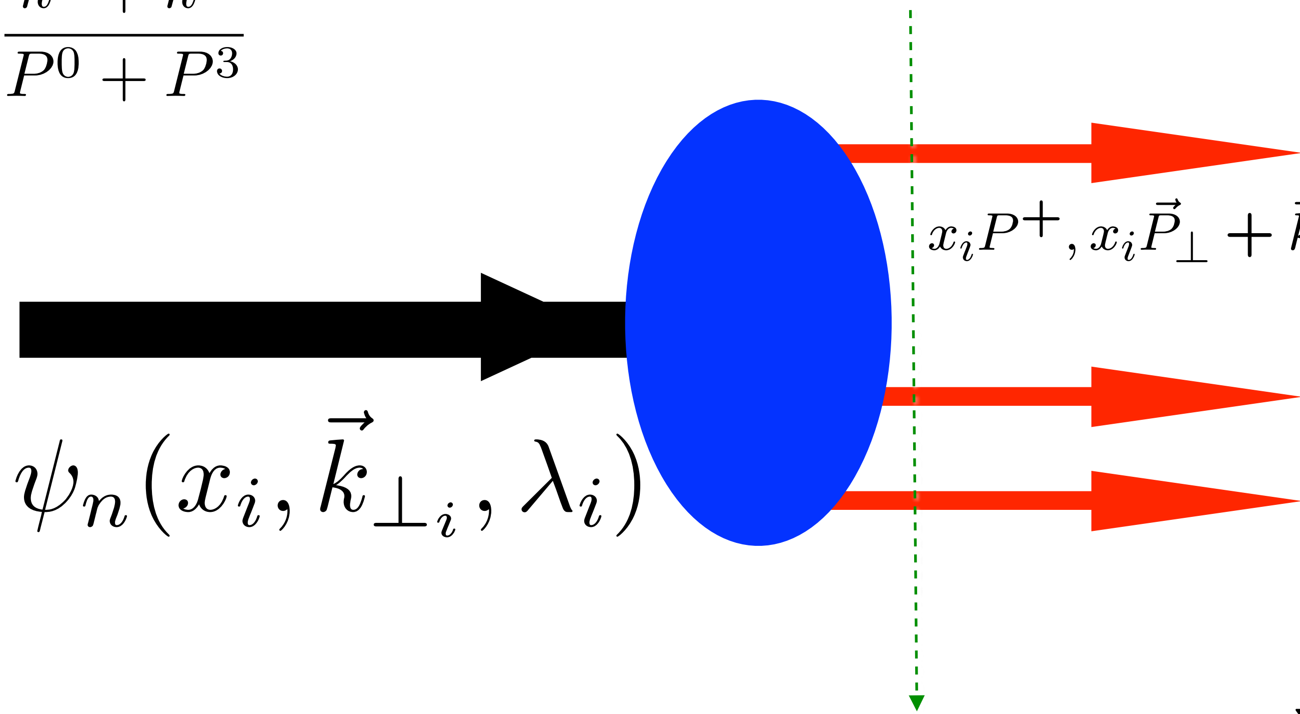
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian : Off-shell in Invariant Mass

Fixed $\tau = t + z/c$

Fixed LF time

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Sum Rules

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

$$|p,S_z> = \sum_{n=3} \Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

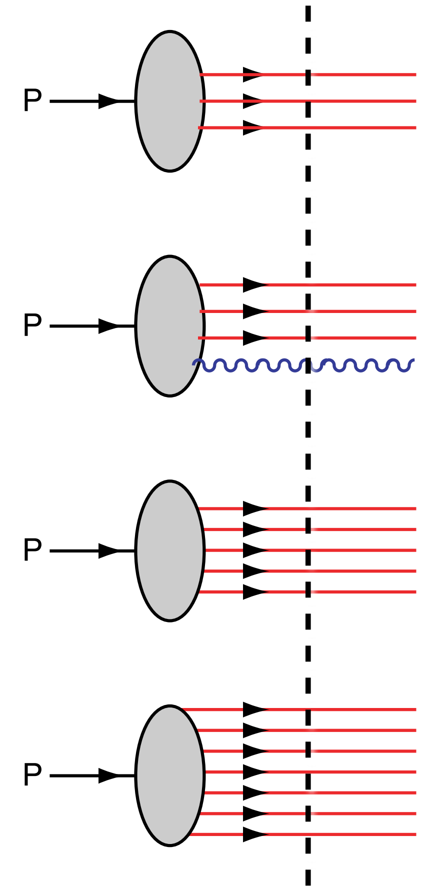
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+,\ \sum_i^n x_i = 1,\ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks

s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

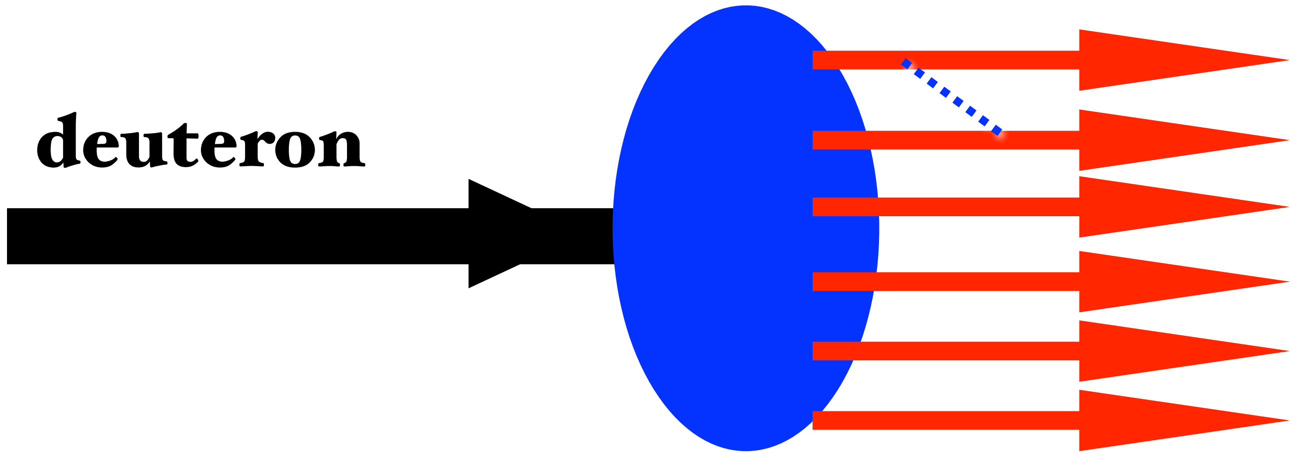
Hidden Color

Mueller: gluon Fock states BFKL

Evolution of 5 color-singlet Fock states

Lepage, Ji, sjb

$$\Psi^{\mathbf{d}}_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$\sum_i^n x_i = 1$

$$\Phi_n(x_i, Q) = \int^{k^2_{\perp i} < Q^2} \Pi' d^2 k_{\perp j} \psi_n(x_i, \vec{k}_{\perp j})$$

5 X 5 Matrix Evolution Equation for deuteron distribution amplitude

Hidden Color of Deuteron

Deuteron six-quark state has five color - singlet configurations, only one of which is n-p.

Asymptotic Solution has Expansion

$$\psi_{[6]\{33\}} = \left(\frac{1}{9}\right)^{1/2} \psi_{NN} + \left(\frac{4}{45}\right)^{1/2} \psi_{\Delta\Delta} + \left(\frac{4}{5}\right)^{1/2} \psi_{CC}$$

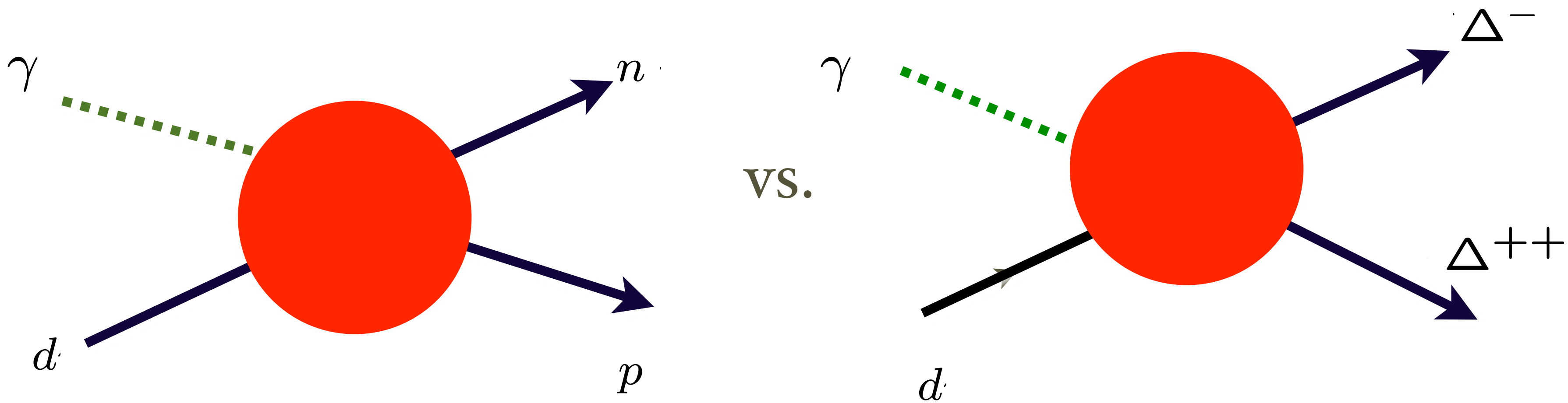
Look for transition to Delta-Delta

Test of Hidden Color in Deuteron Photodisintegration

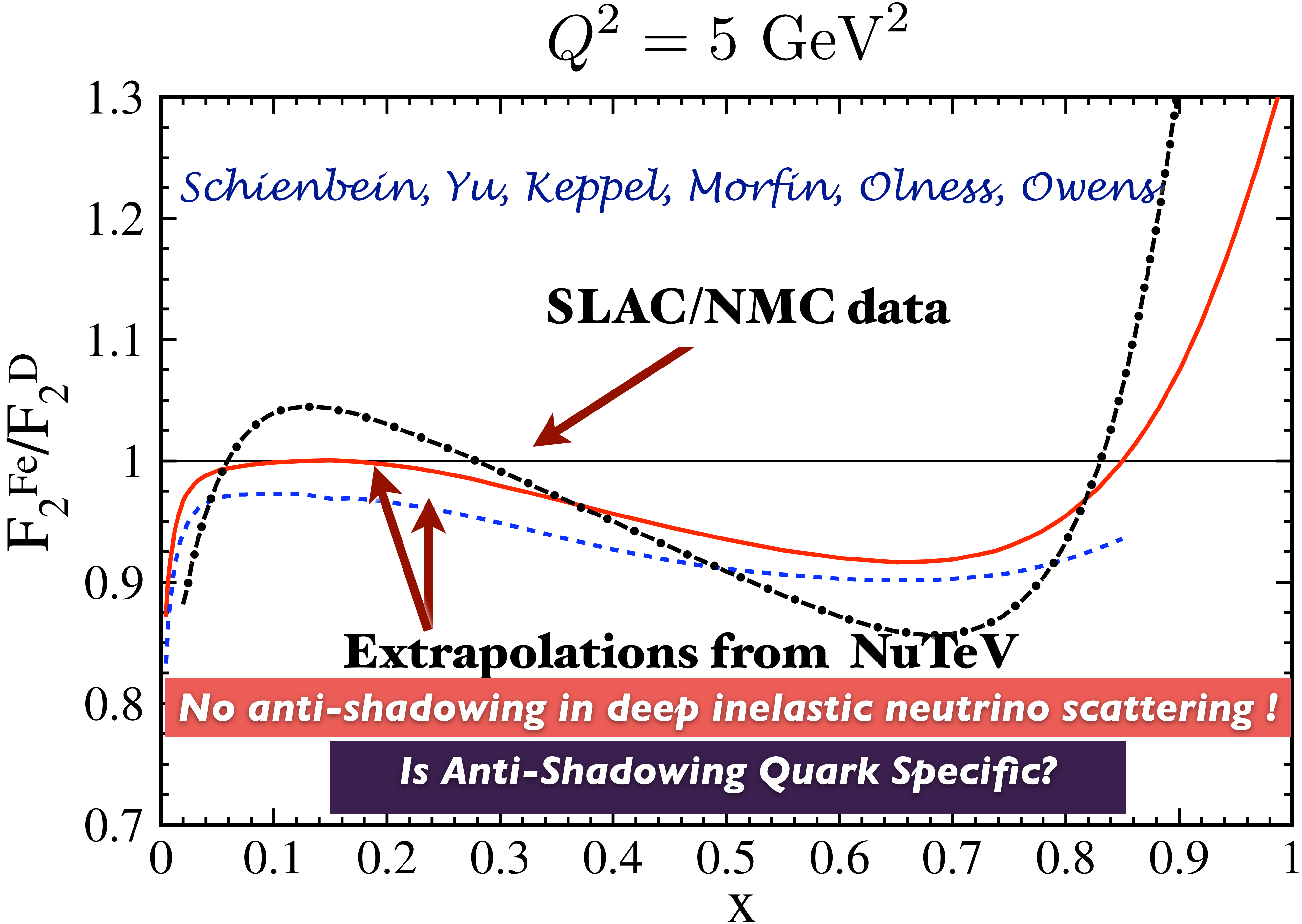
$$R = \frac{\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{--})}{\frac{d\sigma}{dt}(\gamma d \to pn)}$$

Ratio predicted to approach 2:5

Ratio should grow with transverse momentum as the hidden color component of the deuteron grows in strength.

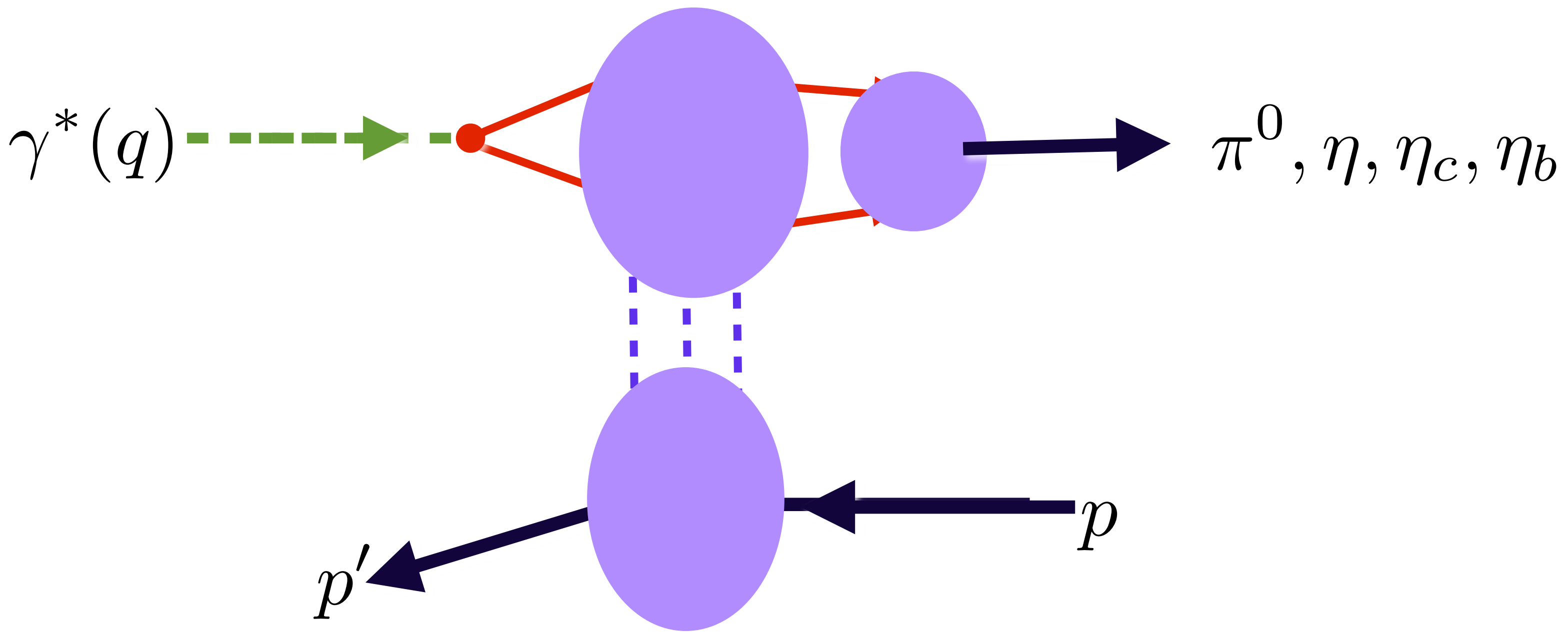


Possible contribution from pion charge exchange at small t.



$Q^2 = 5$ GeV^2
Schienbein, Yu, Keppel, Morfin, Olness, Owens
SLAC/NMC data
Extrapolations from NuTeV
No anti-shadowing in deep inelastic neutrino scattering !
Is Anti-Shadowing Quark Specific?
F_2^{Fe}/F_2^D
x
1.3
1.2
1.1
1
0.9
0.8
0.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

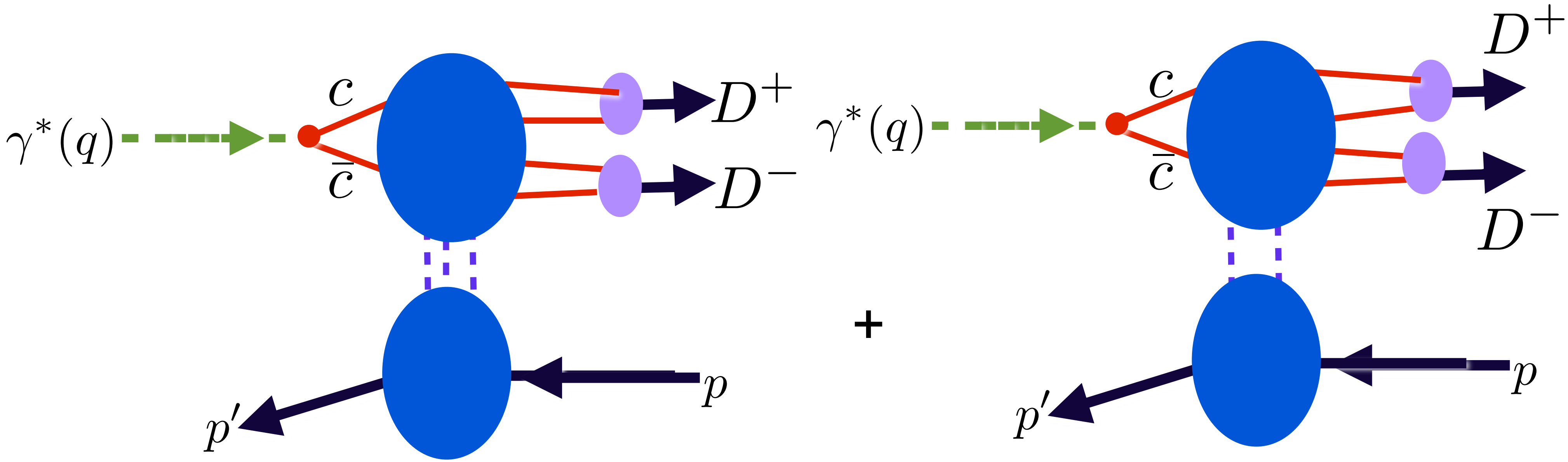
Is Antishadowing in DIS Non-Universal, Flavor-Dependent?



Odderon has never been observed!

Look for Charge Asymmetries from Odderon-Pomeron Interference

Merino, Rathsman, sjb



Odderon-Pomeron Interference leads to $K^+ K^-$, $D^+ D^-$ and $B^+ B^-$ charge and angular asymmetries

Odderon at amplitude level

Merino, Rathsman, sjb

Strong enhancement at heavy-quark pair threshold from QCD Sakharov-Schwinger-Sommerfeld effect

$$\frac{\pi \alpha_s(\beta^2 s)}{\beta}$$

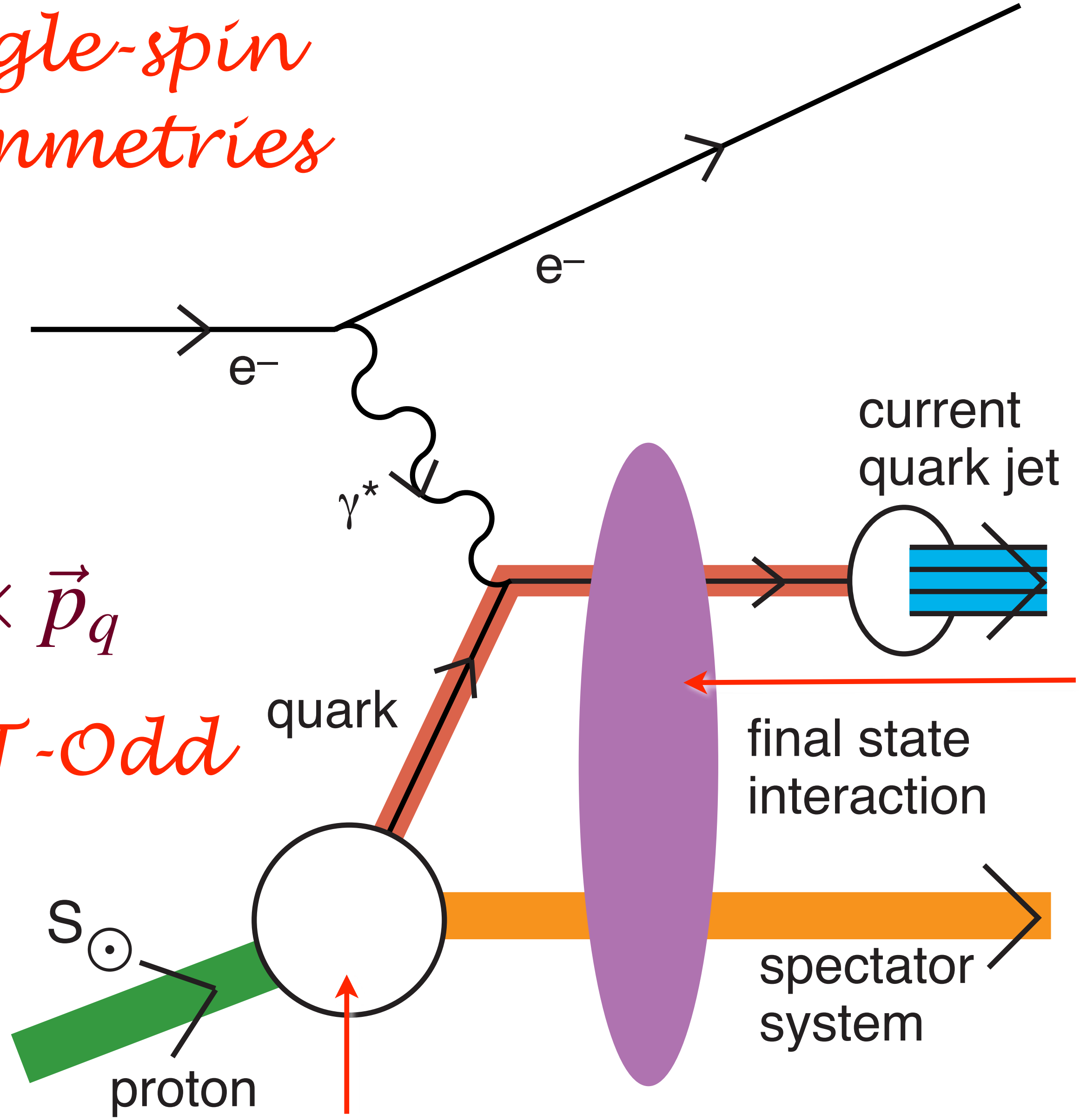
Hoang, Kuhn, sjb

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Xiao, Pasquini, ...



$$i \ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd

QED: Lensing involves soft scales

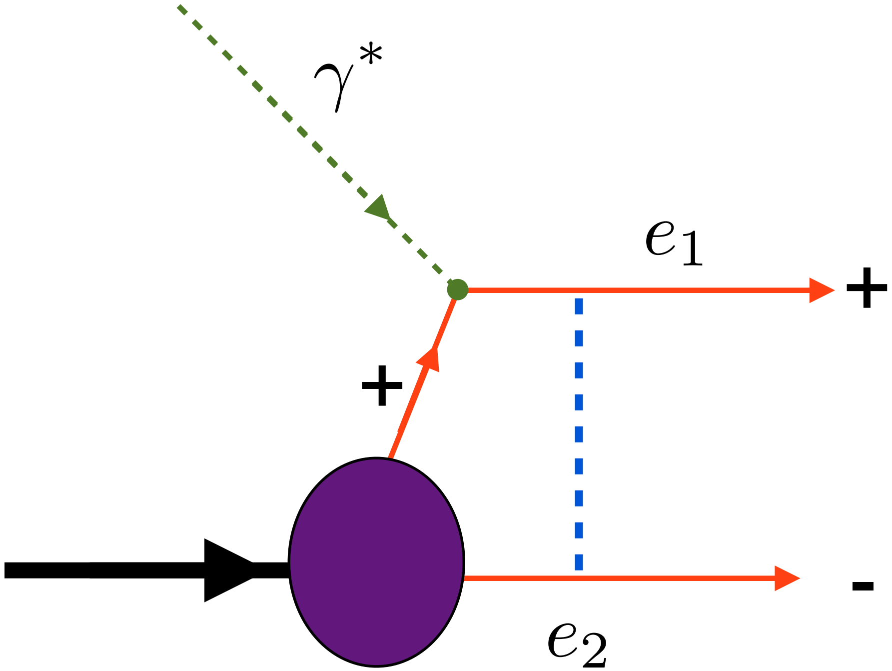
QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

Light-Front Wavefunction S and P- Waves!

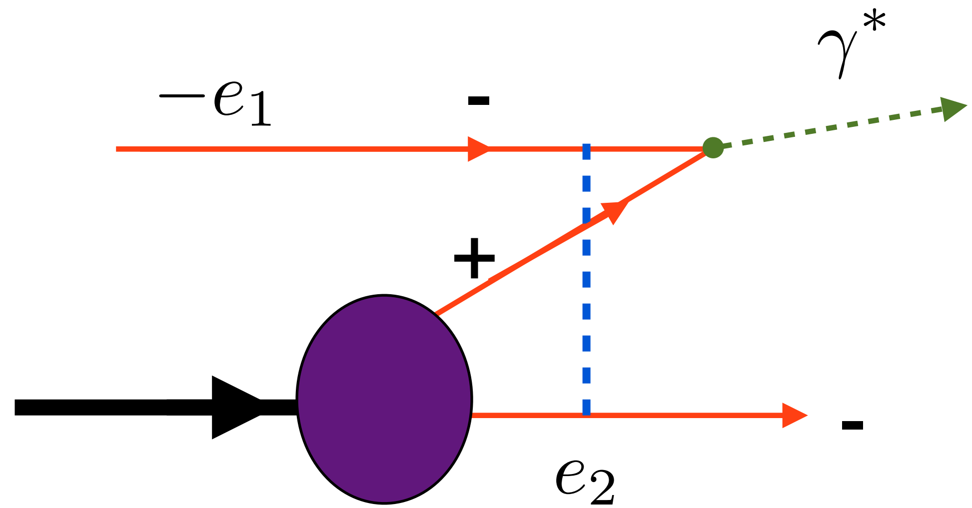
Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!



DIS

Attractive, opposite-sign rescattering potential



DY

Repulsive, same-sign scattering potential

Dae Sung Hwang, Yuri V. Kovchegov, Ivan Schmidt, Matthew D. Sievert, sjb

What is measured!

Static	Dynamic
• Square of Target LFWFs	Modified by Rescattering: ISI & FSI
• No Wilson Line	Contains Wilson Line, Phases
• Probability Distributions	No Probabilistic Interpretation
• Process-Independent	Process-Dependent - From Collision
• T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
• No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
• Sum Rules: Momentum and J^z	Sum Rules Not Proven !
• DGLAP Evolution; mod. at large x	DGLAP Evolution
• No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

Hwang, Schmidt, sjb,

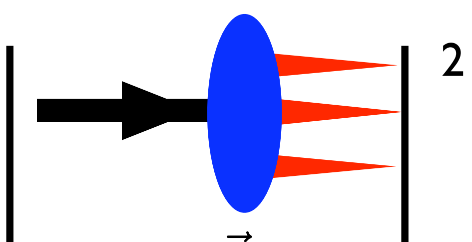
Mulders, Boer

Qiu, Sterman

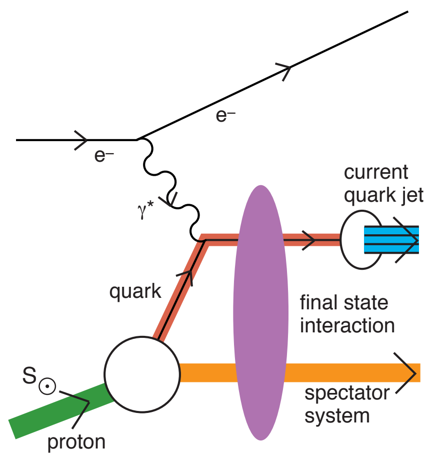
Collins, Qiu

Pasquini, Xiao, Yuan, sjb

Liuti, sjb



$$\left| \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2$$



Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Unique confinement potential!

