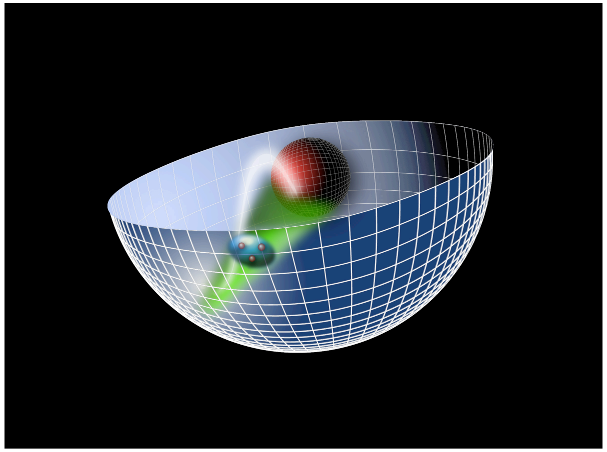
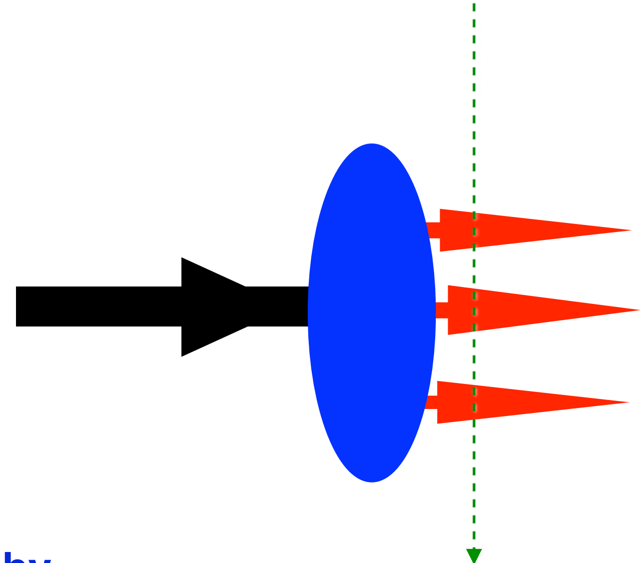
Scattering Theory and Light-Front QCD

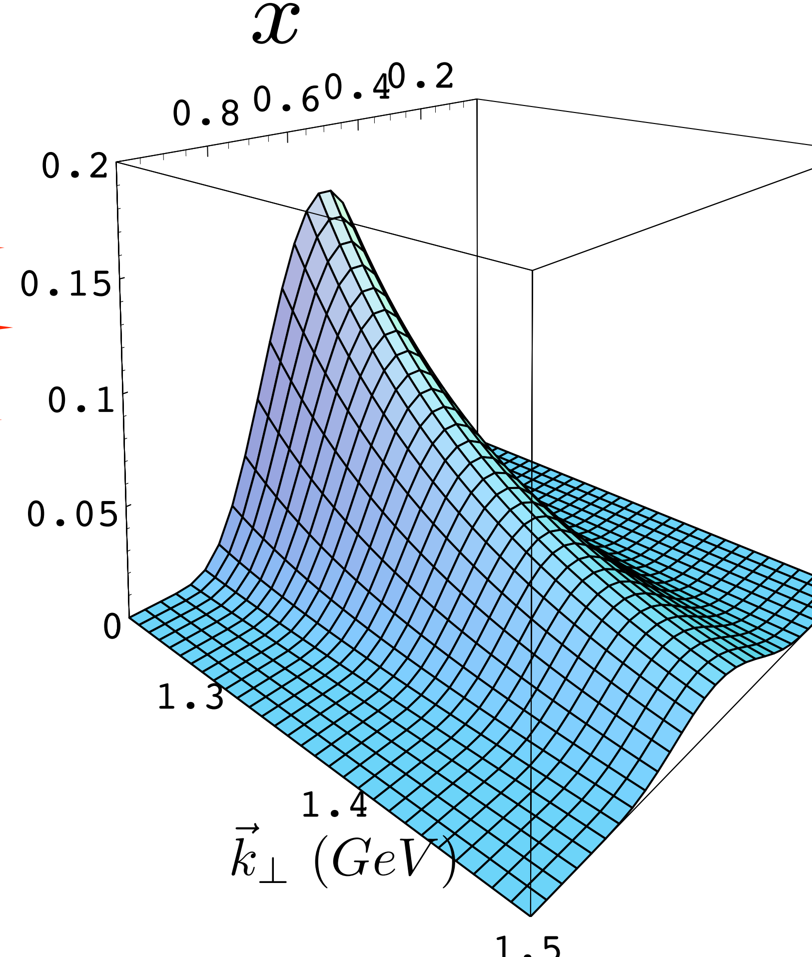


AdS/QCD : Light-Front Holography



Fixed $\tau = t + z/c$

$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$



2015 International Summer Workshop on Reaction Theory

June 11, 2015

INDIANA UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY

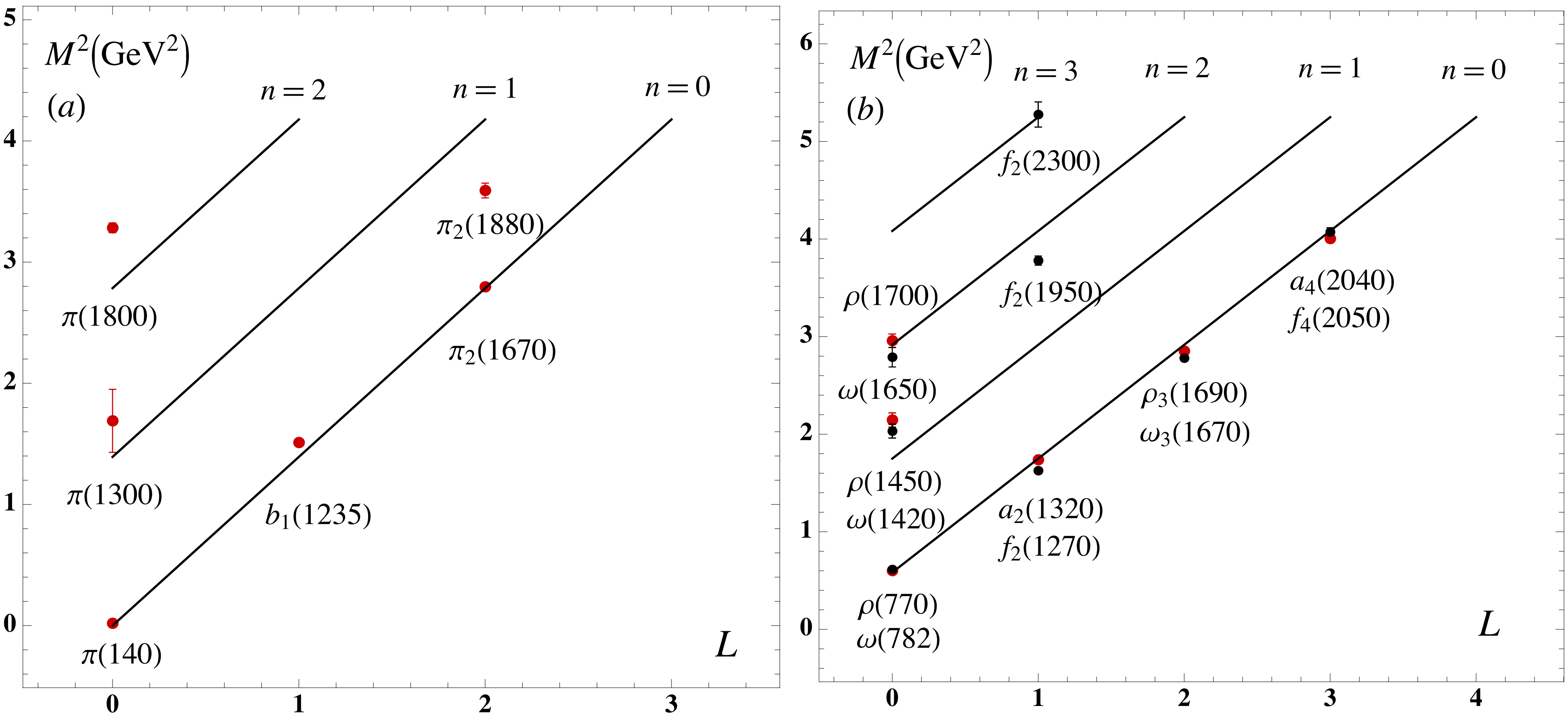
Regge Theory Revisited

The Workshop is dedicated in memory of Tullio Regge who passed away on October 23, 2014.

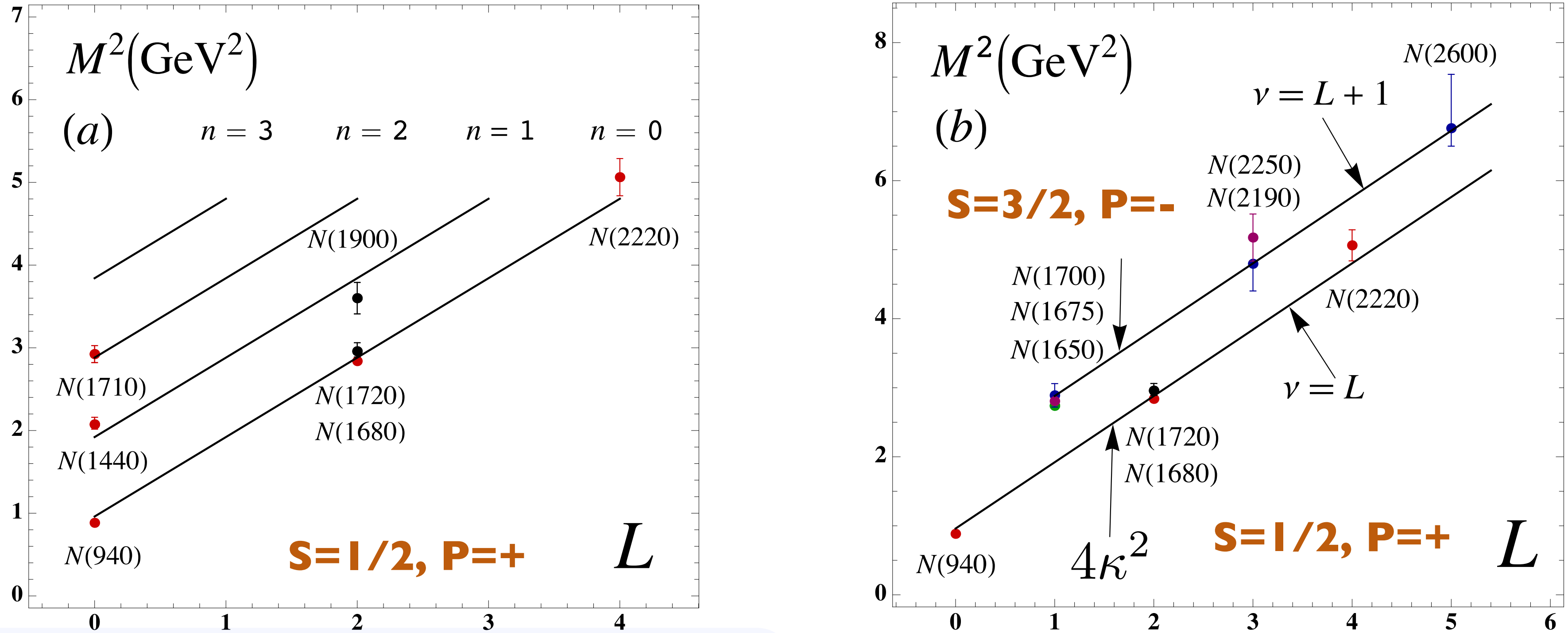
He discovered the role of complex angular momentum singularities. Named after him, Regge poles and cuts, determine asymptotic behavior of relativistic scattering amplitudes, and the discovery led to the most successful phenomenology of high energy collisions.

$m_u = m_d = 0$

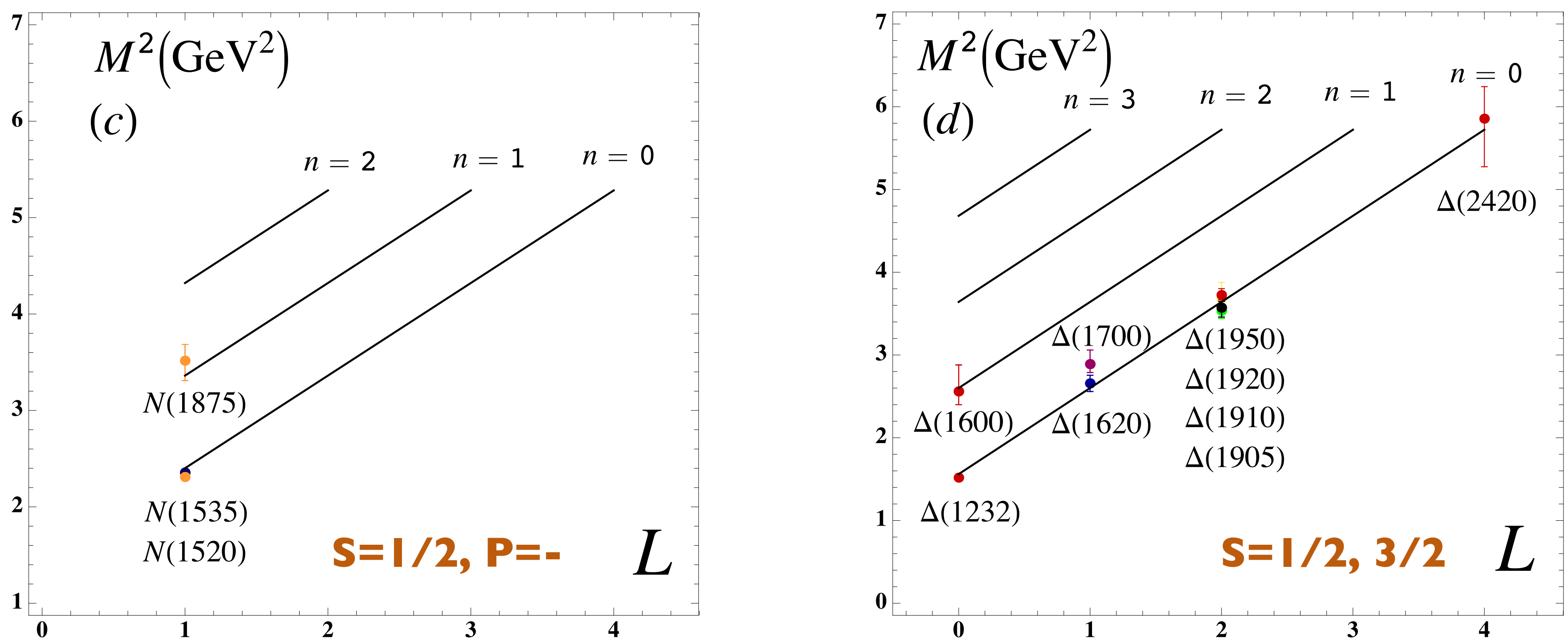
Regge Trajectories from AdS/QCD



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

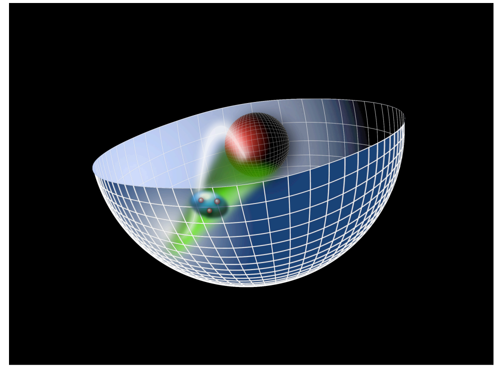


Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

de Teramond, Dosch, sjb



AdS/QCD Soft-Wall Model

Light-Front Holography

Semi-Classical Approximation to QCD

Relativistic, frame-independent
Unique color-confining potential
Zero mass pion for massless quarks
Regge trajectories with equal slopes in n and L
Light-Front Wavefunctions

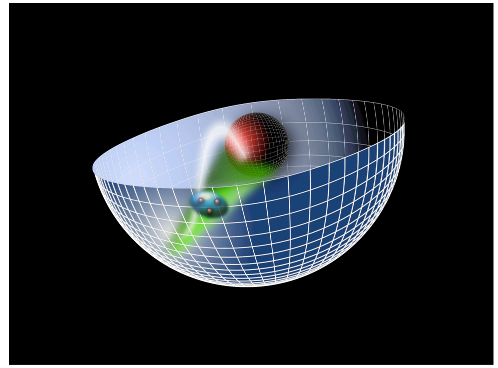
Conformal Symmetry of the action

Light-Front Schrödinger Equation

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

Preserves Conformal Symmetry
of the action

Confinement scale:

$$\kappa \simeq 0.6 \ GeV$$

$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:**
  - **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!

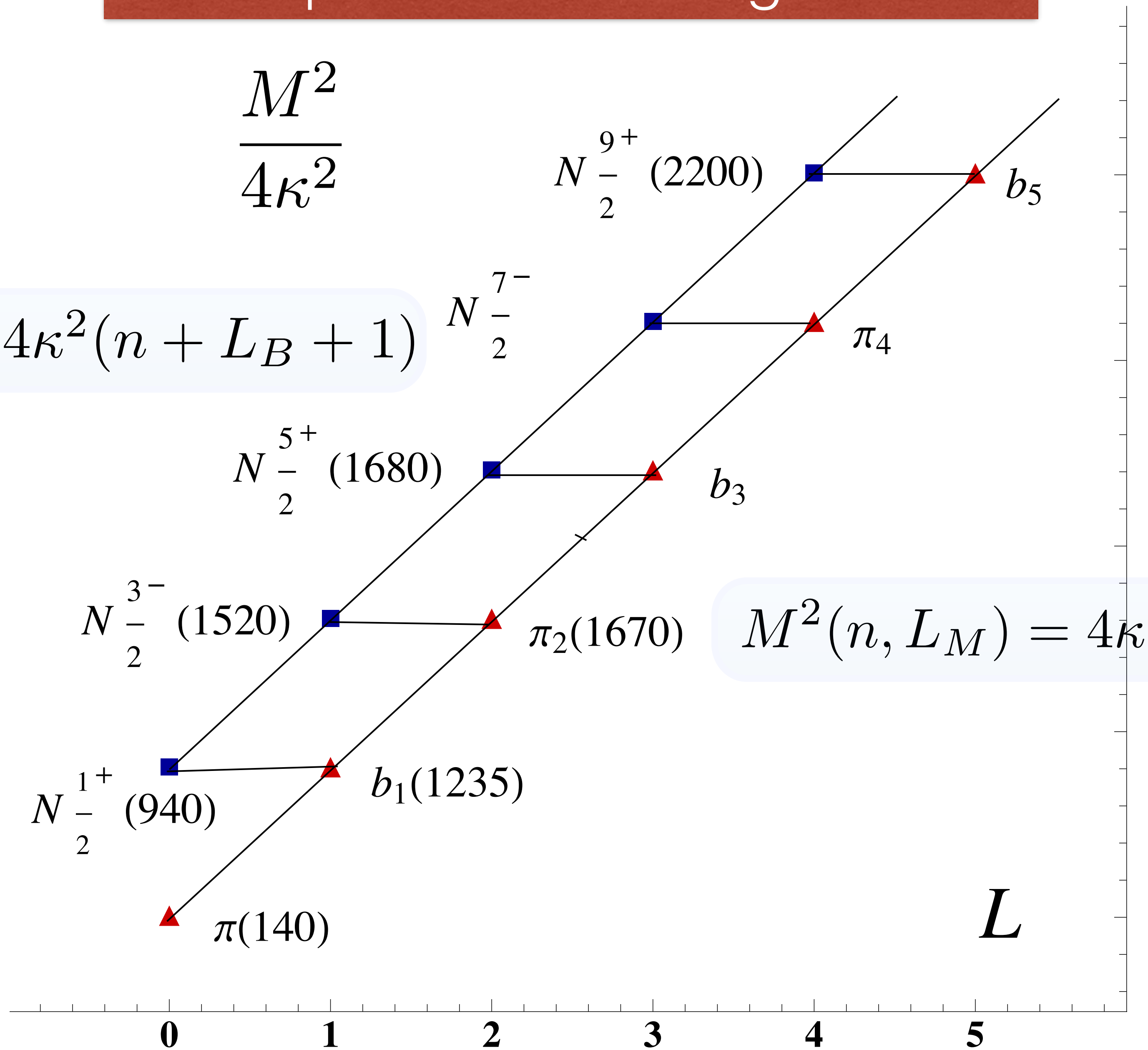
Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons, baryons***
- ***Chiral features for m_q=0: m_π=0, chiral-invariant proton***
- ***Hadronic Frame-Independent Wavefunctions***
- ***Counting Rules for fall-off in momentum transfer***
- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

Superconformal AdS Light-Front Holographic QCD:

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

Superconformal Algebra



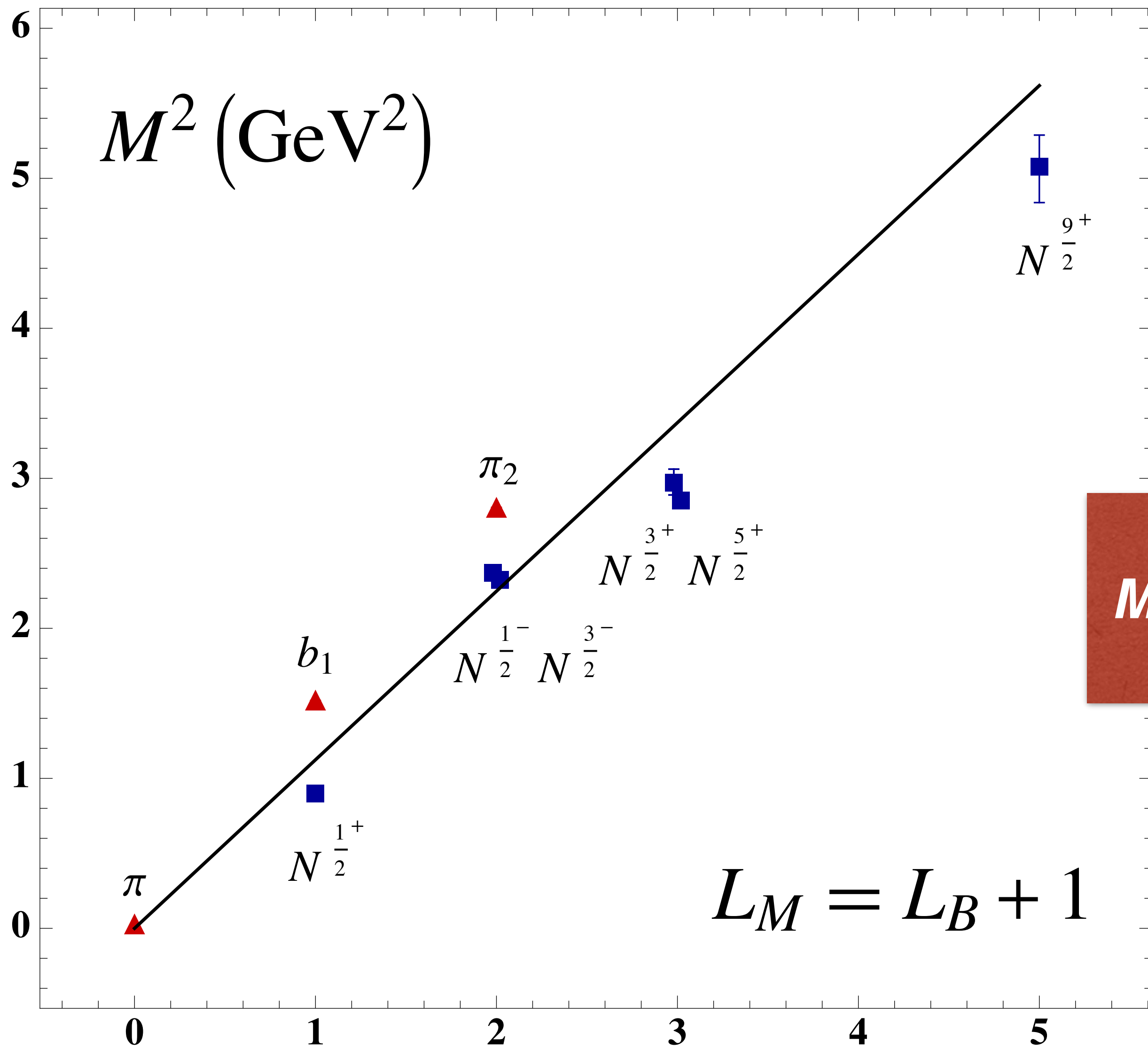
$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