Goal: An analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **Confinement in QCD -- What is the analytic form of the confining interaction?**
- **What sets the QCD mass scale?**
- **QCD Running Coupling at all scales**
- **Hadron Spectroscopy-Regge Trajectories**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **Constituent Counting Rules**
- **Hadronization at the Amplitude Level**
- **Insights into QCD Condensates**

Atomic Physics from First Principles

$\mathcal{L}_{QED} \rightarrow$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) \, |\Psi> = E \, |\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})] \; \psi(\vec{r}) = E \; \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)] \; \psi(r) = E \; \psi(r)$$

Spherical Basis r, θ, ϕ

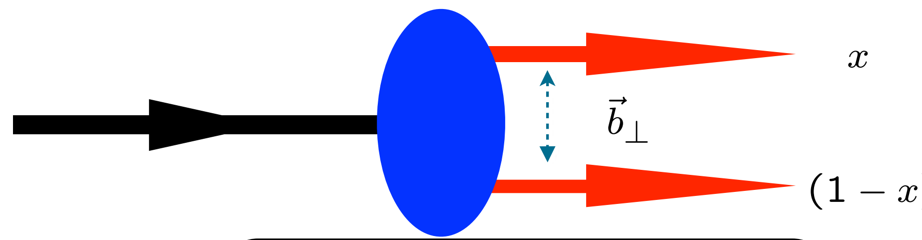
$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x) b_\perp^2$$

$$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\text{eff}}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)]\psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Azimuthal Basis

$$\zeta, \phi$$

$$m_q = 0$$

AdS/QCD:

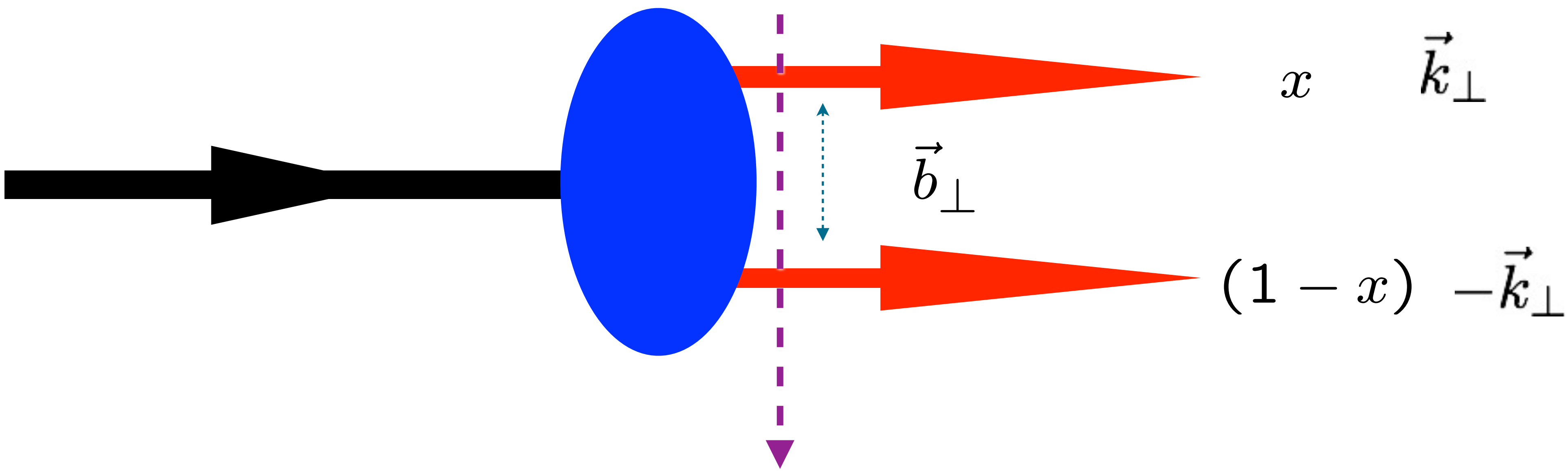
$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

Fixed $\tau = t + z/c$



$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

Invariant transverse separation

$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}^2_{q+\bar{q}}$$

$$\int dk^- \Psi_{BS}(P,k) \to \psi_{LF}(x, \vec{k}_\perp)$$

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp\, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables $(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

$$+ \int d\zeta\, \phi^*(\zeta) U(\zeta)\phi(\zeta)$$

$$= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

Light-Front Schrödinger Equation

G. de Teramond, sjb

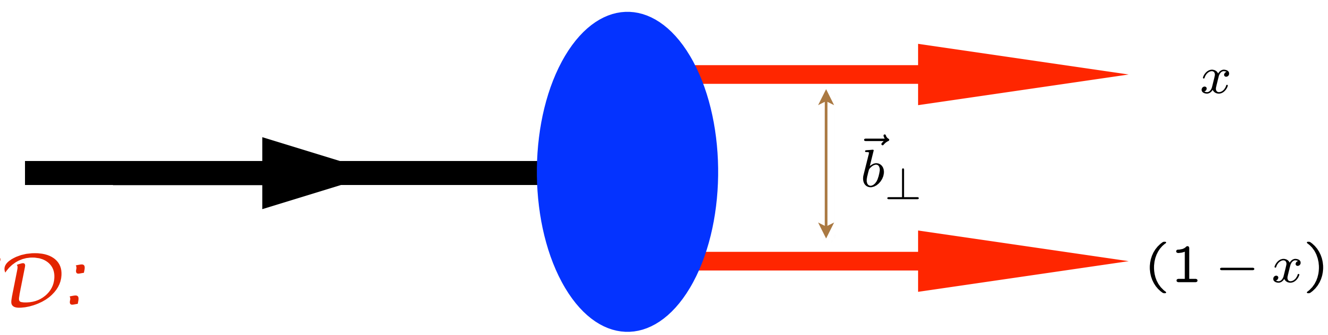
Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

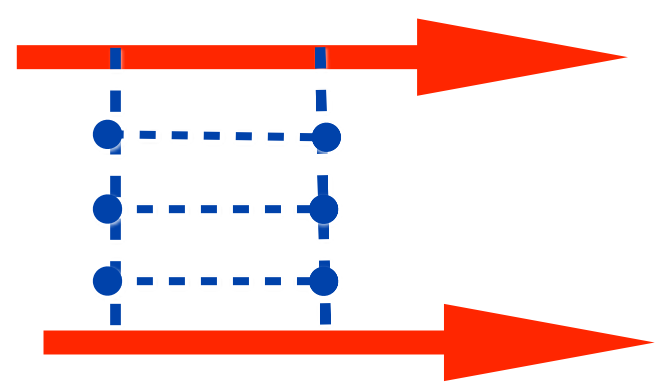
$$m_q \sim 0$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



AdS/QCD:

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$



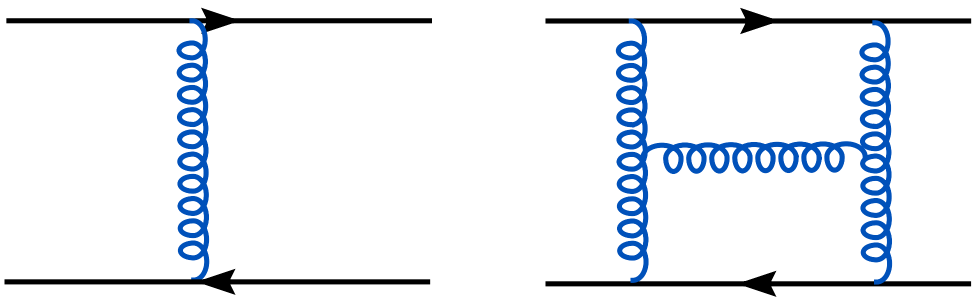
U is the exact QCD potential
Conjecture: 'H'-diagrams generate U?

Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \Big[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2$$

$$+ (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3 \Big]$$

Smirnov, Smirnov, Steinhauser, 2010



$\log \kappa^2 \zeta^2$

Summation of H graphs: confining potential

Confinement eliminates IR divergences
Self-consistent mass scale κ

Light-Front Schrödinger Equation

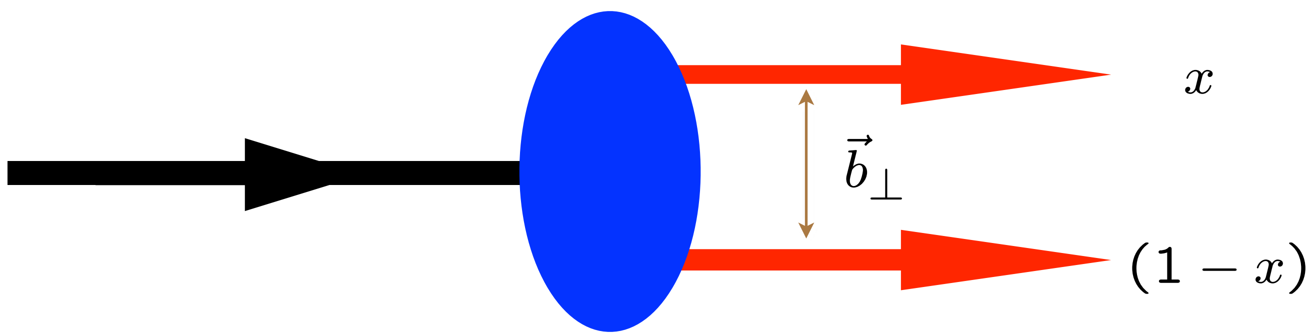
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

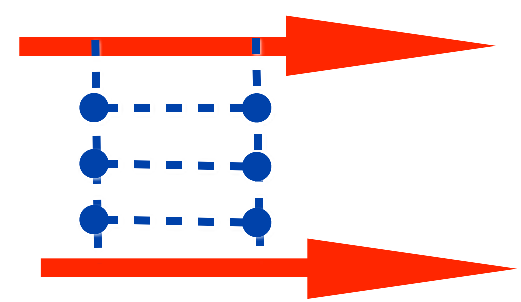
Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \, \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



U is the confining QCD potential
Conjecture: 'H'-diagrams generate

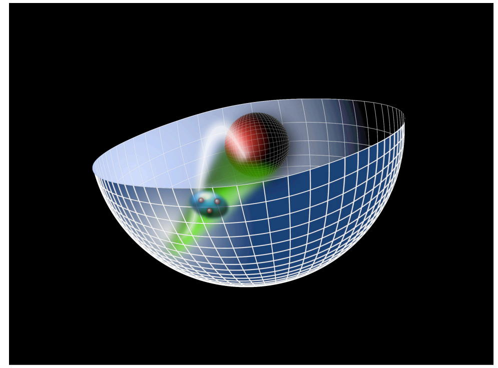


$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

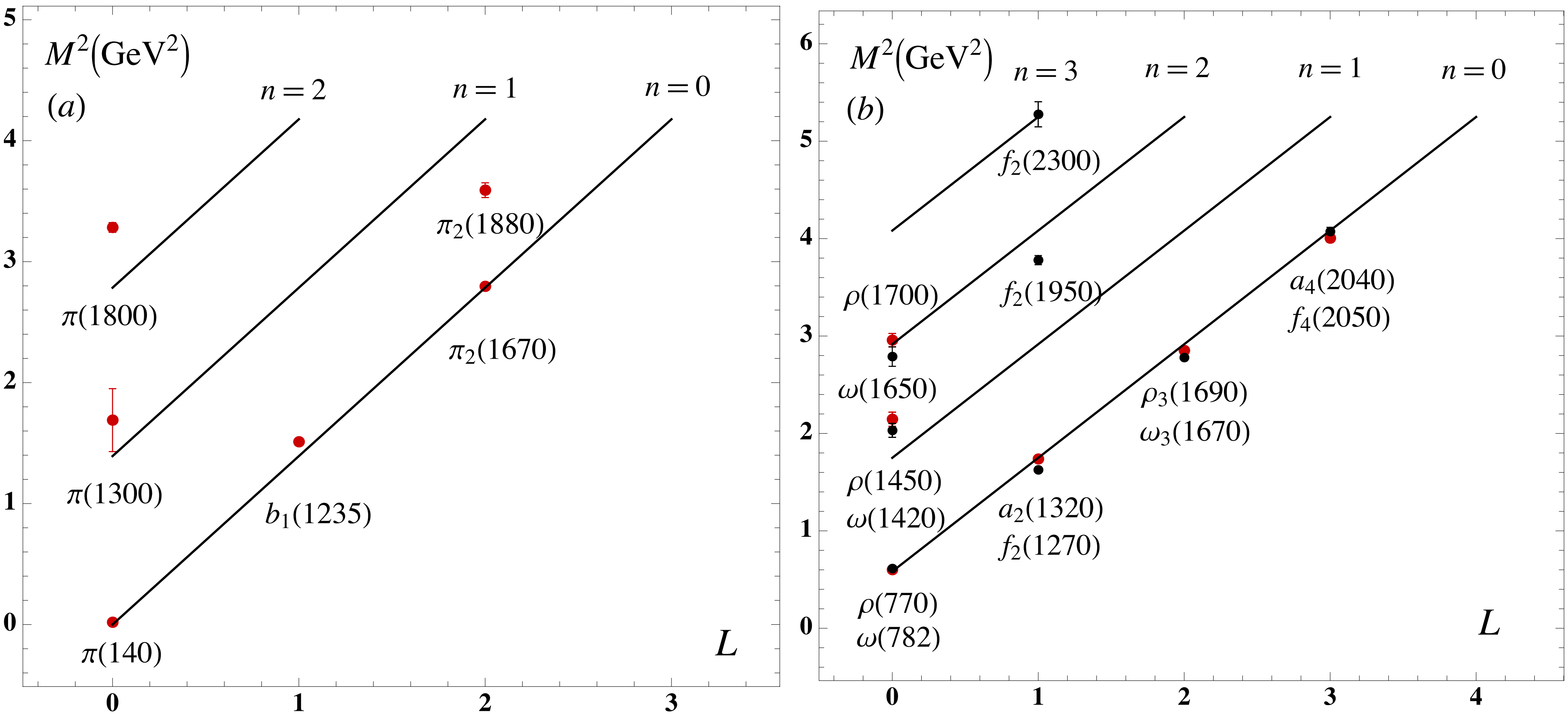
$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

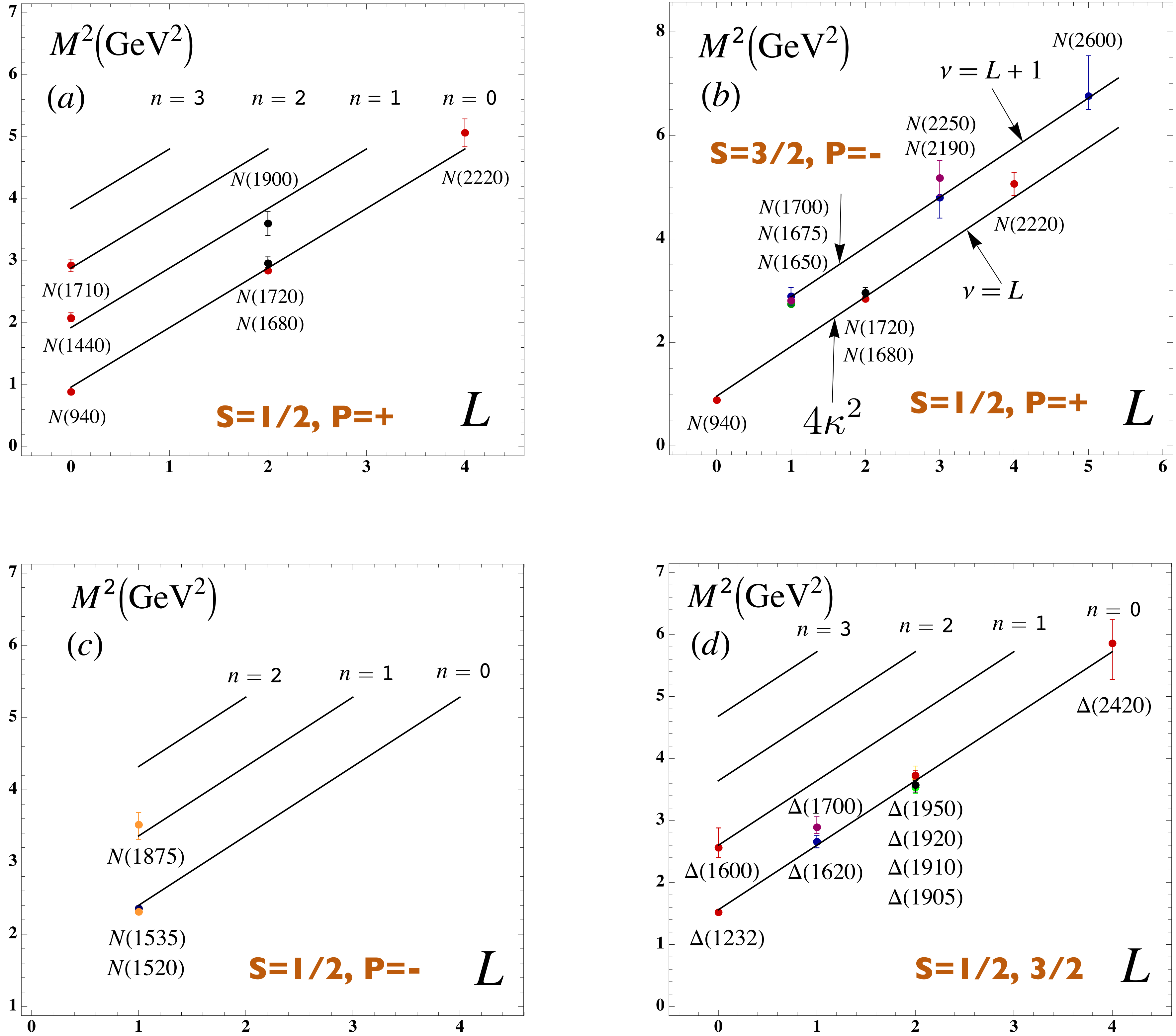
Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

$m_u = m_d = 0$

Preview

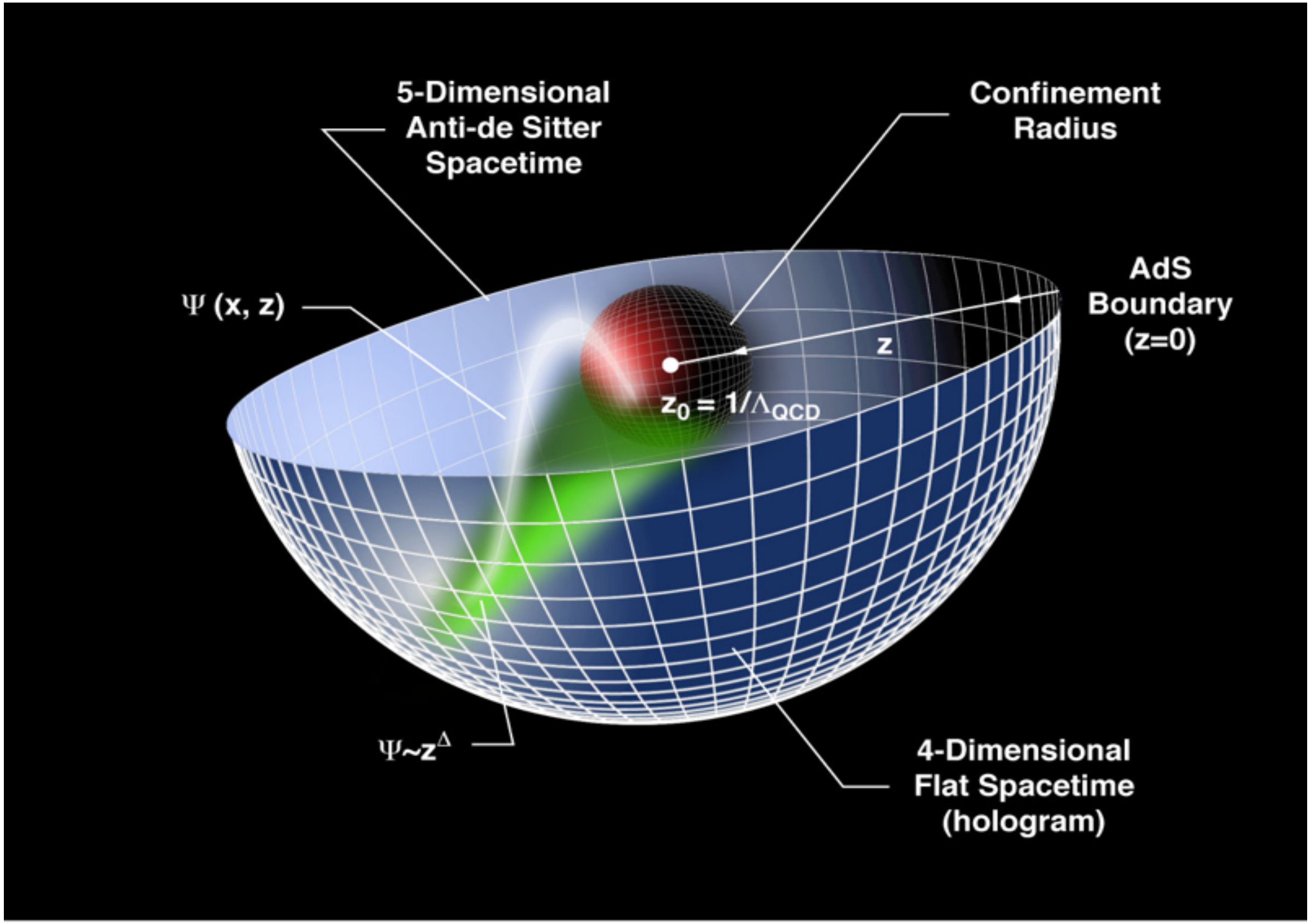


$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$



M^2(GeV^2)
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
M^2(GeV^2)
(b)
ν = L + 1
N(2600)
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
ν = L
N(1720)
N(1680)
N(940)
4κ^2
S=1/2, P=+
L
M^2(GeV^2)
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
M^2(GeV^2)
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

Applications of AdS/CFT to QCD



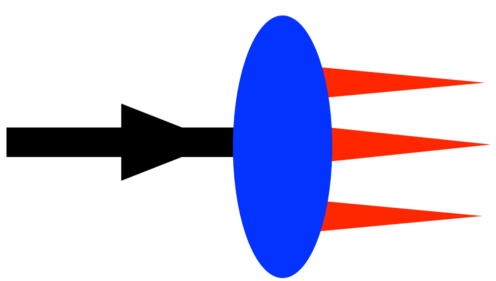
Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond and H. Guenter Dosch

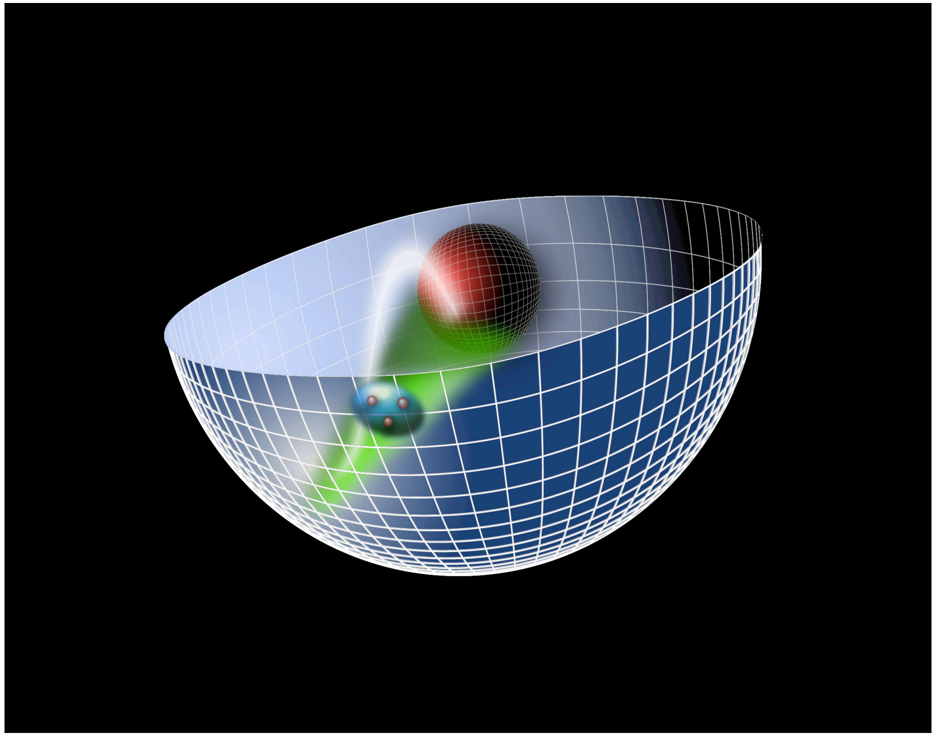
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with Guy de Teramond and H. Guenter Dosch

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

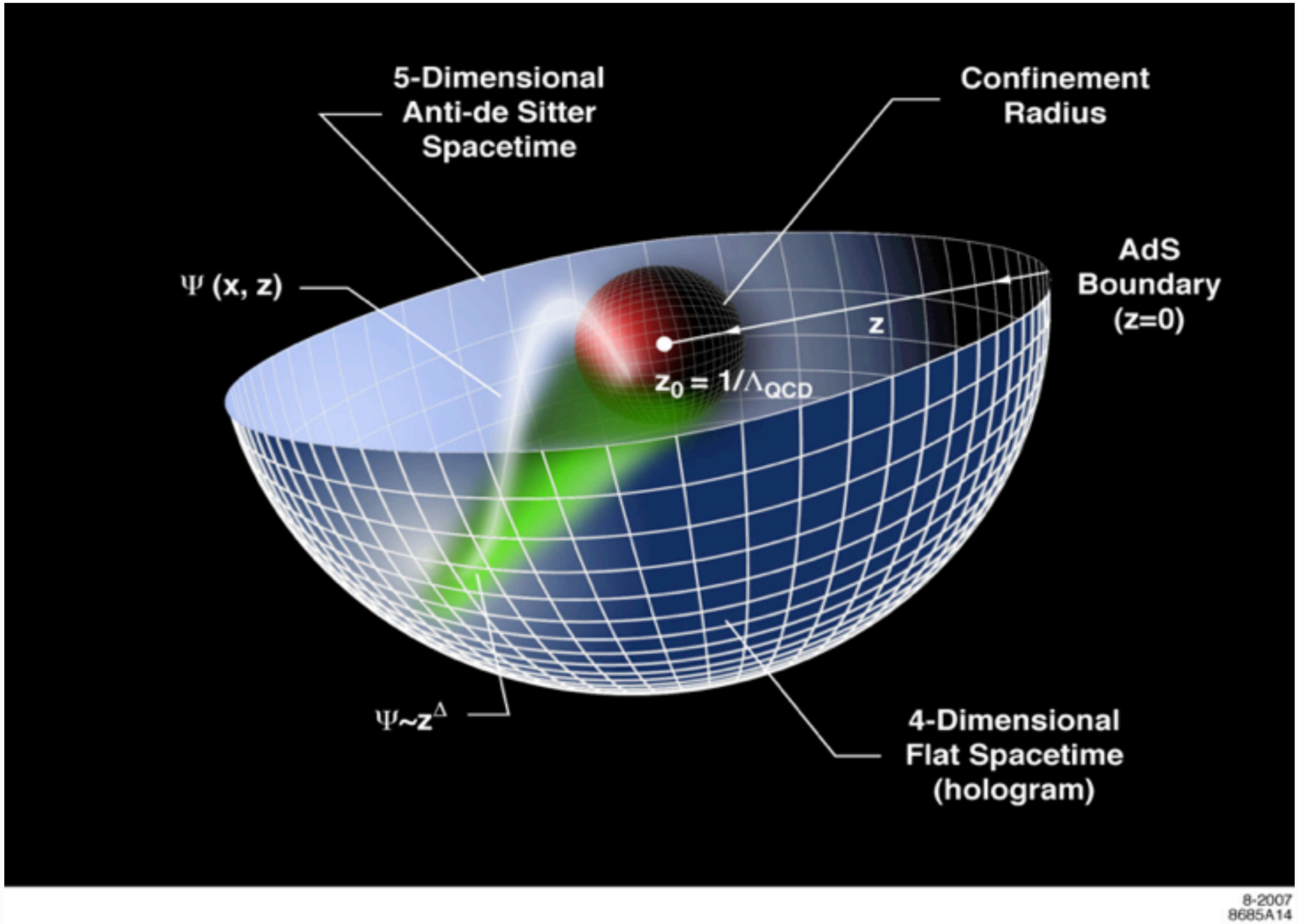
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.



Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g}\, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x, z) = e^{-iP\cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z \mathcal{M}) \qquad \Delta = \tfrac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS_5 as template for conformal theory**

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

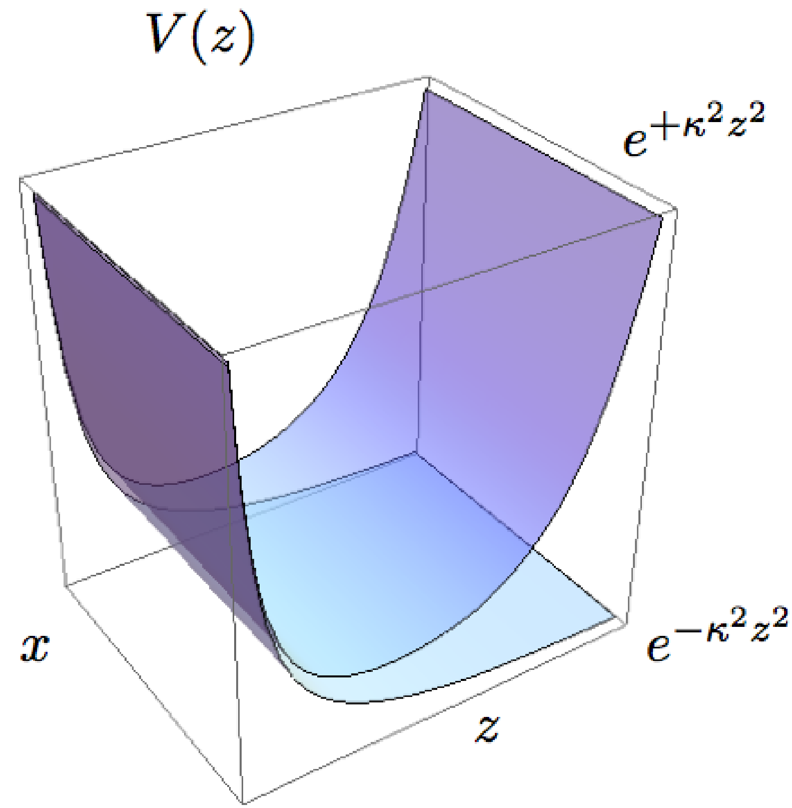
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

de Tèramond, Dosch, sjb

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

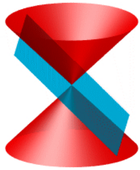
$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L+S-1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}$$



with $(\mu R)^2 = -(2-J)^2 + L^2$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Quark separation increases with L

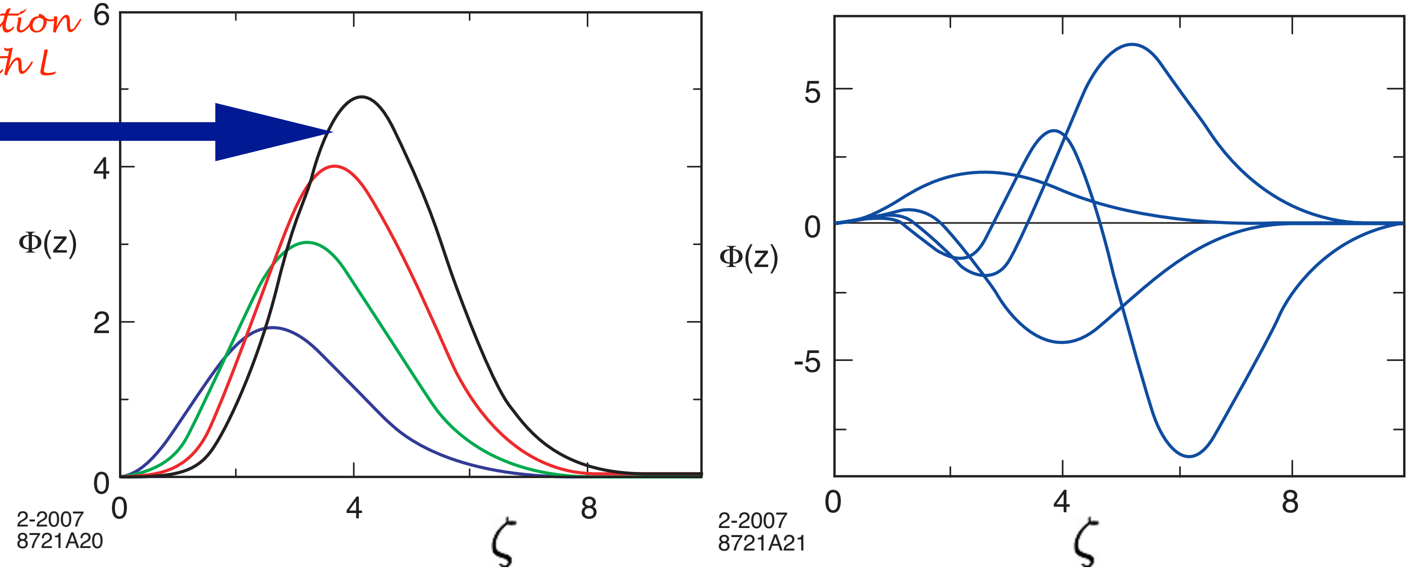
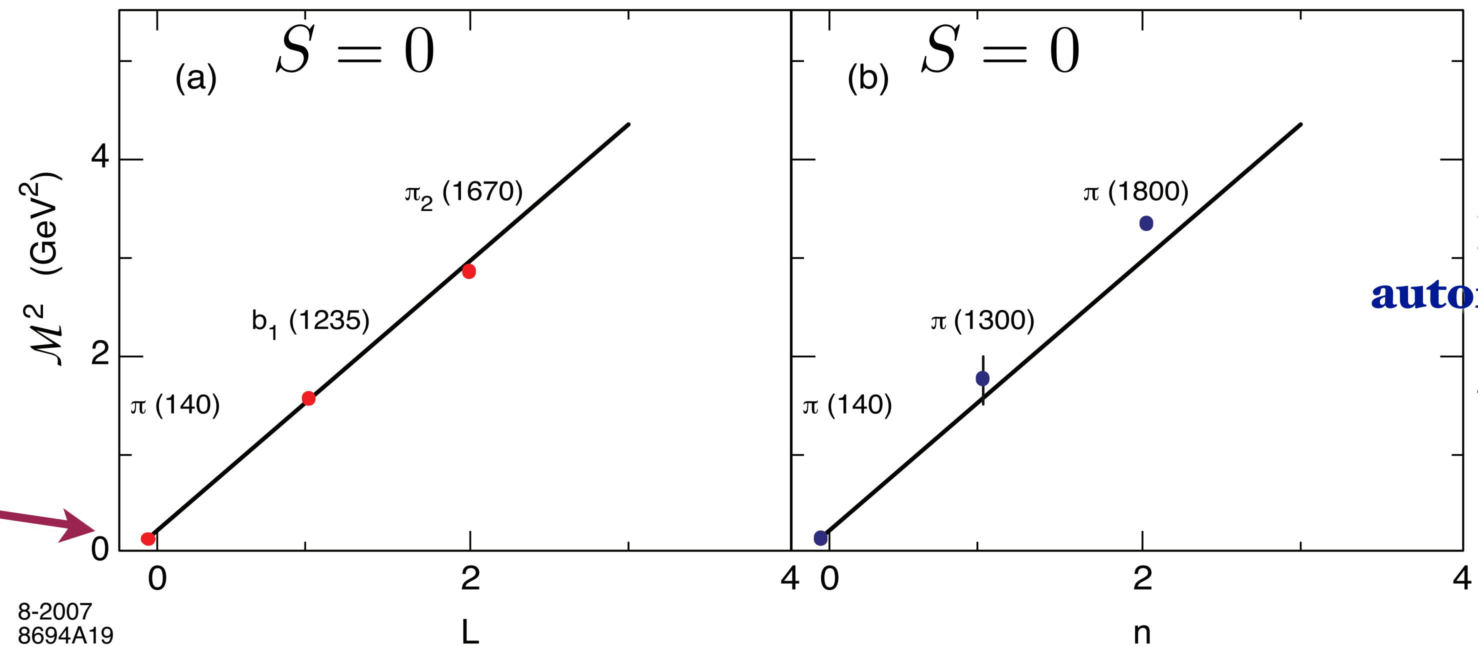


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

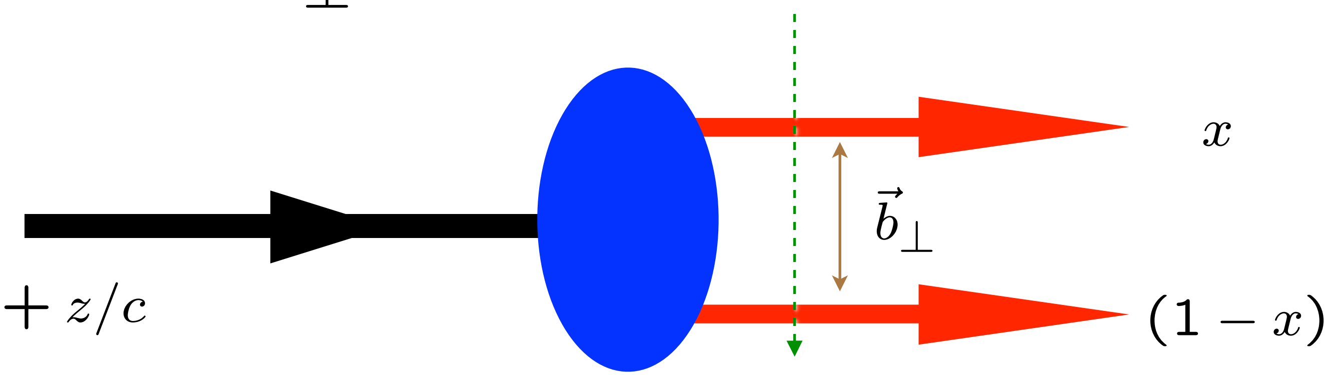
de Teramond, sjb

LF(3+1) ↔ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

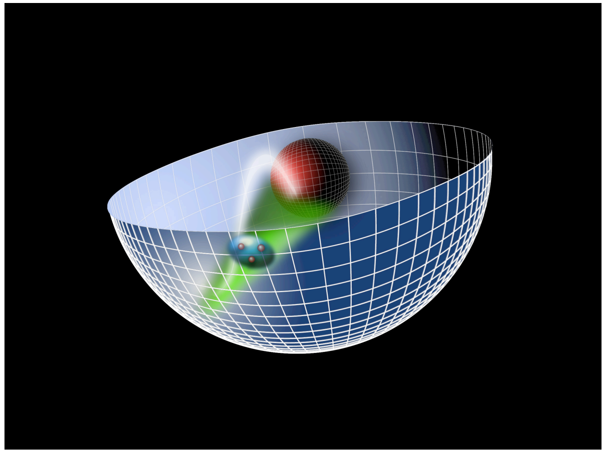
- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

G. de Teramond, H. G. Dosch, sjb

$\phi(z)$

AdS5: Conformal Template for QCD

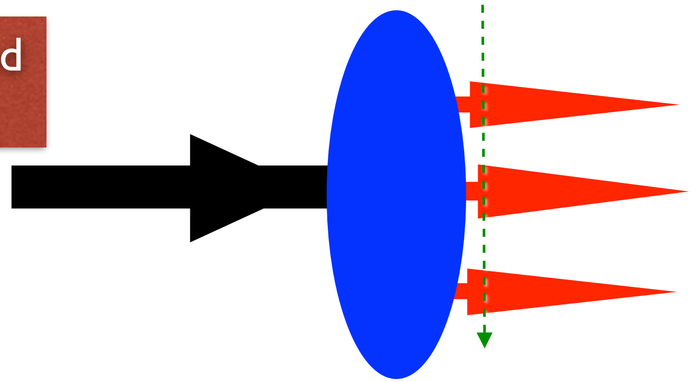


with Guy de Teramond and Hans Guenter Dosch

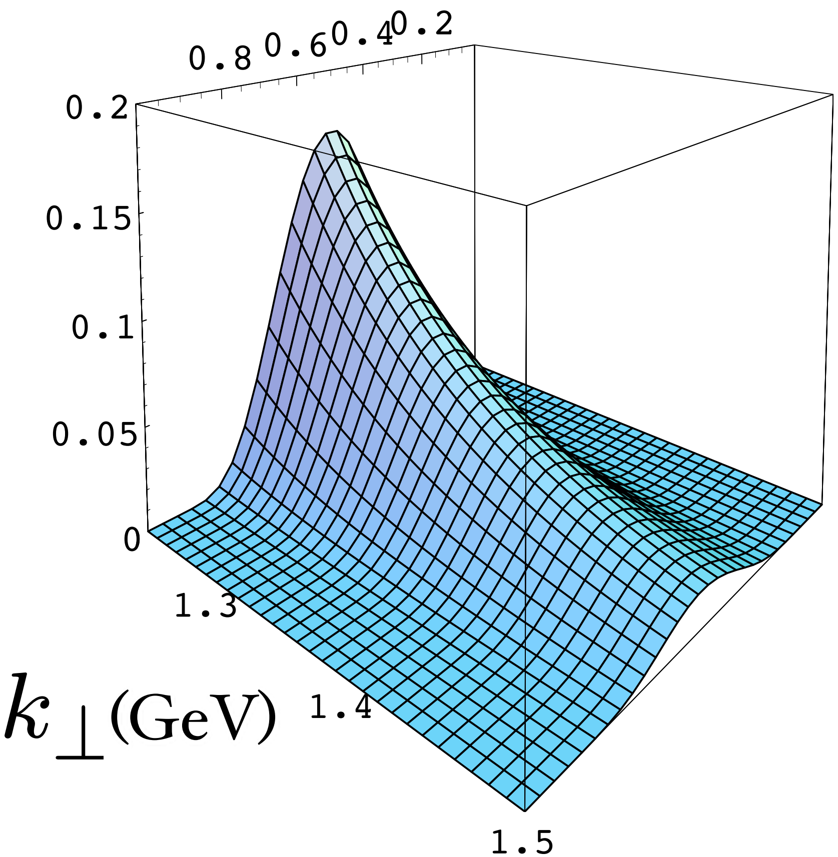
- *Light-Front Holography*

Duality of AdS_5 with LF Hamiltonian Theory

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



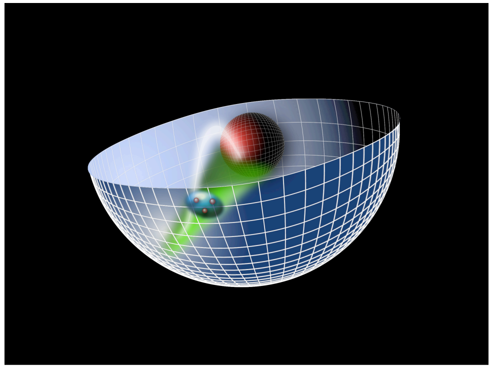
- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation

Spectroscopy and Dynamics

de Teramond, Dosch, sjb

AdS/QCD
Soft-Wall Model



Light-Front Holography

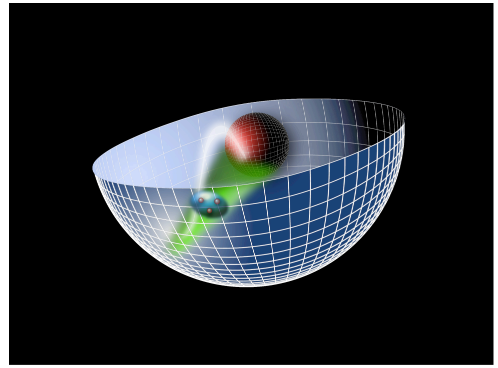
Semi-Classical Approximation to QCD

Relativistic, frame-independent
Unique color-confining potential
Zero mass pion for massless quarks
Regge trajectories with equal slopes in n and L
Light-Front Wavefunctions

Conformal Symmetry
of the action

Light-Front Schrödinger Equation

de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

Preserves Conformal Symmetry
of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

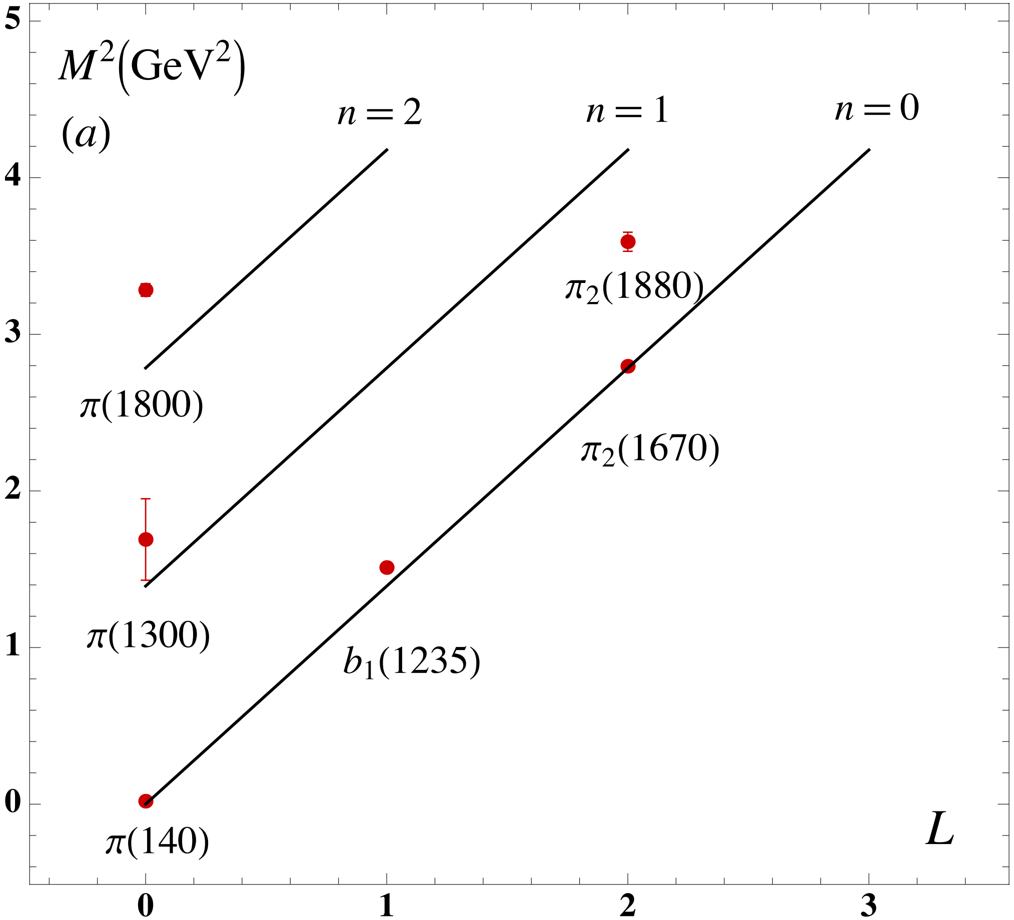
$$1/\kappa \simeq 1/3\ fm$$

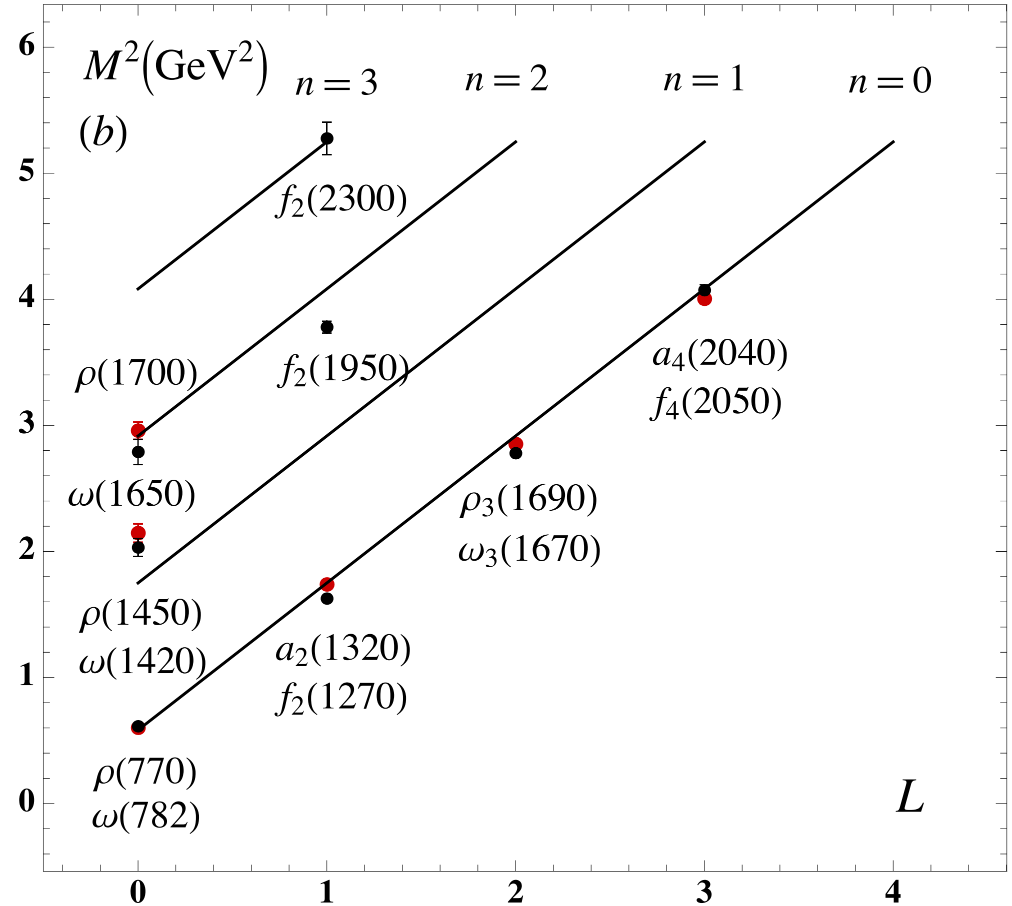
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

$$m_u = m_d = 0$$

Preview





$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

De Teramond, Dosch, sjb

- Results easily extended to light quarks masses (Ex: K-mesons)

- First order perturbation in the quark masses

$$\Delta M^2 = \langle\psi| \sum_a m_a^2/x_a |\psi\rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x,\zeta) \sim \sqrt{x(1-x)}\, e^{-\frac{1}{2\lambda}\left(\frac{m_q^2}{x}+\frac{m_q^2}{1-x}\right)} e^{-\frac{1}{2}\lambda\zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA
  [J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

- For the K^*

$$M^2_{n,L,S} = M^2_{K^\pm} + 4\lambda\left(n + \frac{J+L}{2}\right)$$

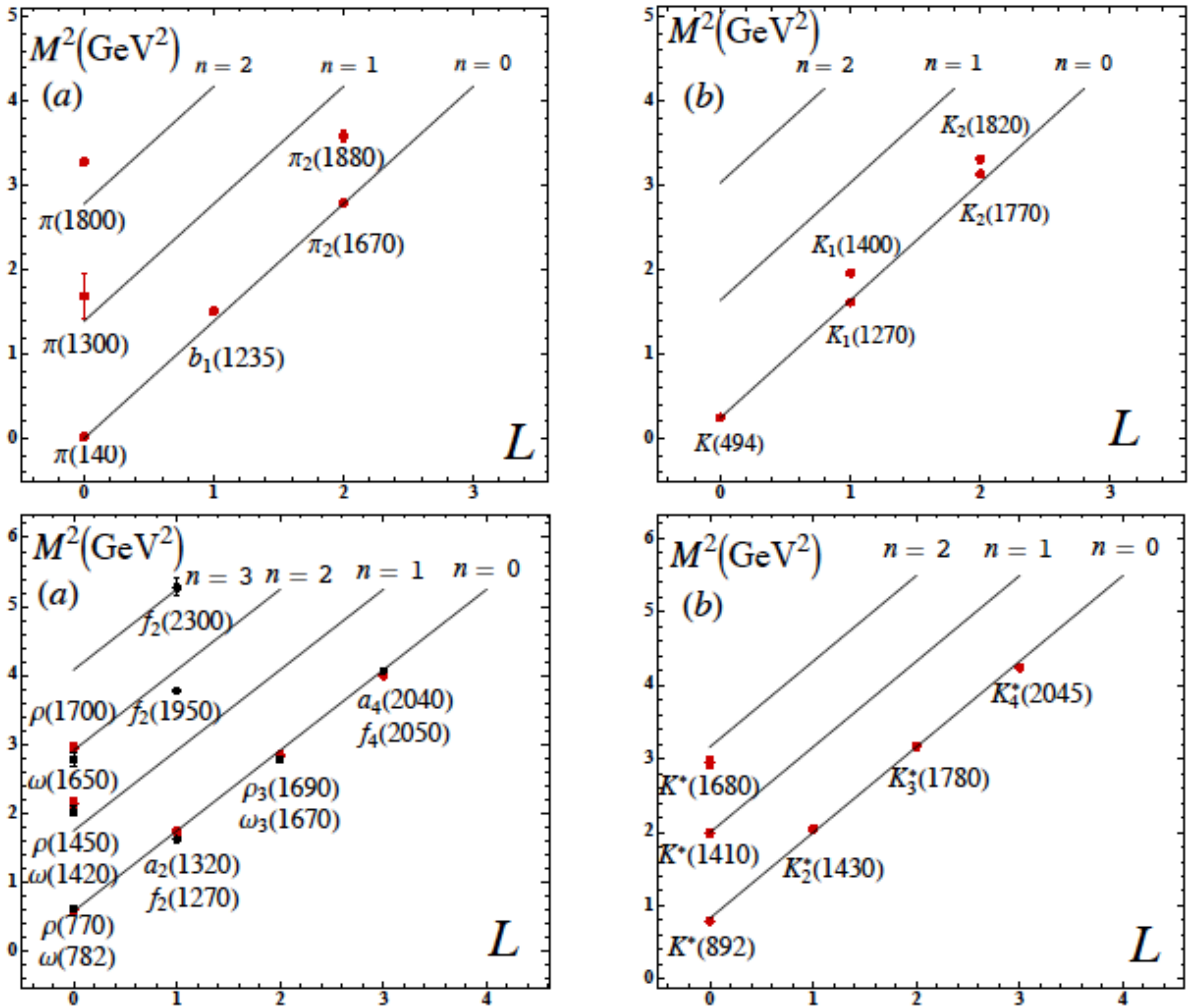
- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

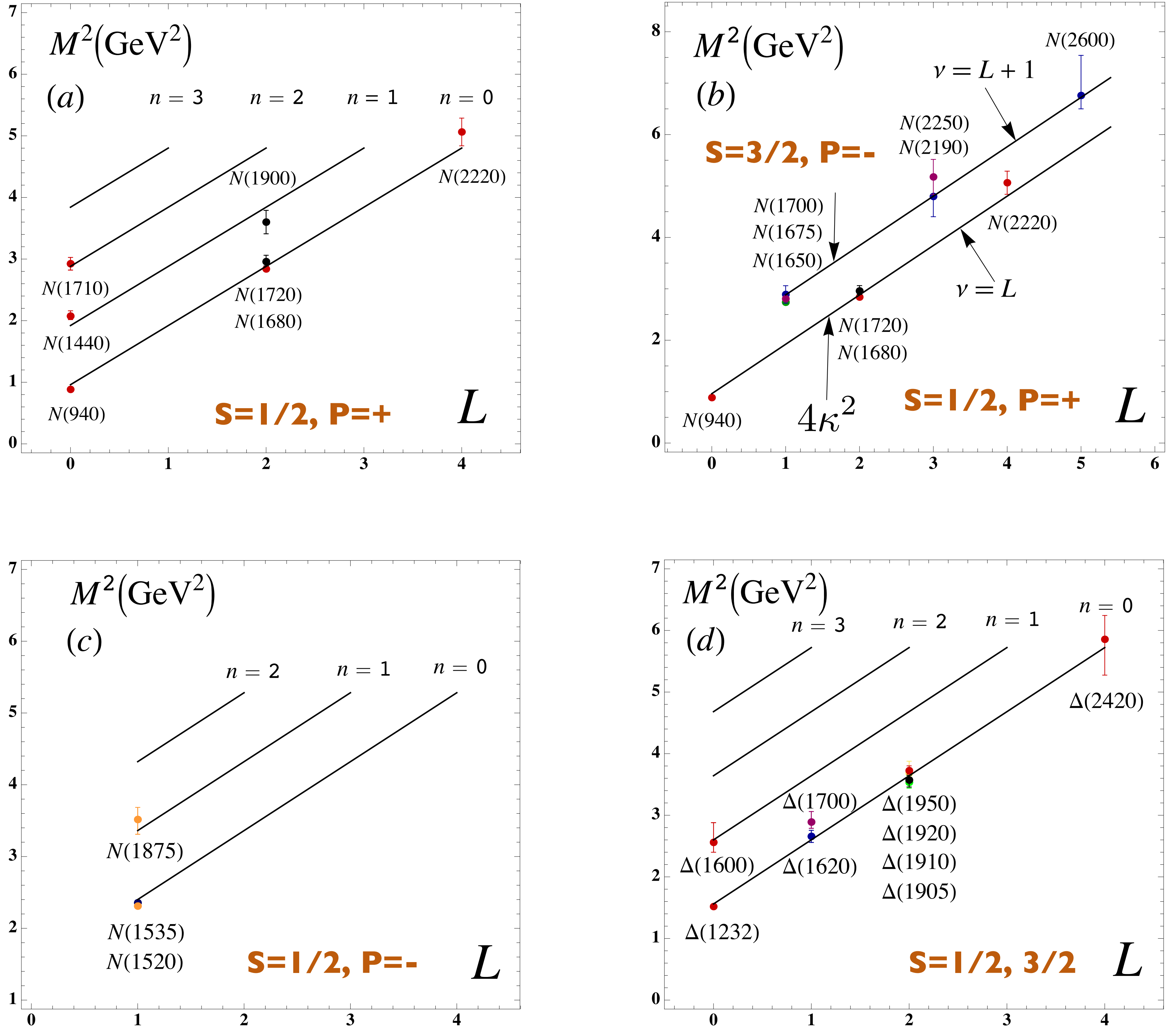
$$m_u = m_d = 46\ \mathrm{MeV}, \quad m_s = 357\ \mathrm{MeV}$$

De Tèramond, Dosch, sjb

$$m_u = m_d = 46\ \mathrm{MeV}, \quad m_s = 357\ \mathrm{MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar q}^2}{1-x} \right| X \right\rangle$$



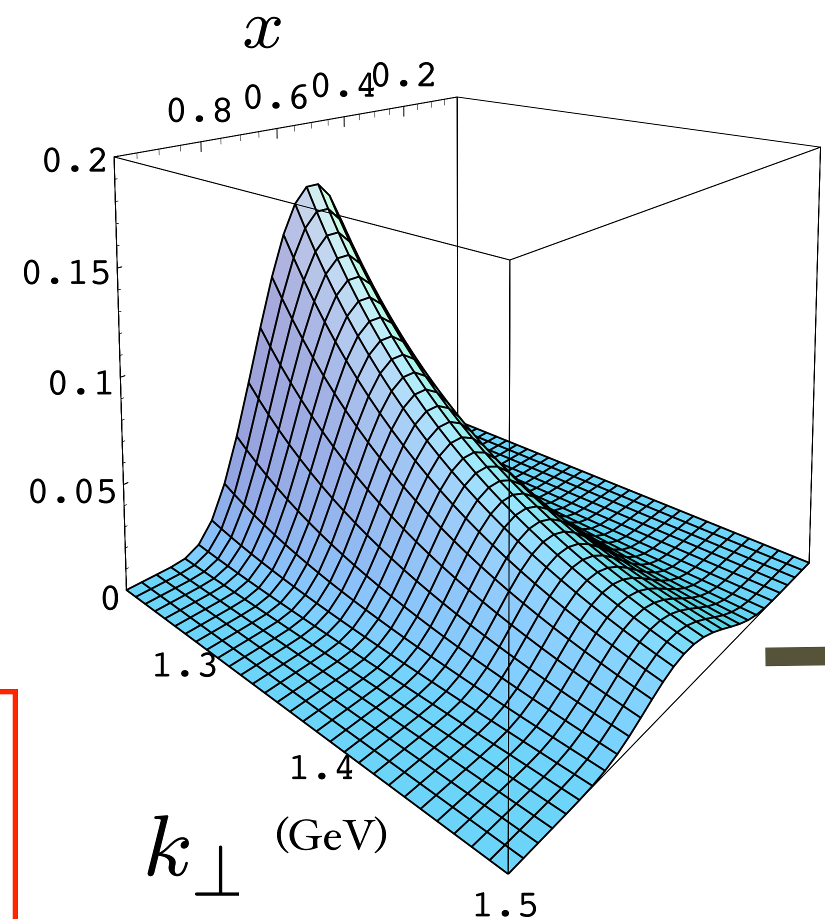


$M^2(\mathrm{GeV}^2)$
(a)
$n=3$
$n=2$
$n=1$
$n=0$
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
$\nu = L+1$
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
$n=2$
$n=1$
$n=0$
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
$n=3$
$n=2$
$n=1$
$n=0$
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

Prediction from AdS/QCD: Meson LFWF

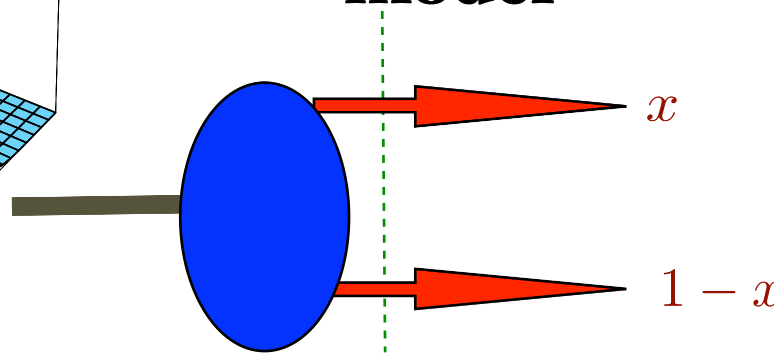
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



Note coupling

$$k_\perp^2, x$$

massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE!

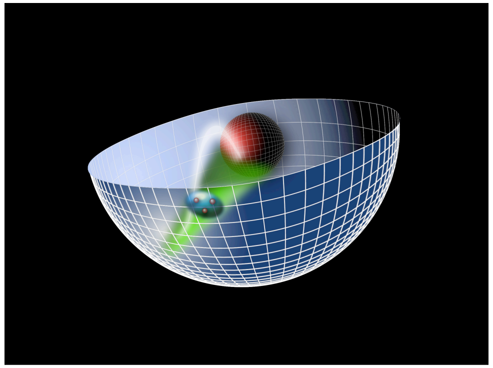
Provides Connection of Confinement to Hadron Structure

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Single scheme-independent
fundamental mass scale



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

Light-Front Holography

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

Preserves Conformal Symmetry
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Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

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$$m_q = 0$$

- de Alfaro, Fubini, Furlan:
- Fubini, Rabinovici:

Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!