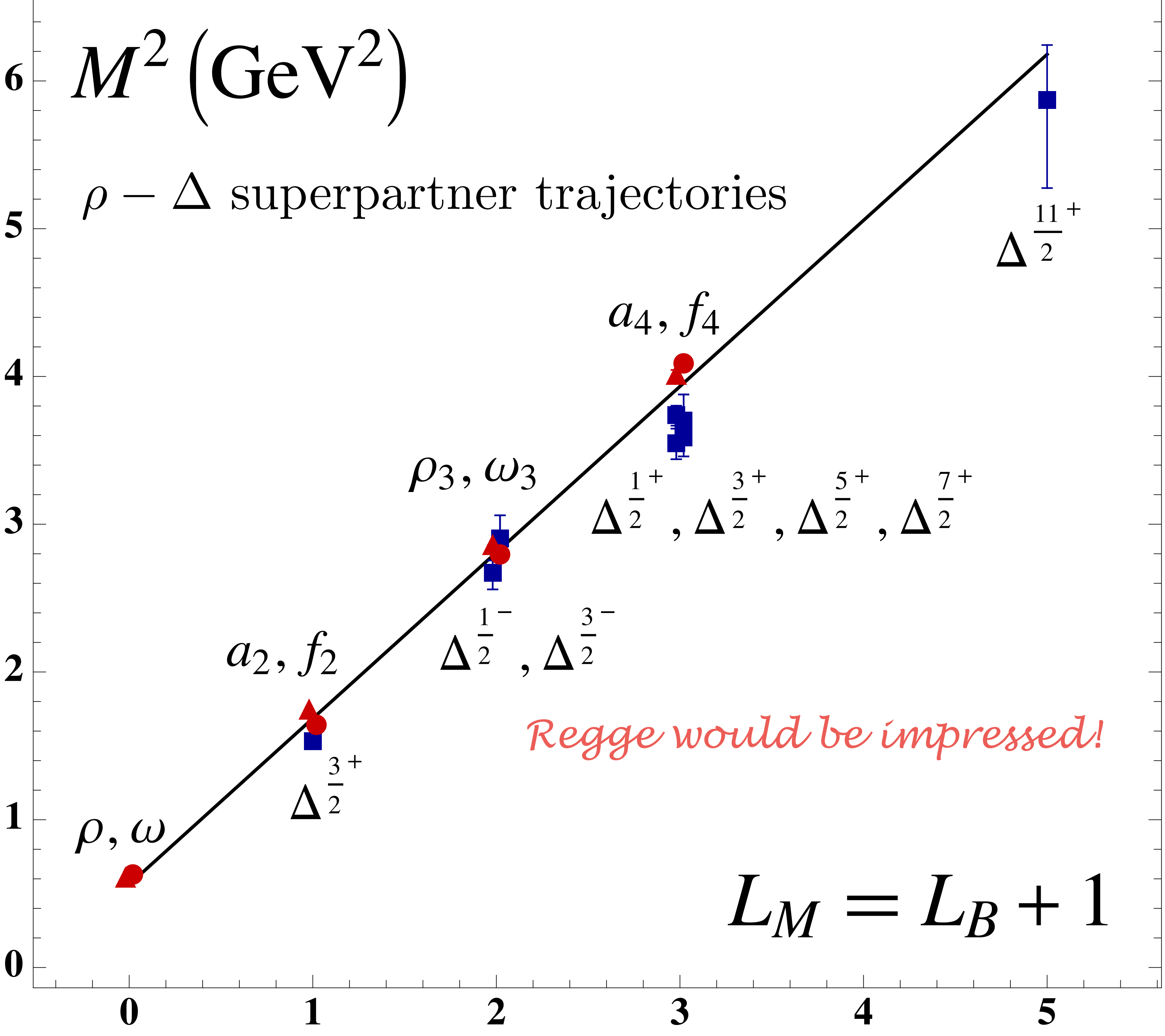
Similar slopes

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



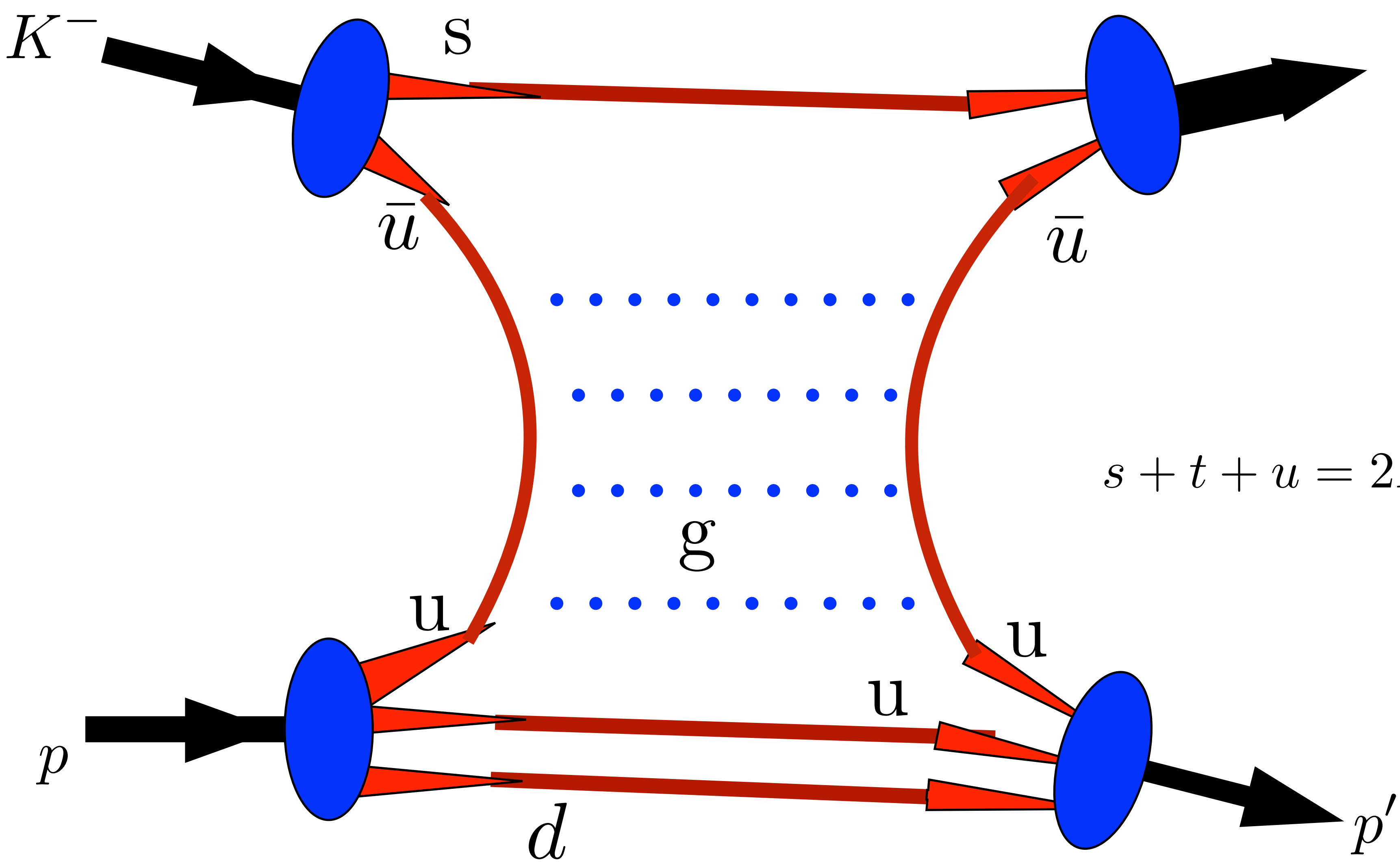
Meson-Baryon Mass Degeneracy for L_M=L_B+1

S=0, I=1 Meson is "superpartner" of S=1/2, I=1 Baryon



Dosch, de Teramond, sjb

Reggeon Exchange in the t-channel



$$s + t + u = 2M_p^2 + 2M_K^2$$

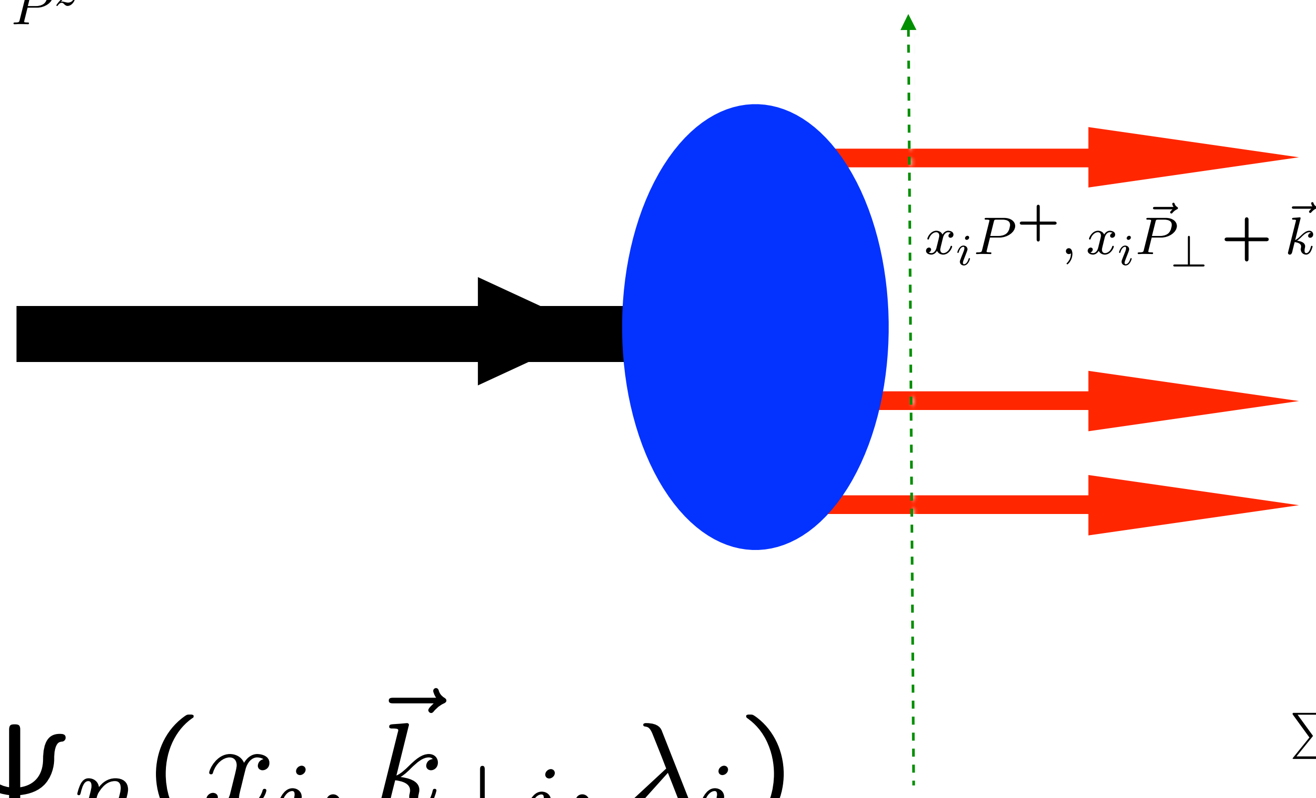
$$\mathcal{M}_R \sim s^{\alpha_R(t)} F_R(t) \frac{1}{2} [e^{-i\pi\alpha_R(t)} \pm 1]$$

Signature factor $C = \pm 1$

Need Hadron Wavefunctions

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$P^+ = P^0 + P^z$



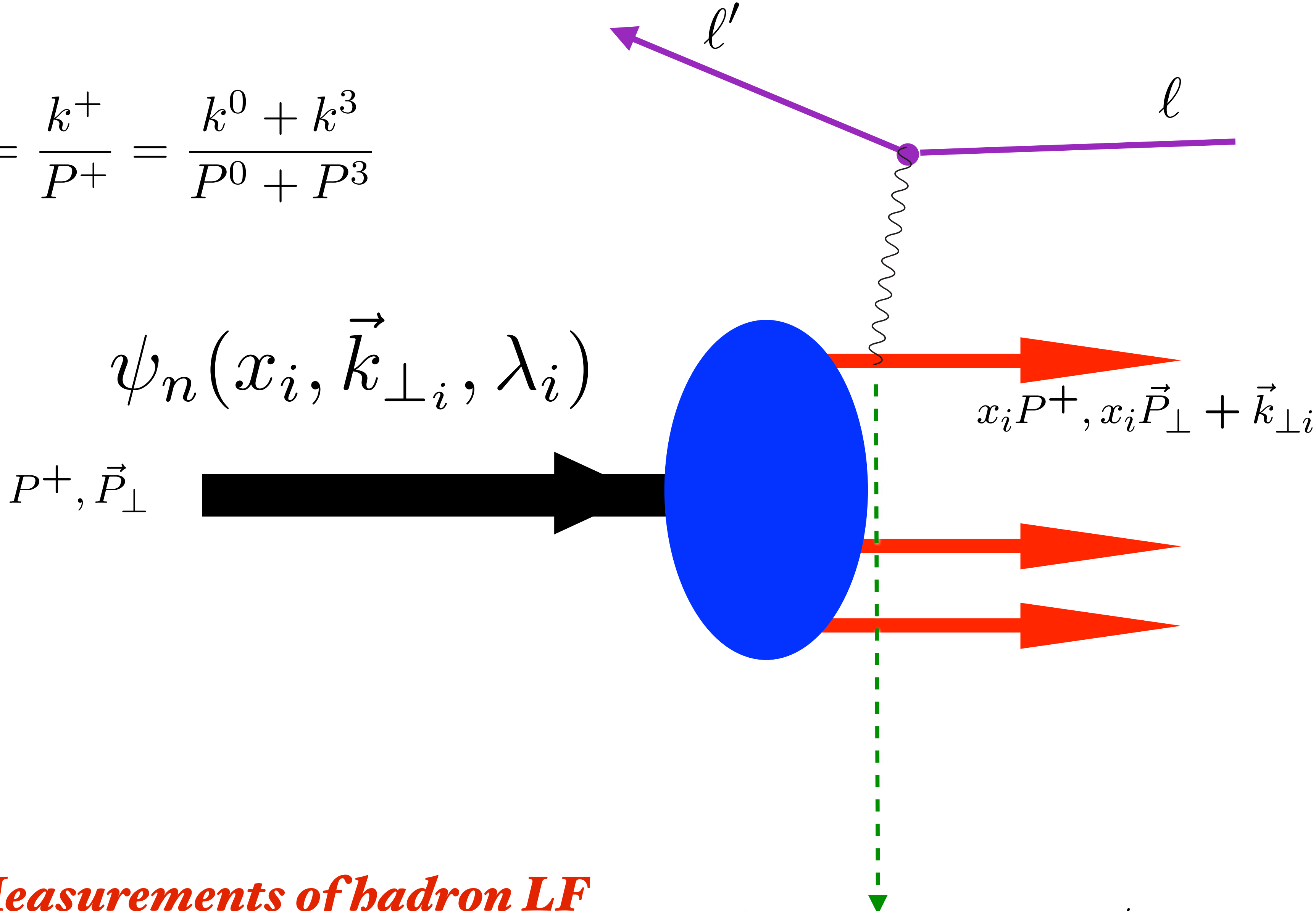
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Fixed $\tau = t + z/c$

Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Each element of flash photograph illuminated along the light front

at a fixed

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenvalue

$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

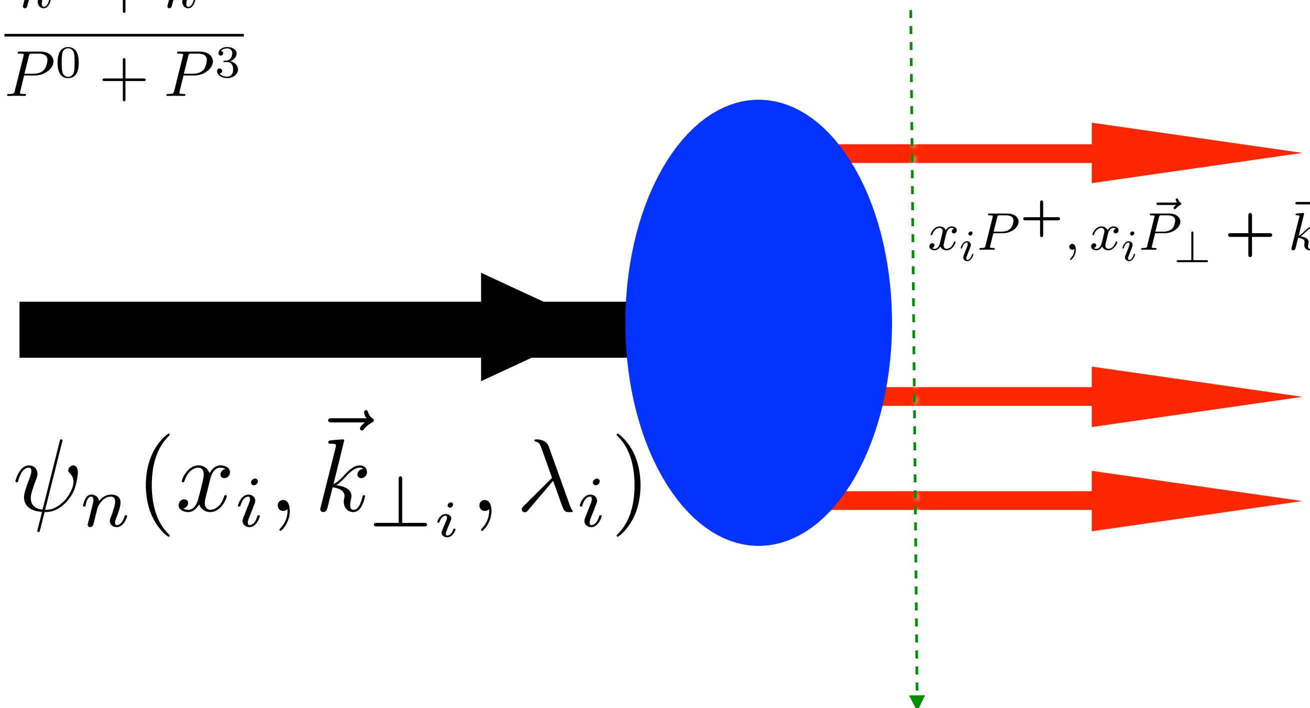
$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian : Off-shell in Invariant Mass

Fixed $\tau = t + z/c$ *Fixed LF time*

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

$\sum_i^n x_i = 1$

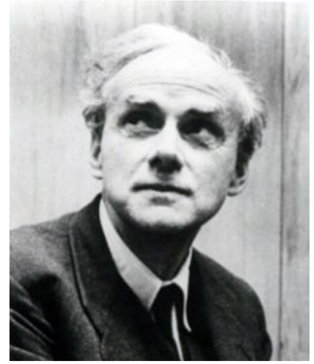
$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Sum Rules

Invariant under boosts! Independent of P^μ

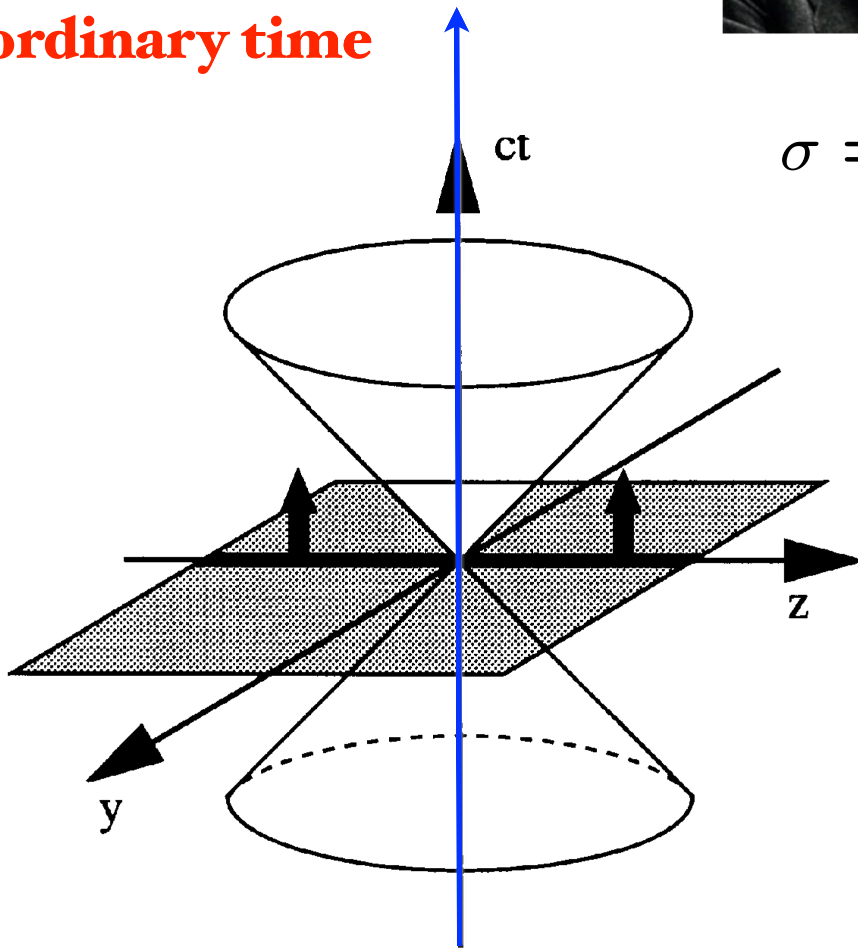
Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Dirac's Amazing Idea: The "Front Form"



P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

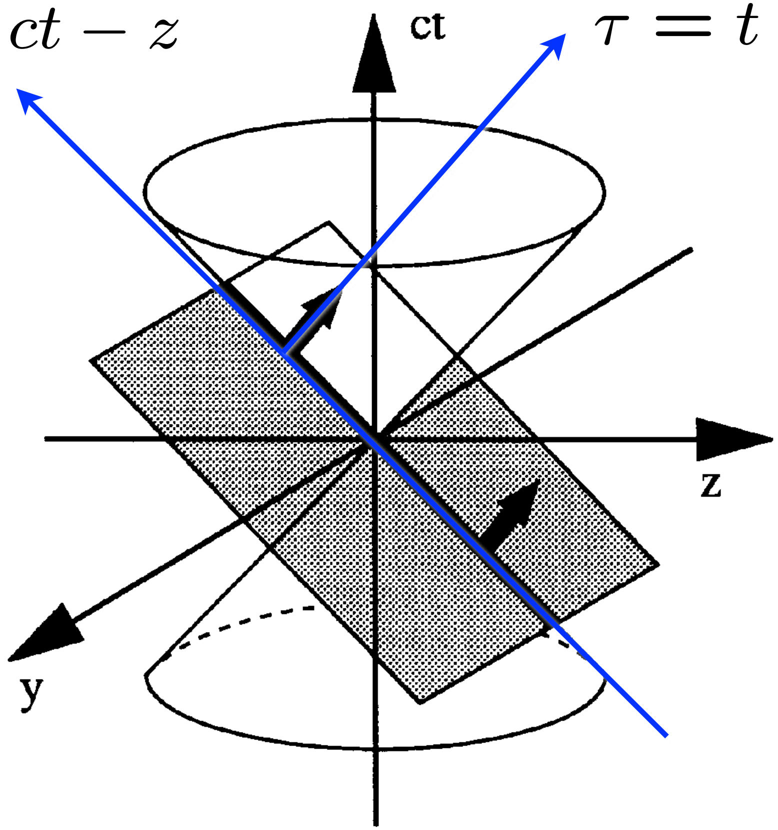
Evolve in ordinary time



Instant Form

Evolve in light-front time!

$\sigma = ct - z$ $\quad$ $\tau = t + z/c$



Front Form

Boost Invariant!

Angular Momentum on the Light-Front

LC gauge A$^+$=0

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

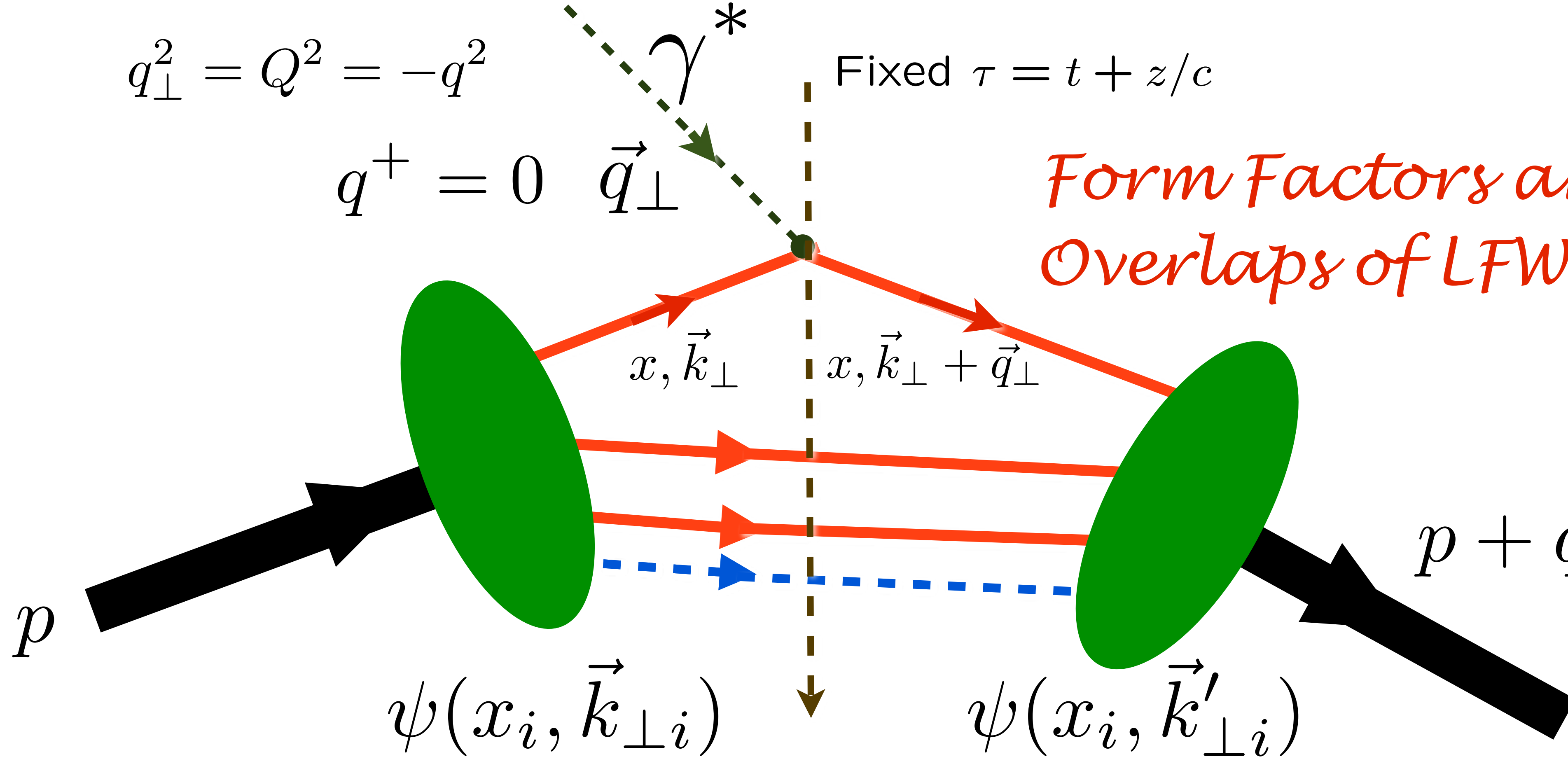
Orbital Angular Momentum is a property of LFWFS

Nonzero Anomalous Moment -->
Nonzero quark orbital angular momentum!

$$< p+q|j^+(0)|p > = 2p^+ F(q^2)$$

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

Drell &Yan, West
Exact LF formula

Exact LF Formula for Pauli Form Factor

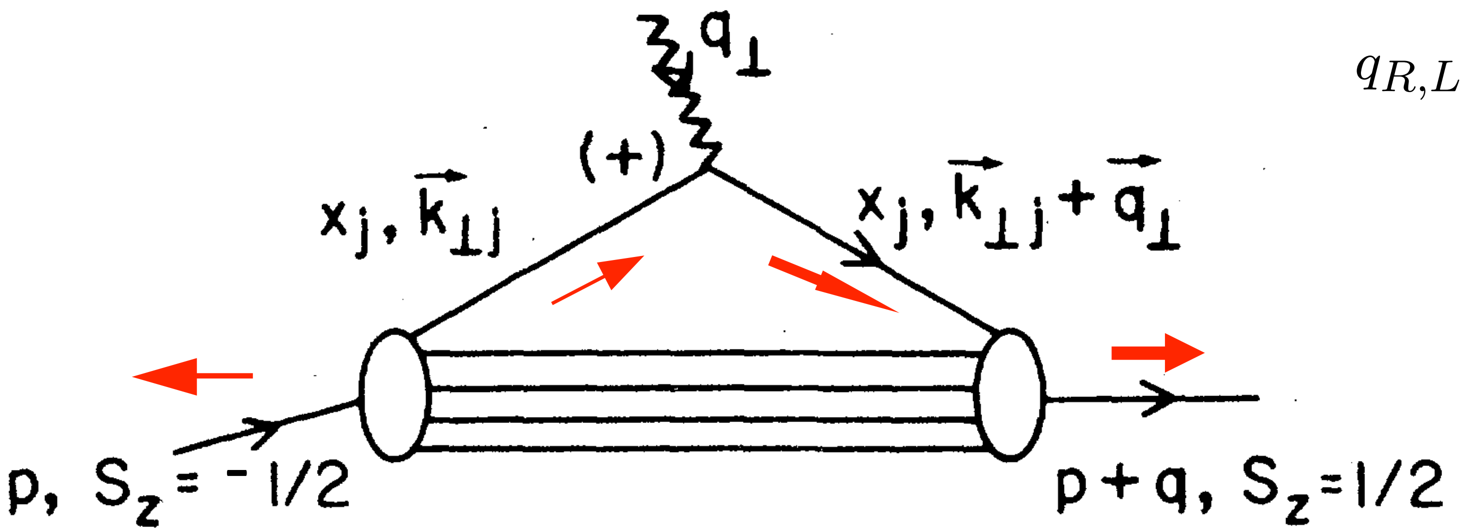
Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

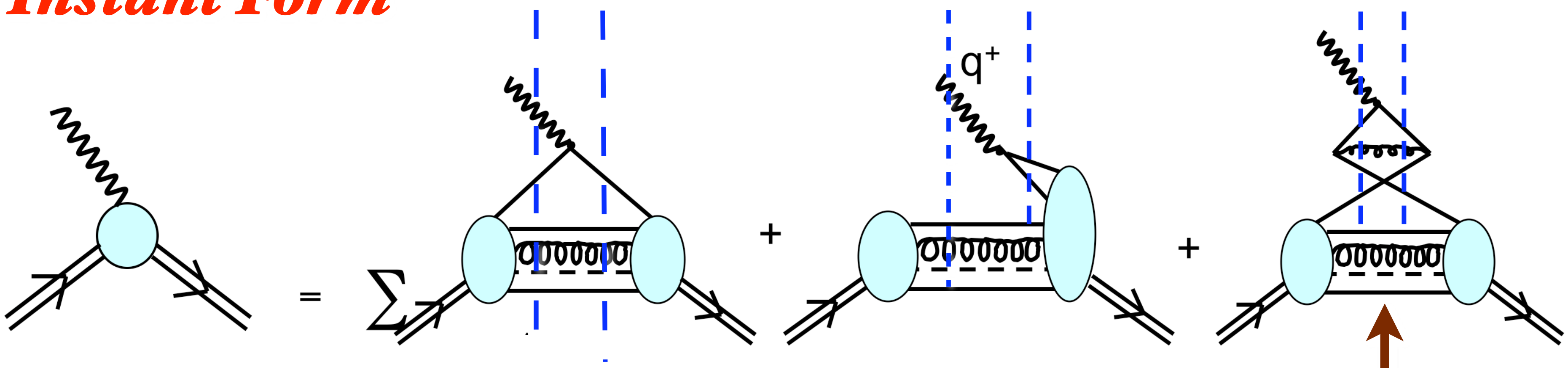


Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Calculation of Form Factors in Equal-Time Theory

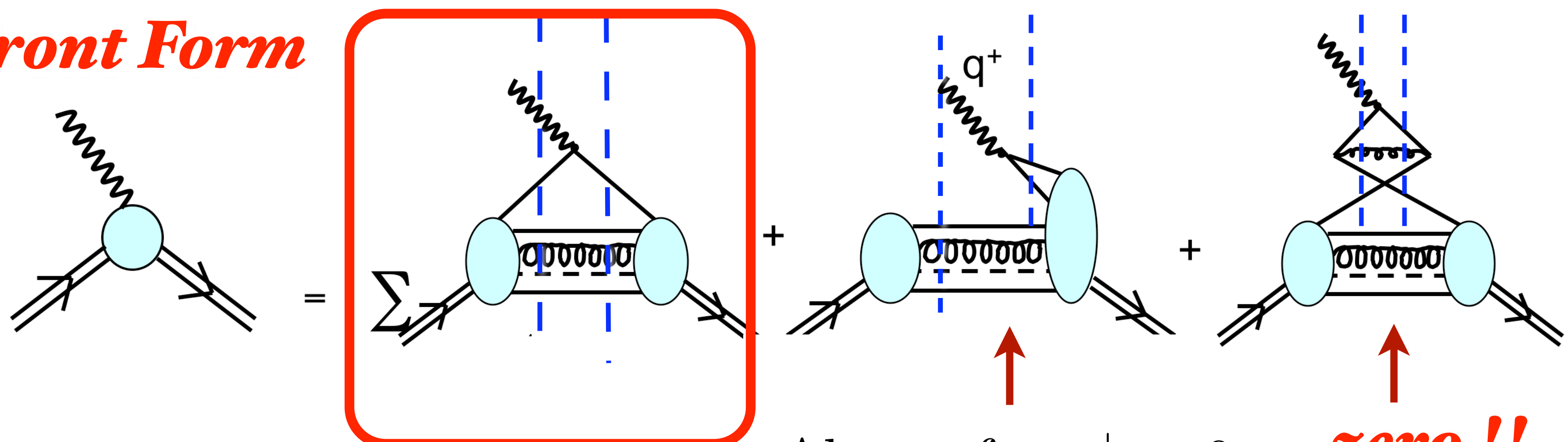
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



Complete Answer

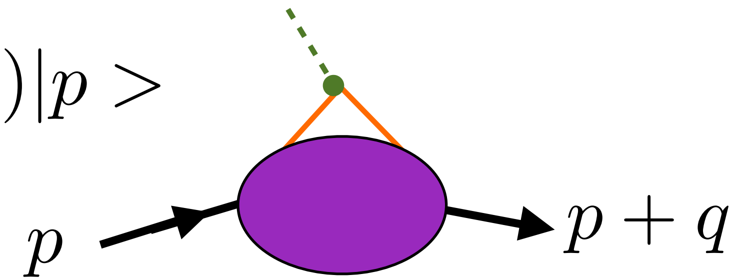
Absent for $q^+ = 0$

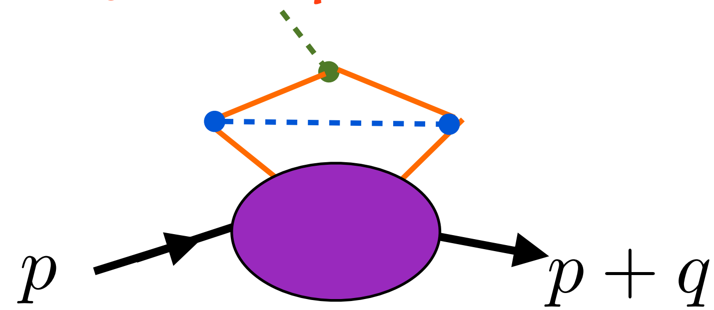
zero !!

No vacuum graphs

Calculation of proton form factor in Instant Form

$$< p+q|J^{\mu}(0)|p >$$





- **Need to boost proton wavefunction from p to p+q: Extremely complicated dynamical problem; even the particle number changes**

- **Need to couple to all currents arising from vacuum!! Remains even after normal-ordering**

- **Each time-ordered contribution is frame dependent**

- **Divide by disconnected vacuum diagrams**

- **Instant form: acausal boundary conditions**

Other Features of Light-Front Wavefunctions

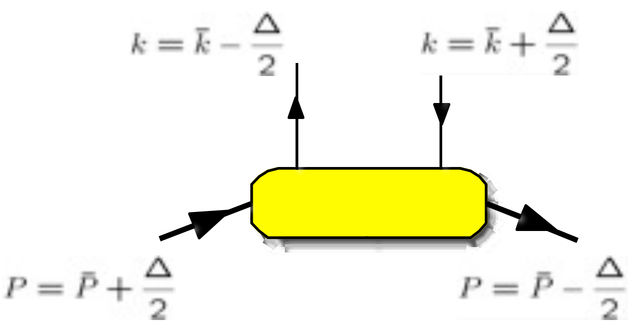
- **Cluster Decomposition Theorem**
- **Zero Anomalous Gravitomagnetic Moment**
- **Angular Momentum J^z**
- **J^z Momentum Sum Rule**
- **Bethe-Salpeter WF integrated over k^-**
- **Electron WFs reproduce pQED results**
- **Parke-Taylor (Stasto)**
- **Gauge Dependent WF but observables are GI**
- **Stable hadron: Real LFWF**

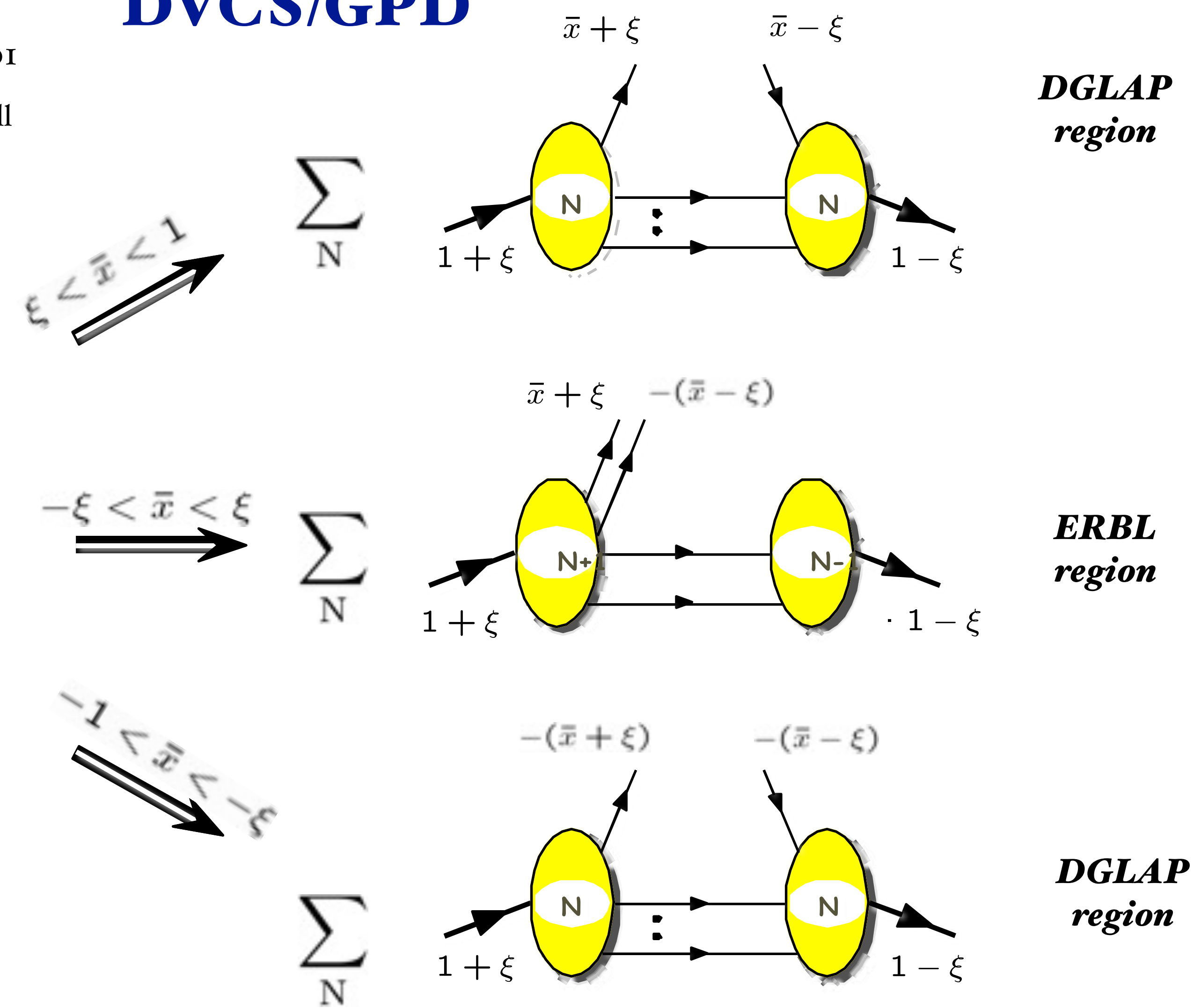
Light-Front Wave Function Overlap Representation

DVCS/GPD

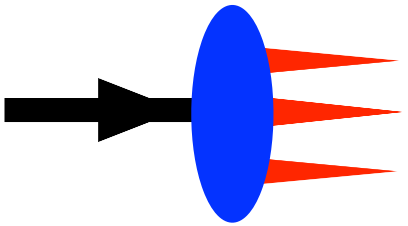
Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll





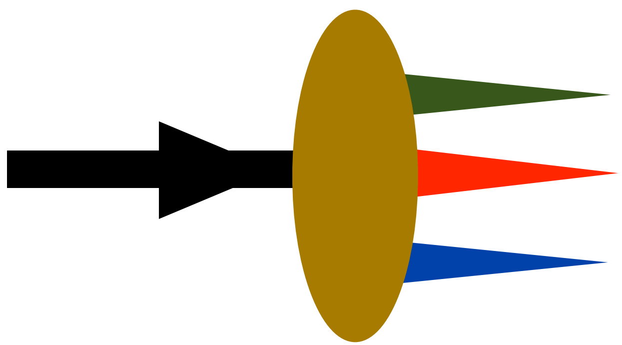
Bakker & Ji
Lorce



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**
- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**
- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs**
- **Cannot compute current matrix elements using instant or point form from eigensolutions alone -- need to include vacuum currents!**
- **Hadron Physics without LFWFs is like Biology without DNA!**

- *Hadron Physics without LFWFs is like Biology without DNA!*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Exact LF Formula for Pauli Form Factor

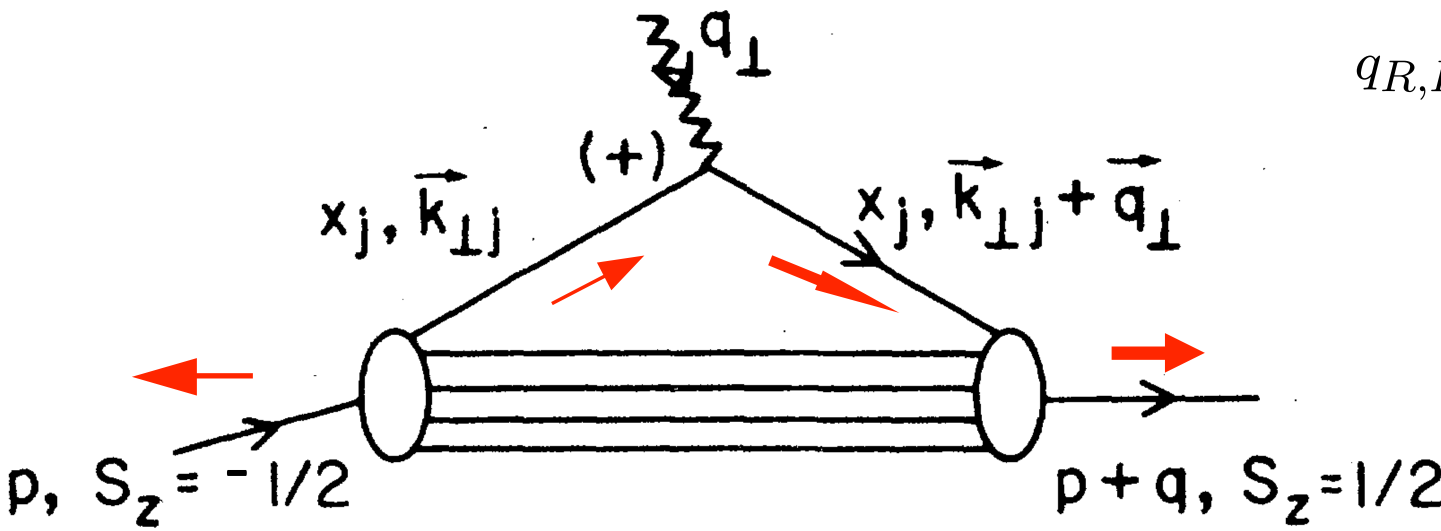
Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

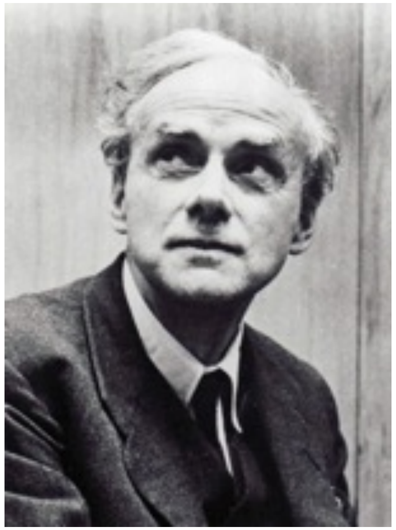
$$q_{R,L} = q^x \pm i q^y$$

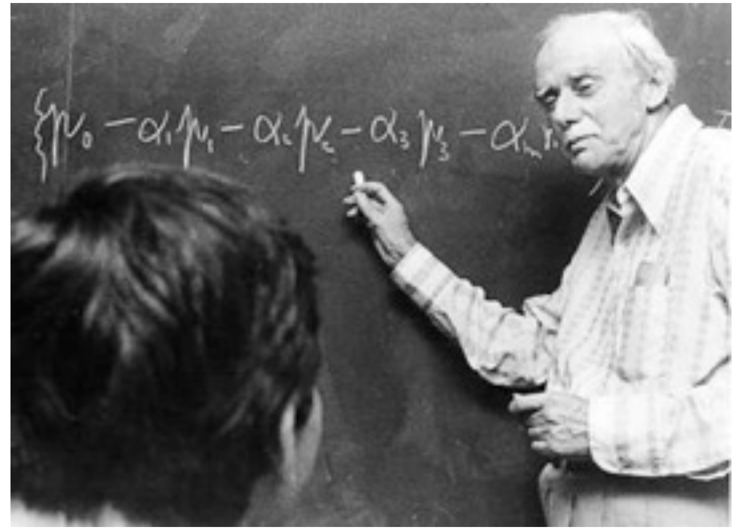


Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)





"Working with the light front is a process that is unfamiliar to physicists.

But still I feel that the mathematical simplification that it introduces is all-important.

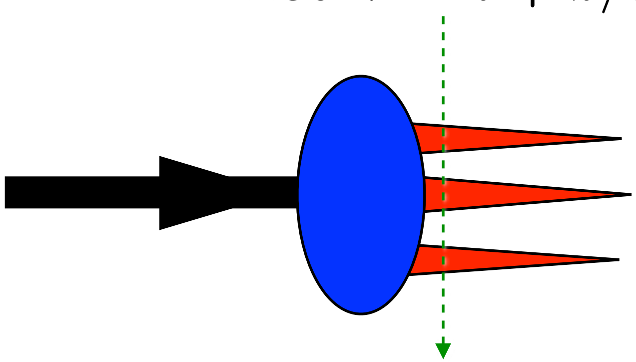
I consider the method to be promising and have recently been making an extensive study of it.

It offers new opportunities, while the familiar instant form seems to be played out " -
P.A.M. Dirac (1977)

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

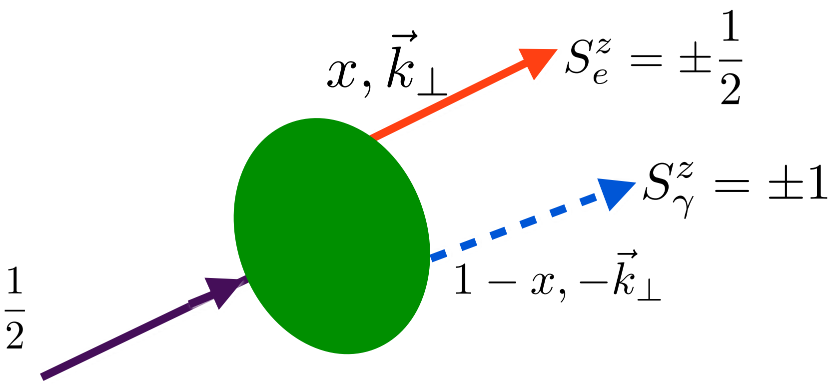
$$H_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

The two-particle Fock state for an electron with $J^z = +\frac{1}{2}$ has four possible spin combinations:

$$\left|\Psi^{\uparrow}_{\text{two particle}}(P^+, \vec{P}_\perp = \vec{0}_\perp)\right\rangle$$
$$= \int \frac{\mathrm{d}^2\vec{k}_\perp\, \mathrm{d}x}{\sqrt{x(1-x)}\, 16\pi^3} \Big[\psi^{\uparrow}_{+\frac{1}{2}+1}(x, \vec{k}_\perp) \left| +\tfrac{1}{2} + 1;\, xP^+, \vec{k}_\perp \right\rangle$$
$$+ \psi^{\uparrow}_{+\frac{1}{2}-1}(x, \vec{k}_\perp) \left| +\tfrac{1}{2} - 1;\, xP^+, \vec{k}_\perp \right\rangle + \psi^{\uparrow}_{-\frac{1}{2}+1}(x, \vec{k}_\perp) \left| -\tfrac{1}{2} + 1;\, xP^+, \vec{k}_\perp \right\rangle$$
$$+ \psi^{\uparrow}_{-\frac{1}{2}-1}(x, \vec{k}_\perp) \left| -\tfrac{1}{2} - 1;\, xP^+, \vec{k}_\perp \right\rangle \Big],$$



$$\begin{cases} \psi^{\uparrow}_{+\frac{1}{2}+1}(x, \vec{k}_\perp) = -\sqrt{2}\, \dfrac{(-k^1 + \mathrm{i}k^2)}{x(1-x)}\, \varphi, \\ \psi^{\uparrow}_{+\frac{1}{2}-1}(x, \vec{k}_\perp) = -\sqrt{2}\, \dfrac{(+k^1 + \mathrm{i}k^2)}{1-x}\, \varphi, \\ \psi^{\uparrow}_{-\frac{1}{2}+1}(x, \vec{k}_\perp) = -\sqrt{2} \left(M - \dfrac{m}{x} \right) \varphi, \\ \psi^{\uparrow}_{-\frac{1}{2}-1}(x, \vec{k}_\perp) = 0, \end{cases}$$

$$\varphi = \varphi(x, \vec{k}_\perp) = \frac{e/\sqrt{1-x}}{M^2 - (\vec{k}^2_\perp + m^2)/x - (\vec{k}^2_\perp + \lambda^2)/(1-x)}.$$

Hwang, Schmidt, Ma, sjb

$$
\begin{aligned}
F_2(q^2) &= \frac{-2M}{(q^1 - \mathrm{i}q^2)} \left\langle \Psi^{\uparrow}\left(P^+, \vec{P}_\perp = \vec{q}_\perp\right) \middle| \Psi^{\downarrow}\left(P^+, \vec{P}_\perp = \vec{0}_\perp\right) \right\rangle \\
&= \frac{-2M}{(q^1 - \mathrm{i}q^2)} \int \frac{\mathrm{d}^2\vec{k}_\perp\, \mathrm{d}x}{16\pi^3} \Big[\psi^{\uparrow\,*}_{+\frac{1}{2}-1}\left(x, \vec{k}'_\perp\right) \psi^{\downarrow}_{+\frac{1}{2}-1}\left(x, \vec{k}_\perp\right) \\
&\qquad\qquad + \psi^{\uparrow\,*}_{-\frac{1}{2}+1}\left(x, \vec{k}'_\perp\right) \psi^{\downarrow}_{-\frac{1}{2}+1}\left(x, \vec{k}_\perp\right) \Big] \\
&= 4M \int \frac{\mathrm{d}^2\vec{k}_\perp\, \mathrm{d}x}{16\pi^3} \frac{(m - Mx)}{x} \varphi\left(x, \vec{k}'_\perp\right)^* \varphi\left(x, \vec{k}_\perp\right) \\
&= 4Me^2 \int \frac{\mathrm{d}^2\vec{k}_\perp\, \mathrm{d}x}{16\pi^3} \frac{(m - xM)}{x(1-x)} \\
&\quad \times \frac{1}{M^2 - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + m^2)/x - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + \lambda^2)/(1-x)} \\
&\quad \times \frac{1}{M^2 - (\vec{k}^2_\perp + m^2)/x - (\vec{k}^2_\perp + \lambda^2)/(1-x)}. \qquad (30)
\end{aligned}
$$

$$
F_2(q^2) = \frac{Me^2}{4\pi^2} \int_0^1 \mathrm{d}\alpha \int_0^1 \mathrm{d}x \, \frac{m - xM}{\alpha(1-\alpha)\frac{1-x}{x}\vec{q}^{\,2}_\perp - M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}}.
$$

The anomalous moment is obtained in the limit of zero momentum transfer:

$$F_2(0) = 4Me^2 \int \frac{\mathrm{d}^2\vec{k}_\perp\, \mathrm{d}x}{16\pi^3} \frac{(m - xM)}{x(1-x)} \frac{1}{[M^2 - (\vec{k}_\perp^{\,2} + m^2)/x - (\vec{k}_\perp^{\,2} + \lambda^2)/(1-x)]^2}$$

$$= \frac{Me^2}{4\pi^2} \int_0^1 \mathrm{d}x \frac{m - xM}{-M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}}, \qquad (32)$$

which is the result of Ref. [8]. For zero photon mass and $M = m$, it gives the correct order α Schwinger value $a_e = F_2(0) = \alpha/2\pi$ for the electron anomalous magnetic moment for QED.