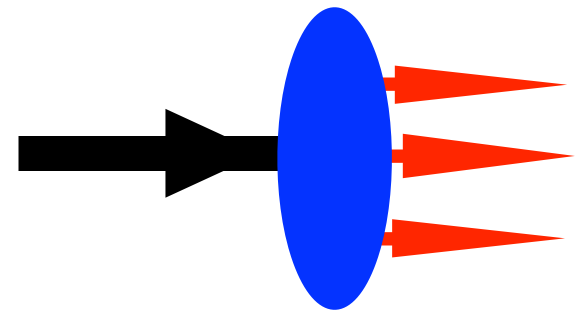
Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons,***
- ***Chiral features for m_q=0: m_π=0, chiral-invariant proton***
- ***Hadronic LFWFs***
- ***Counting Rules***
- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

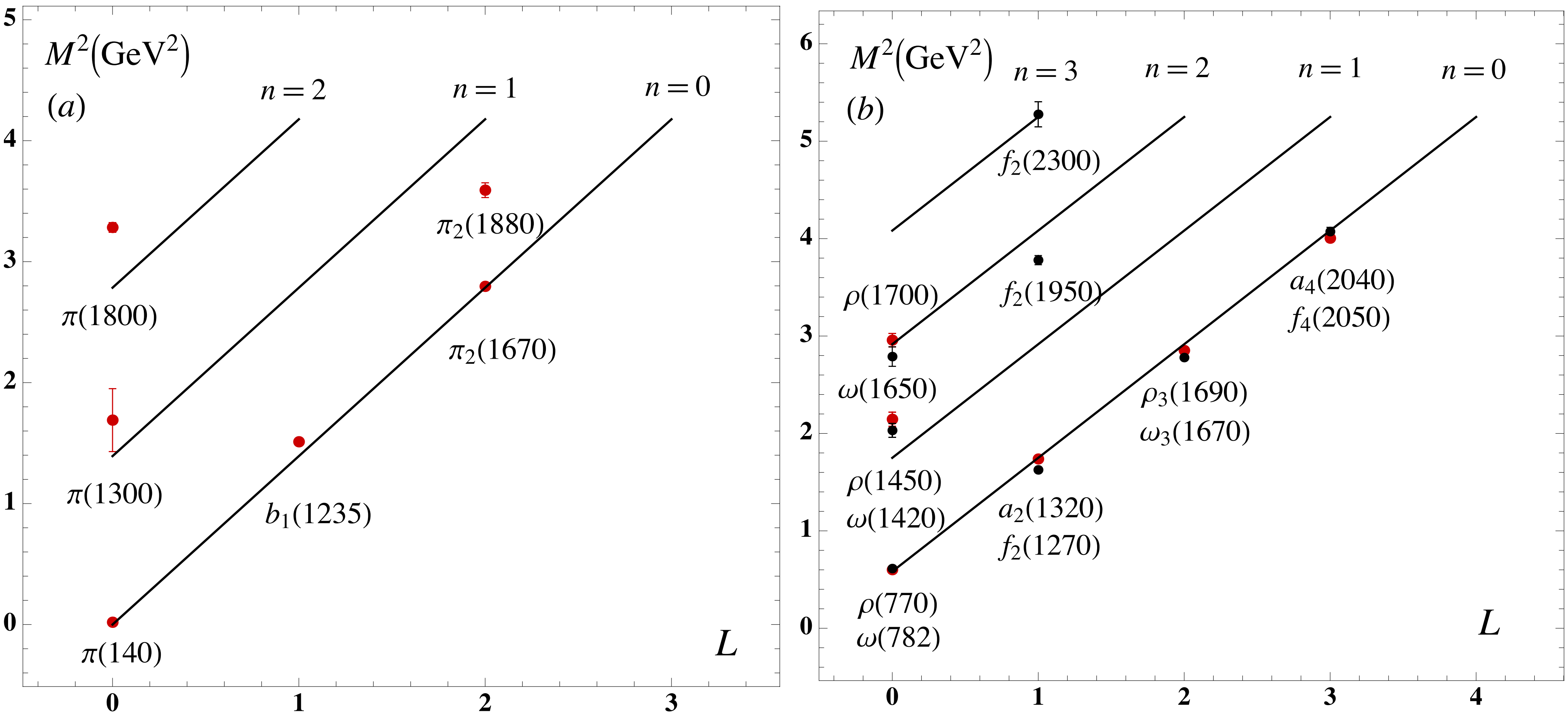
AdS/QCD and Light-Front Holography



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- A first, semi-classical approximation to nonpertubative QCD
- Hadron Spectroscopy and LF Dynamics
- Color Confinement Potential
- Running QCD Coupling $\alpha(Q^2)$ at All Scales Q^2
- What sets the QCD Mass Scale?
- Connection of Hadron Masses to $\Lambda_{\overline{MS}}$

Prediction from AdS/QCD



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

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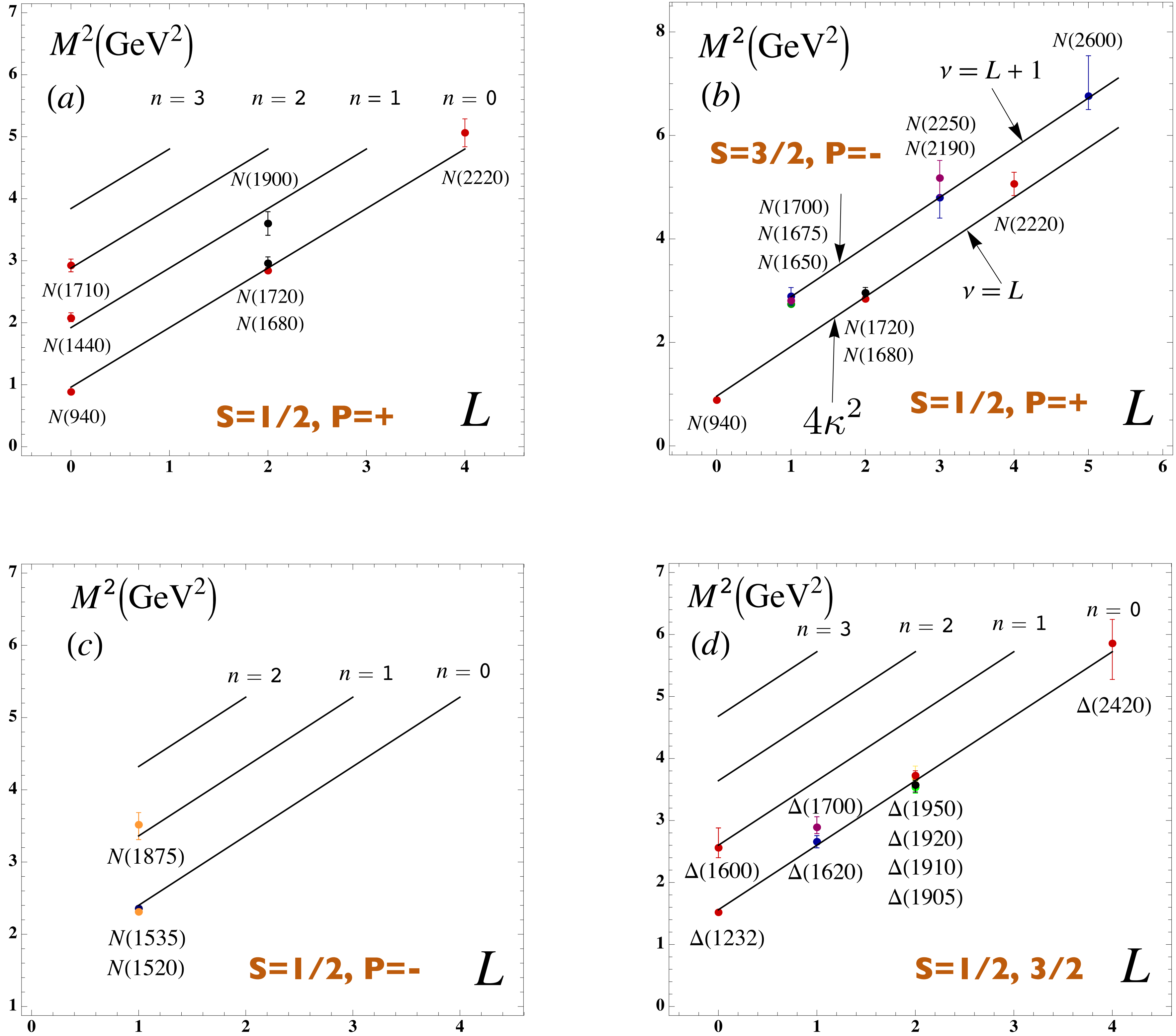

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L
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ν = L + 1
N(2600)
N(2250)
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N(1700)
N(1675)
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ν = L
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N(1680)
N(940)
4κ²
S=1/2, P=+
L
M²(GeV²)
(c)
n = 2
n = 1
n = 0
N(1875)
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De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

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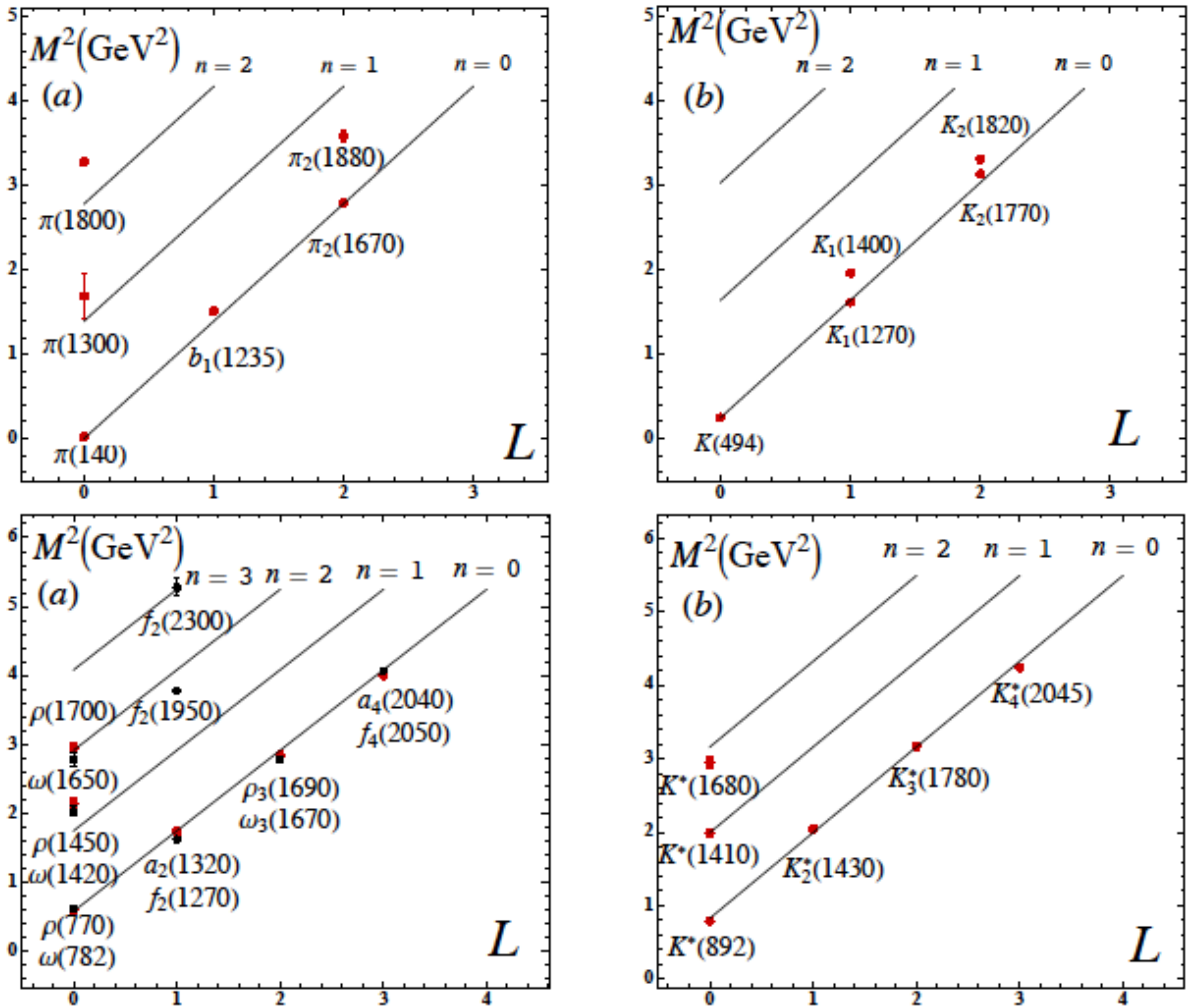


Uniqueness

de Tèramond, Dosch, sjb

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \qquad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**

- **Linear Regge trajectories in n and L: same slope!**

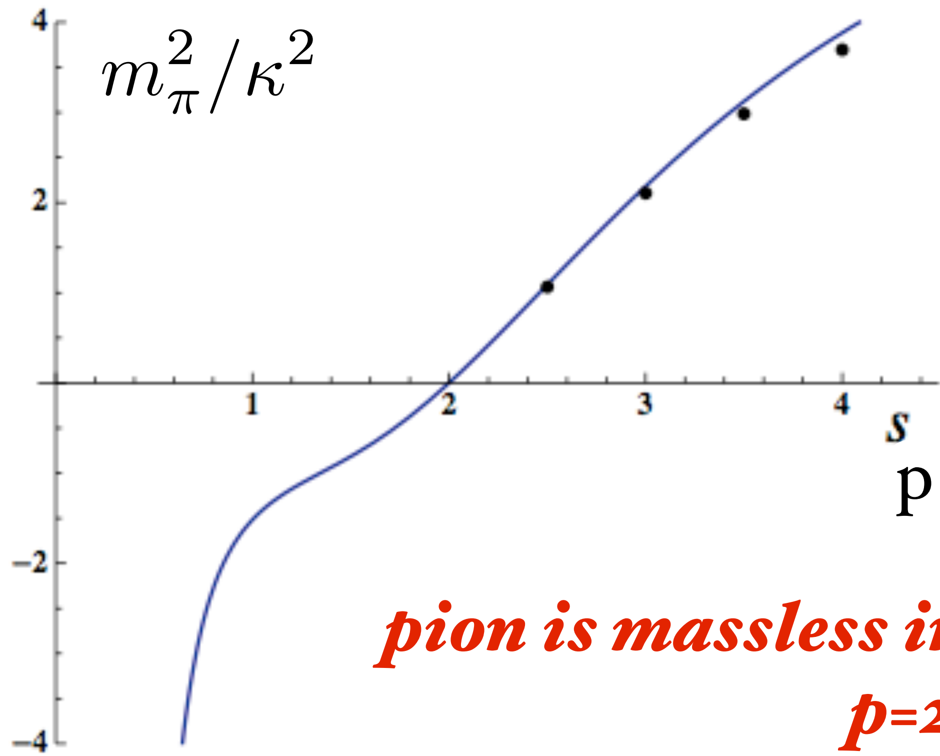
- **Massless pion in chiral limit! No vacuum condensate!**

- **Conformally invariant action for massless quarks retained despite mass scale**

- **Same principle, equation of motion as de Alfaro, Furlan, Fubini, Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976) 569**

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**

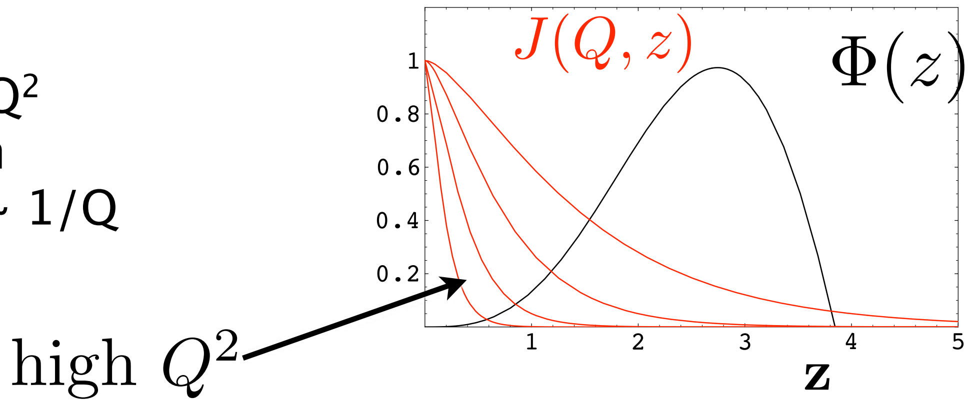
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q,z) \Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$.

$$\text{Twist } \tau = n + L$$

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

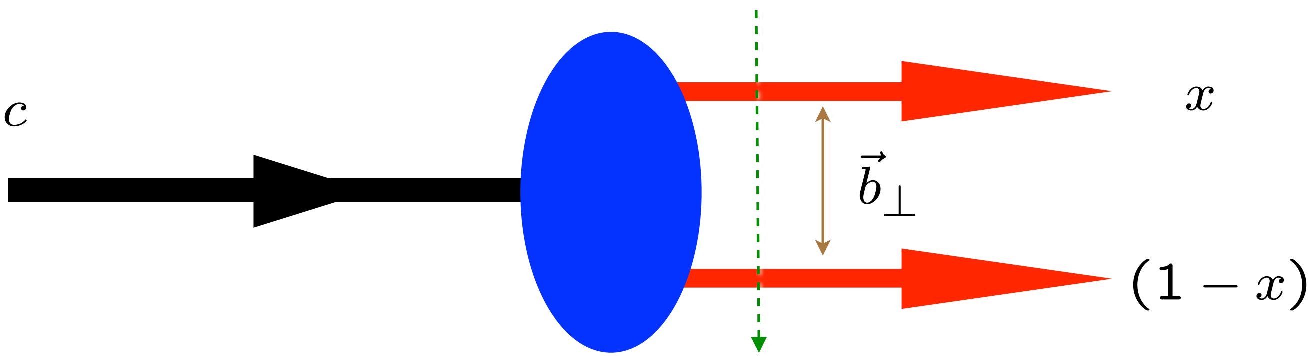
Identical to Polchinski-Strassler Convolution of AdS Amplitudes de Teramond, sjb

$LF(3+1) \longleftrightarrow AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

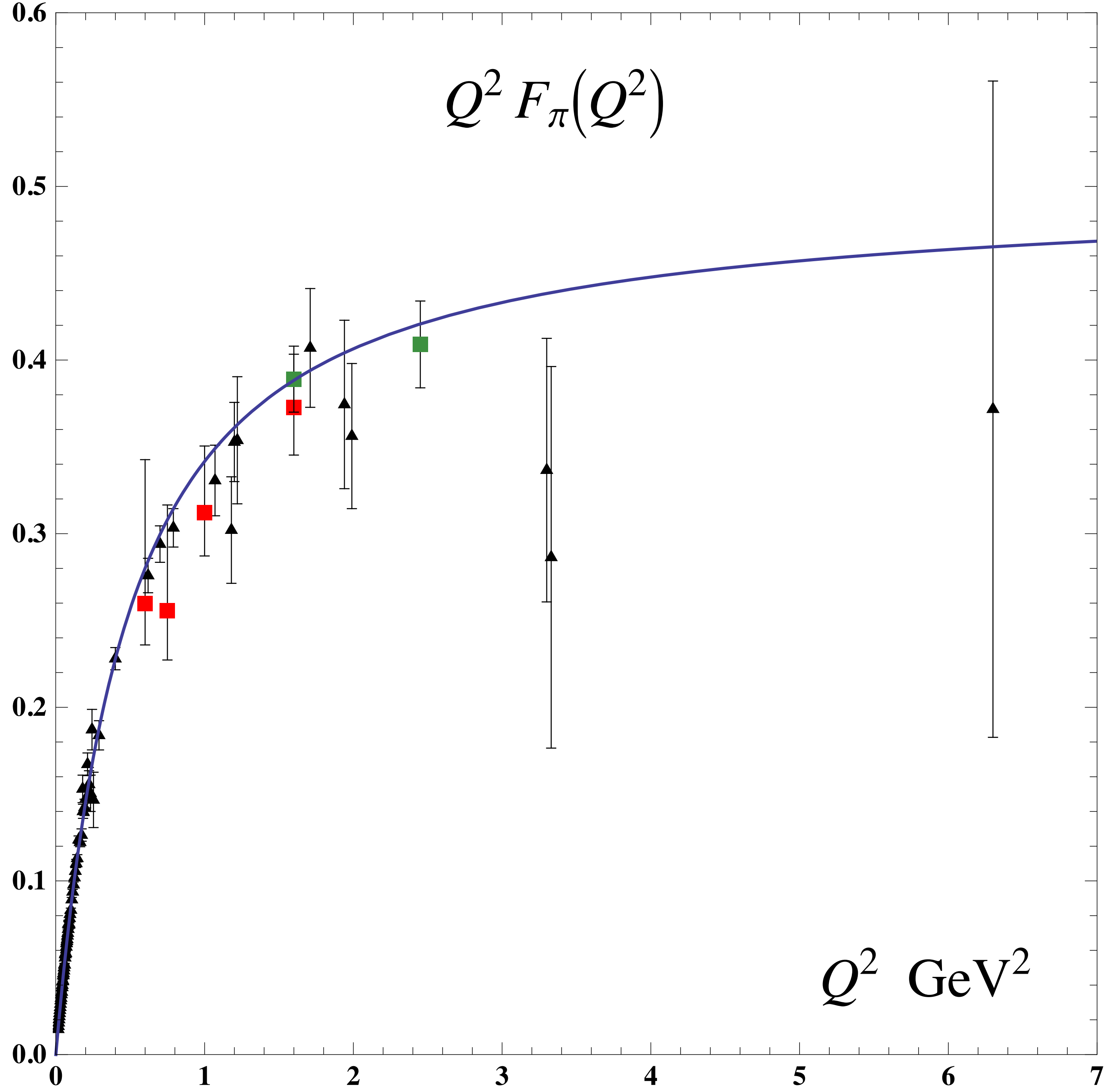
$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

Fixed $\tau = t + z/c$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Current Matrix Elements in AdS Space (SW)

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

Dressed Current in Soft-Wall Model

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

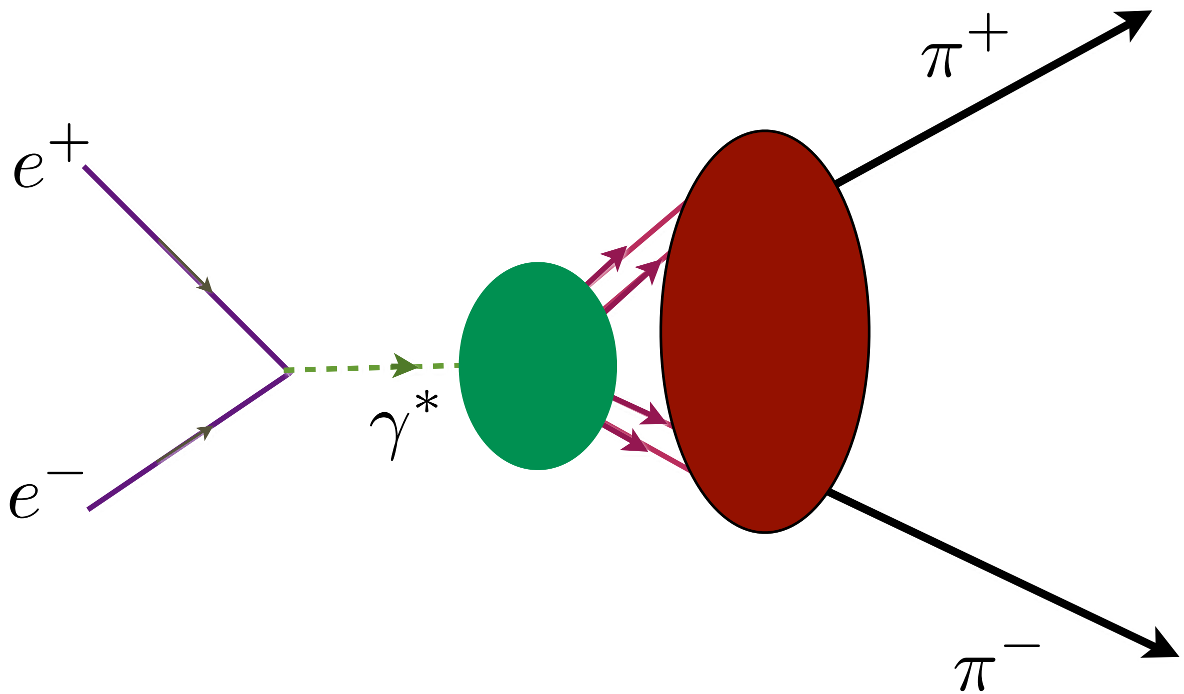
- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

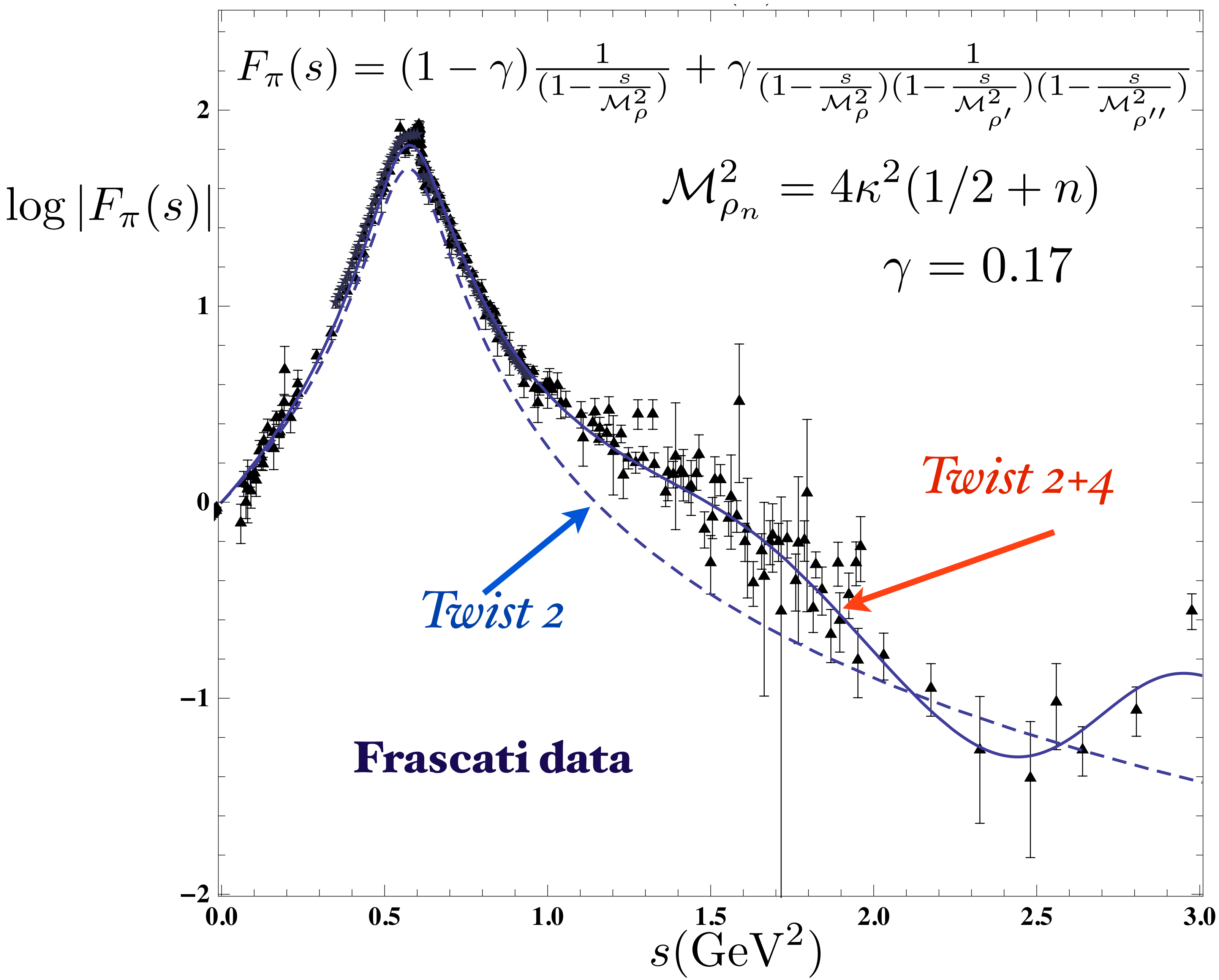
the external current decouples from the dilaton field.

de Tèramond & sjb
Grigoryan and Radyushkin

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



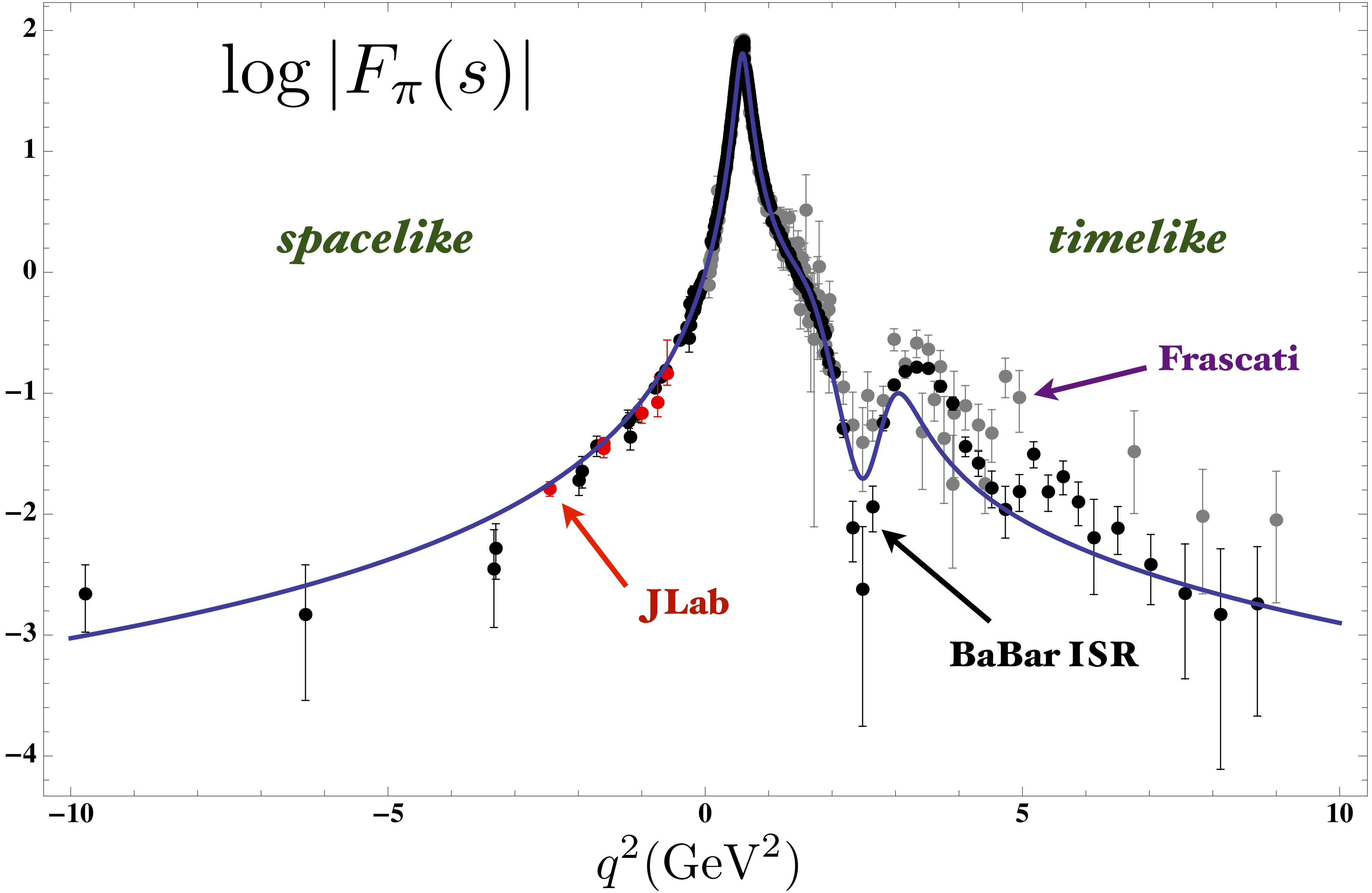
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



Remarkable Features of Light-Front Schrödinger Equation

- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

To Appear in Physics Reports

Light-Front Holographic QCD and Emerging Confinement

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Superconformal Baryon-Meson Symmetry and Light-Front Holographic QCD

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LF Holography

Baryon Equation

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$ G_{22}

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$ G_{11}

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$ G_{11}

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ !