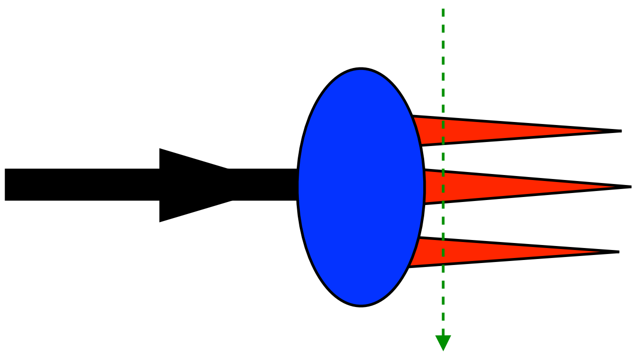
$$a_e = F_2(0) = \frac{\alpha}{2\pi}$$

Simpler than Feynman/Schwinger

Anomalous gravitomagnetic moment vanishes: $\frac{\alpha}{3\pi} - \frac{\alpha}{3\pi} = 0$

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

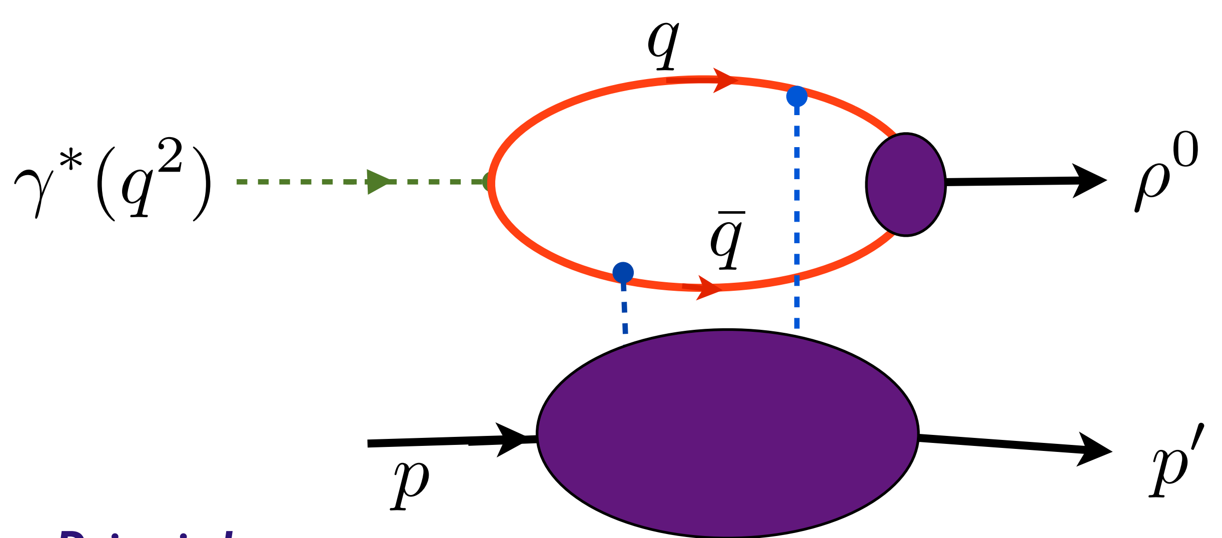
Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Diffractive leptoproduction of vector mesons in QCD

Stanley J. Brodsky (SLAC) , L. Frankfurt (Tel Aviv U.) , J.F. Gunion (UC, Davis) , Alfred H. Mueller (Columbia U.) , M. Strikman (Penn State U.)

Jan 1994 - 34 pages

Phys.Rev. D50 (1994) 3134-3144
DOI: 10.1103/PhysRevD.50.3134
SLAC-PUB-6412, CU-TP-617, UCD-93-36
e-Print: hep-ph/9402283 | PDF

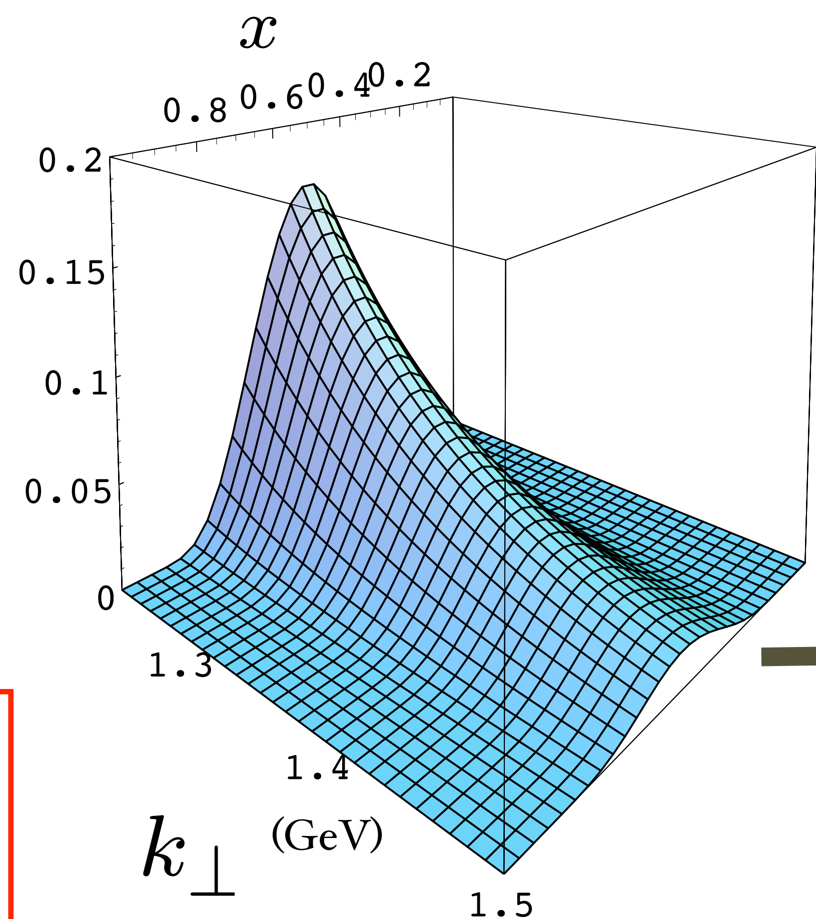


- *Factorization Principle*
- *LF Wave Function, Distribution Amplitude*
- *s, 1/Q⁶ dependence,* σ_L/σ_T

Prediction from AdS/QCD: Meson LFWF

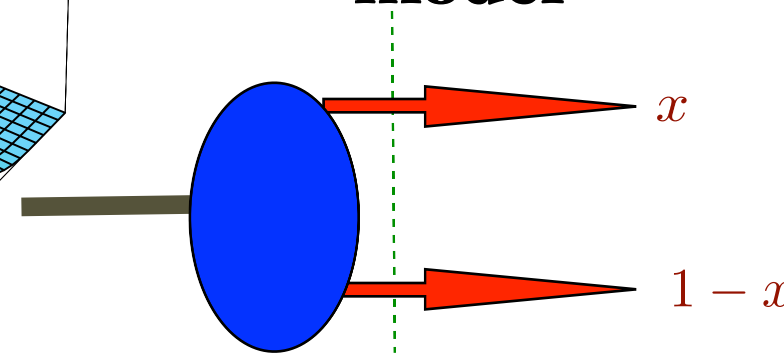
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Same as DSE!

Provides Connection of Confinement to Hadron Structure

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

J. R. Forshaw*

Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester, Oxford Road, Manchester M13 9PL, United Kingdom

R. Sandapen†

Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A3E9, Canada

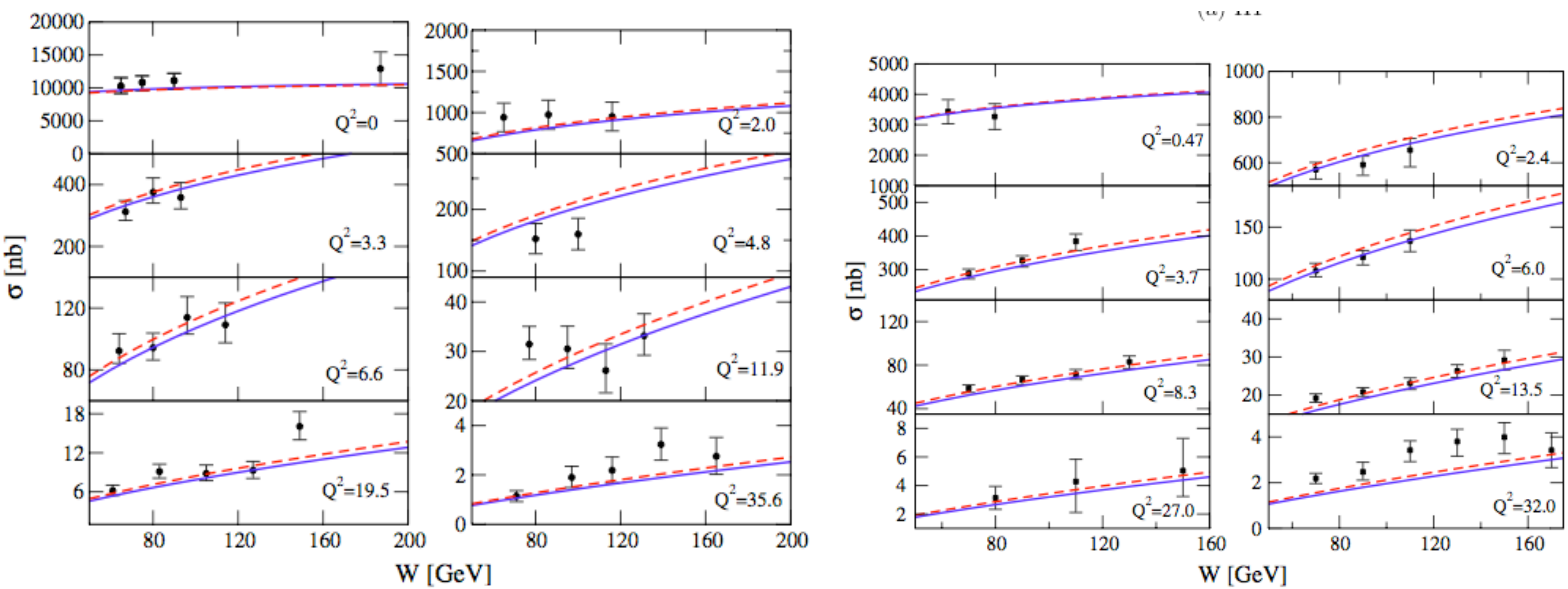
(Received 5 April 2012; published 20 August 2012)

We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

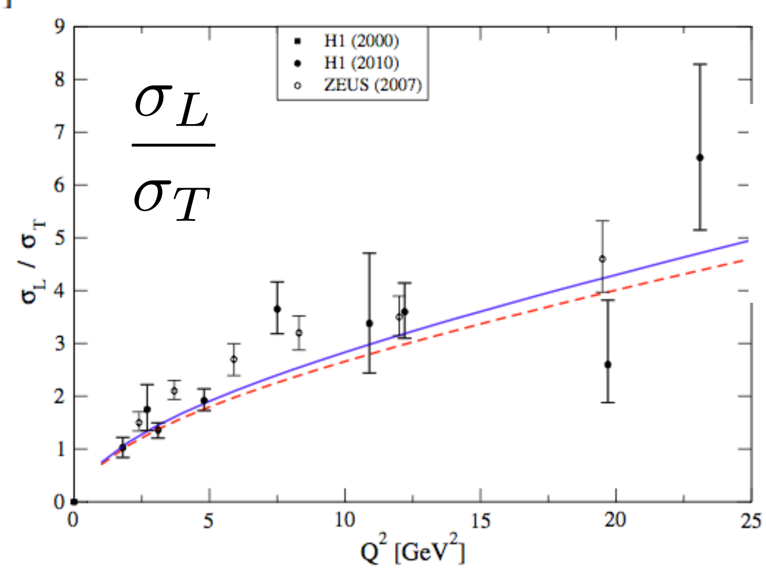
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

See also Ferreira and Dosch

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H (b) ZEUS



$$\frac{\sigma_L}{\sigma_T}$$

J. R. Forshaw, R. Sandapen

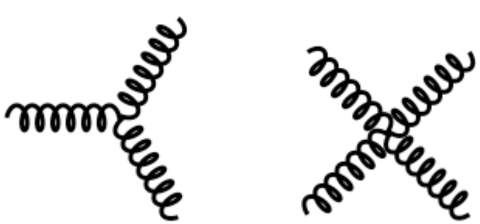
$$\gamma^* p \rightarrow \rho^0 p'$$

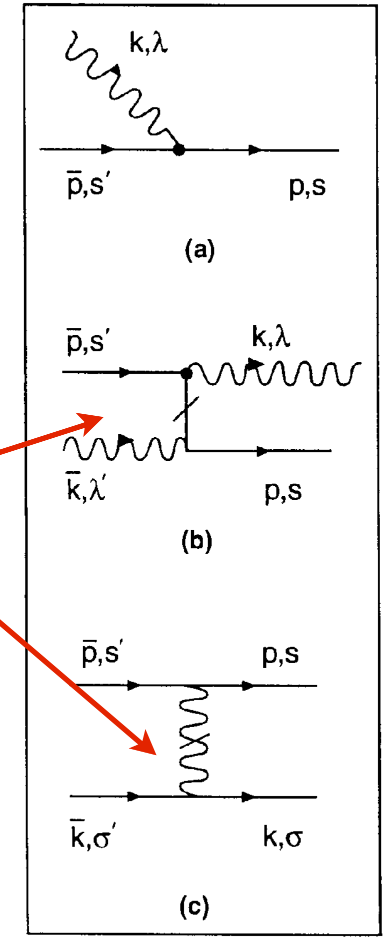
$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

See also Ferreira and Dosch

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$\begin{aligned} H^{LF}_{QCD} &= \frac{1}{2}\int d^3x \overline{\tilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\tilde{\psi} - A^i_a(\mathrm{i}\partial^\perp)^2 A_{ia} \\ &- \frac{1}{2}g^2 \int d^3x \mathbf{Tr}\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\left[\tilde{A}_\mu, \tilde{A}_\nu\right] \\ &+ \frac{1}{2}g^2 \int d^3x \overline{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \frac{1}{(\mathrm{i}\partial^+)^2} \overline{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \\ &- g^2 \int d^3x \overline{\tilde{\psi}}\gamma^+ \left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right)\tilde{\psi} \\ &+ g^2 \int d^3x \mathbf{Tr}\left(\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right) \\ &+ \frac{1}{2}g^2 \int d^3x \overline{\tilde{\psi}}\tilde{\not{A}}\frac{\gamma^+}{\mathrm{i}\partial^+}\tilde{\not{A}}\tilde{\psi} \\ &+ g \int d^3x \overline{\tilde{\psi}}\tilde{\not{A}}\tilde{\psi} \\ &+ 2g \int d^3x \mathbf{Tr}\left(\mathrm{i}\partial^\mu \tilde{A}^\nu \left[\tilde{A}_\mu, \tilde{A}_\nu\right]\right) \end{aligned}$$





Physical gauge: $A^+ = 0$

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

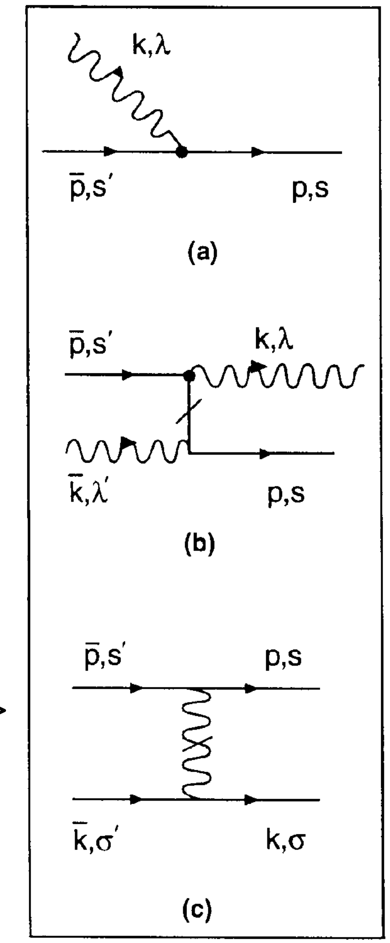
$$L^{QCD} \to H^{QCD}_{LF}$$

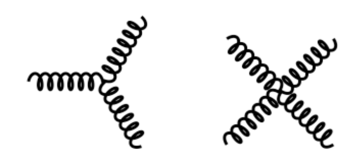
$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$





H^{int}_{LF}

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

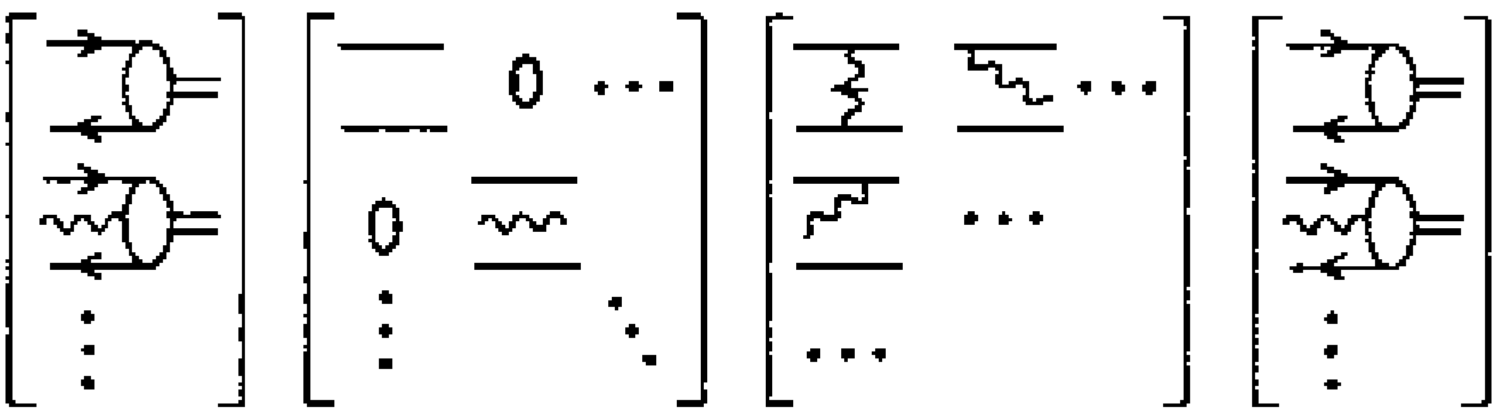
LFWFs: Off-shell in P- and invariant mass

LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\,V\,|q\bar{q}\rangle & \langle q\bar{q}|\,V\,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\,V\,|q\bar{q}\rangle & \langle q\bar{q}g|\,V\,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

$A^+ = 0$



Minkowski space; frame-independent; no fermion doubling; no ghosts

- Light-Front Vacuum = vacuum of free Hamiltonian!

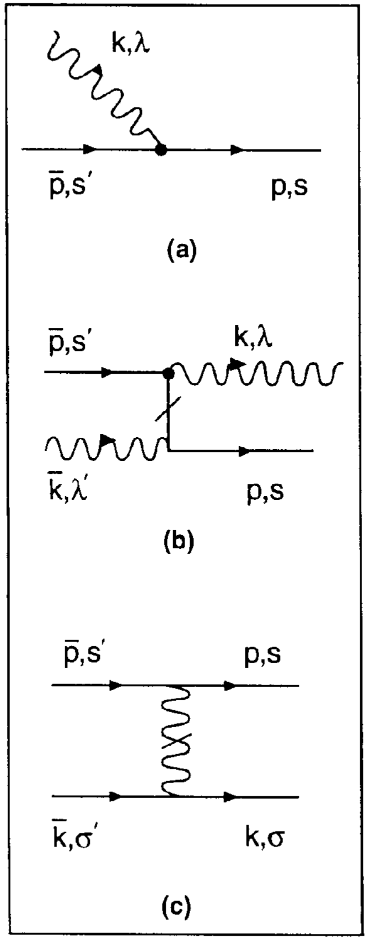
Light-Front QCD

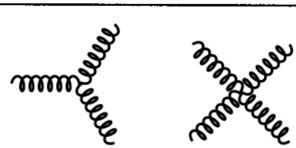
Heisenberg Equation

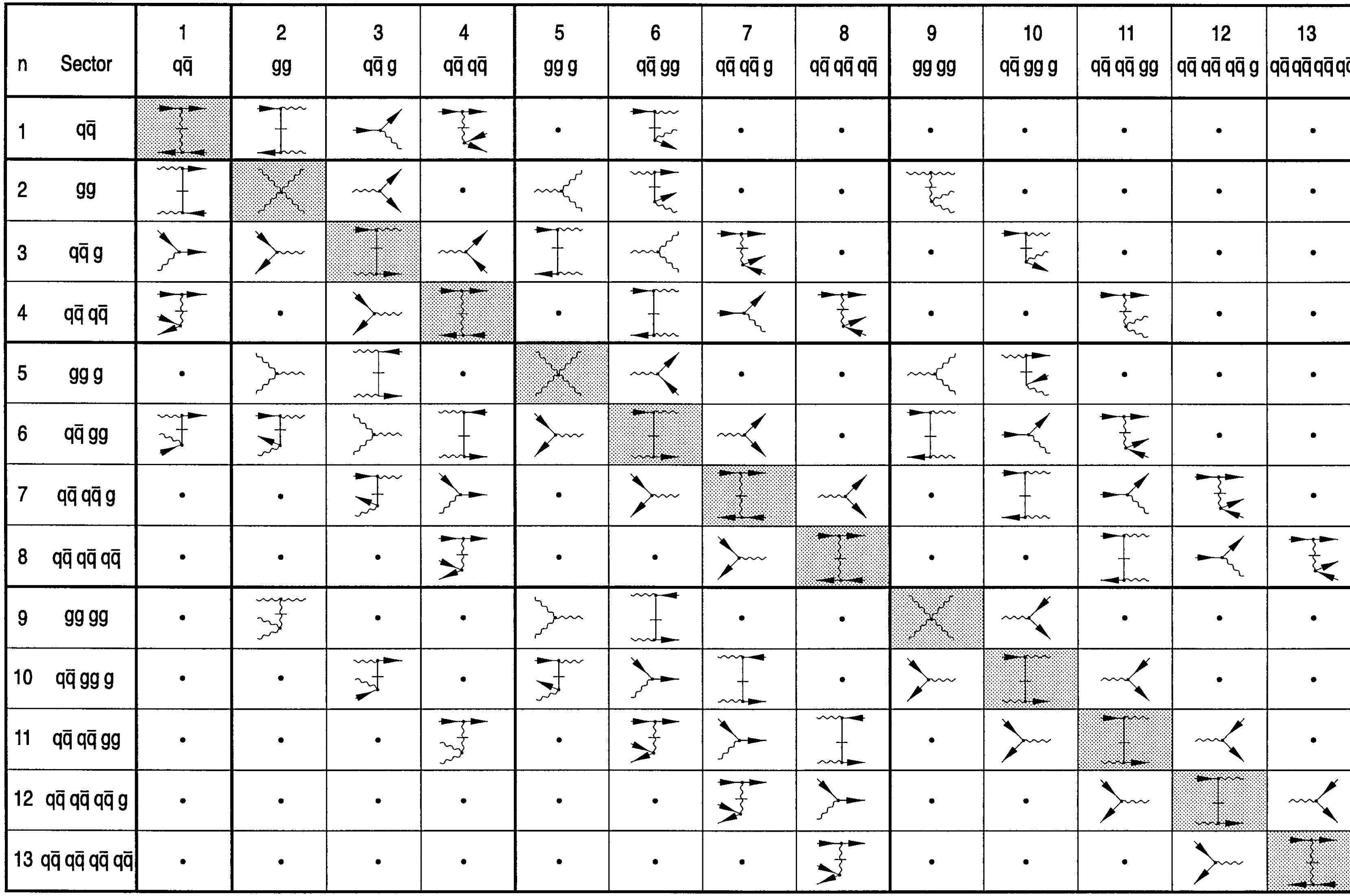
$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 \, |\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb



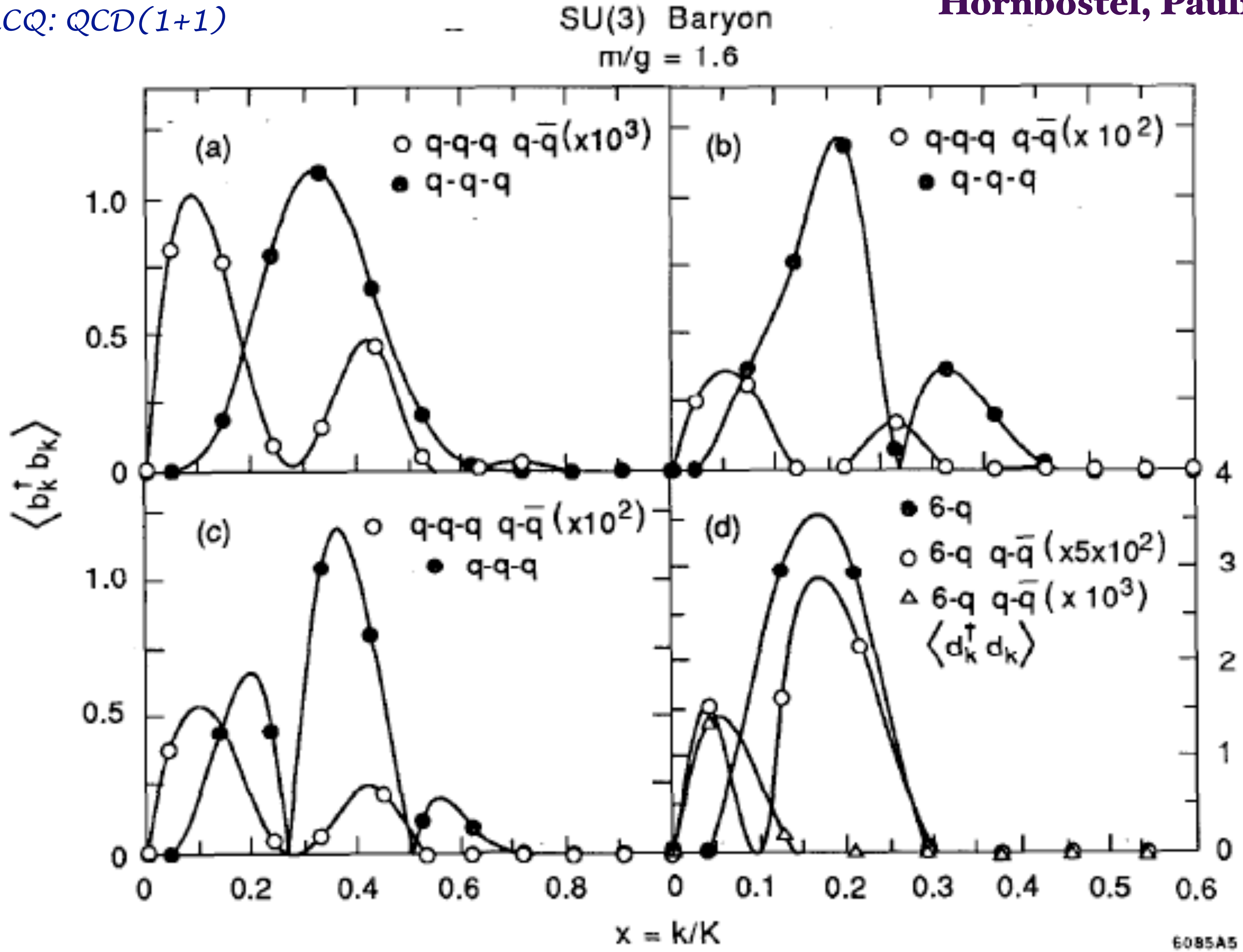




Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

DLCQ: QCD(1+1)

Hornbostel, Pauli, sjb



a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Light-Front Perturbative QCD

- Calculate T matrix $$T = V + V \frac{1}{D + i\epsilon} T$$

$$D = \mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate}$$

Sum over intermediate states

All propagating particles have positive $k^+ = k^0 + k^3 > 0$

PHYSICAL REVIEW D VOLUME 22, NUMBER 9 1 NOVEMBER 1980

Exclusive processes in perturbative quantum chromodynamics

G. Peter Lepage
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Stanley J. Brodsky
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 27 May 1980)

We present a systematic analysis in perturbative quantum chromodynamics (QCD) of large-momentum-transfer exclusive processes. Predictions are given for the scaling behavior, angular dependence, helicity structure, and normalization of elastic and inelastic form factors and large-angle exclusive scattering amplitudes for hadrons and photons. We prove that these reactions are dominated by quark and gluon subprocesses at short distances, and thus that the dimensional-counting rules for the power-law falloff of these amplitudes with momentum transfer are rigorous predictions of QCD, modulo calculable logarithmic corrections from the behavior of the hadronic wave functions at short distances. These anomalous-dimension corrections are determined by evolution equations for process-independent meson and baryon "distribution amplitudes" $\phi(x_i,Q)$ which control the valence-quark distributions in high-momentum-transfer exclusive reactions. The analysis can be carried out systematically in powers of $\alpha_s(Q^2)$, the QCD running coupling constant. Although the calculations are most conveniently carried out using light-cone perturbation theory and the light-cone gauge, we also present a gauge-independent analysis and relate the distribution amplitude to a gauge-invariant Bethe-Salpeter amplitude.

Matrix element $\bar{u}_{\lambda'} \cdots u_{\lambda}$	Helicity ($\lambda \to \lambda'$) $\uparrow \to \uparrow$, $\downarrow \to \downarrow$	$\uparrow \to \downarrow$, $\downarrow \to \uparrow$
$\frac{\bar{u}(p)}{(p^+)^{1/2}} \gamma^+ \frac{u(q)}{(q^+)^{1/2}}$	2	0
$\frac{\bar{u}(p)}{(p^+)^{1/2}} \gamma^- \frac{u(q)}{(q^+)^{1/2}}$	$\frac{2}{p^+q^+}(p_\perp \cdot q_\perp \pm i p_\perp \times q_\perp + m^2)$	$\mp \frac{2m}{p^+q^+}\left[(p^1 \pm i p^2) - (q^1 \pm i q^2)\right]$
$\frac{\bar{u}(p)}{(p^+)^{1/2}} \gamma_\perp^i \frac{u(q)}{(q^+)^{1/2}}$	$\frac{p_\perp^i \mp i\epsilon^{ij} p_\perp^j}{p^+} + \frac{q_\perp^i \pm i\epsilon^{ij} q_\perp^j}{q^+}$	$\mp m \left(\frac{p^+ - q^+}{p^+q^+}\right)(\delta^{il} \pm i\delta^{i2})$
$\frac{\bar{u}(p)}{(p^+)^{1/2}} \frac{u(q)}{(q^+)^{1/2}}$	$m\left(\frac{p^+ + q^+}{p^+q^+}\right)$	$\mp\left(\frac{p^1 \pm i p^2}{p^+} - \frac{q^1 \pm i q^2}{q^+}\right)$
$\frac{\bar{u}(p)}{(p^+)^{1/2}} \gamma^- \gamma^+ \gamma^- \frac{u(q)}{(q^+)^{1/2}}$	$\frac{8}{p^+q^+}(p_\perp \cdot q_\perp \pm i p_\perp \times q_\perp + m^2)$	$\mp \frac{8m}{p^+q^+}[(p^1 \pm i p^2) - (q^1 \pm i q^2)]$
$\frac{\bar{u}(p)}{(p^+)^{1/2}} \gamma^- \gamma^+ \gamma_\perp^i \frac{u(q)}{(q^+)^{1/2}}$	$4\left(\frac{p_\perp^i \mp i\epsilon^{ij} p_\perp^j}{p^+}\right)$	$\pm \frac{4m}{p^+}(\delta^{il} \pm i\delta^{i2})$
$\frac{\bar{u}(p)}{(p^+)^{1/2}} \gamma_\perp^i \gamma^+ \gamma^- \frac{u(q)}{(q^+)^{1/2}}$	$4\left(\frac{q_\perp^i \pm i\epsilon^{ij} q_\perp^j}{q^+}\right)$	$\mp \frac{4m}{q^+}(\delta^{il} \pm i\delta^{i2})$
$\frac{\bar{u}(p)}{(p^+)^{1/2}} \gamma_\perp^i \gamma^+ \gamma_\perp^j \frac{u(q)}{(q^+)^{1/2}}$	$2(\delta^{ij} \pm i\epsilon^{ij})$	0

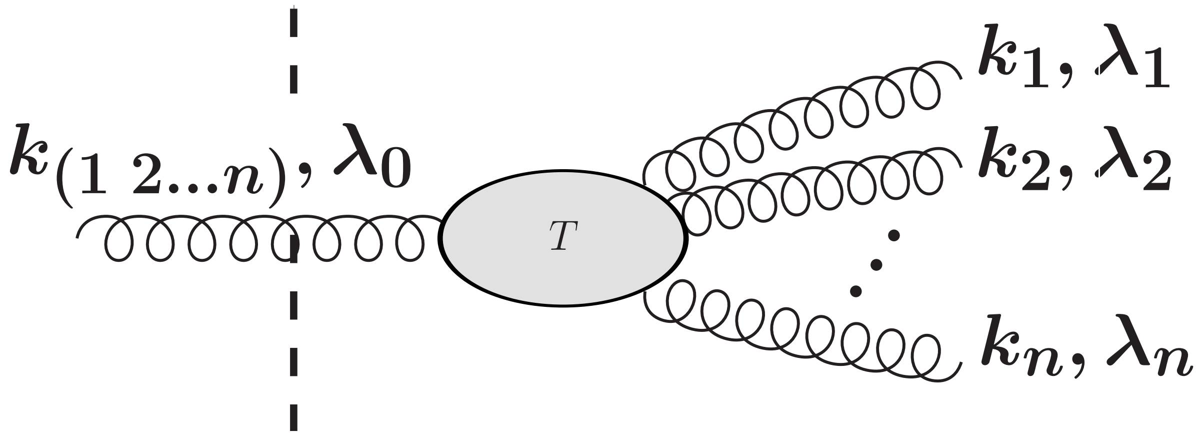
$$\bar{v}_\mu(p)\gamma^\alpha v_\nu(q) = \bar{u}_\nu(q)\gamma^\alpha u_\mu(p) \qquad \bar{v}_\mu(p) v_\nu(q) = -\bar{u}_\nu(q) u_\mu(p)$$

$$\bar{v}_\mu(p)\gamma^\alpha\gamma^\beta\gamma^\delta v_\nu(q) = \bar{u}_\nu(q)\gamma^\delta\gamma^\beta\gamma^\alpha u_\mu(p)$$

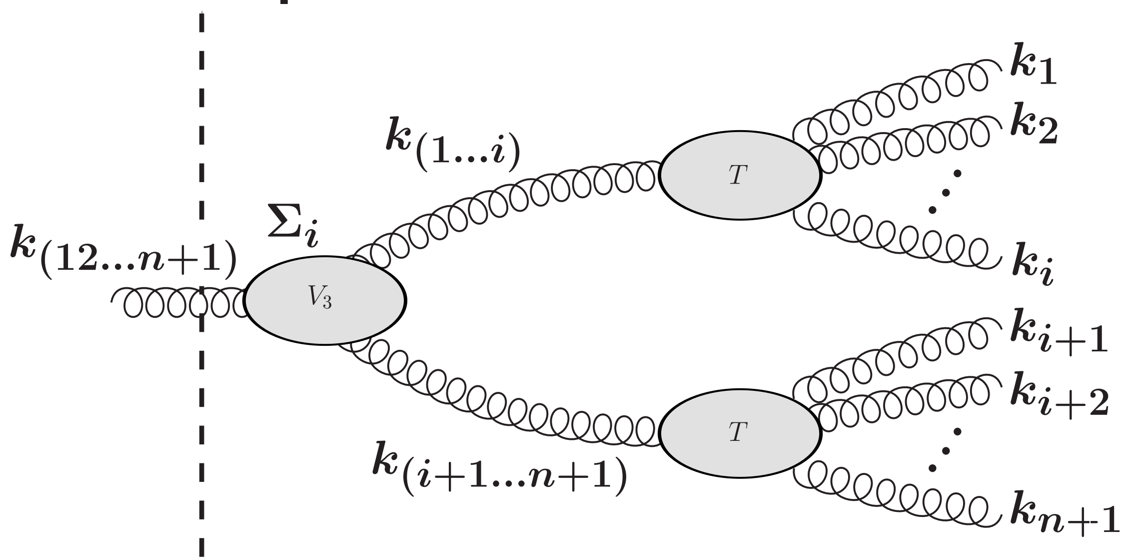
Recursion Relations and Scattering Amplitudes in the Light-Front Formalism

Cruz-Santiago & Stasto

Cluster Decomposition Theorem for relativistic systems: **C. Ji & sjb**



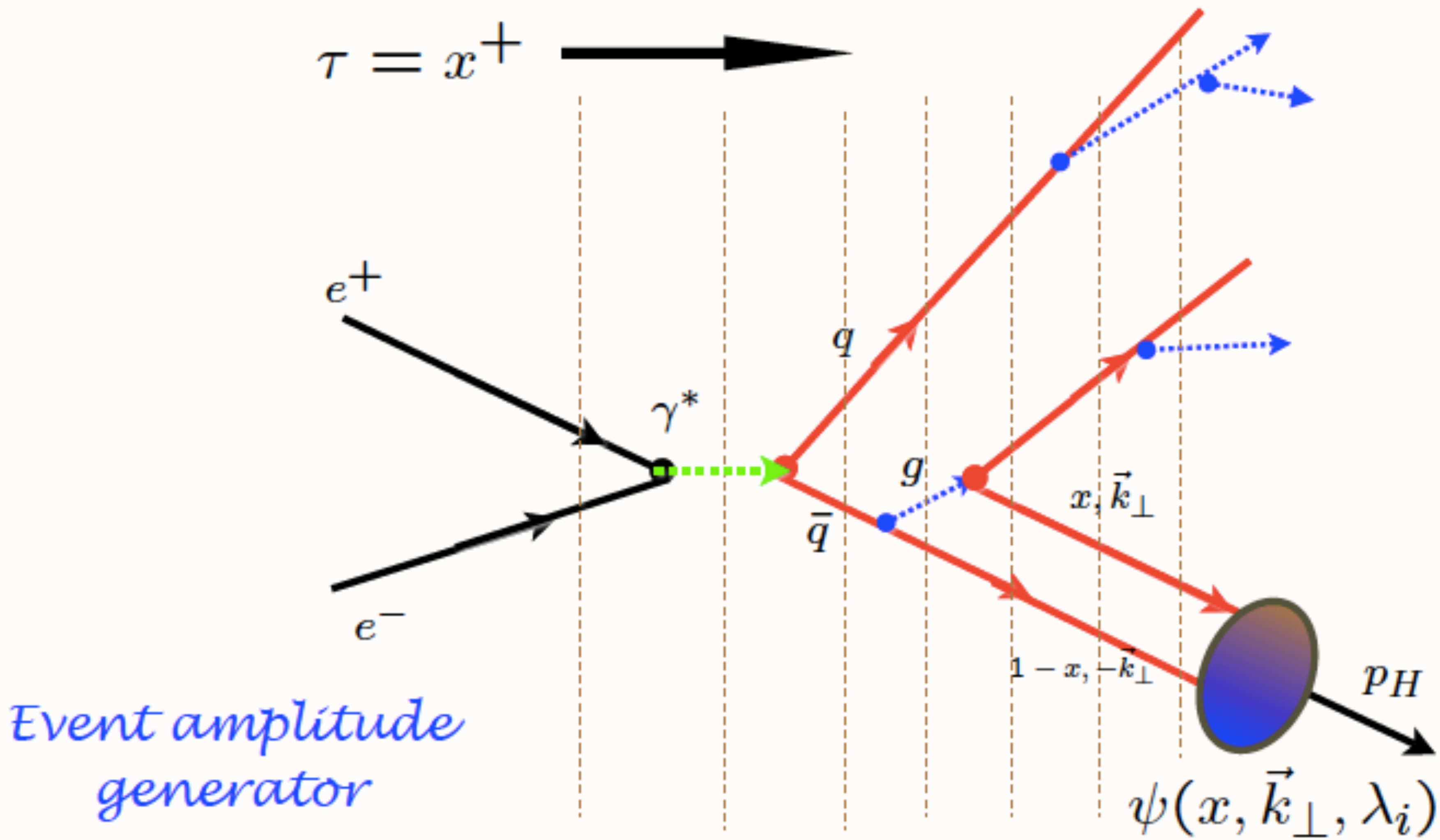
$T_{n+1} \quad =$



Parke-Taylor amplitudes reflect LF angular momentum conservation

$$\langle i j \rangle = \sqrt{z_i z_j}\, \underline{\epsilon}^{(-)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right) =$$

Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

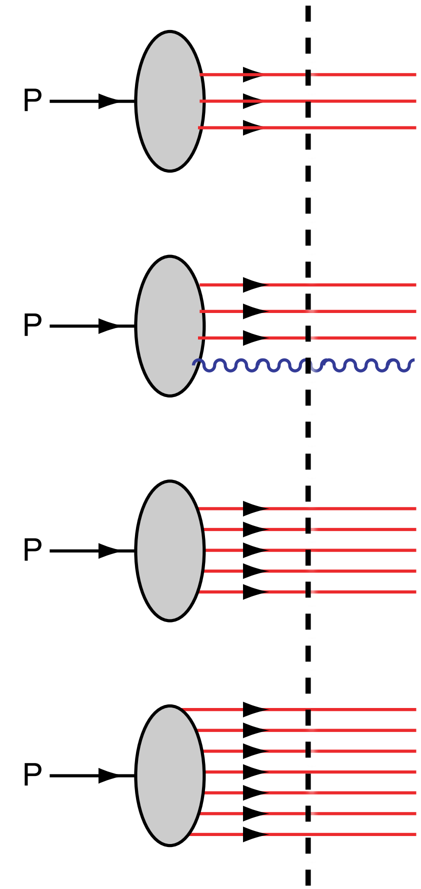
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \; \sum_i^n x_i = 1, \; \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

$$\tau = t + z/c$$

Intrinsic heavy quarks

s(x), c(x), b(x) at high x !

$$\bar{s}(x) \neq s(x)$$
$$\bar{u}(x) \neq \bar{d}(x)$$

Deuteron: Hidden Color

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six quark wavefunction:

- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>

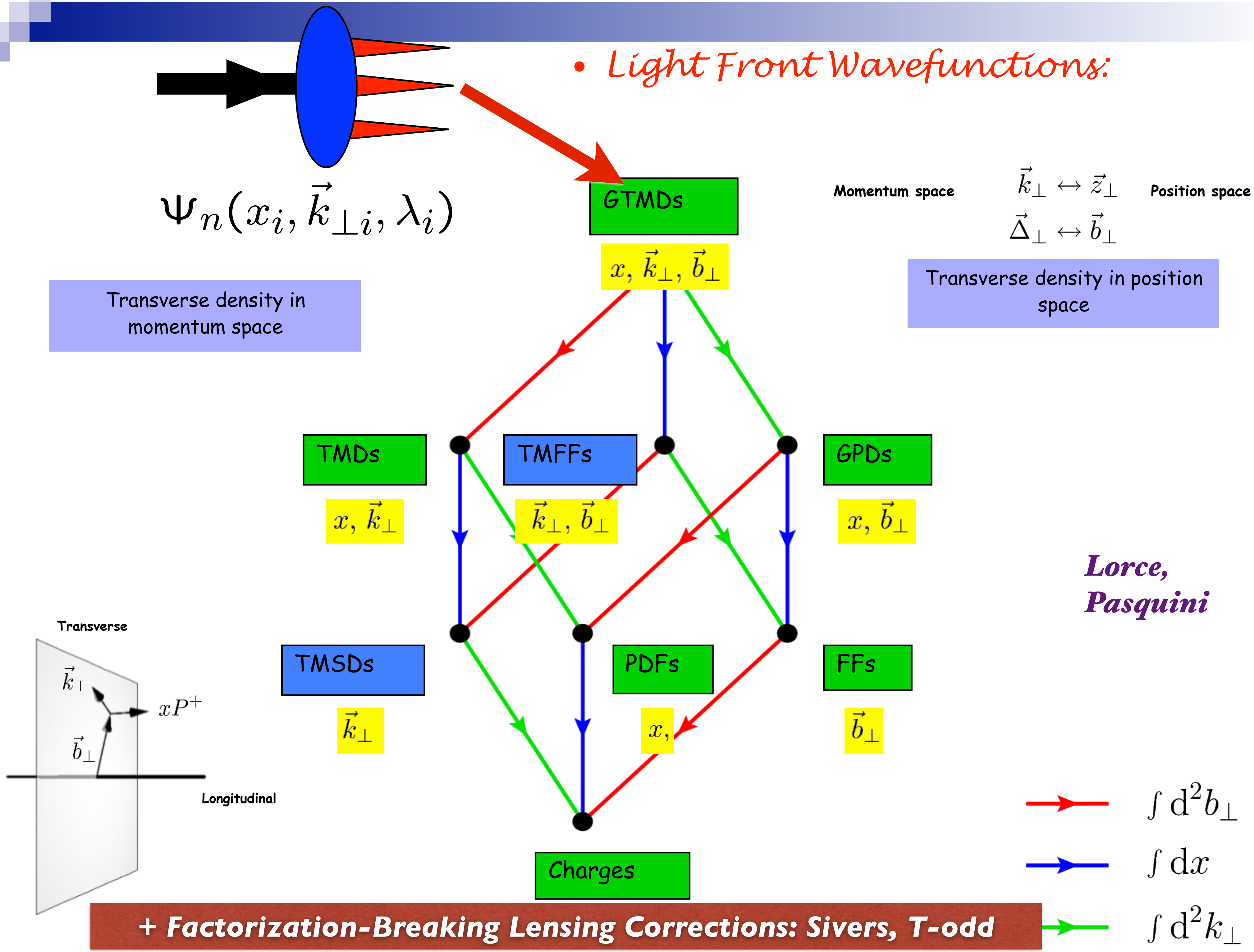
- Components evolve towards equality at short distances

- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer

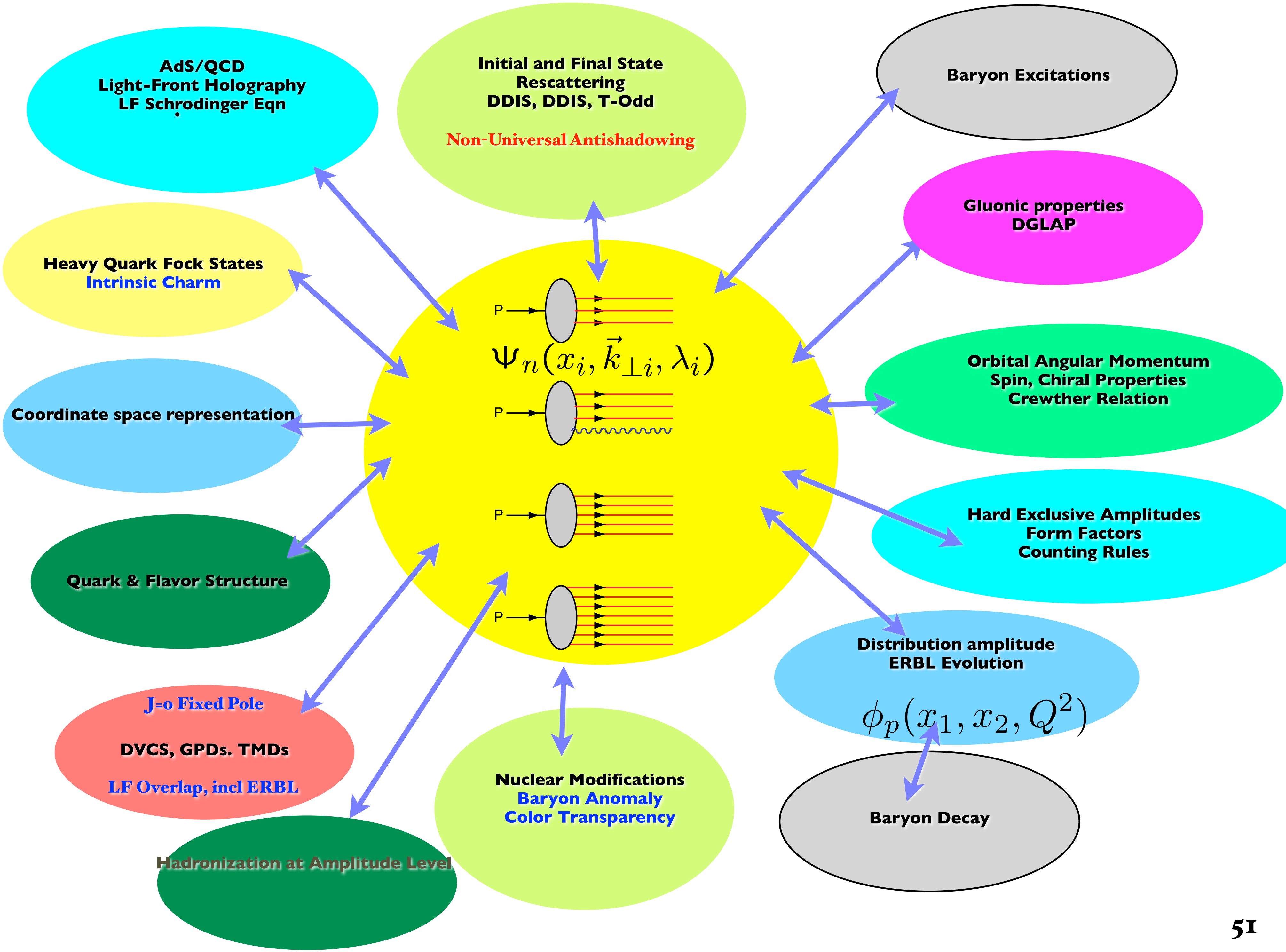
- Predict

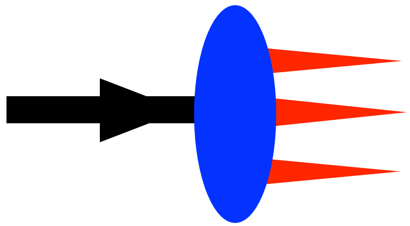
$$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn) \text{ at high } Q^2$$