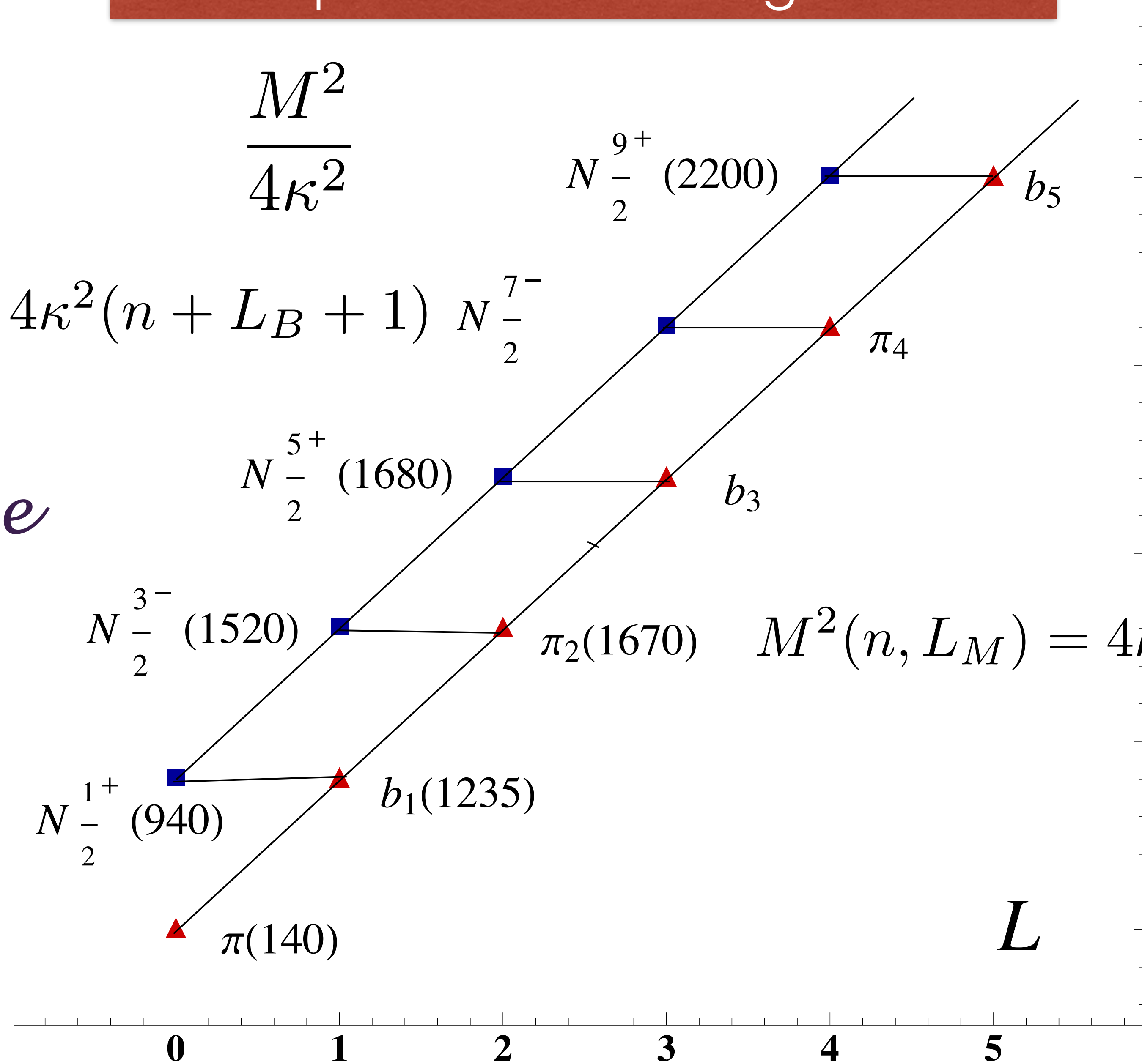
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for L_M=L_B+1

Superconformal Algebra

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

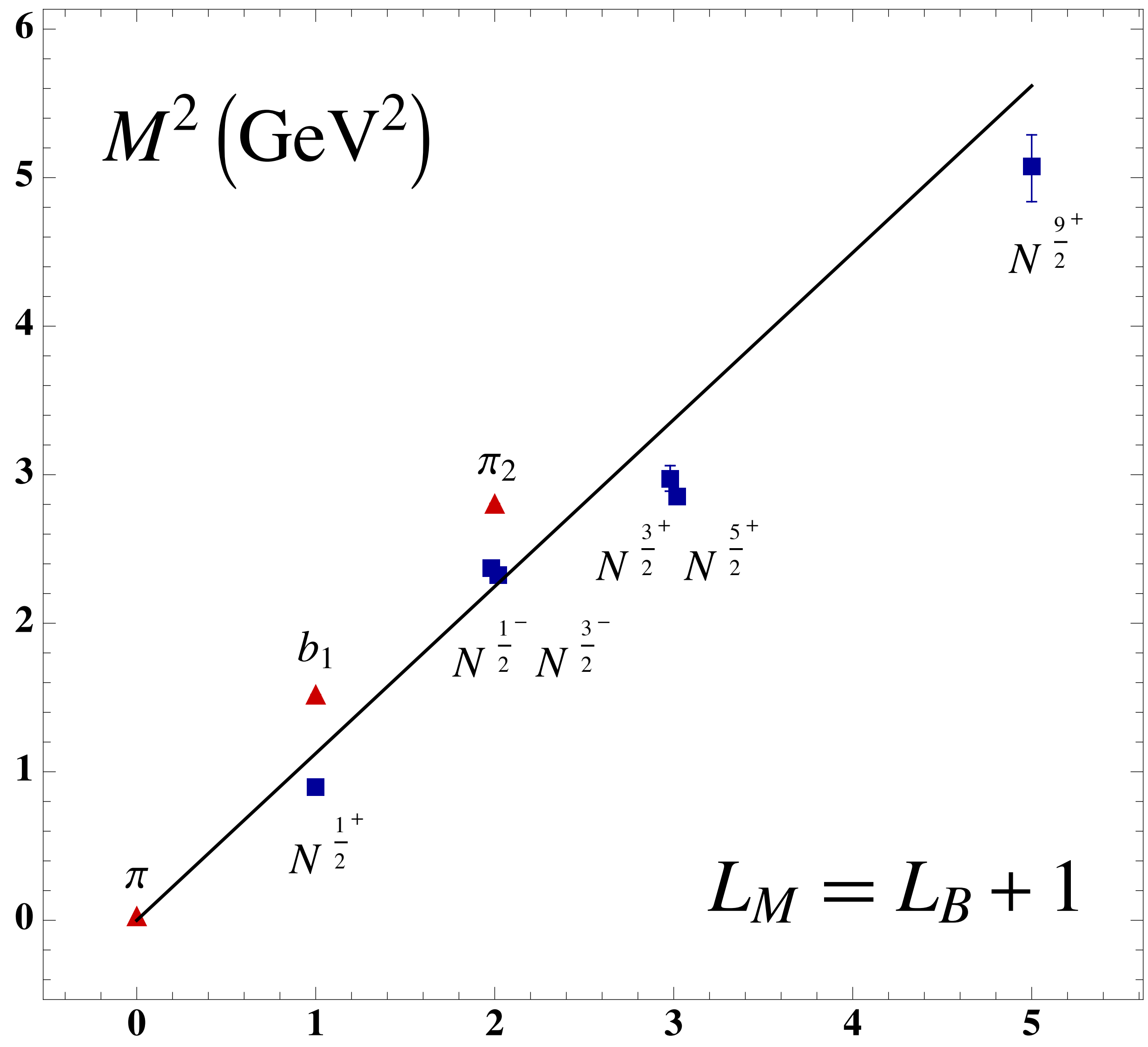


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

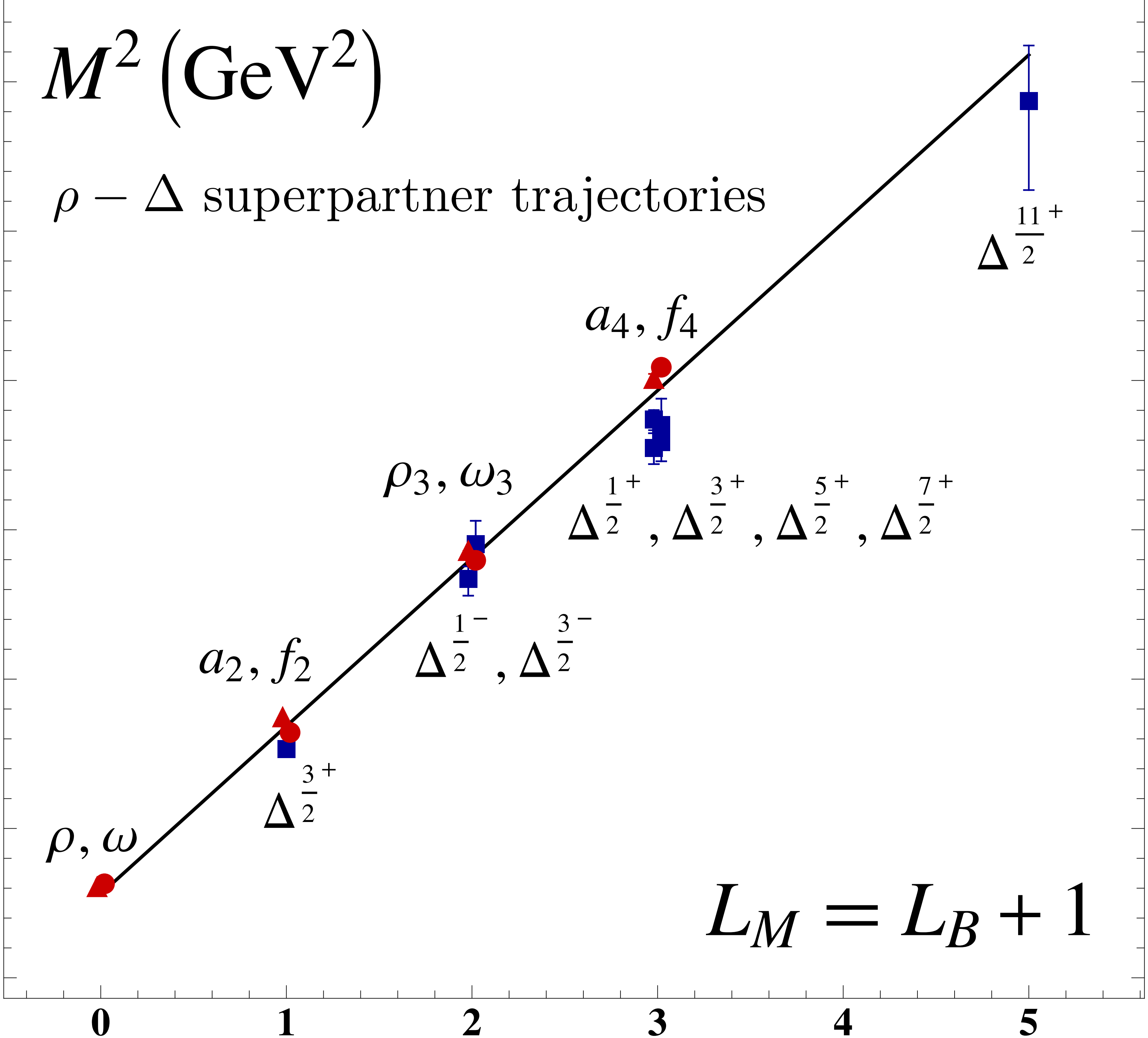
Dosch, de Teramond, sjb

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon



Dosch, de Teramond, sjb

Features of Supersymmetric Equations

- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 for same mass eigenvalue

- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $$S^z = \pm 1/2$$

- Baryon spin carried by quark orbital angular momentum: $\langle J^z \rangle = L^z + 1/2$

- Mass-degenerate meson "superpartner" with $L_M = L_B + 1$. *"Shifted meson-baryon Duality"*

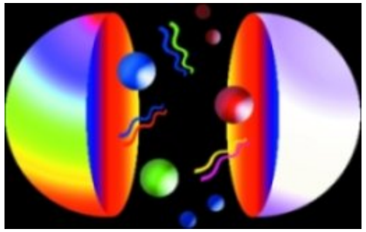
Meson and baryon have same κ !

Counting Rules Obeyed

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensates, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z >= 0$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons,***
- ***Chiral features for m_q=0: m_π =0, chiral-invariant proton***
- ***Hadronic LFWFs : Single dynamical LF radial variable*** ζ
- ***Counting Rules***
- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for L_M=L_B+1

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

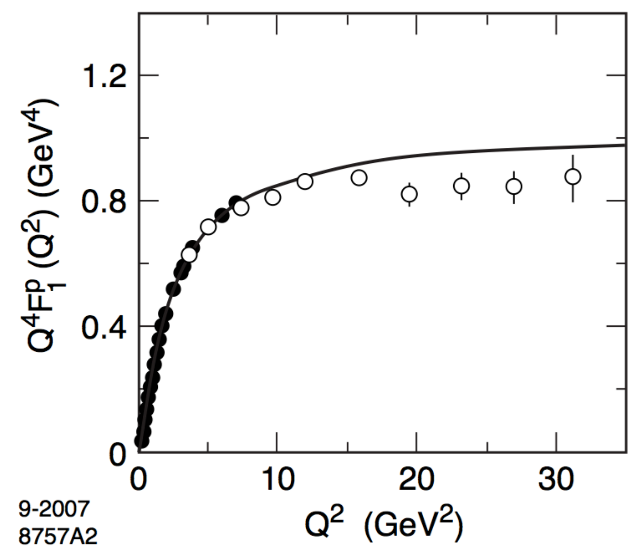
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

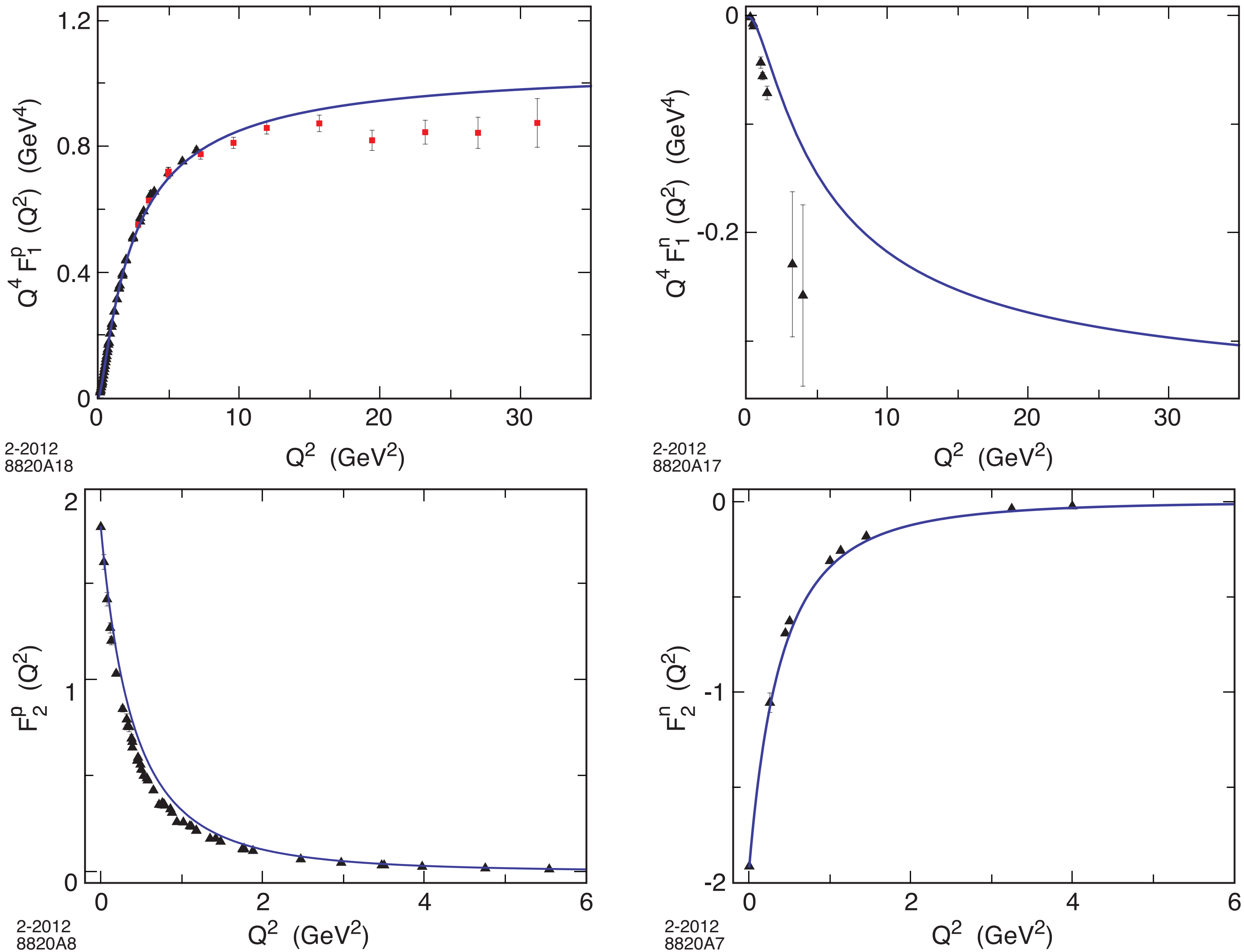
- Find

$$F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n+1/2)$

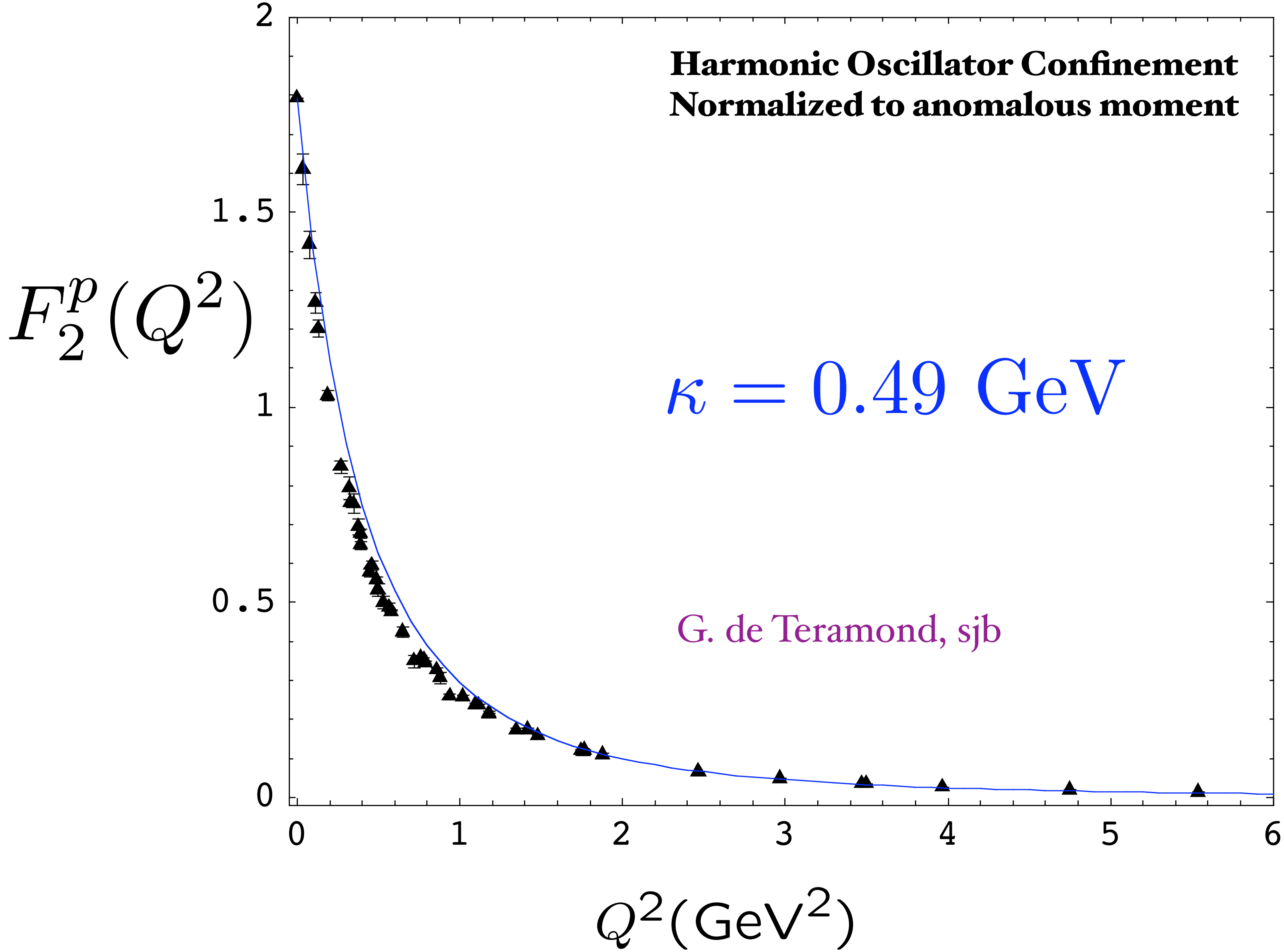


Using $SU(6)$ flavor symmetry and normalization to static quantities

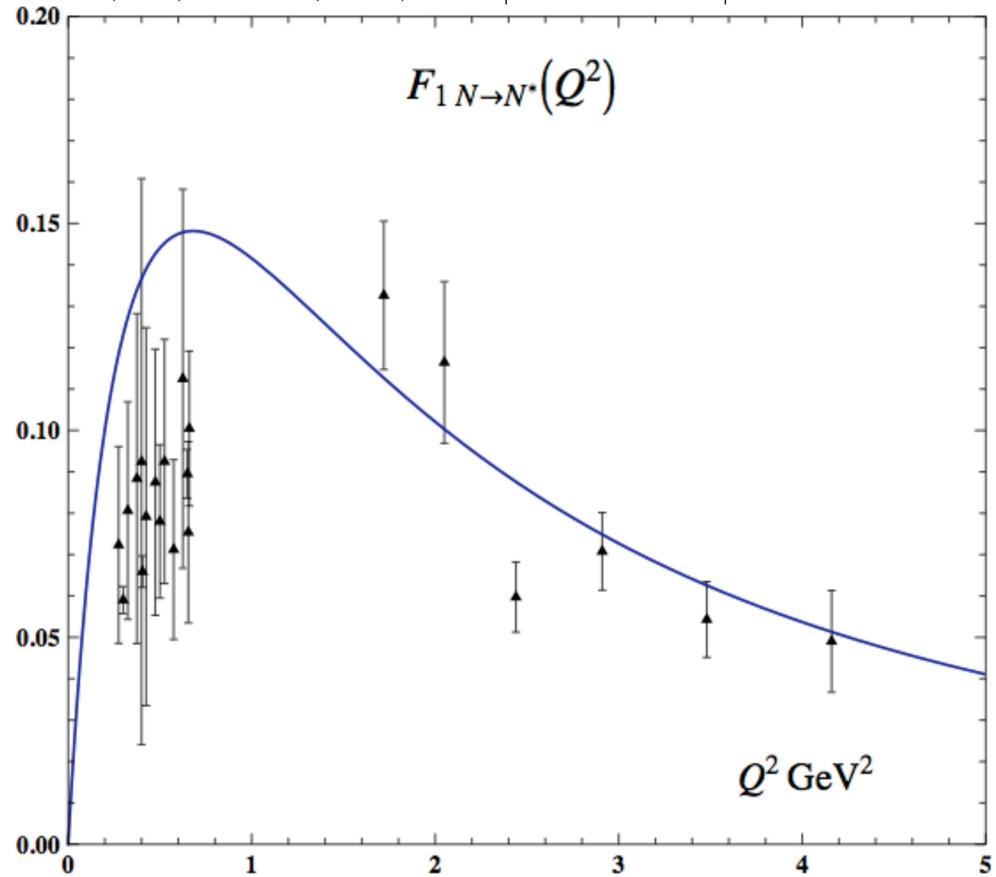


Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



$N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$



Data from I. Aznauryan, *et al.* CLAS (2009)

$$F_{1}{}^{p}_{N\to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n + 1/2)$

Predictions from AdS Holographic QCD

Dosch, Deur, de Teramond, sjb

- Zero-Mass pion for zero quark mass
- Regge Spectroscopy $$M_\pi^2(n, L) = 4\kappa^2(n + L)$$
- Same slope in n, L
- LFWFs, Distribution Amplitudes $$\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$$
- Form Factors, Structure Functions, GPDs
- Non-perturbative running coupling $$\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$$
- Meson-Baryon Supersymmetry for $L_M = L_{B+1}$

$$\lambda = \kappa^2$$

Interpretation of Mass Scale κ

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of κ
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Unique confinement potential!

• **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau) >= i\frac{\partial}{\partial\tau}|\psi(\tau) >$$

New term

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

• **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)

- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$

- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK \quad 4uw - v^2 = \kappa^4 = [M]^4$

- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ

- Only dimensionless mass ratios (and M times R) predicted

- Mass and time units [GeV] and [sec] from physics external to QCD

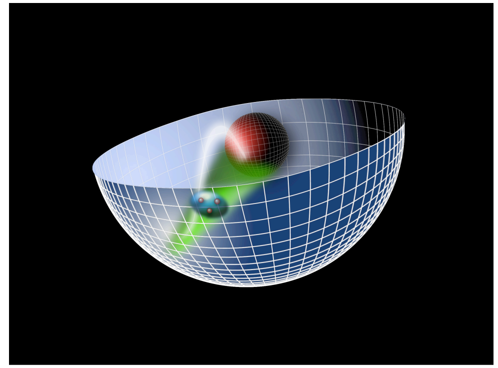
- New feature: bounded frame-independent relative time

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

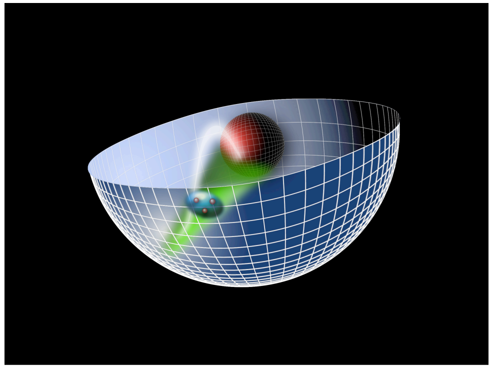
$$\kappa \simeq 0.6 \ GeV$$

$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

de Teramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

Semi-Classical Approximation to QCD

Relativistic, frame-independent
Unique color-confining potential
Zero mass pion for massless quarks
Regge trajectories with equal slopes in n and L
Light-Front Wavefunctions

Conformal Symmetry
of the action

Light-Front Schrödinger Equation

AdS/QCD and Light-Front Holography

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

- **Zero mass pion for m_q =0 (n=J=L=0)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2: Dimensional counting** $[Q^2]^{n-1} F(Q^2) \to \text{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$

Connecting the Hadron Mass Scale to the Fundamental Mass Scale of Quantum Chromodynamics

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Grunberg

Deur, de Teramond, sjb

Bjorken sum rule defines effective charge

$$\alpha_{g1}(Q^2)$$

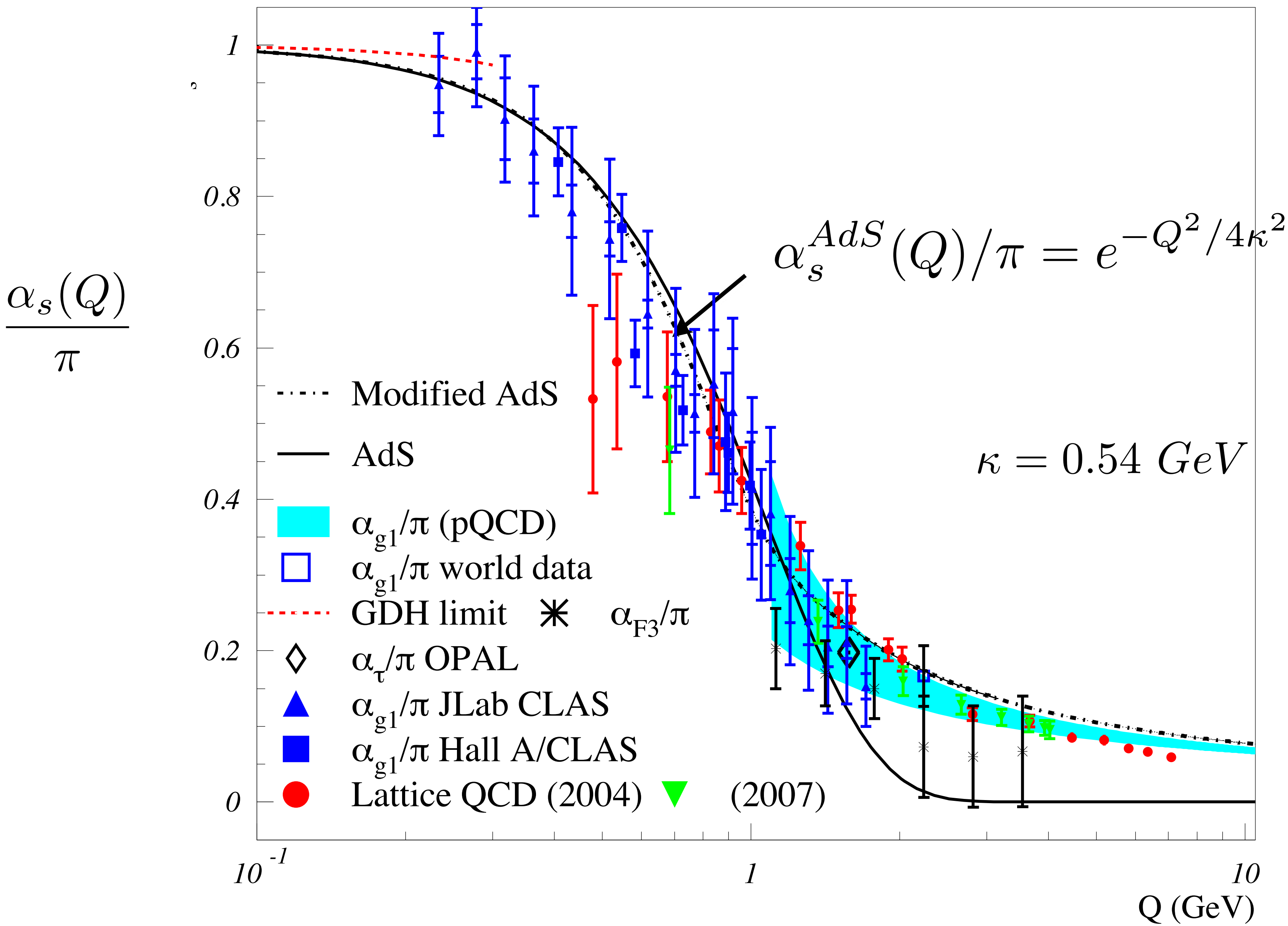
$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

$$\alpha_s^{AdS}(Q)/\pi = e^{-Q^2/4\kappa^2}$$

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

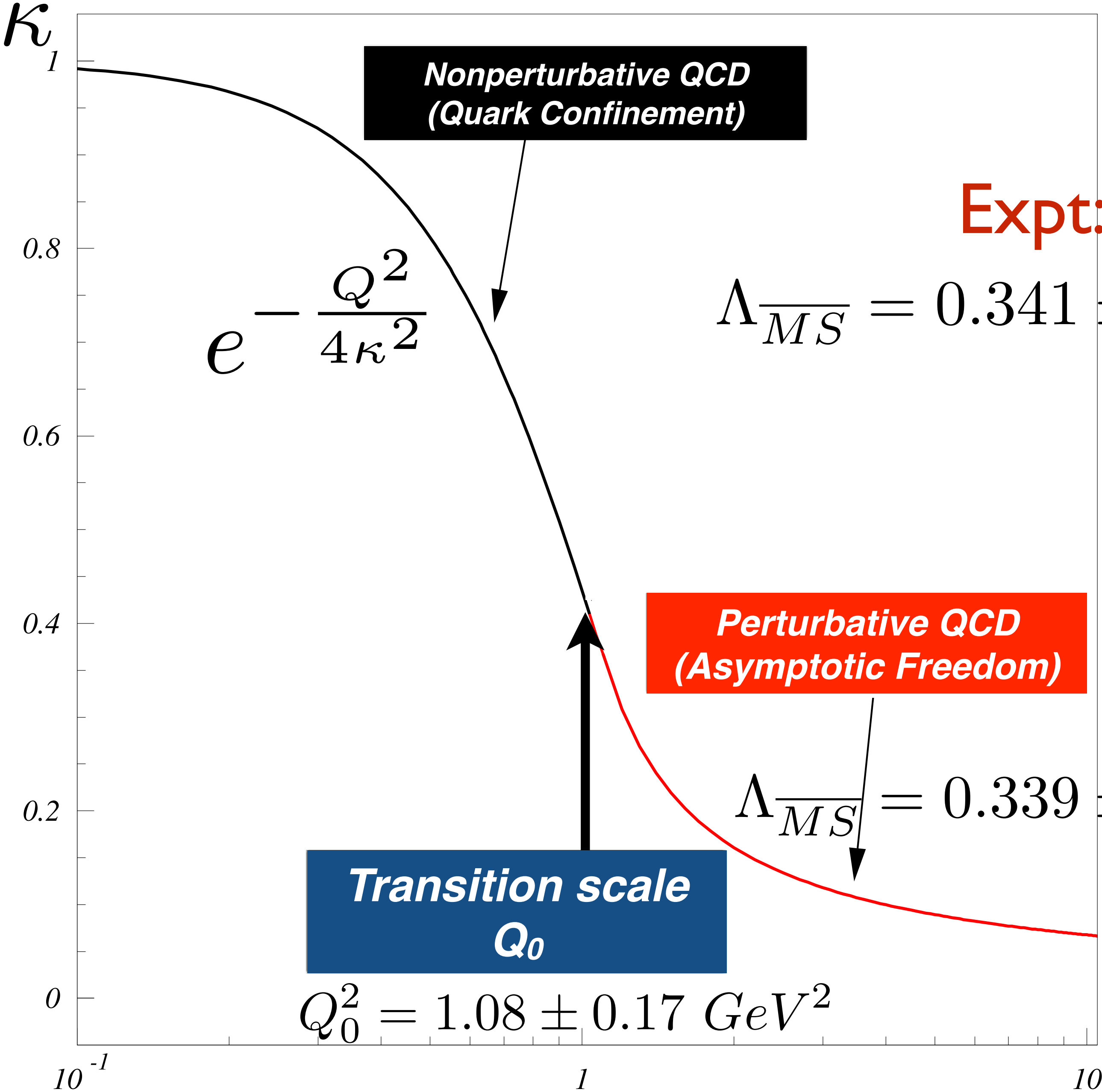
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Tèramond, sjb



Expt:

$$\Lambda_{\overline{MS}} = 0.341 \pm 0.024 \; GeV$$

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016 \; GeV$$

$$Q_0^2 = 1.08 \pm 0.17 \; GeV^2$$

$$\lambda \equiv \kappa^2$$

Connect npQCD to pQCD!