- *Light Front Wavefunctions:*



QCD and the LF Hadron Wavefunctions





$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**

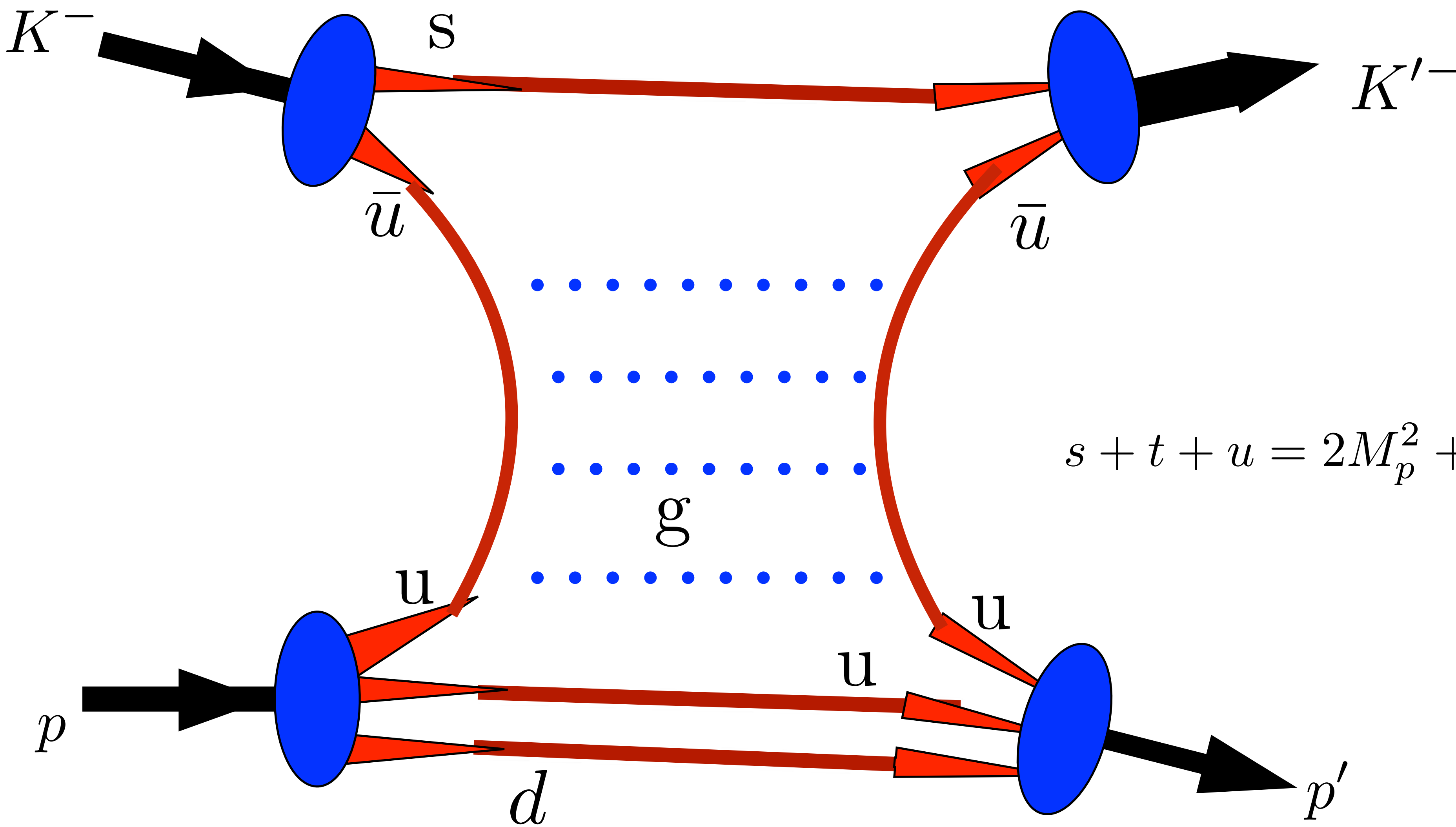
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**

- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs**

- **Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!**

- **Hadron Physics without LFWFs is like Biology without DNA!**

Reggeon Exchange in the t-channel

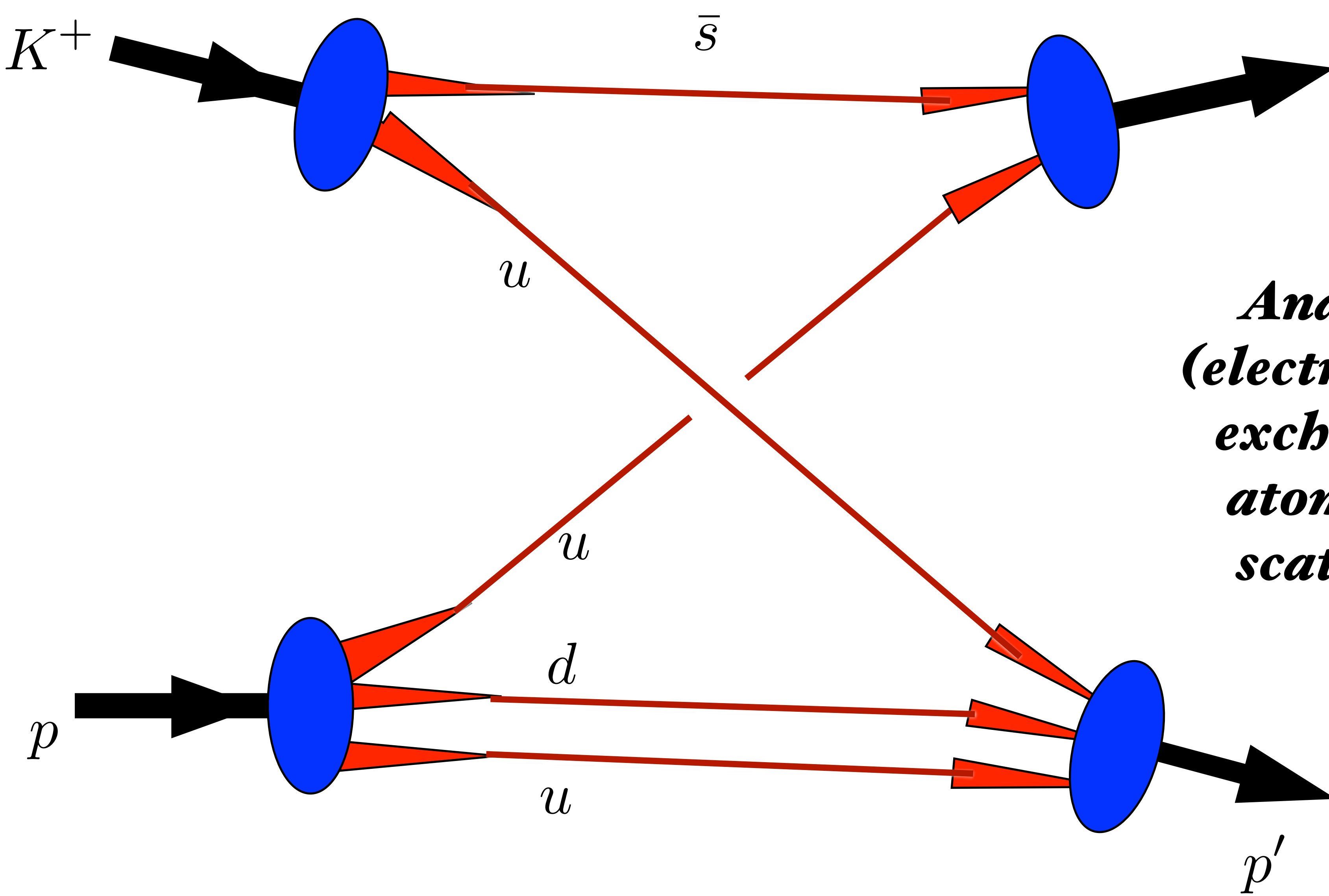


$$s + t + u = 2M_p^2 + 2M_K^2$$

$$\mathcal{M}_R \sim s^{\alpha_R(t)} F_R(t) \frac{1}{2}[e^{-i\pi\alpha_R(t)} \pm 1]$$

Signature factor $C = \pm 1$

easiest to compute u-channel exchange



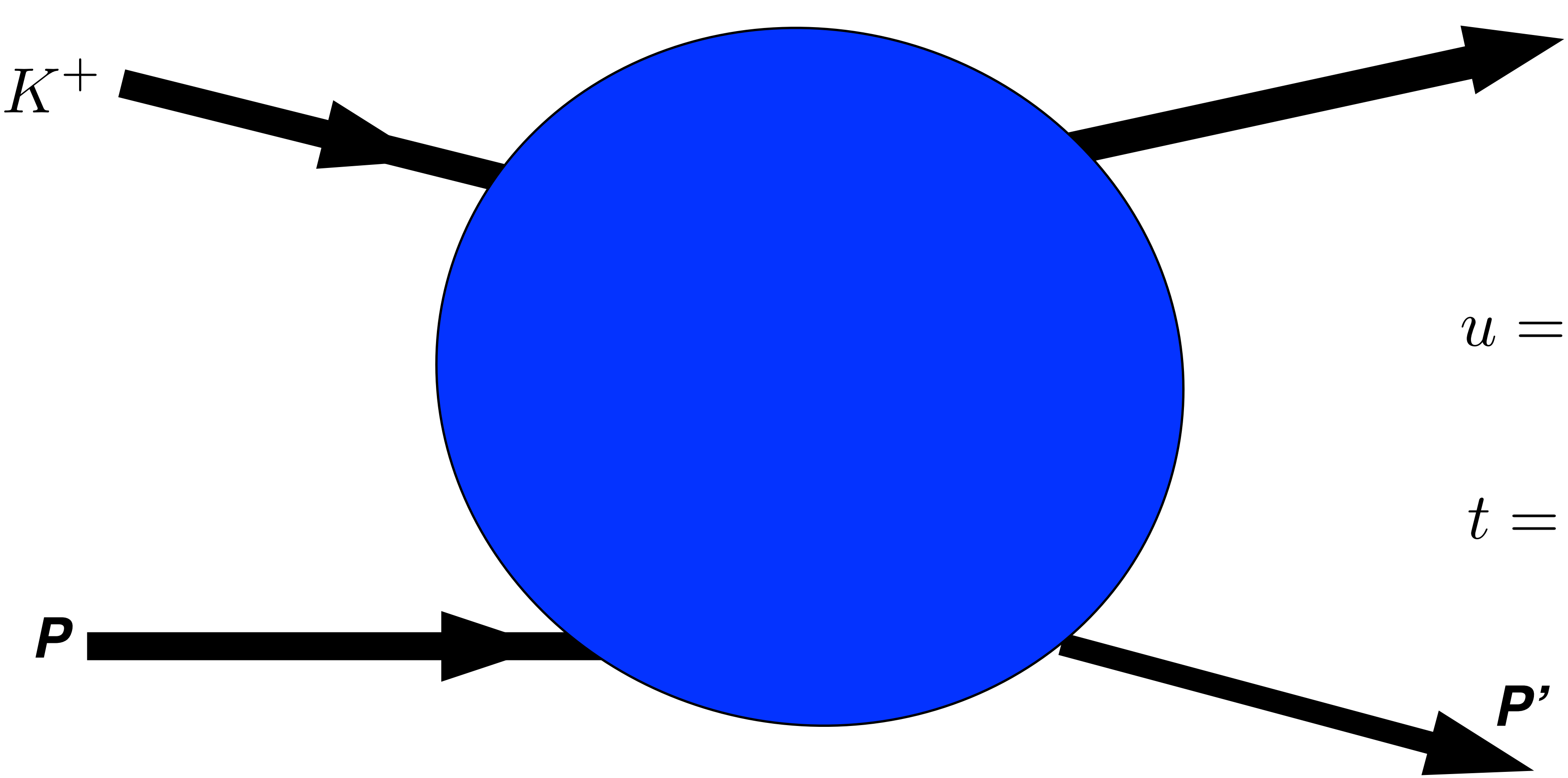
Analog of (electron) spin exchange in atom atom scattering

Constituent Interchange Model (CIM)

Blankenbecler, Gunion, sjb (1972)

$$K^\mu = (P^+, \frac{M_K^2 + r_\perp^2 + q_\perp^2}{P^+}, \vec{q}_\perp + \vec{r}_\perp)$$

$$K^{\mu\prime} = (P^+, \frac{M_K^2 + r_\perp^2}{P^+}, \vec{r}_\perp)$$



$$u = -\vec{r}_\perp^{\,2}$$

$$t = -\vec{q}_\perp^{\,2}$$

$$P^\mu = (P^+, P^-, \vec{P}_\perp) = (P^+, \frac{M_p^2}{P^+}, \vec{0}_\perp)$$

$$P^{\mu\prime} = (P^+, \frac{M_p^2 + q_\perp^2}{P^+}, \vec{q}_\perp)$$

Remarkable Light-Front Frame

Bj: "Fool's ISR Frame"

Ideal for
QCD factorization proofs
Single A+=0 Gauge

$$P^{\pm} \equiv P^0 \pm P^3$$

$$P^2 = P^+P^- - \vec{P}_\perp^2 = M^2$$

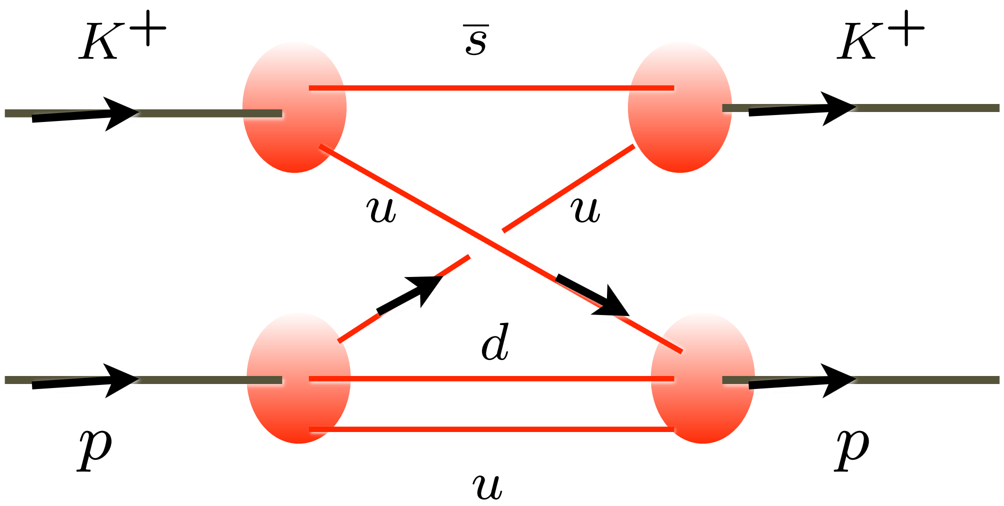
$$P^- = \frac{\vec{P}_\perp^2 + M^2}{P^+} \qquad P^\mu = [P^+, \frac{\vec{P}_\perp^2 + M^2}{P^+}, \vec{P}_\perp]$$

$$(\vec{q}_\perp)^2 = -t \qquad (\vec{r}_\perp)^2 = -u \qquad \vec{q}_\perp \cdot \vec{r}_\perp = 0$$

0

$$\begin{aligned} s = (p+K)^2 &= M_p^2 + M_K^2 + P^+(P_K^- + P_p^-) - 2\vec{p}_\perp \cdot \vec{K}_\perp \\ &= M_p^2 + M_K^2 + P^+(P_K^- + P_p^-) \\ &= 2M_p^2 + 2M_K^2 - t - u \end{aligned}$$

CIM: Blankenbecler, Gunion, sjb



$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

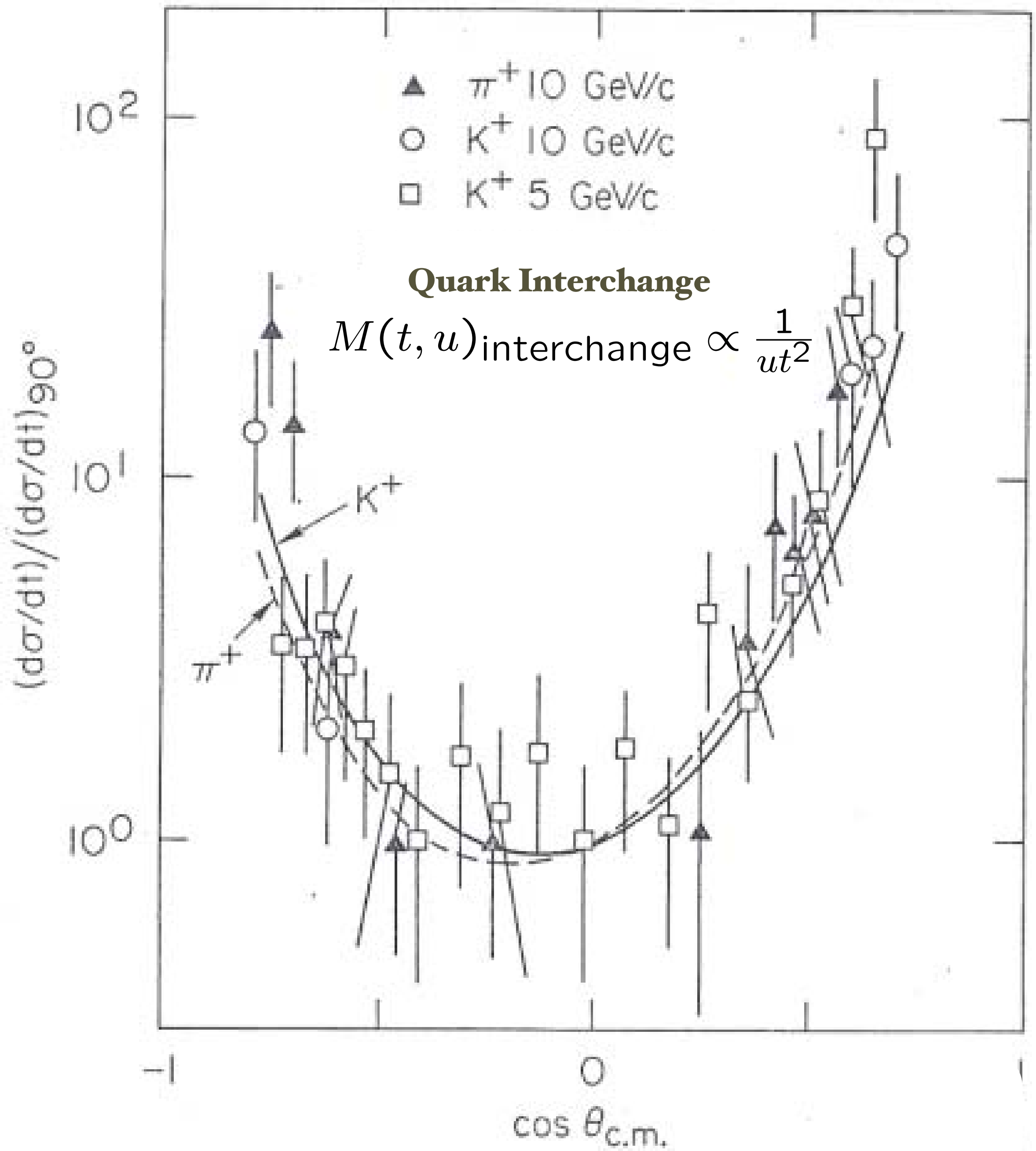
$$M(s,t)_{A+B\to C+D}$$

$$= \frac{1}{2(2\pi)^3}\int d^2k \int_0^1 \frac{dx}{x^2(1-x)^2}\,\Delta\psi_C(\vec{k}_\perp - x\vec{r}_\perp, x)\psi_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x)\psi_A(\vec{k}_\perp - x\vec{r}_\perp + (1-x)\vec{q}_\perp, x)\psi_B(\vec{k}_\perp, x)$$

$$\Delta = s - \sum_i \frac{k_{\perp i}^2 + m_i^2}{x_i}$$

Agrees with electron exchange in atom-atom scattering in nonrelativistic limit

$$K^+ p \to K^+ p$$



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$\frac{d\sigma}{dt}(K^+ p \to K^+ p) \propto \frac{1}{s^2 u^2 t^4}$$

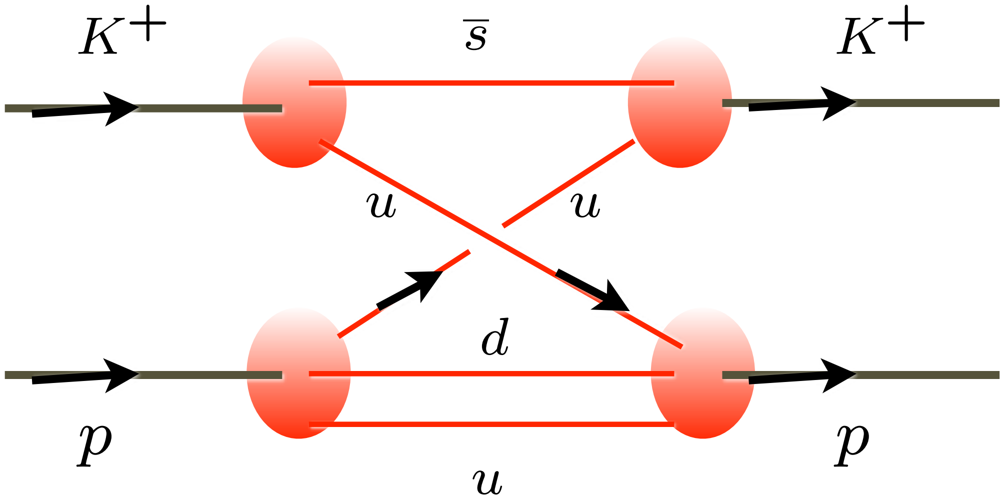
Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

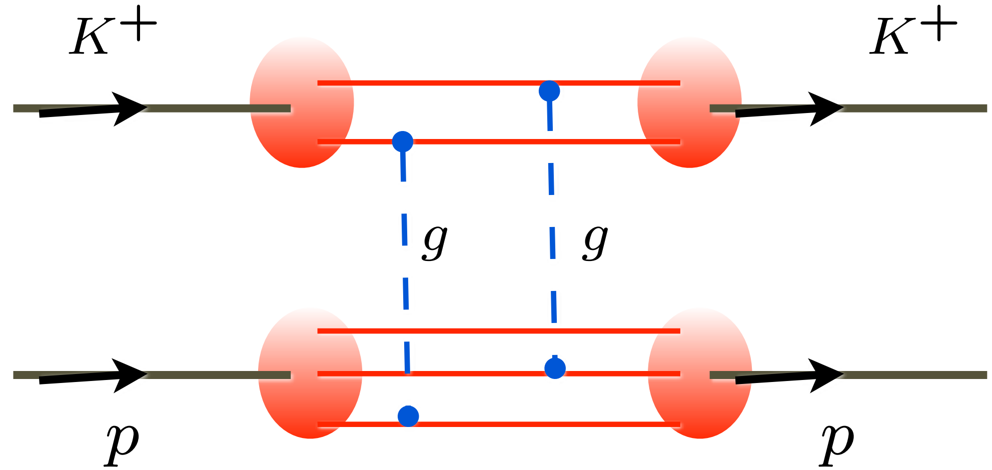
$$\frac{d\sigma}{dt}(MB \to MB) = \frac{F(\theta_{cm})}{s^8} \text{ at fixed } \theta_{cm}$$

Test of Quark Interchange Mechanism

CIM: Blankenbecler, Gunion, sjb



Quark Interchange
(Spin exchange in atom-atom scattering)



Gluon Exchange
Landshoff

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s,t)_{\text{gluonexchange}} \propto sF(t)$$

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
Brookhaven National Laboratory, Upton, New York 11973

and

S. Gushue[e] and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \to p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^0/\Sigma^0)K^0$; $K^{\pm}p \to pK^{\pm}$; $p^{\pm}p \to pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \to p\pi^{\mp}$,

$K^{\pm}p \to pK^{\pm}$,

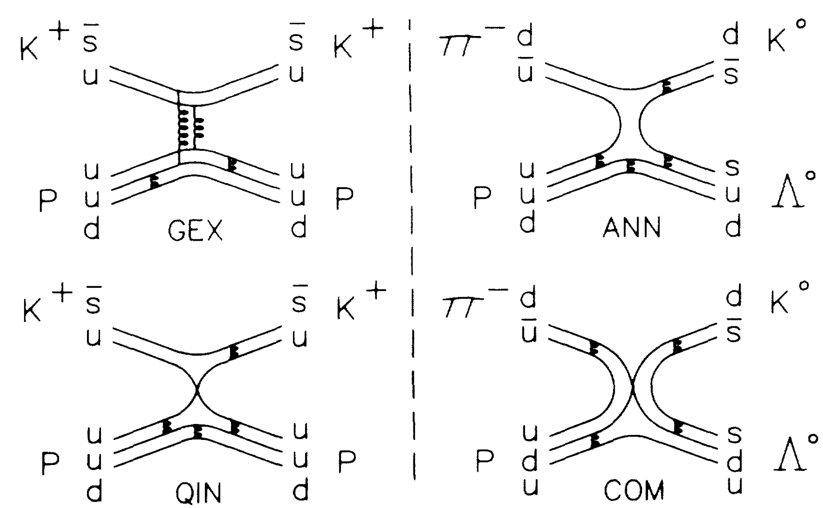
$\pi^{\pm}p \to p\rho^{\pm}$,

$\pi^{\pm}p \to \pi^{+}\Delta^{\pm}$,

$\pi^{\pm}p \to K^{+}\Sigma^{\pm}$,

$\pi^{-}p \to \Lambda^0K^0, \Sigma^0K^0$,

$p^{\pm}p \to pp^{\pm}$.



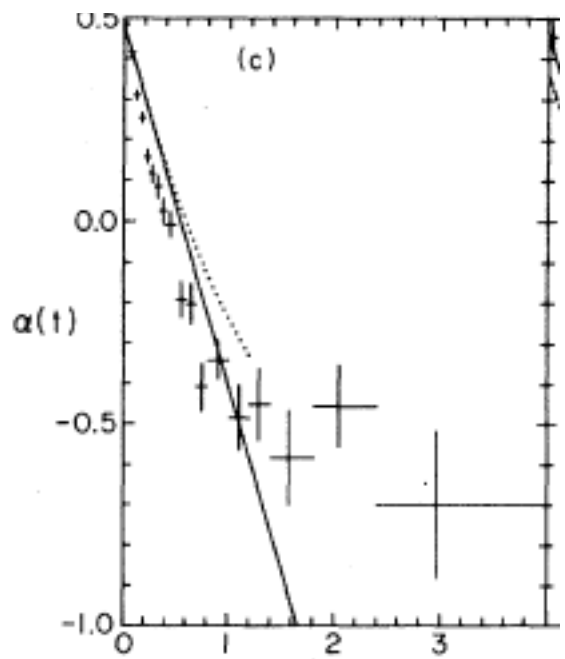
Precise Tests of Triple-Regge Theory from π^0 and η Inclusive Production in 100-GeV/c $\pi \pm p$ Collisions

A. V. Barnes, G. C. Fox, R. G. Kennett, and R. L. Walker
California Institute of Technology, Pasadena, California 91125

and

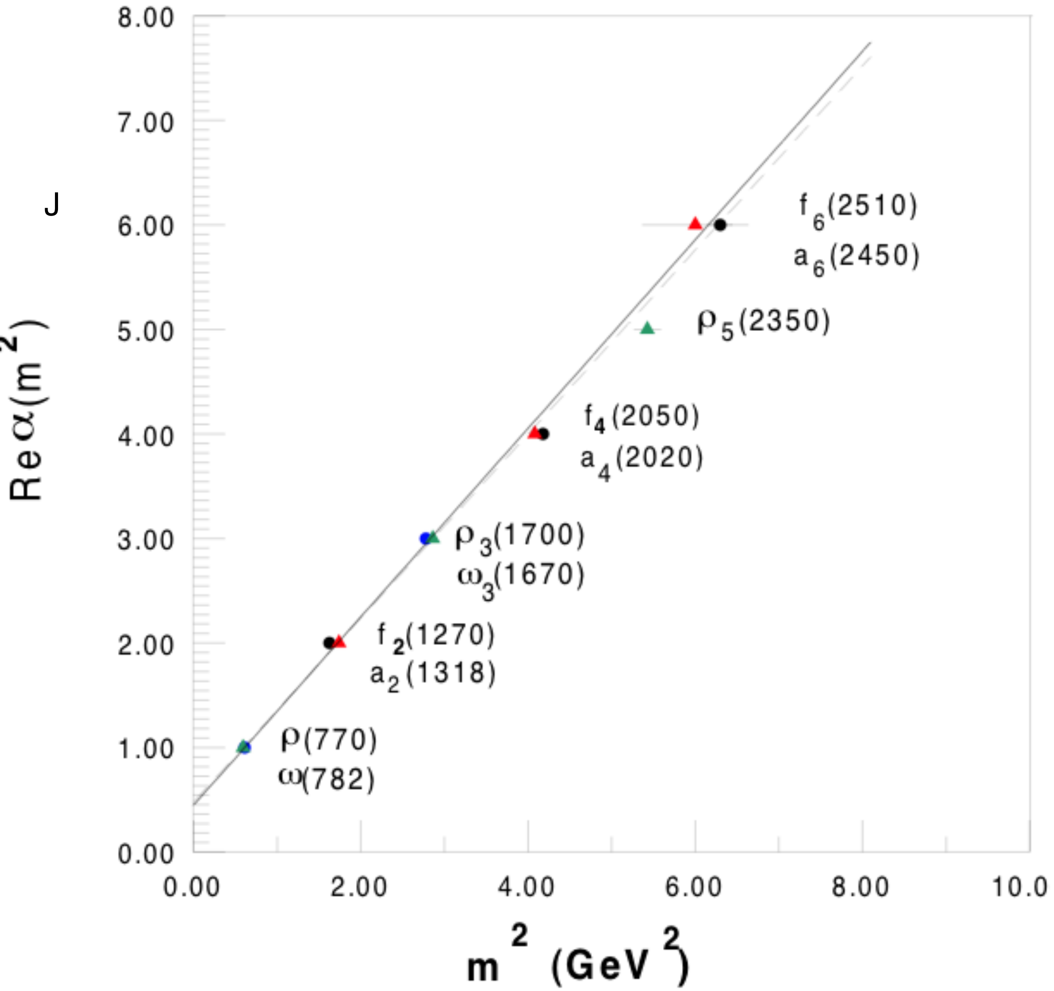
O. I. Dahl, R. W. Kenney, A. Ogawa, and M. Pripstein
Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 21 August 1978)

We present data on π^0 and η inclusive production from 100-GeV/c $\pi^{\pm}p$ collisions in the kinematic region $x \geqslant 0.7$ and $0 < -t \lesssim 4$ (GeV/c)^2. The results are in excellent agreement with the predictions of triple-Regge theory and we have extracted the ρ and A_2 trajectories out to $-t = 4$ (GeV/c)^2.



$-t(GeV)^2$

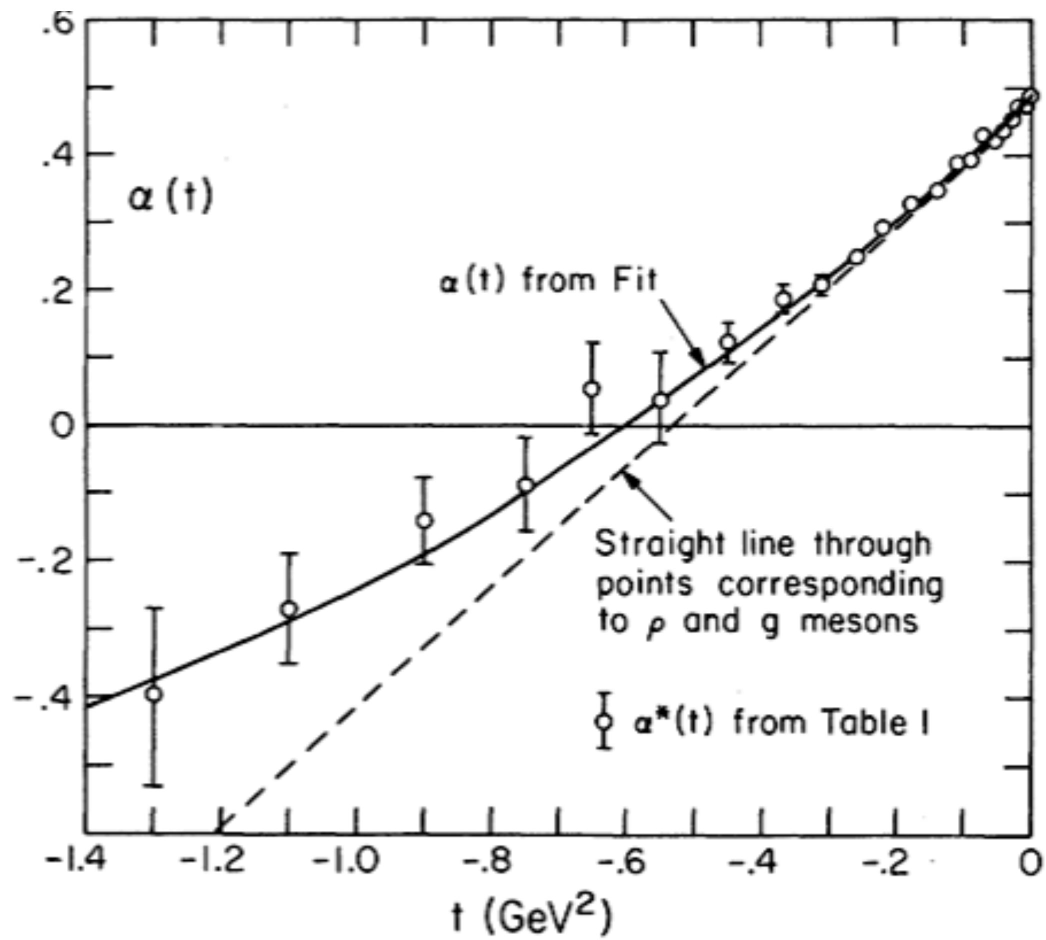
$\alpha_\rho(t)$ from $\pi^- p \to \pi^0 X$ with $0.71 < x_F < 0.98$



Desgrolard et al. '00 (PDG,RPP data)

t > 0: spectrum (s-channel pole)

Oleg Andreev & Warren Siegel



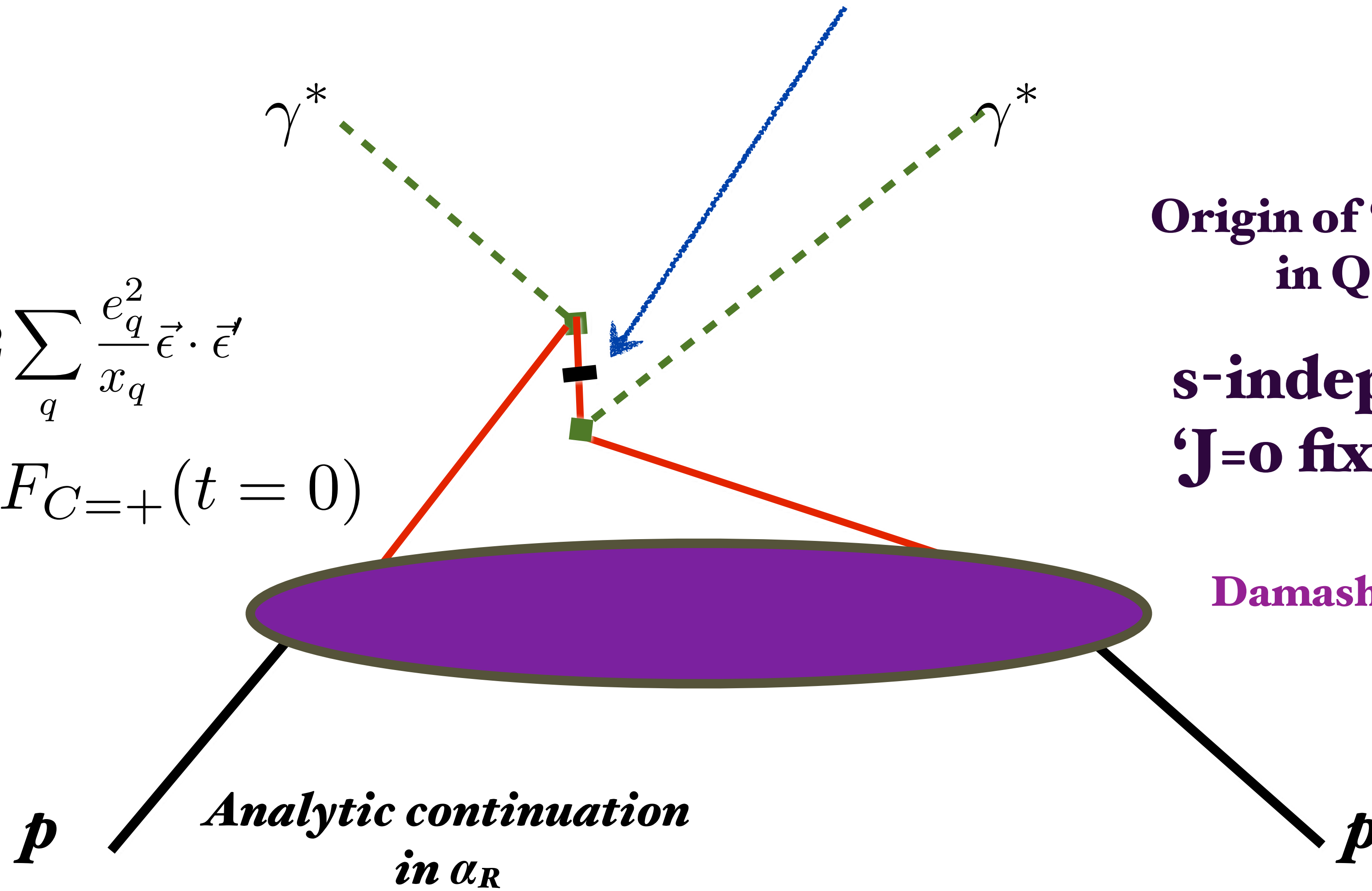
Regge Trajectories nonlinear for spacelike momenta

Barnes et al. '76 (Fermilab data)

Leading-Twist Contribution to Real Part of DVCS

Close, Gunion, sjb
Szczepaniak, Llanes Estrada, sjb

LF Instantaneous interaction



$$T = -2 \sum_q \frac{e_q^2}{x_q} \vec{\epsilon} \cdot \vec{\epsilon}'$$

$$T \propto s^0 F_{C=+}(t=0)$$

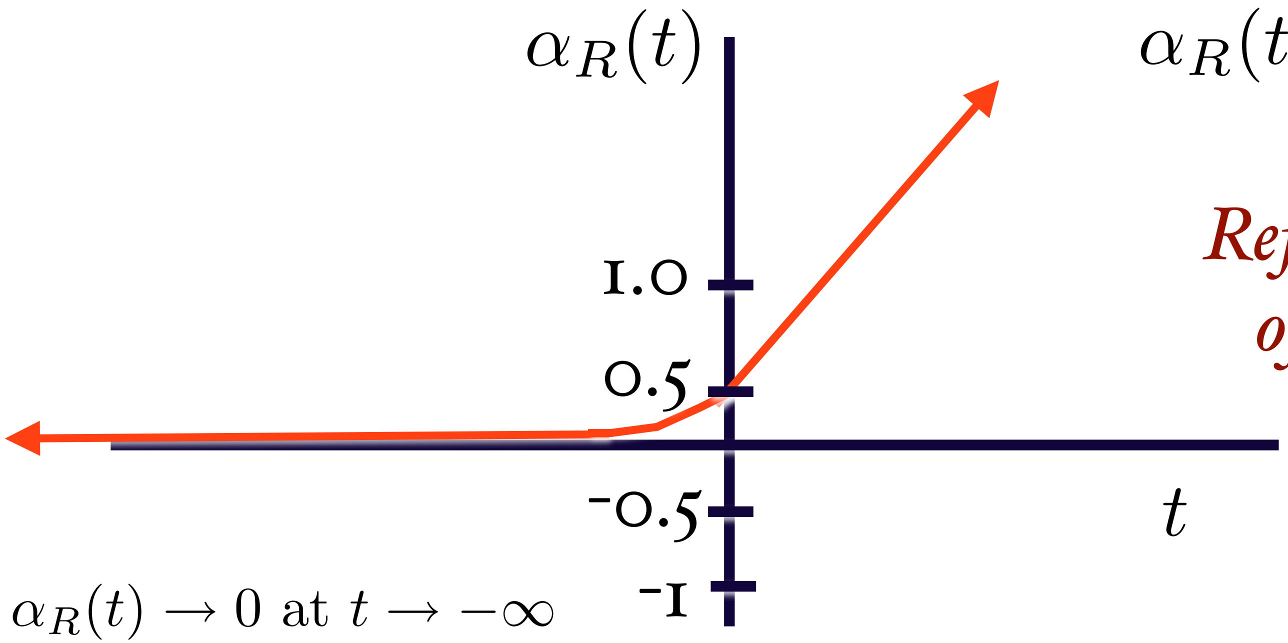
Origin of 'D-Term' in QCD

s-independent 'J=0 fixed pole'

Damashek, Gilman

Regge domain virtual Compton Scattering

$$M(\gamma^* p \to \gamma p) \sim s^{\alpha_R(t)} \beta_R(t) \qquad s >> -t, Q^2$$



$\alpha_R(t) \to 0$ at $t \to -\infty$

J=0 fixed pole

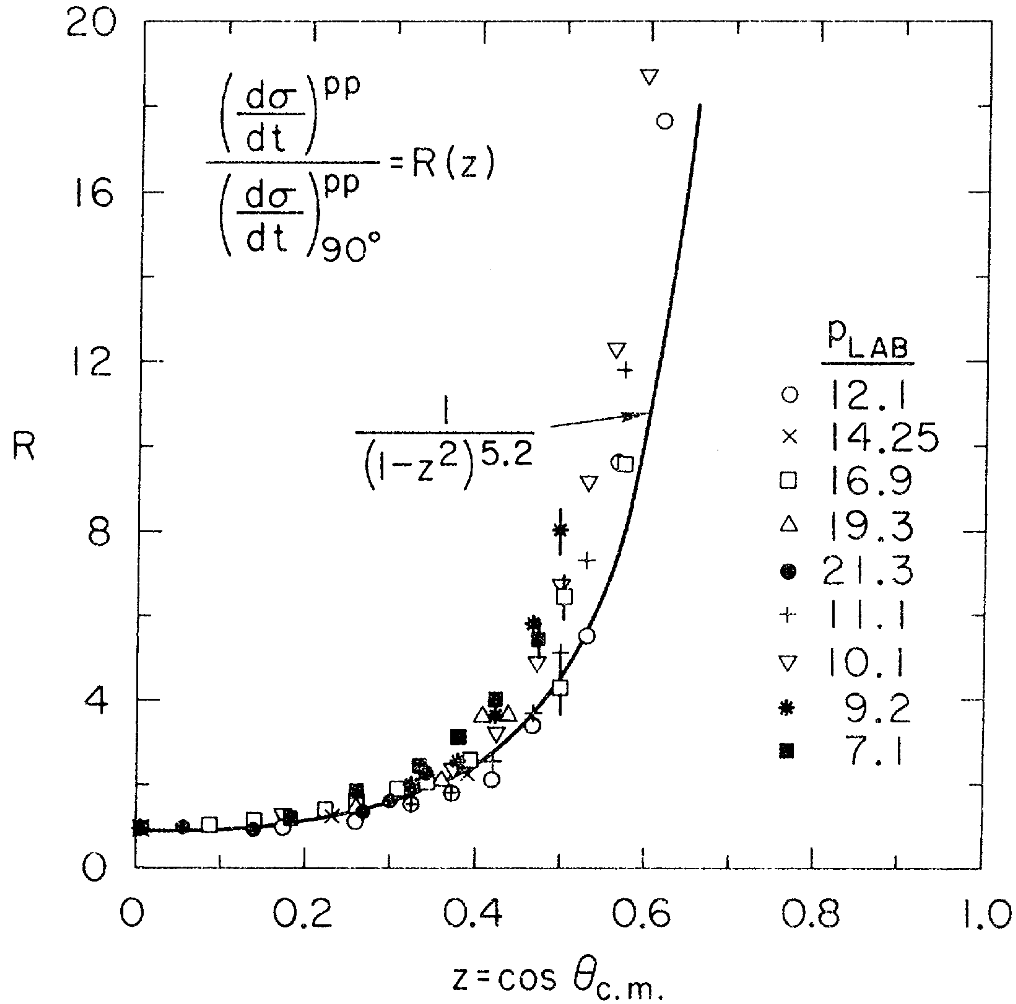
Reflects elementary coupling of two photons to quarks

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt}(\gamma^* p \to \gamma p) \to \frac{1}{s^2}\beta_R^2(t) \sim \frac{1}{s^2 t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{t}{s}, \frac{Q^2}{s}$$

Fundamental test of QCD

Test of BBG Quark Interchange Mechanism in $pp \to pp$



$$\frac{d\sigma}{dt}(pp \to pp) \propto \frac{1}{s^2 u^4 t^4}$$

$$\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{cm})}{s^{10}} \text{ at fixed } \theta_{cm}$$

$$\alpha_R(t) \to -2$$

PHYSICAL REVIEW D **79,** 033012 (2009)

Local two-photon couplings and the $J = 0$ fixed pole in real and virtual Compton scattering

Stanley J. Brodsky*

Theory Group, SLAC National Accelerator Laboratory, 2575 Sand Hill Road, 94025 Menlo Park, California, USA

Felipe J. Llanes-Estrada†

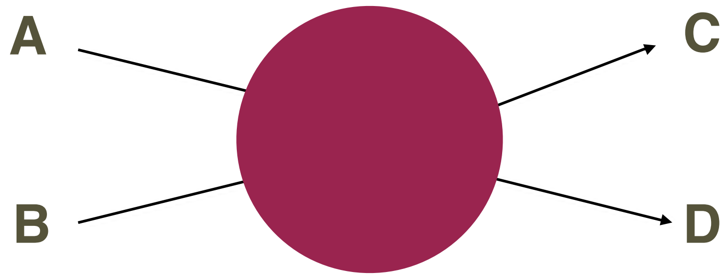
Departmento Física Teórica I, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, 28040 Madrid, Spain

Adam P. Szczepaniak‡

Department of Physics and Nuclear Theory Center, Indiana University, Bloomington, Indiana 47405, USA
(Received 5 December 2008; published 20 February 2009)

The local coupling of two photons to the fundamental quark currents of a hadron gives an energy-independent contribution to the Compton amplitude proportional to the charge squared of the struck quark, a contribution which has no analog in hadron scattering reactions. We show that this local contribution has a real phase and is universal, giving the same contribution for real or virtual Compton scattering for any photon virtuality and skewness at fixed momentum transfer squared t. The t dependence of this $J = 0$ fixed Regge pole is parameterized by a yet unmeasured even charge-conjugation form factor of the target nucleon. The $t = 0$ limit gives an important constraint on the dependence of the nucleon mass on the quark mass through the Weisberger relation. We discuss how this $1/x$ form factor can be extracted from high-energy deeply virtual Compton scattering and examine predictions given by models of the H generalized parton distribution.

Counting Rules:



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\text{cm}})}{s^{[n_{\text{tot}}-2]}} \qquad s = E^2_{\text{cm}}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos\theta_{cm}$

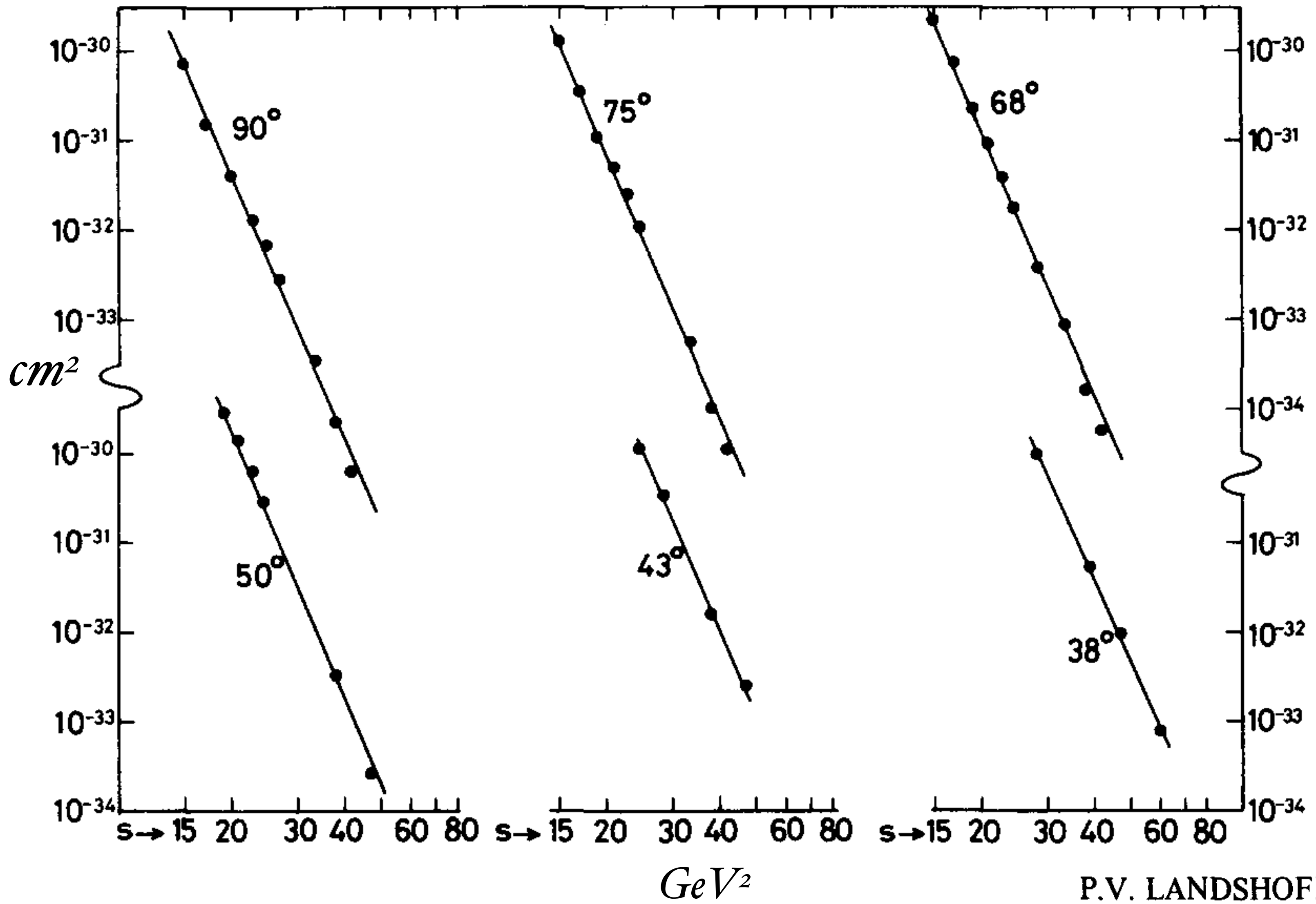
Farrar & sjb;
Matveev, Muradyan, Tavkhelidze

pQCD predicts the leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

$$s, -t >> m_\ell^2$$

Non-Perturbative Proof from AdS/CFT: **Polchinski and Strassler**

Quark-Counting: $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $\quad n = 4 \times 3 - 2 = 10$