Tests of AdS/QCD and LF Holography at JLab 12 GeV

- **Compare Spacelike-Transition Form Factors, Counting Rules**

$$F_{\pi \to b_1}(Q^2) \quad \textbf{vs.} \quad F_{p \to N^*}(Q^2)$$

- **Supersymmetric QCD Relations: Spectra, Dynamics**
- **Baryons: q + diquark:** $[q]_{3C}[qq]_{\bar{3}C}$
- **Pentaquarks: diquark-antidiquark?:** $[qq]_{\bar{3}C}[\bar{q}\bar{q}]_{3C}$

New Directions: AdS/QCD and LF Holography

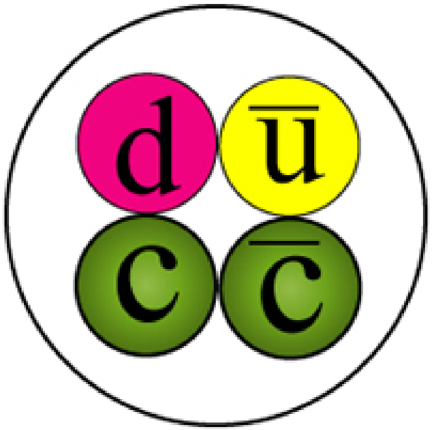
- **Hadronization at Amplitude Level: Calculate Fragmentation Functions from LFWFs**

- **Higher-Fock States of Proton: Intrinsic Heavy Quarks; s(x) vs. $\bar{s}$(x) asymmetry**

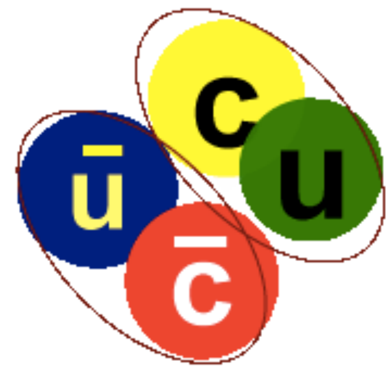
- **Hidden Color of Deuteron**

- **Predict Spectrum of Tetraquarks, Exotic Hadrons**

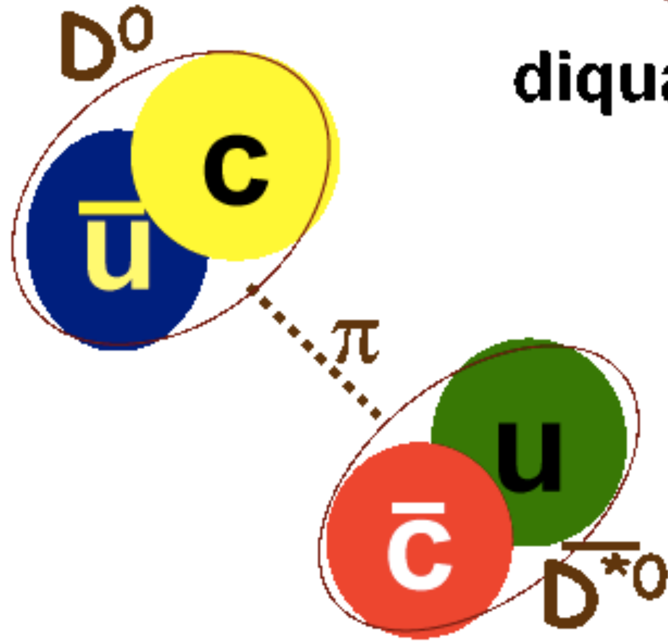
$$pA \to \text{Jet } [\text{Jet Jet}]\ A' \to pA \to \text{Jet Jet Jet } A'$$



$[\bar{u}\bar{c}]_{3C}[uc]_{\bar{3}C}$ diquarks



diquark-diantiquark



$D^0 - \overline{D^{*0}}$ "molecule"

Four-Quark Hadrons: an Updated Review

arXiv:1411.5997v2

A. ESPOSITOA, L. GUERRIERI, F. PICCININI, A. PILLONI and A. POLOSA

State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)
$X(3823)$	3823.1 ± 1.9	< 24	$?^{?-}$	$B \to K(\chi_{c1}\gamma)$	Belle[23] (4.0)
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle[24,25] (>10), *BABAR*[26] (8.6)
				$p\bar{p} \to (\pi^+\pi^- J/\psi)\ldots$	CDF[27,28] (11.6), D0[29] (5.2)
				$pp \to (\pi^+\pi^- J/\psi)\ldots$	LHCb[30,31] (np)
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle[32] (4.3), *BABAR*[33] (4.0)
				$B \to K(\gamma\, J/\psi)$	Belle[34] (5.5), *BABAR*[35] (3.5)
					LHCb[36] (> 10)
				$B \to K(\gamma\, \psi(2S))$	*BABAR*[35] (3.6), Belle[34] (0.2)
					LHCb[36] (4.4)
				$B \to K(D\bar{D}^*)$	Belle[37] (6.4), *BABAR*[38] (4.9)
$Z_c(3900)^+$	3888.7 ± 3.4	35 ± 7	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III[39] (np)
				$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III[40] (8), Belle[41] (5.2)
					CLEO data[42] (>5)
$Z_c(4020)^+$	4023.9 ± 2.4	10 ± 6	1^{+-}	$Y(4260) \to \pi^-(\pi^+ h_c)$	BES III[43] (8.9)
				$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III[44] (10)
$Y(3915)$	3918.4 ± 1.9	20 ± 5	0^{++}	$B \to K(\omega J/\psi)$	Belle[45] (8), *BABAR*[33,46] (19)
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle[47] (7.7), *BABAR*[48] (7.6)
$Z(3930)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle[49] (5.3), *BABAR*[50] (5.8)
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle[51,52] (6)
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle[41,53] (7.4)
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle[54] (5.0), *BABAR*[55] (1.1)
$Y(4140)$	4145.6 ± 3.6	14.3 ± 5.9	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF[56,57] (5.0), Belle[58] (1.9),
					LHCb[59] (1.4), CMS[60] (>5)
					D∅[61] (3.1)
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle[52] (5.5)
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle[62] (7.2)

Belle, BaBar:

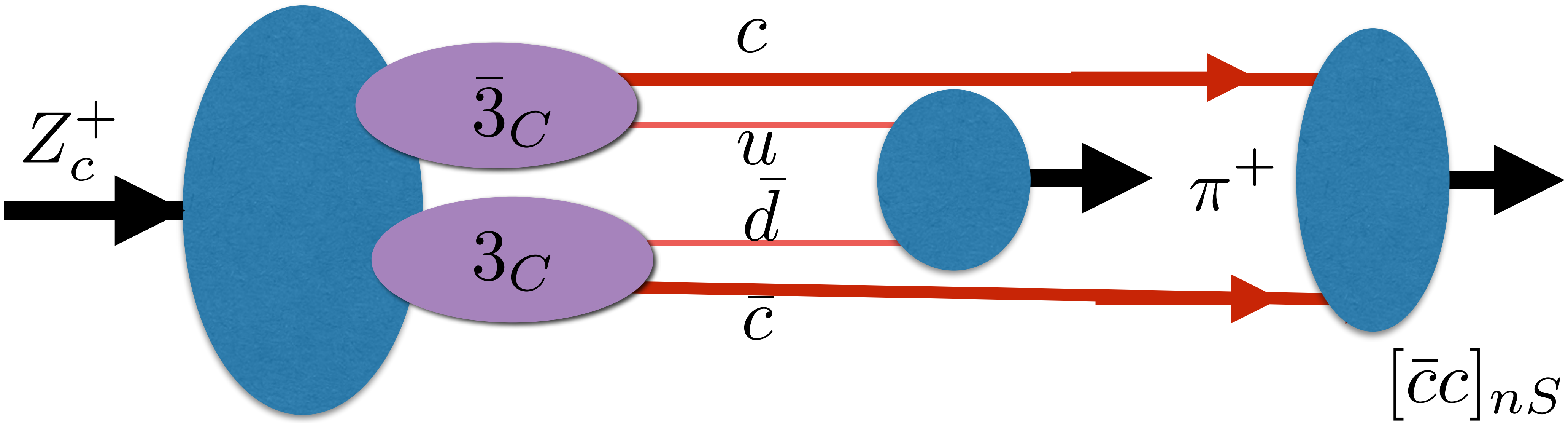
$$\mathcal{B}\left(B^0 \to K^+ Z(4430)^-\right) \times \mathcal{B}\left(Z(4430)^- \to \psi(2S)\pi^-\right) = \left(6.0^{+1.7+2.5}_{-2.0-1.4}\right) \times 10^{-5}.$$

$$\mathcal{B}\left(B^0 \to K^+ Z(4430)^-\right) \times \mathcal{B}\left(Z(4430)^- \to J/\psi\,\pi^-\right) = \left(5.4^{+4.0+1.1}_{-1.0-0.6}\right) \times 10^{-6}.$$

Surprising Result:

Dominance of large size $\psi'(2S)$ vs. J/ψ decays

Diquark Anti-diquark Model



$$Z_c^+([cu]_{\bar{3}C}[\bar{c}\bar{d}]_{3C}) \to \pi^+\psi'$$

Formation of charmonium at large separation:

Dominance of overlap with large-size Ψ' vs J/Ψ decays

Lebed, Hwang, sjb

JLab 12 GeV: An Exotic Charm Factory!

$$\gamma^* p \to J/\psi + p \text{ threshold}$$

$$\text{at } \sqrt{s} \simeq 4 \text{ GeV}, \; E^{\gamma^*}_{\text{lab}} \simeq 7.5 \text{ GeV}.$$

$$\gamma^* p \to X(3872) + p'$$

$|c\bar{c}q\bar{q}>$ *tetraquark*

Produce $[J/\psi + p]$ bound state

$|uudc\bar{c}>$ *pentaquark*

$$\gamma^* d \to J/\psi + d \text{ threshold}$$

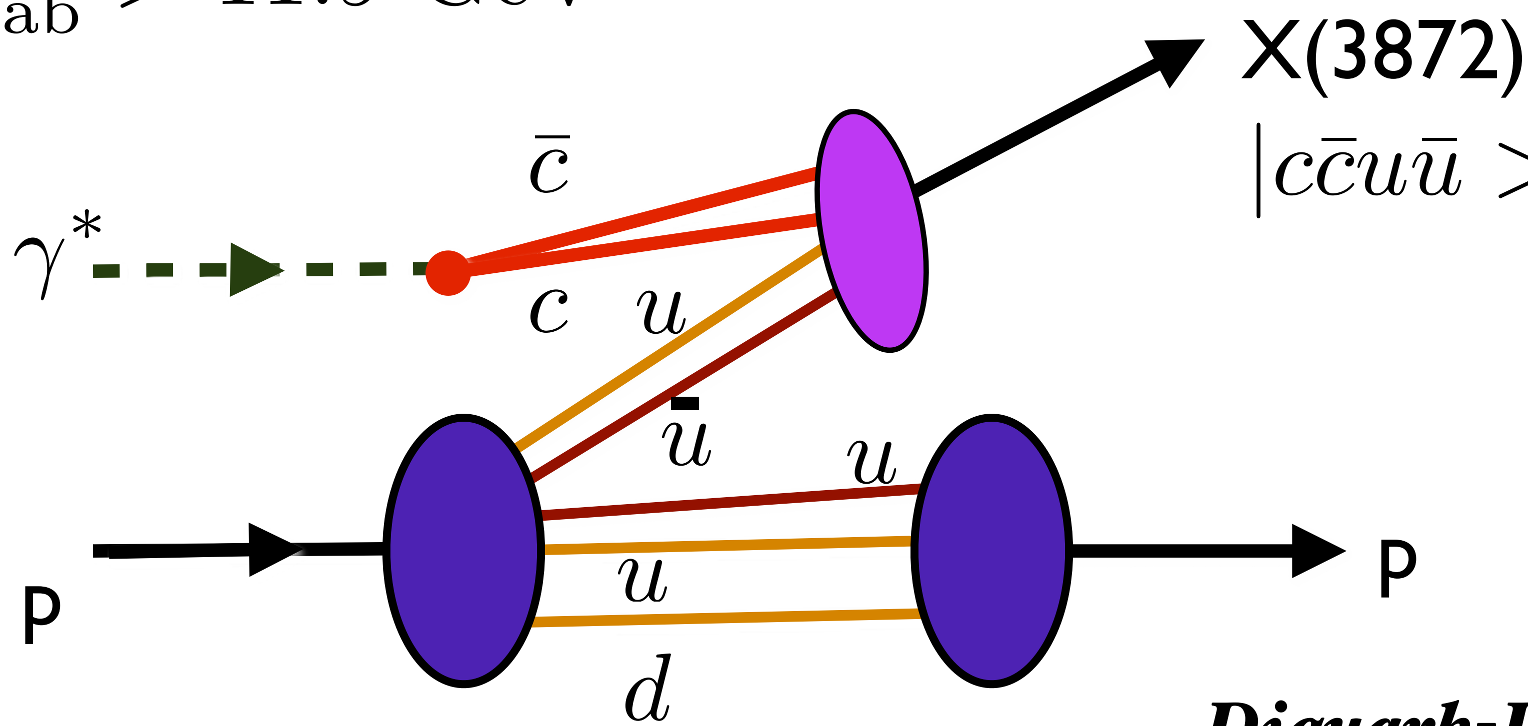
$$\text{at } \sqrt{s} \simeq 5 \text{ GeV}, \; E^{\gamma^*}_{\text{lab}} \simeq 6 \text{ GeV}.$$

Produce $[J/\psi + d]$ nuclear-bound quarkonium state

$|uuddduc\bar{c}>$ *octoquark!*

Tetraquark Production at Threshold

$$E^{\gamma}_{\text{lab}} > 11.9 \text{ GeV}$$



Diquark-Diquark vs Molecular State?

$$\gamma^* p \to X(3872) + p'$$

$$|c\bar{c}q\bar{q}>$$

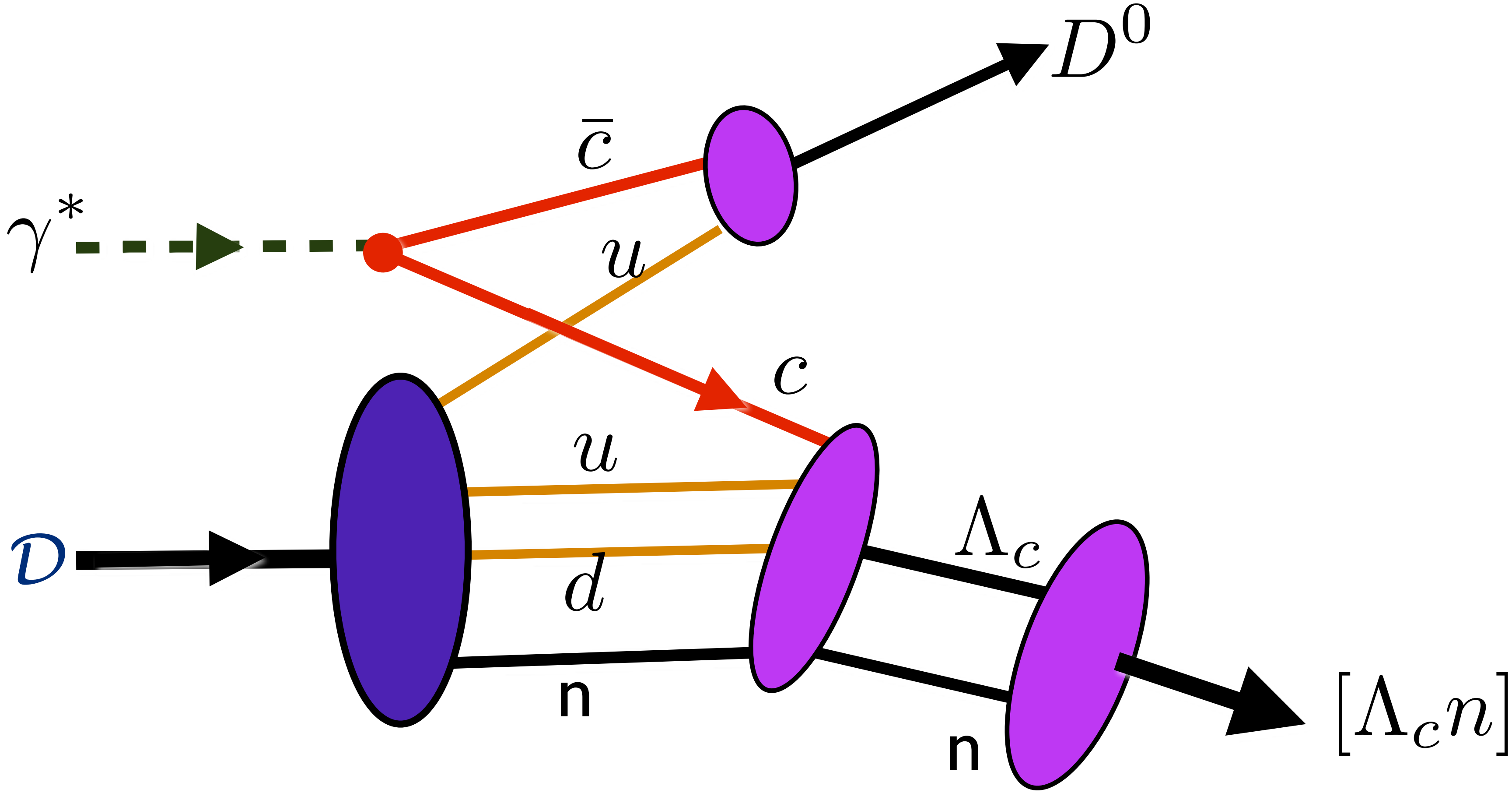
New approach to hadronic decays

Dominance of Ψ' vs J/Ψ decays

Lebed, Hwang, sjb

Open Charm Production at Threshold

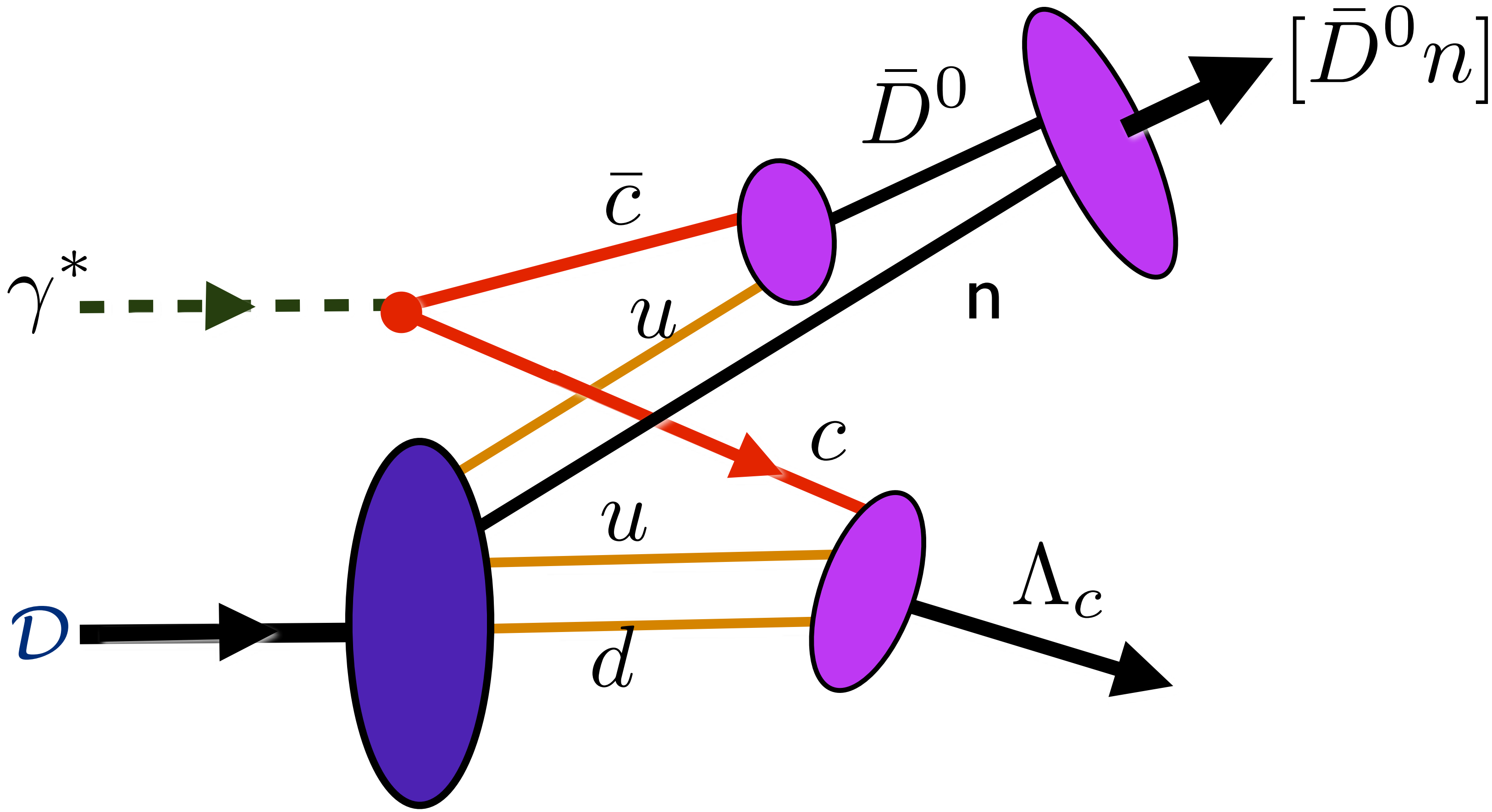
Nuclear binding at low relative velocity



$$\gamma^* d \to \overline{D}^0(\bar{c}u)[\Lambda_c n](cududd)$$

Possible charmed B= 2 nucleus

Open Charm Production at Threshold

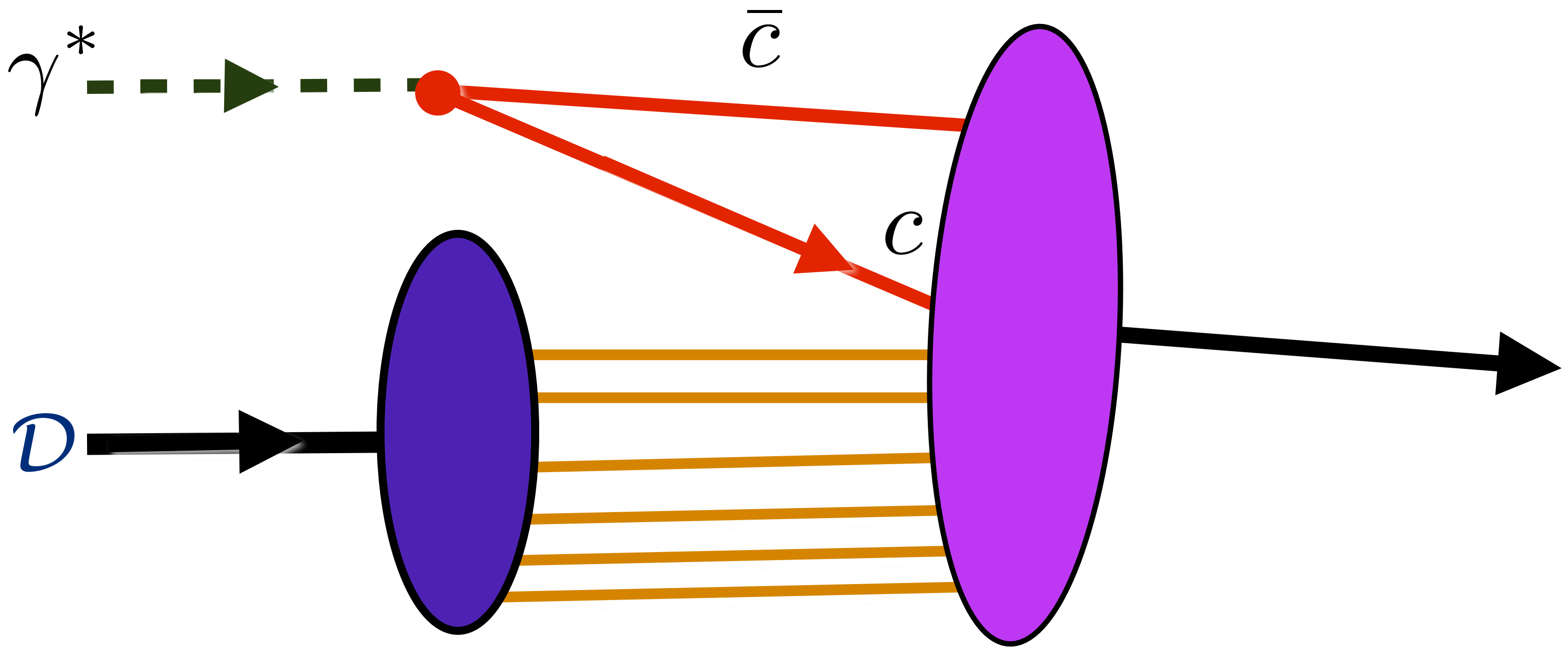


$$\gamma^* d \to \Lambda_c + [\overline{D}^0(\bar{c}u)n](\bar{c}uudd)$$

Create pentaquark on deuteron at low relative velocity

Octoquark Production at Threshold

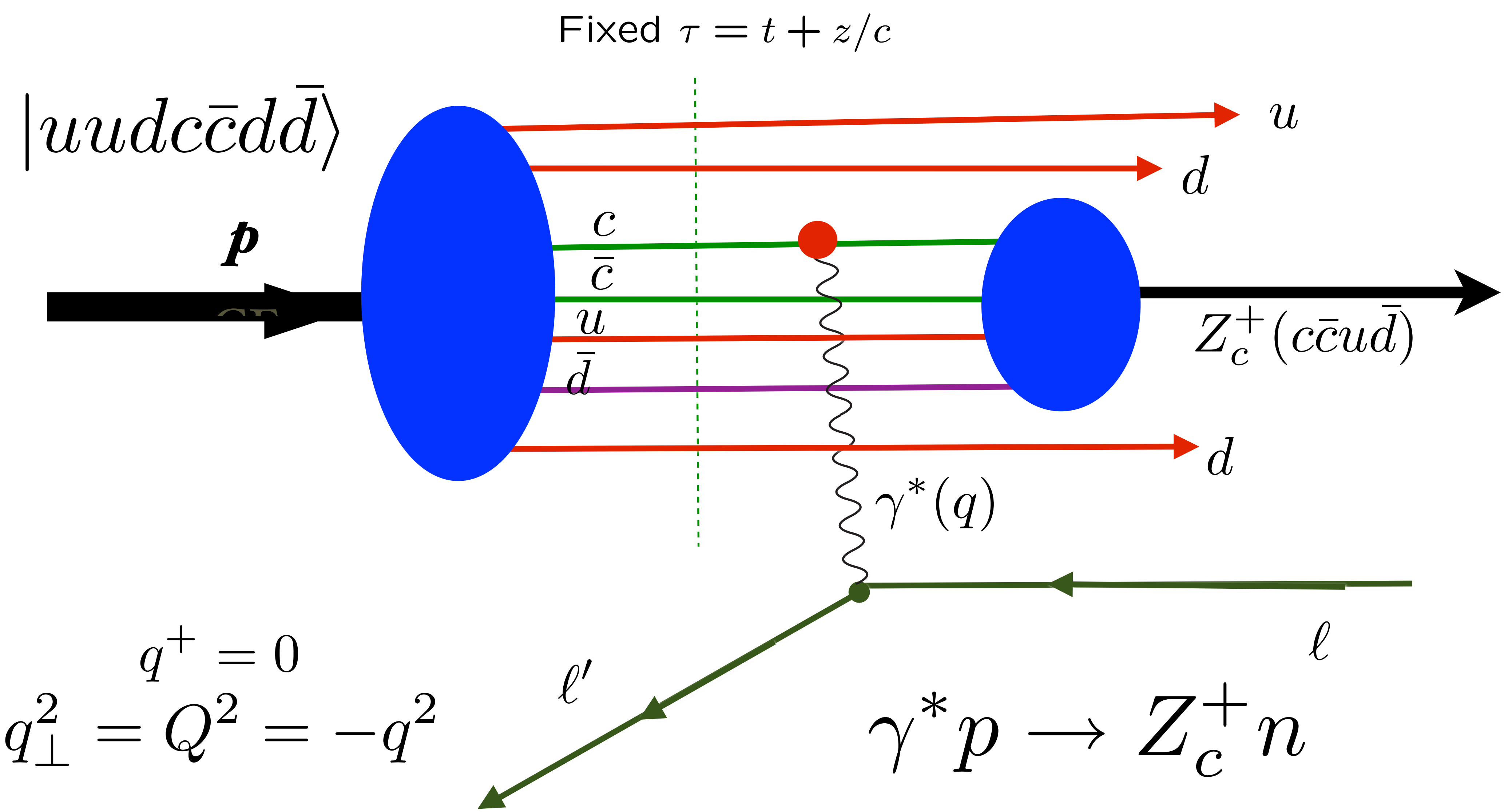
$$M_{\rm octoquark} \sim 5 \text{ GeV}$$



$$\gamma^* D \to |uuduudc\bar{c}>$$

Explains Krisch Effect!

Light-Front Wavefunctions and Heavy-Quark Electroproduction



Produce Charged Tetraquarks at JLab!

Coalescence of comovers at threshold produces Z_c^+ tetraquark resonance

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

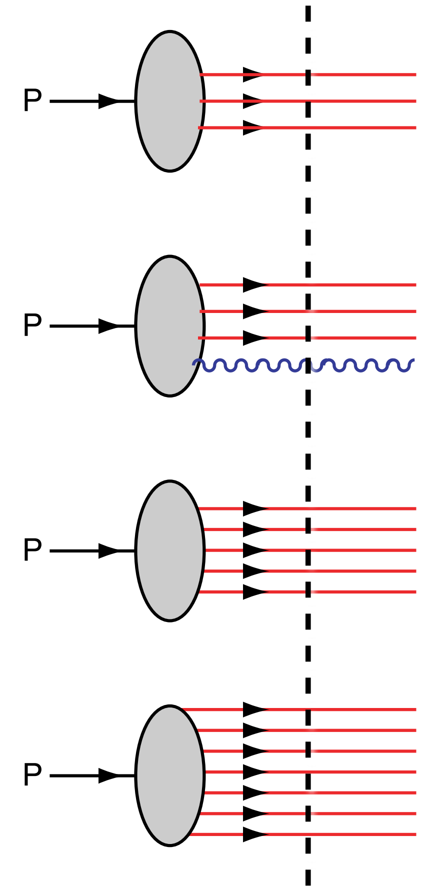
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks

s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

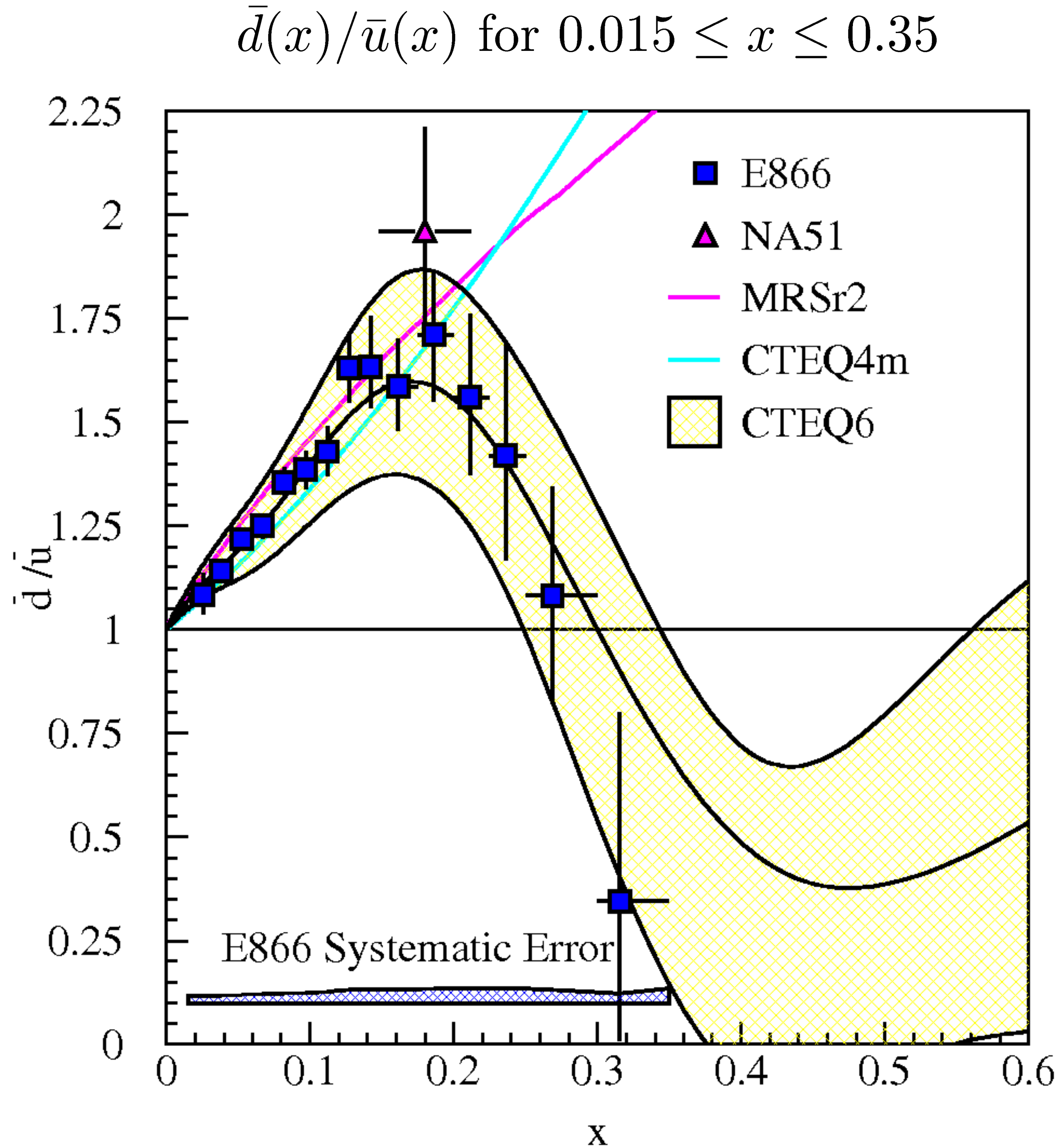
Hidden Color

Mueller: gluon Fock states BFKL

- E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$

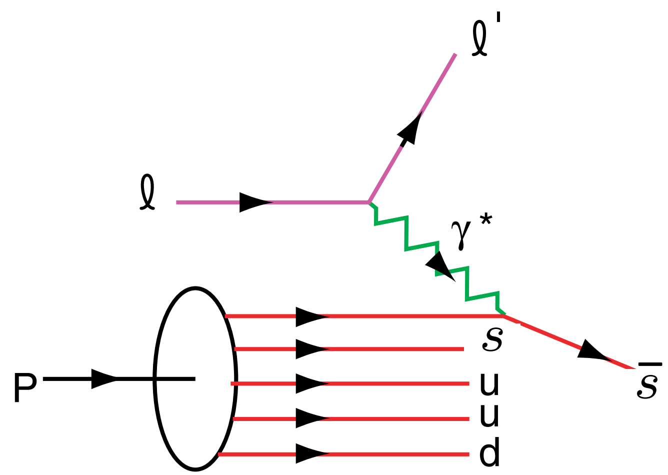
Intrinsic glue, sea, heavy quarks



Measure strangeness distribution in Semi-Inclusive DIS at JLab

$$\text{Is } s(x) = \bar{s}(x)?$$

- **Non-symmetric strange and antistrange sea?**
- **Non-perturbative physics; e.g** $|uuds\bar{s}> \simeq |\Lambda(uds)K^+(\bar{s}u)>$
- **Important for interpreting NuTeV anomaly** **B. Q. Ma, sjb**



Tag struck quark flavor in semi-inclusive DIS $ep \to e' K^+ X$

Do heavy quarks exist in the proton at high x?

Conventional wisdom: impossible!

Standard Assumption: Heavy quarks are generated via DGLAP evolution from gluon splitting

$$s(x,\mu_F^2) = c(x,\mu_F^2) = b(x,\mu_F^2) \equiv 0$$

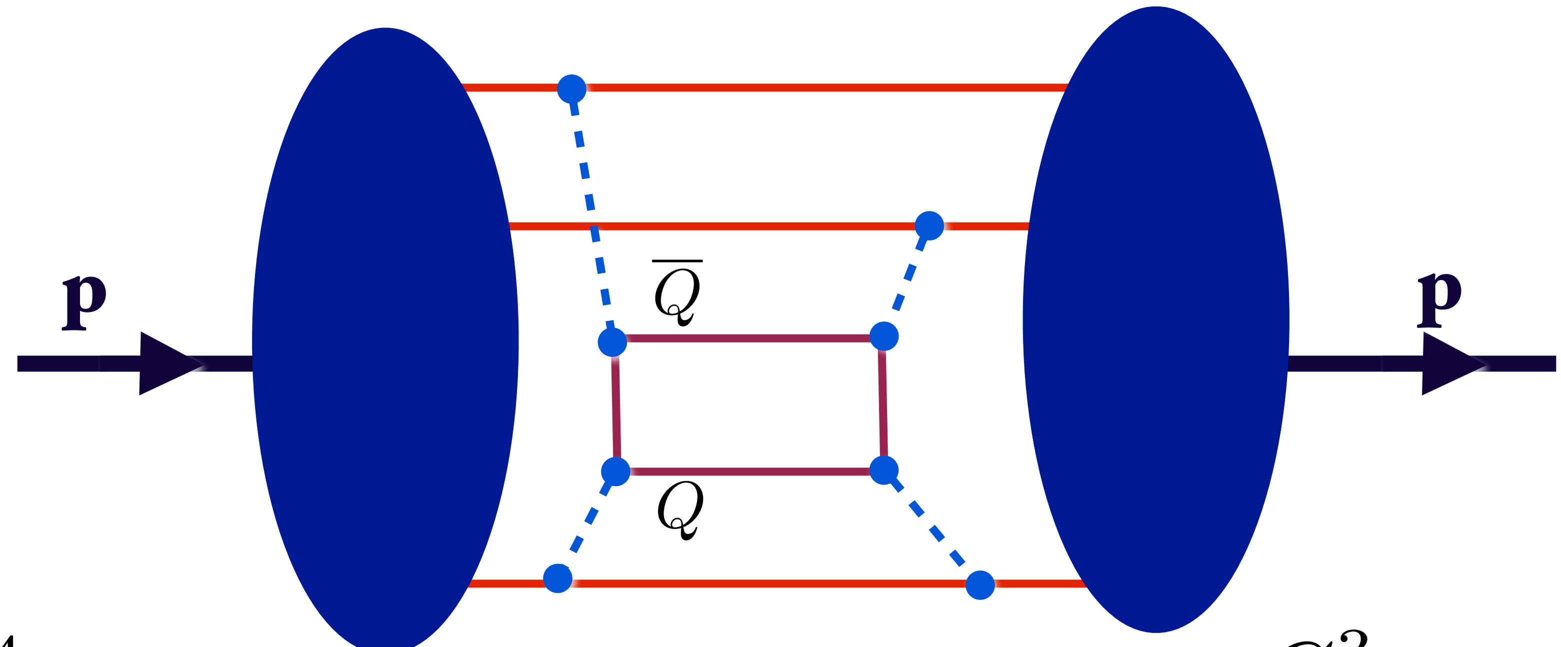
at starting scale μ_F^2

Conventional wisdom is wrong even in QED!

Proton Self Energy from g g to gg scattering

QCD predicts Intrinsic Heavy Quarks!

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$



$$\frac{F_{\mu\nu}^4}{M_\ell^2}$$

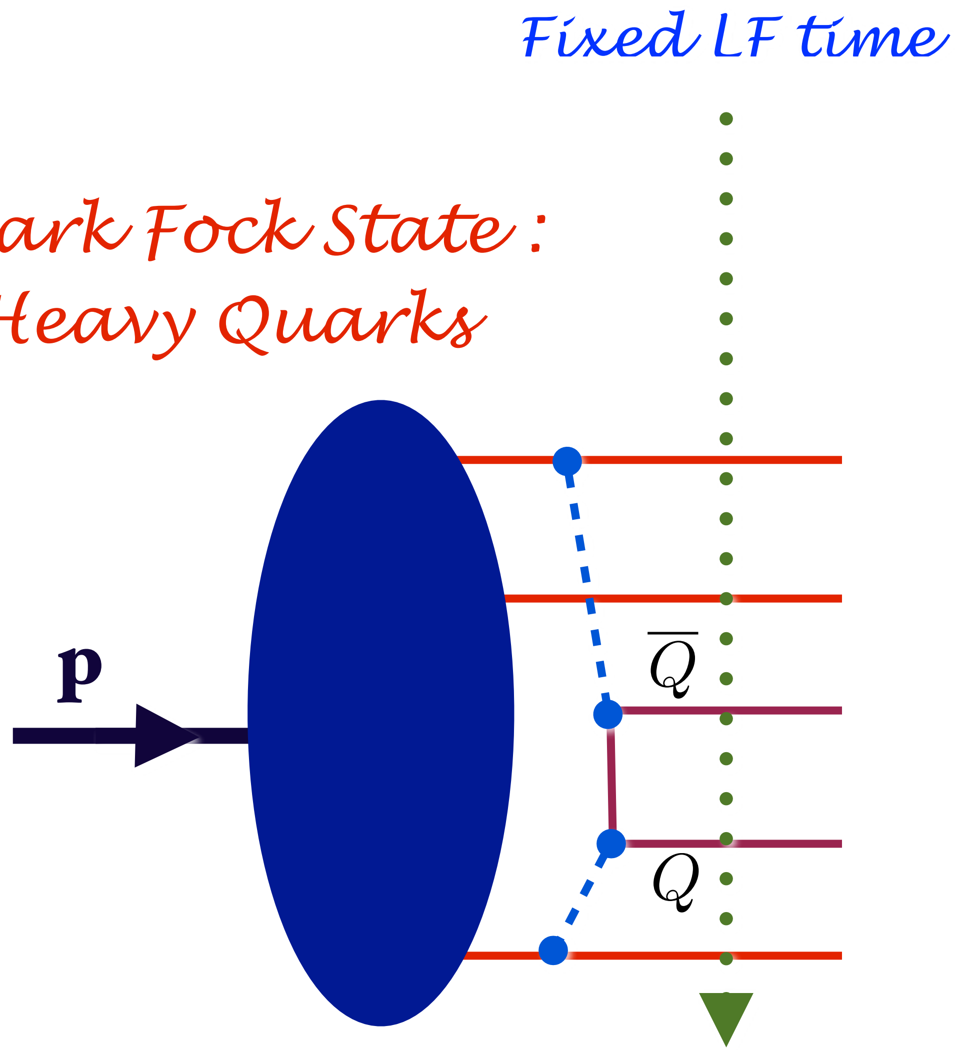
$$\frac{G_{\mu\nu}^3}{M_Q^2}$$

Probability (QED) $\propto \frac{1}{M_\ell^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov, et al.

Proton 5-quark Fock State :
Intrinsic Heavy Quarks



QCD predicts Intrinsic Heavy Quarks at high x

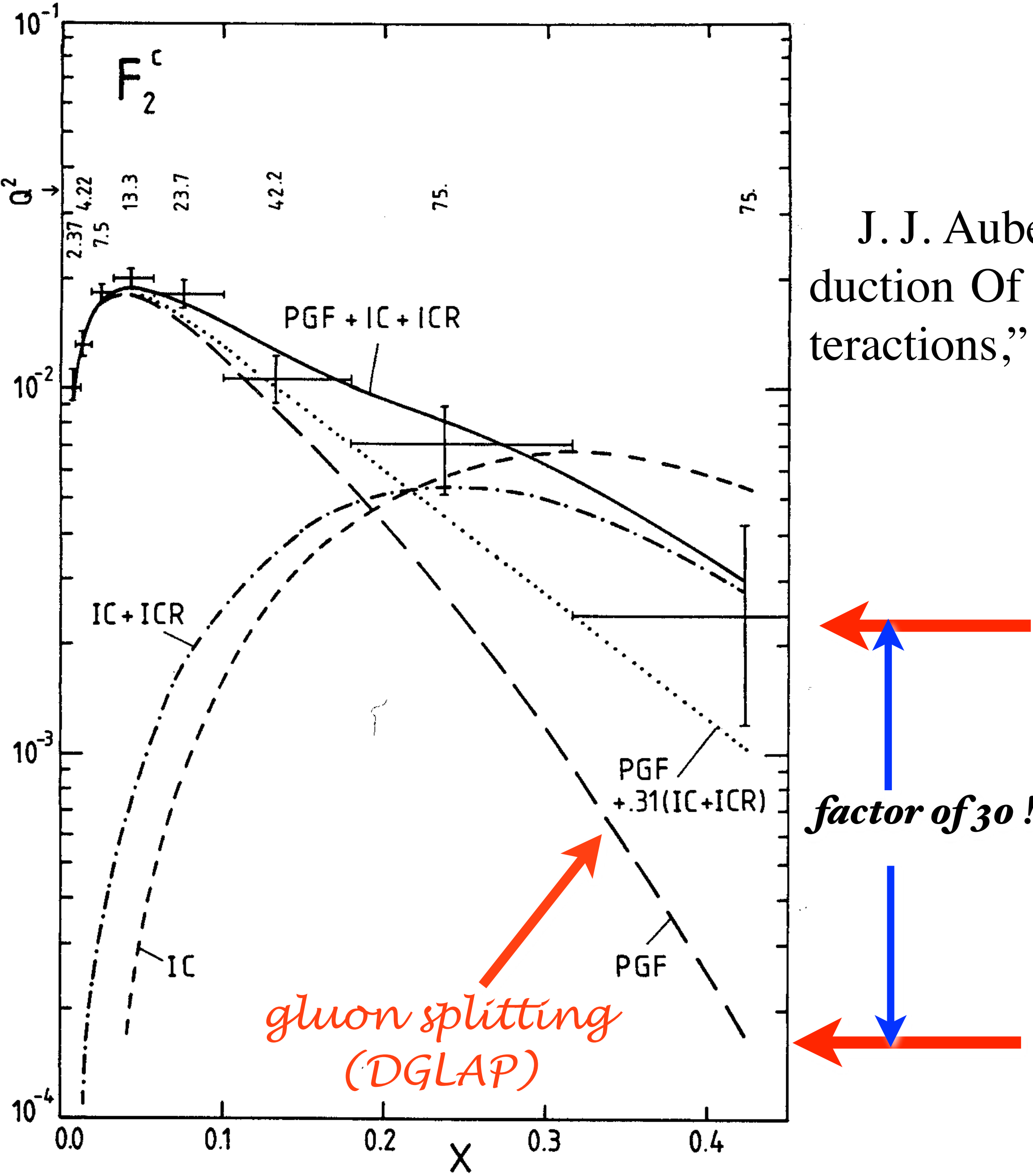
Minimal off-shellness

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4} \qquad \text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

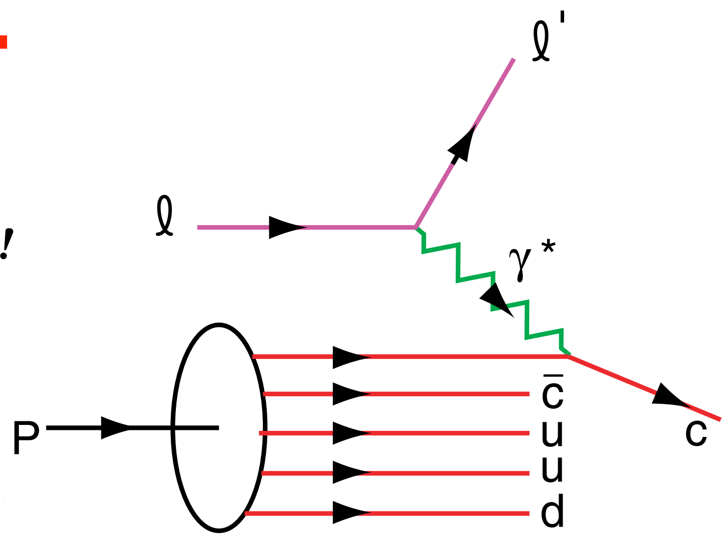
Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov

Measurement of Charm Structure Function



J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm

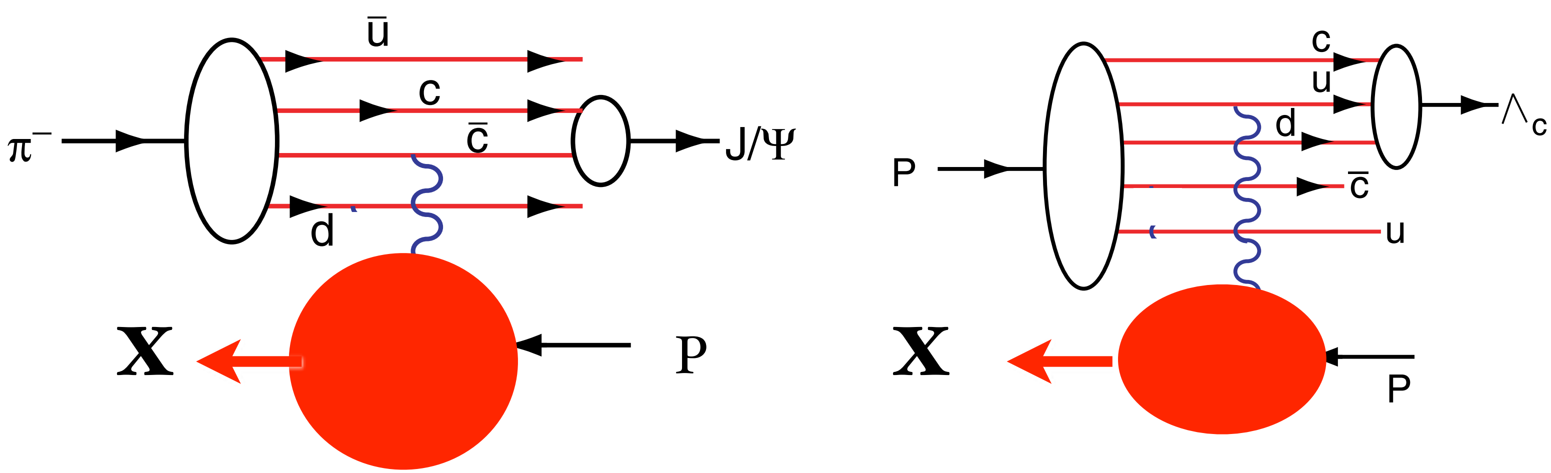


DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

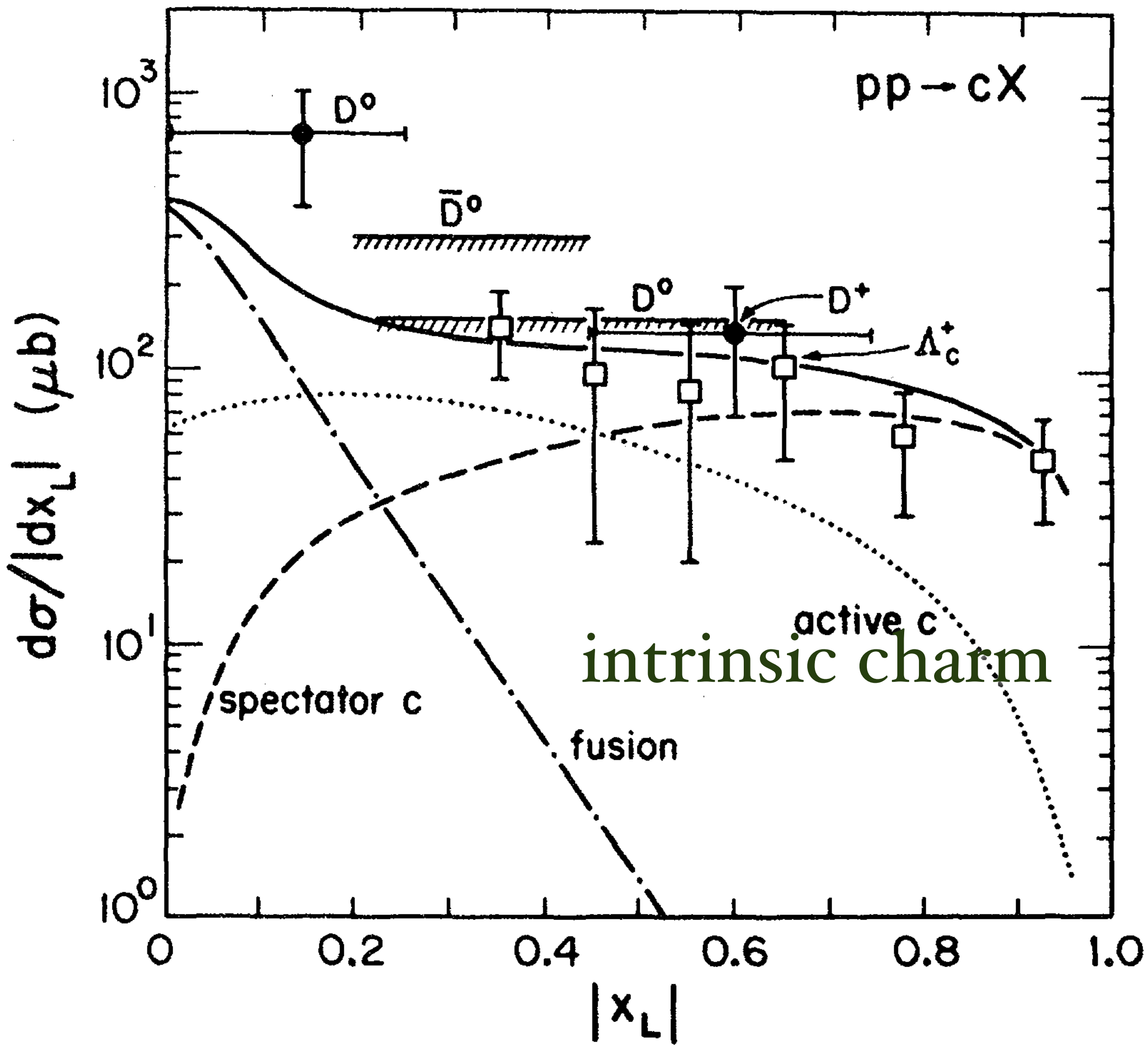
Leading Hadron Production from Intrinsic Charm



Spectator counting rules

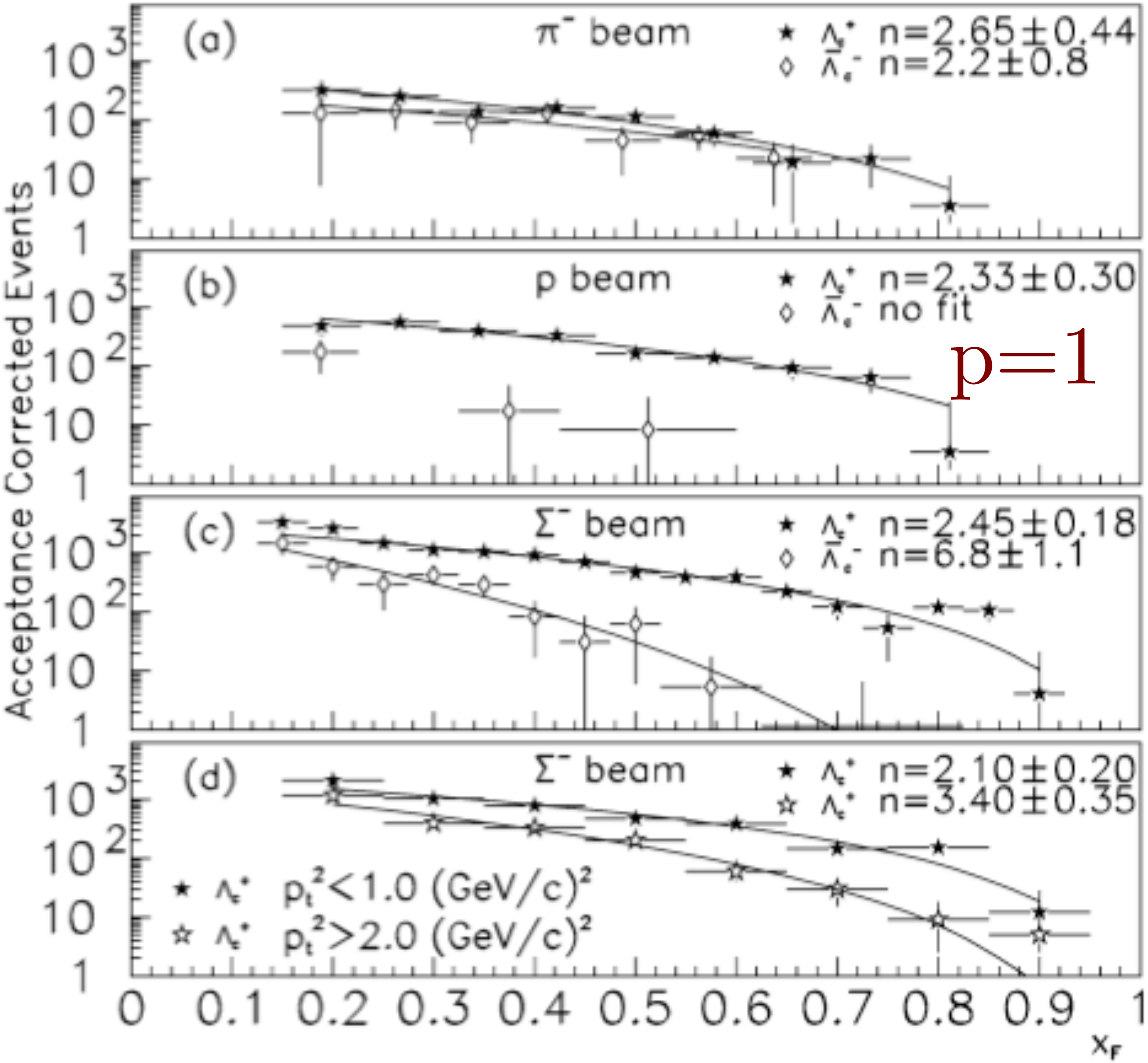
$$\frac{dN}{dx_F} \propto (1 - x_F)^{2n_{spect}-1}$$

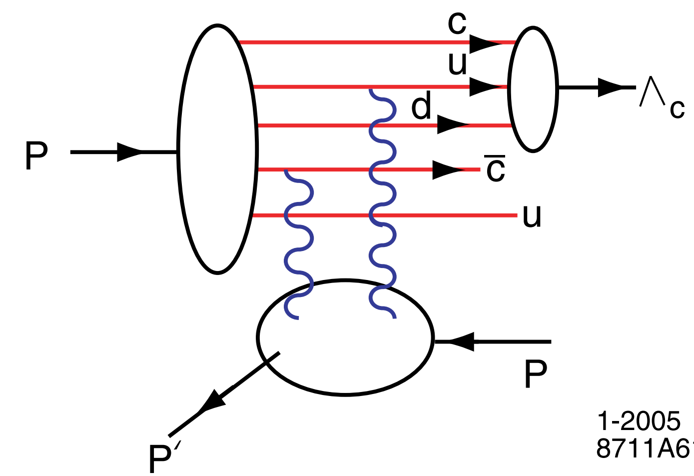
Coalescence of Comoving Charm and Valence Quarks
Produce J/ψ, Λ_c and other Charm Hadrons at High x_F



Barger, Halzen, Keung

Evidence for charm at large x





$$p(uudc\bar{c}) \rightarrow \Lambda_c(cud)$$

$$n_s = 2$$

Phase space alone gives minimum power

$$(1 - x_F)^p, p = n_s - 1$$

Maximum fraction of projectile momentum carried by charm quarks!

- EMC data: $c(x,Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV^2, $x = 0.42$

- High x_F $pp \to J/\psi X$

- High x_F $pp \to J/\psi J/\psi X$

- High x_F $pp \to \Lambda_c X$

- High x_F $pp \to \Lambda_b X$

- High x_F $pp \to \Xi(ccd) X$ (SELEX)

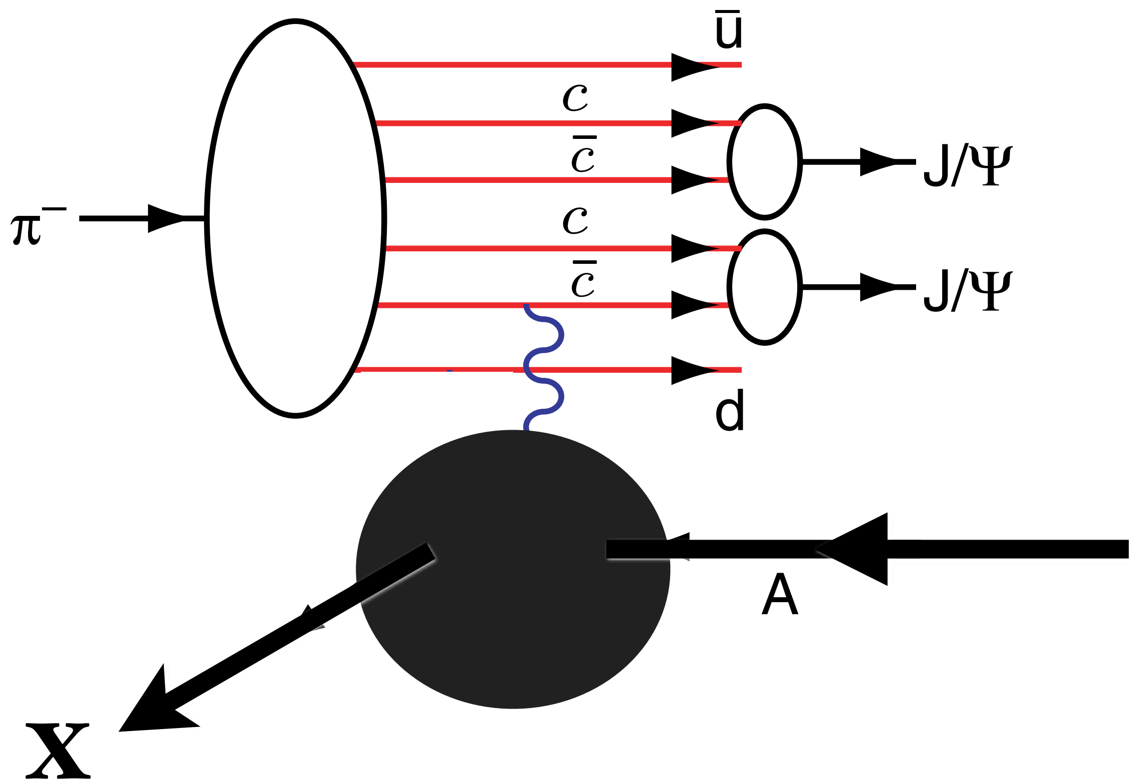
Critical Measurements at threshold for JLab, PANDA

Interesting spin, charge asymmetry, threshold, spectator effects

Important corrections to B decays; Quarkonium decays

Gardner, Karliner, sjb

Production of Two Charmonia at High x_F



NA3: All events at high $x_F = x_\psi + x_\psi$!

Excludes PYTHIA 'color drag' model

All events have $x^F_{\psi\psi} > 0.4$!

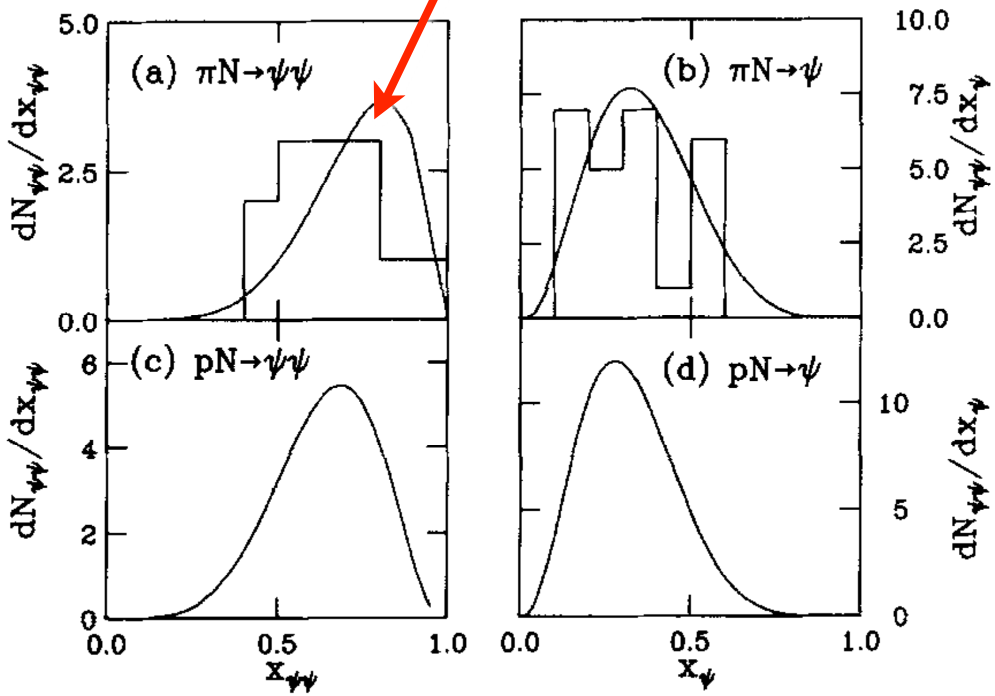


Fig. 3. The $\psi\psi$ pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of J/ψ's from the pairs are shown in (b) and (d). Our calculations are compared with the $\pi^- N$ data at 150 and 280 GeV/c [1]. The $x_{\psi\psi}$ distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single J/ψ's is twice the number of pairs.

NA3 Data

$$\pi A \rightarrow J/\psi J/\psi X$$

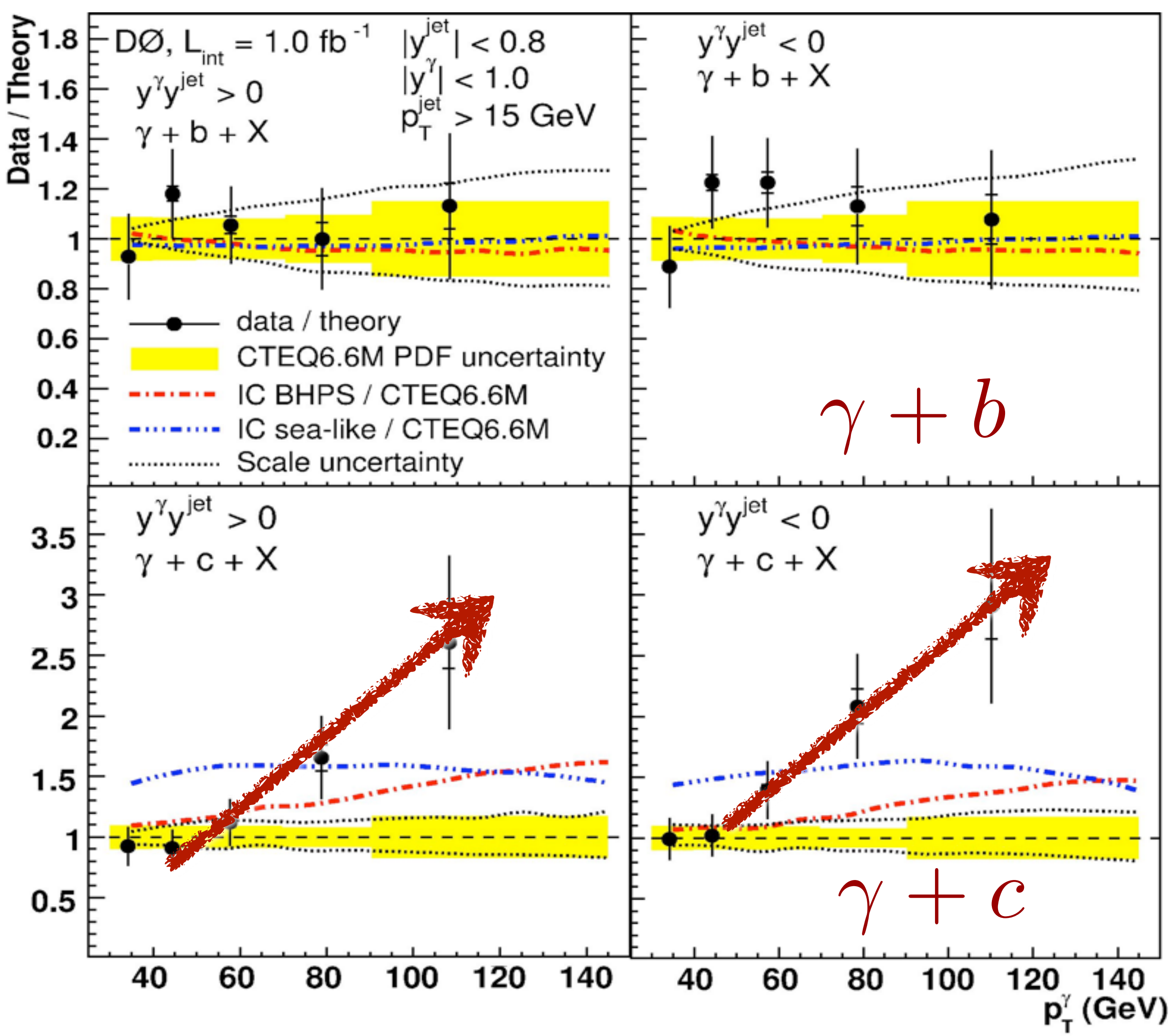
R, Vogt, sjb

The probability distribution for a general n-particle intrinsic $c\bar{c}$ Fock state as a function of x and $\boldsymbol{k}_T$ is written as

$$\frac{dP_{\rm ic}}{\prod_{i=1}^n dx_i d^2 k_{T,i}} = N_n \alpha_s^4(M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^n \boldsymbol{k}_{T,i})\delta(1-\sum_{i=1}^n x_i)}{(m_h^2 - \sum_{i=1}^n (m_{T,i}^2/x_i))^2} ,$$

PRL 102, 192002 (2009) PHYSICAL REVIEW LETTERS week ending 15 MAY 2009

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV



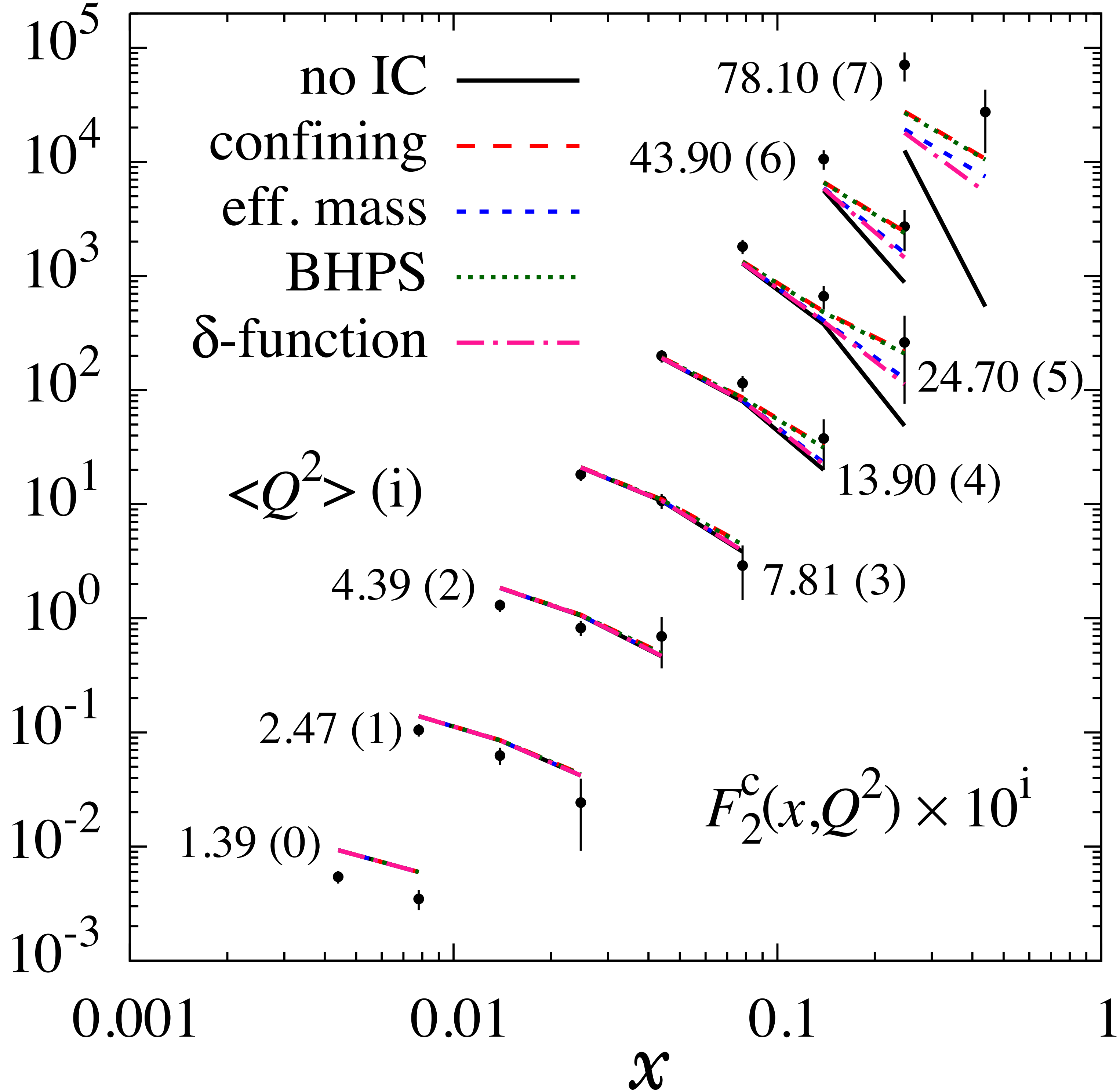
$$\frac{\Delta\sigma(\bar{p}p \to \gamma c X)}{\Delta\sigma(\bar{p}p \to \gamma b X)}$$

Ratio insensitive to gluon PDF, scales

Signal for significant IC at x > 0.1

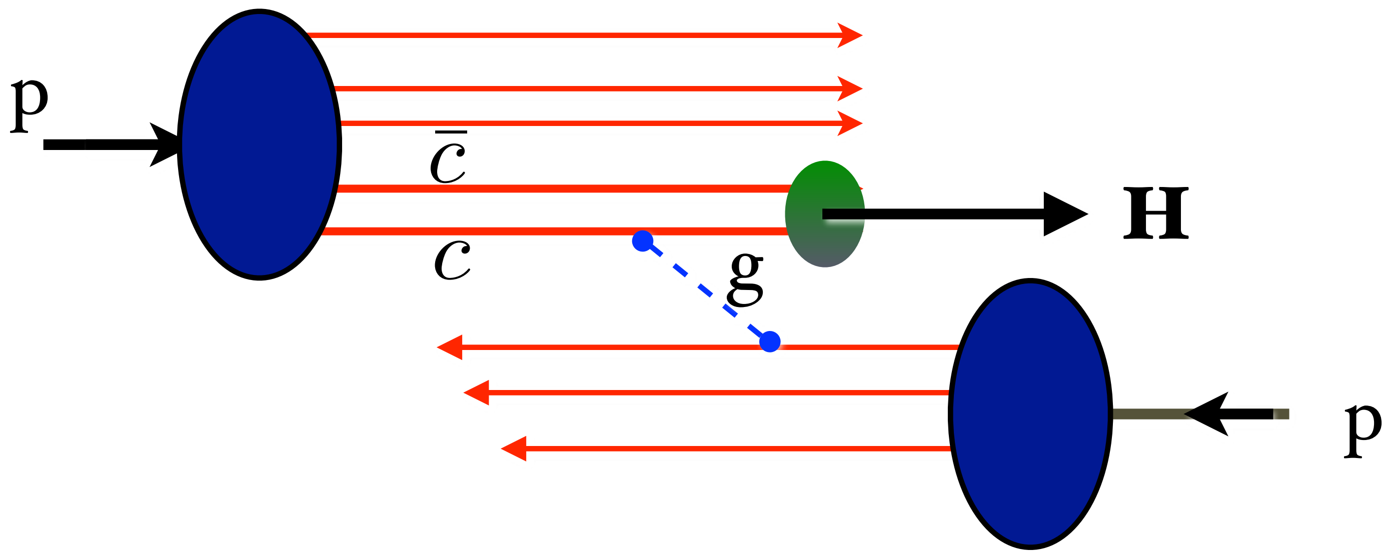
Need COMPASS Measurement of $c(x,Q^2)$!

P. Jimenez-Delgado, T. J. Hobbs, J. T. Londergan, W. Melnitchouk



EMC Data

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



$$pp \to HX$$

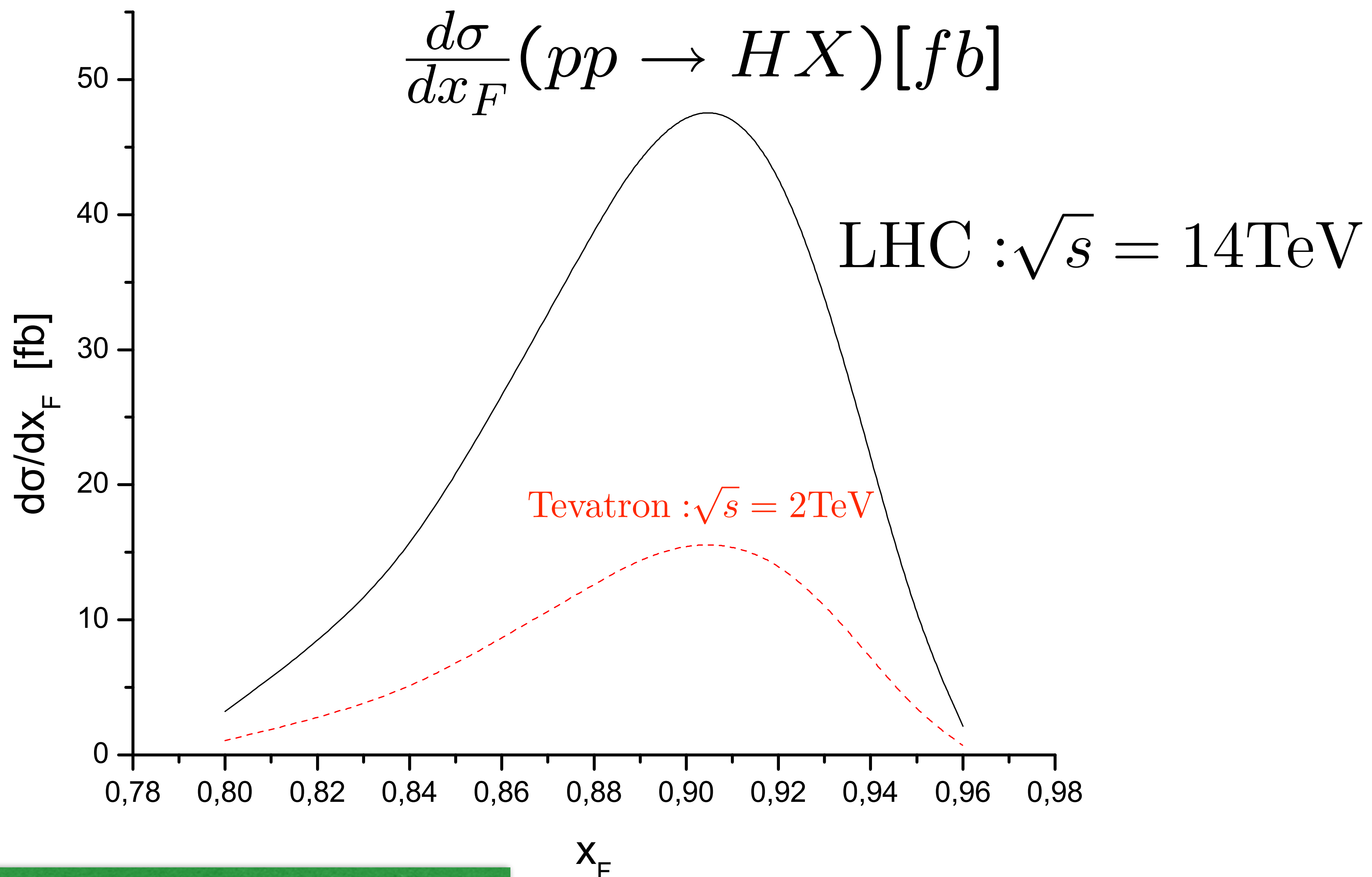
Goldhaber, Soffer, Kopeliovich, Schmidt, sjb

Also: intrinsic bottom, top

Higgs can have 80% of Proton Momentum!

New search strategy for Higgs

AFTER: Higgs production at threshold!



Need High x_F Acceptance

Most practical: Higgs to 2 or 4 muons

Goldhaber, Kopeliovich, Schmidt, Soffer, sjb

Charm at Threshold

- *Intrinsic charm Fock state puts 80% of the proton momentum into the electroproduction process*
- *1/velocity enhancement from FSI*
- *CLEO data for quarkonium production at threshold*
- *Krisch effect shows B=2 resonance*
- *all particles produced at small relative rapidity-- resonance production*
- *Many exotic hidden and open charm resonances will be produced at JLab (12 GeV)*

QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- Heavy quarks only from gluon splitting
- Renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- QCD gives 10^{42} to the cosmological constant

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

QCD $\to$ Abelian Gauge Theory

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\rm init})$

↓

Choose μ_R^{init}*; arbitrary initial renormalization scale*

↓

Identify $\{\beta_i^R\}$ *− terms* **using δ-scheme dependence**

through the PMC − BLM correspondence principle

↓

Shift scale of α_s *to* $\mu_R^{\rm PMC}$ *to eliminate* $\{\beta_i^R\}$ *− terms*

↓

Conformal Series

Result is independent of $\mu_R^{\rm init}$ *and scheme at fixed order*

Principle of Maximum Conformality

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

QED Scale Setting at N_C=0

Eliminates unnecessary systematic uncertainty

Scale fixed at each order

δ-Scheme automatically identifies β-terms!

Xing-Gang Wu, Matin Mojaza Leonardo di Giustino, SJB

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

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(Received 13 January 2013; published 10 May 2013)

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

DOI: 10.1103/PhysRevLett.110.192001 PACS numbers: 12.38.Bx, 11.10.Gh, 11.15.Bt, 12.38.Aw

δ-$\mathcal{R}$enormalization Scheme ($\mathcal{R}_\delta$ scheme)

In dim. reg. $1/\epsilon$ poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln \frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the modified minimal subtraction scheme (MS-bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu^2_{\overline{\rm MS}} = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: $\mathcal{R}_\delta$-scheme

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu^2_\delta = \mu^2_{\overline{\rm MS}} \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

Exposing the Renormalization Scheme Dependence

Observable in the $\mathcal{R}_\delta$-scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$$

$$\mathcal{R}_0 = \overline{\mathrm{MS}} \ , \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \mathrm{MS} \qquad \mu^2 = \mu^2_{\overline{\mathrm{MS}}} \ \exp(\ln 4\pi - \gamma_E) \ , \quad \mu^2_{\delta_2} = \mu^2_{\delta_1} \ \exp(\delta_2 - \delta_1)$$

Note the divergent 'renormalon series' $n! \beta^n \alpha_s^n$

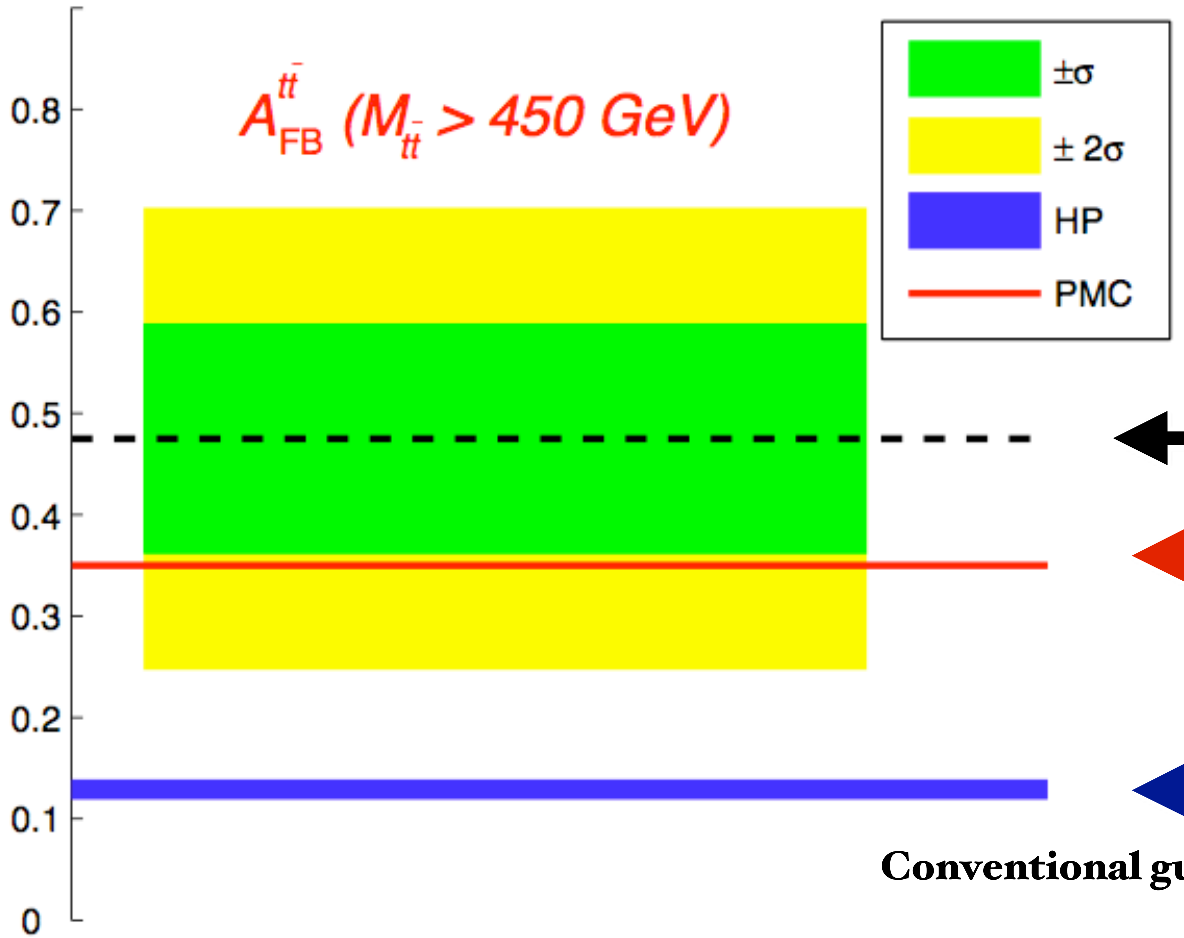
Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \mathrm{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$-term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p. Grouping the different δ_k-terms, one recovers in the $N_c \to 0$ Abelian limit the dressed skeleton expansion.

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the ‘Principle of Maximum Conformality’ (PMC)



Xing-Gang Wu
SJB

Experimental asymmetry

PMC Prediction

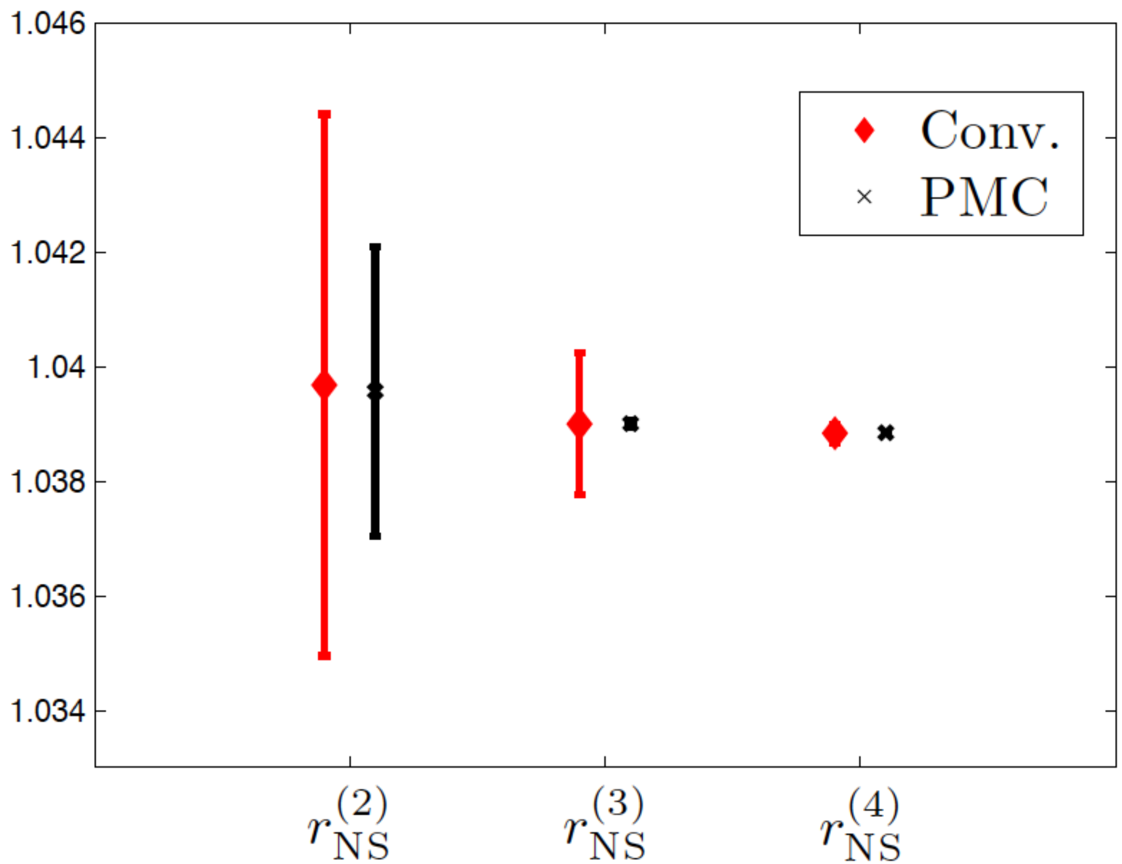
Conventional guess for renormalization scale and range

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

Reanalysis of the Higher Order Perturbative QCD corrections to Hadronic Z Decays using the Principle of Maximum Conformality

S-Q Wang, X-G Wu, sjb

P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, and J. Rittinger, Phys. Rev. Lett. 108, 222003 (2012).



The values of $r_{\rm NS}^{(n)} = 1 + \sum_{i=1}^{n} C_i^{\rm NS} a_s^i$ and their errors $\pm|C_n^{\rm NS} a_s^n|_{\rm MAX}$. The diamonds and the crosses are for conventional (Conv.) and PMC scale settings, respectively. The central values assume the initial scale choice $\mu_r^{\rm init} = M_Z$.

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52 Q$$

Conformal relation true to all orders in perturbation theory

No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

No renormalization scale ambiguity!

Both observables go through new quark thresholds at commensurate scales!

Principle of Maximum Conformality (PMC)

- **Sets pQCD renormalization scale correctly at every finite order**
- **Predictions are scheme-independent**
- **Satisfies all principles of the renormalization group**
- **Agrees with Gell Mann-Low procedure for pQED in Abelian limit**
- **Shifts all β terms into α_s, leaving conformal series**
- **Automatic procedure: R_δ scheme**
- **Number of flavors n_f set**

Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

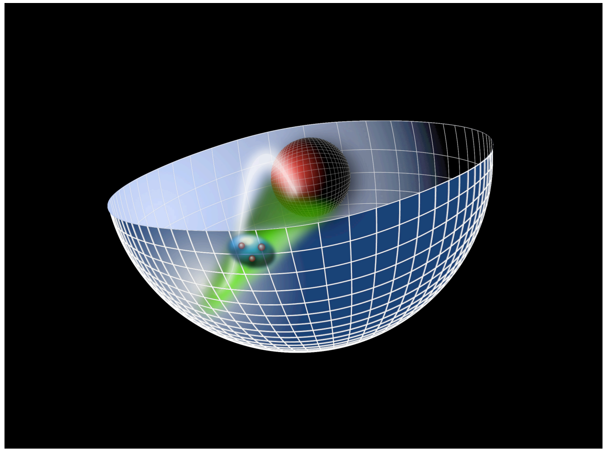
$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

- Although we know the QCD Lagrangian, we have just begun to understand its remarkable properties.

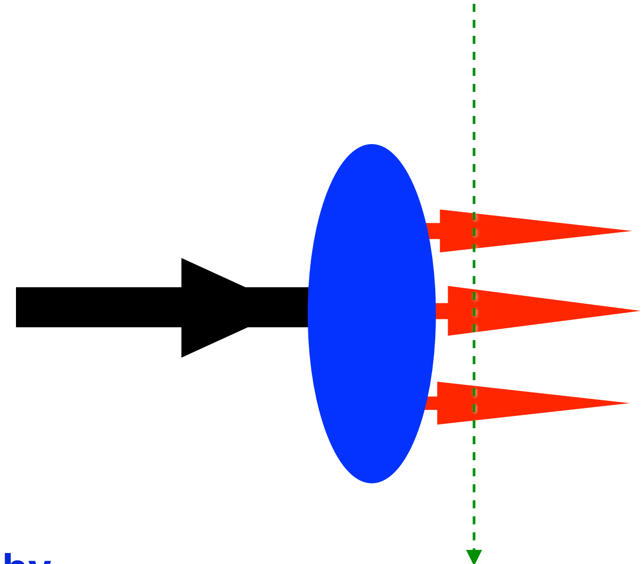
- Novel Phenomena: Color Confinement, Color Transparency, Intrinsic Heavy Quarks, Hidden Color, Tetraquarks, Octoquarks, Nuclear Bound Quarkonium...

- *"Truth is stranger than fiction, because fiction is obliged to stick to possibilities"* — *Mark Twain*

Scattering Theory and Light-Front QCD

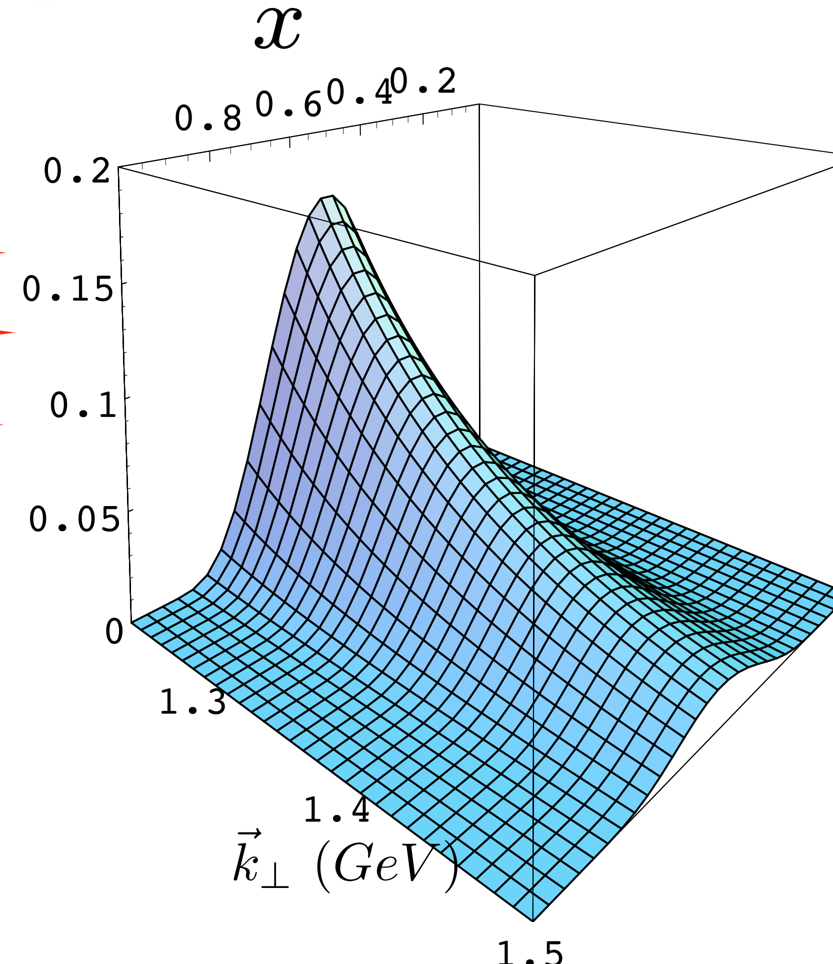


AdS/QCD : Light-Front Holography



Fixed $\tau = t + z/c$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Thanks!!!

2015 International Summer Workshop on Reaction Theory

June 12, 2015

INDIANA UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

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