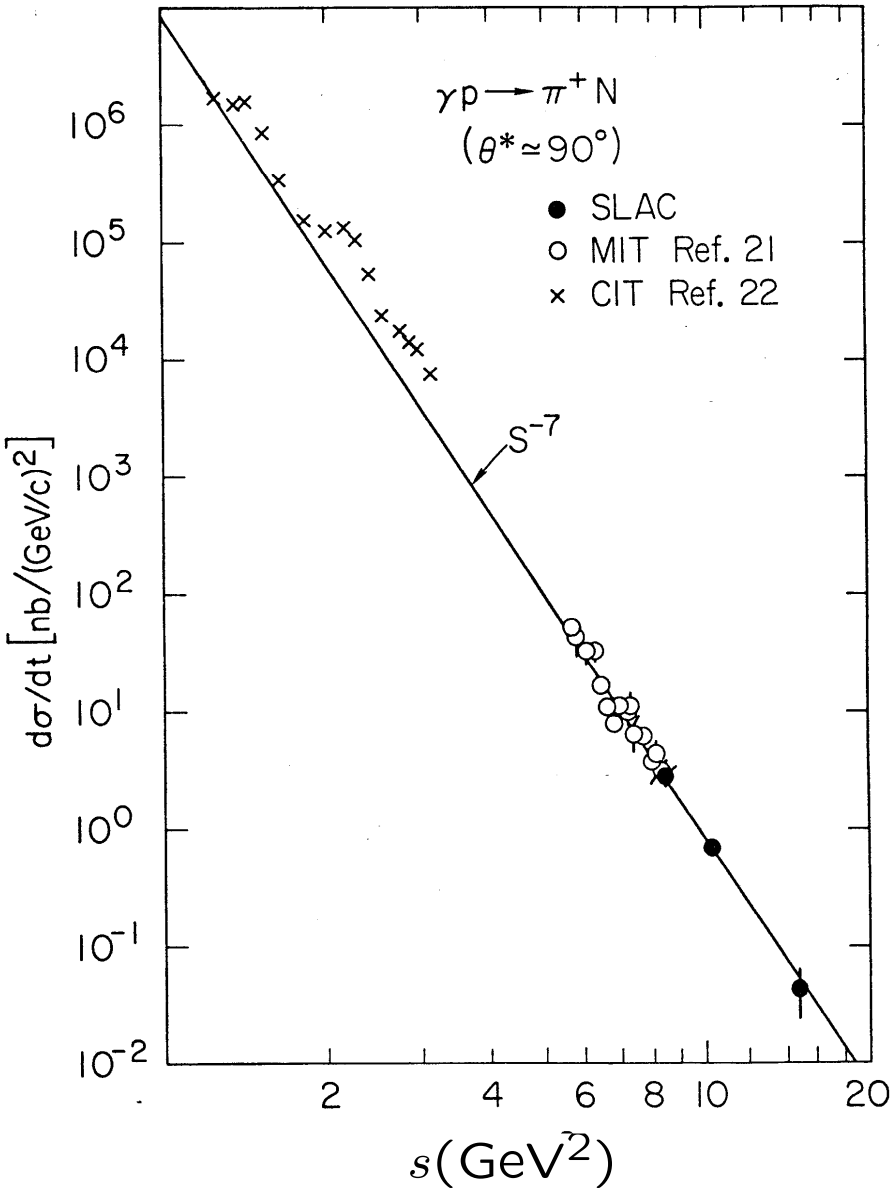


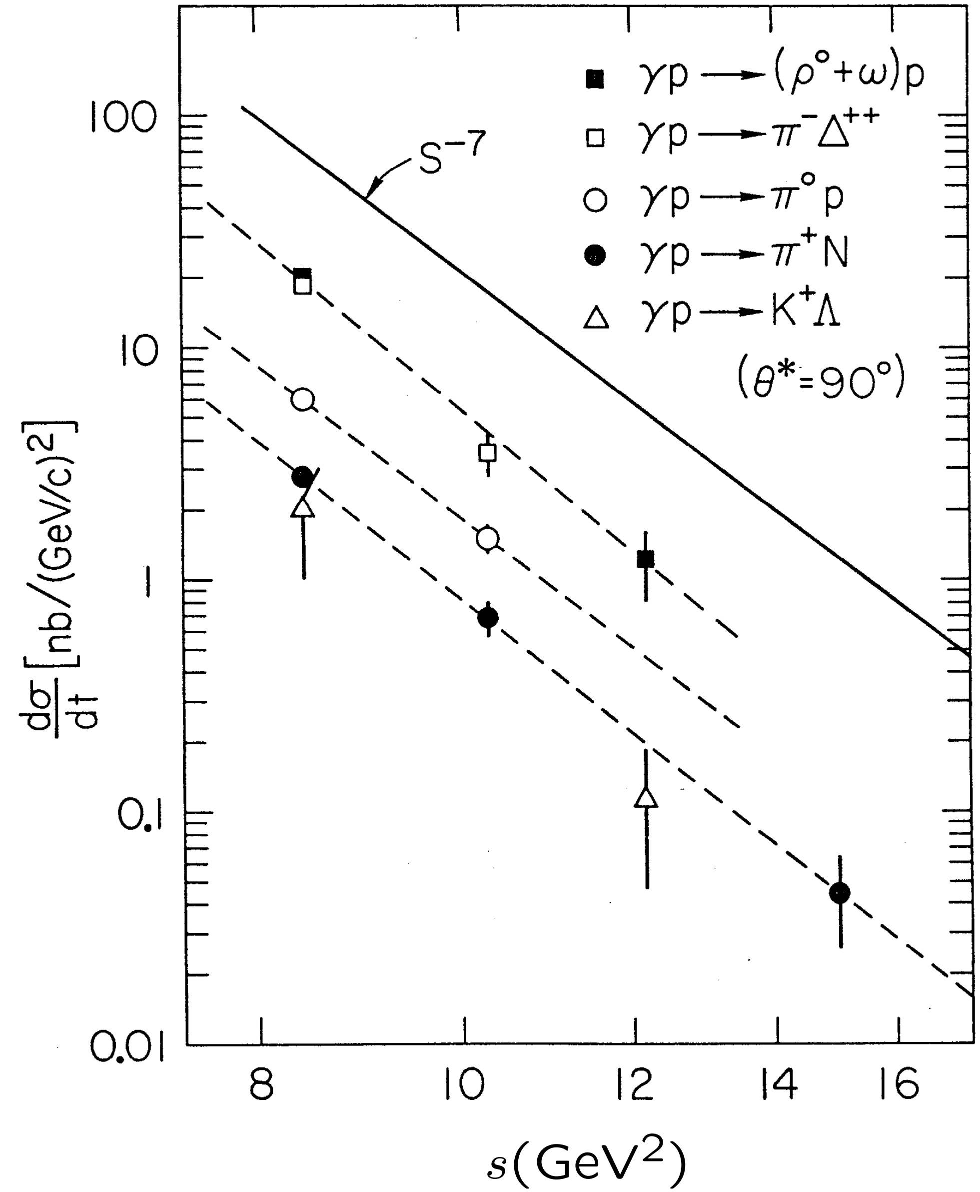
P.V. LANDSHOFF and J.C. POLKINGHORNE

Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions



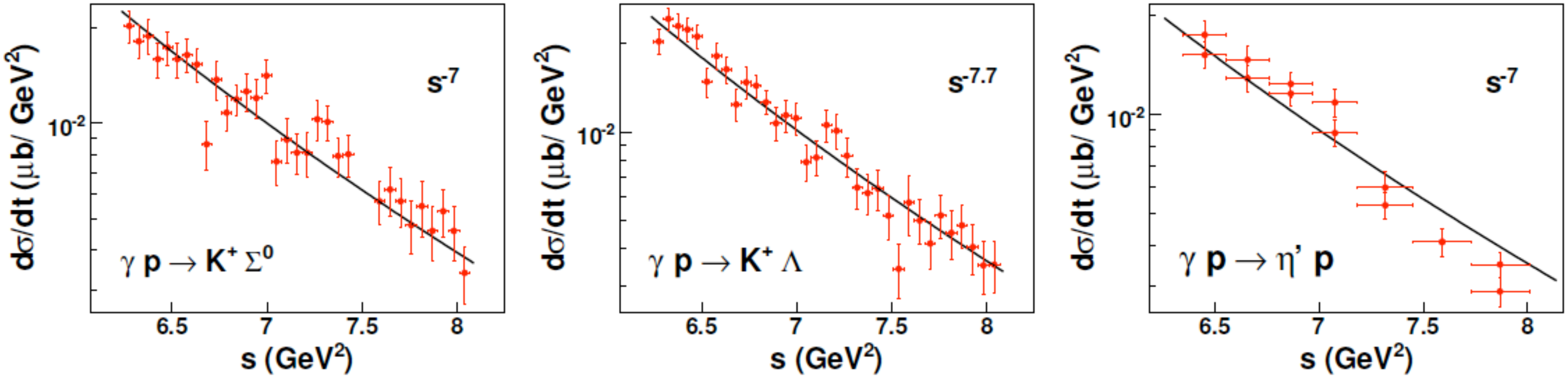


Counting Rules: $n = 9 - 2 = 7$

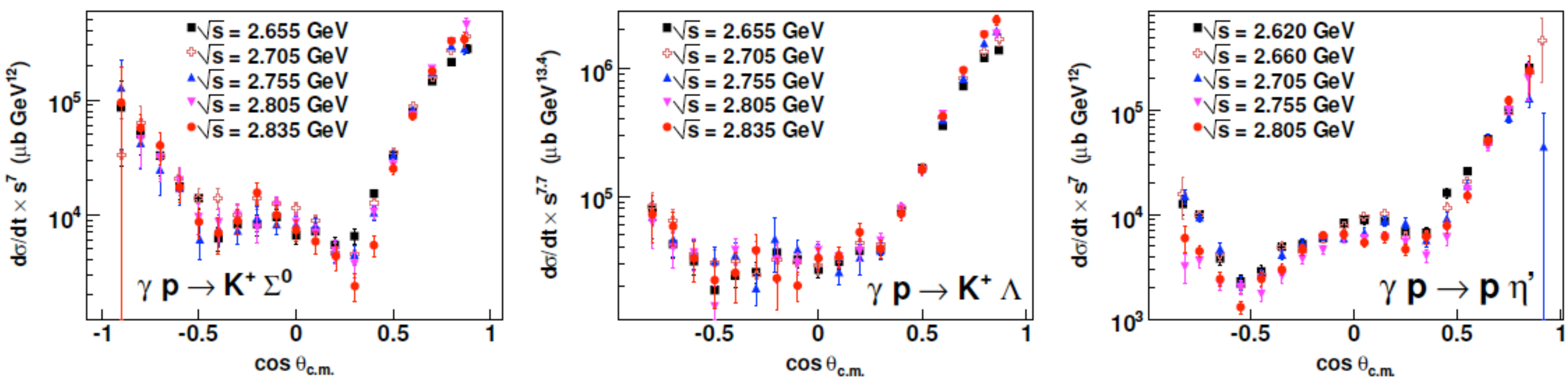
$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

Scaling behavior in exclusive meson photoproduction from Jefferson Lab at large momentum transfers

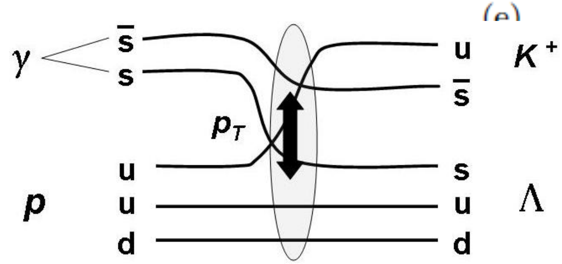
$-0.95 \leq \cos\theta_{c.m.} \leq 0.95.$



(a) (b) (c)

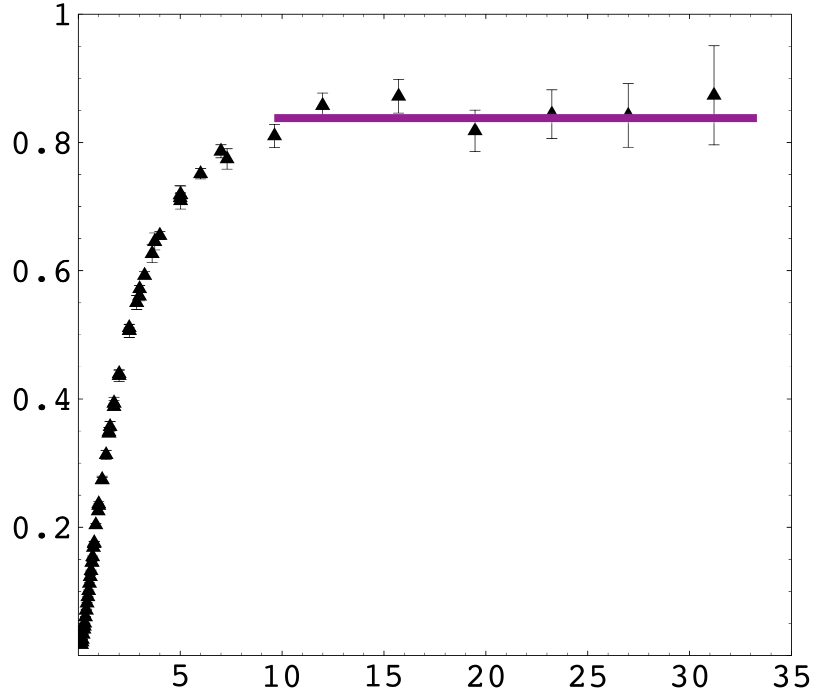


(d) (e) (f)



Biplab Dey

$Q^4 F_1^p(Q^2)$ [GeV^4]



Q^2 [GeV^2]

$$F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \quad n = 3$$

measured in electron-proton elastic scattering

From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

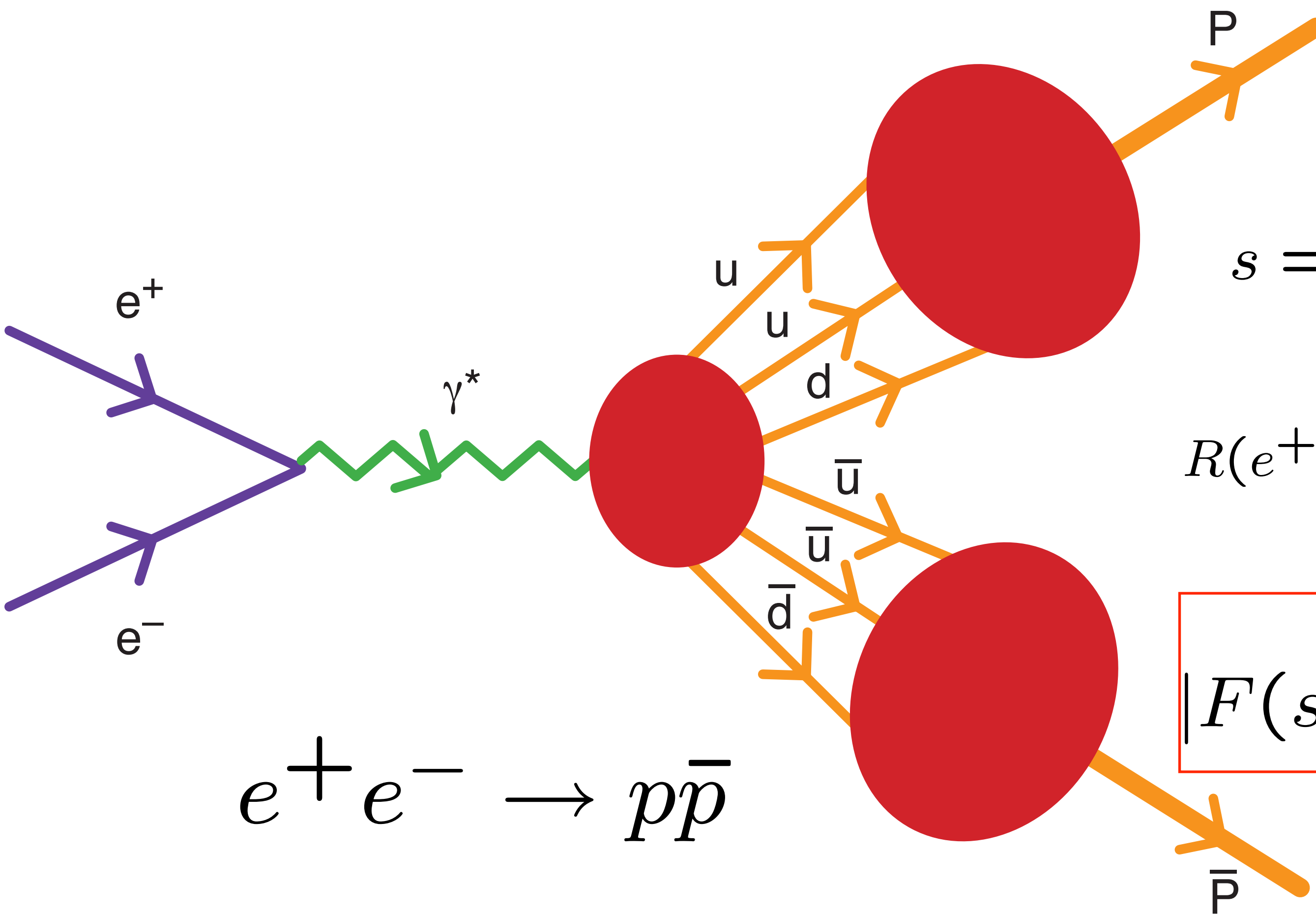
implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev *et al.* (1973).

- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).

Exclusive Processes

What if we ask for a specific final state?



$$s = (E_{e^+} + E_{e^-})^2$$

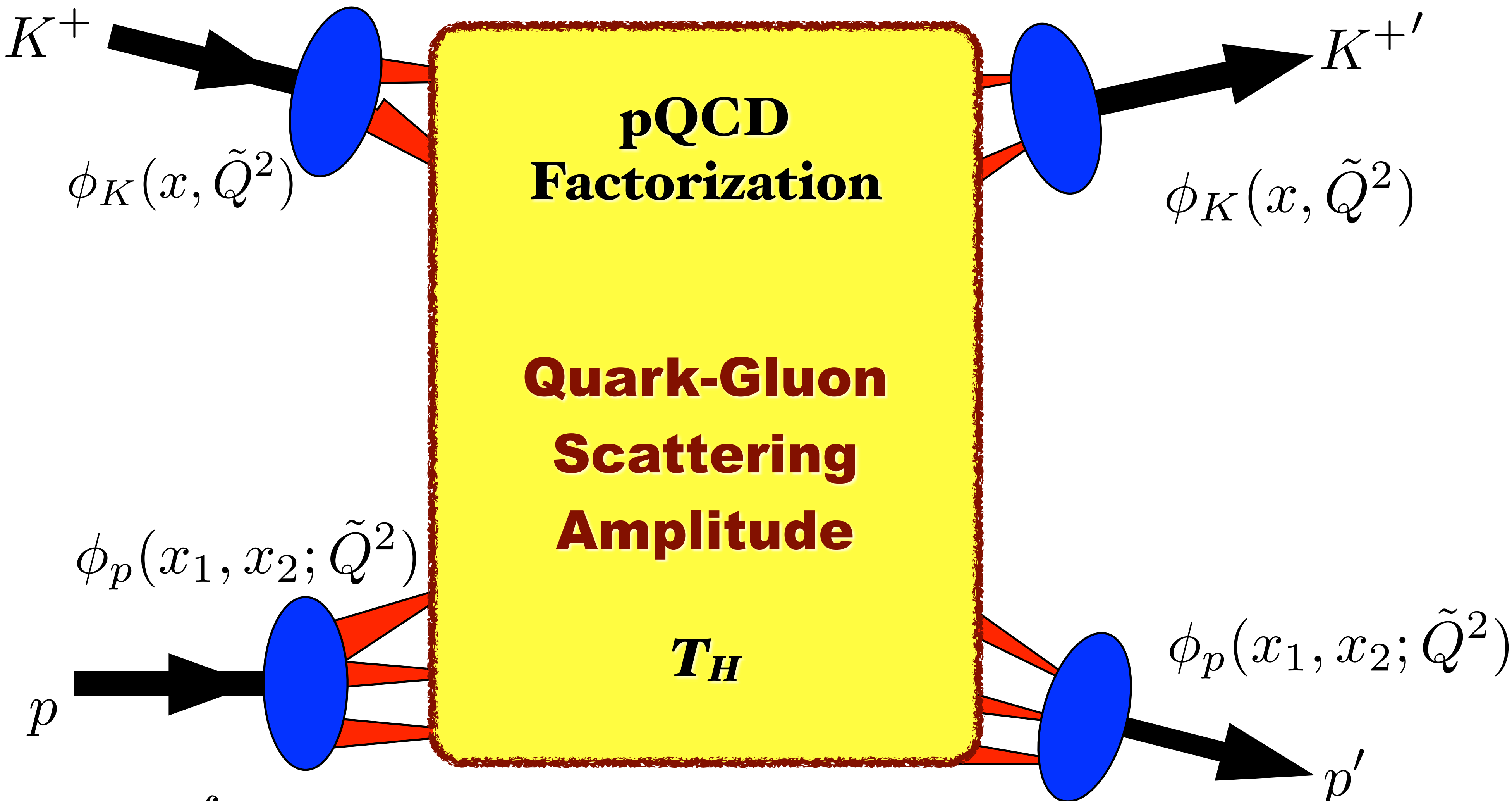
$$R(e^+e^- \rightarrow H\bar{H}) \propto |F(s)|^2$$

$$|F(s)| \propto [\frac{1}{s}]^{n_q - 1}$$

$$e^+e^- \rightarrow p\bar{p}$$

Probability decreases with number of constituents!

Lepage, sjb; Efremov and Radyushkin



$$M = \int \prod dx_i dy_i \phi_F(x,\tilde{Q}) \times T_H(x_i,y_i,\tilde{Q}) \phi_I(y_i,Q)$$

Distribution Amplitudes
(gauge and frame-independent)

PQCD and Exclusive Processes

Lepage; SJB
Efremov, Radyuskin

$$M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q)$$

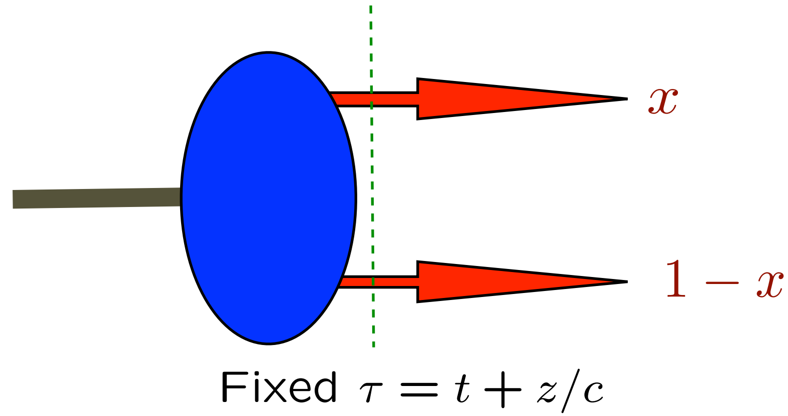
- Iterate kernel of LFWFs when at high virtuality; distribution amplitude contains all physics below factorization scale
- Rigorous Factorization Formulae: Leading twist
- Underly Exclusive B-decay analyses
- Distribution amplitude: gauge invariant, OPE, evolution equations, conformal expansions
- BLM/PMC scale setting: sum nonconformal contributions in scale of running coupling
- Derive Dimensional Counting Rules/ Conformal Scaling

Farrar; SJB
Matveev, Muradyan, Tavkhelidze

Inspired by BBG Factorization

Hadron Distribution Amplitudes

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

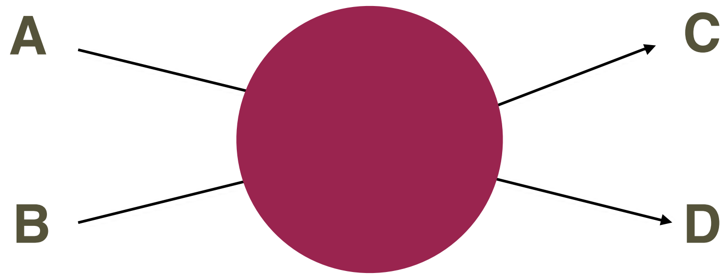
Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

Counting Rules:



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\text{cm}})}{s^{[n_{\text{tot}}-2]}} \qquad s = E^2_{\text{cm}}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos\theta_{cm}$

Farrar & sjb;
Matveev, Muradyan, Tavkhelidze

pQCD predicts the leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

$$s, -t >> m_\ell^2$$

Non-Perturbative Proof from AdS/CFT: **Polchinski and Strassler**

ERBL Evolution of Meson Distribution Amplitude

$$x_1 x_2 Q^2 \frac{\partial}{\partial Q^2} \tilde{\phi}(x_i, Q) = C_F \frac{\alpha_s(Q^2)}{4\pi} \left\{ \int_0^1 [dy] V(x_i, y_i) \tilde{\phi}(y_i, Q) - x_1 x_2 \tilde{\phi}(x_i, Q) \right\}$$

where $\tilde{\phi} = x_1 x_2 \phi$

$$V(x_i, y_i) = 2\left[x_1 y_2 \theta(y_1 - x_1) \left(\delta_{h_1 \bar{h}_2} + \frac{\Delta}{y_1 - x_1} \right) + (1 \leftrightarrow 2) \right] = V(y_i, x_i),$$

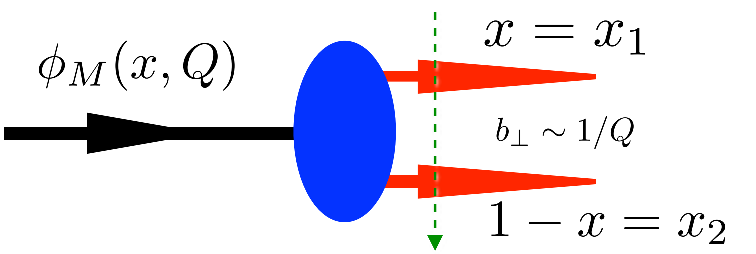
and $\Delta \tilde{\phi}(y_i, Q) \equiv \tilde{\phi}(y_i, Q) - \tilde{\phi}(x_i, Q)$.

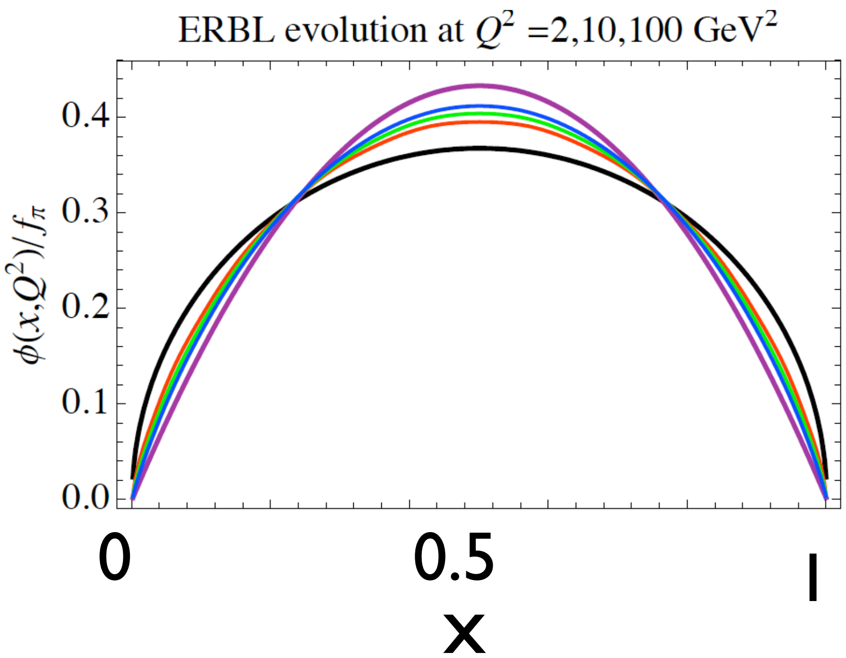
$$\phi(x_i, Q) = x_1 x_2 \sum_{n=0}^{\infty} a_n C_n^{3/2}(x_1 - x_2) \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-\gamma_n}$$

where

$$\gamma_n = \frac{C_F}{\beta} \left(1 + 4 \sum_2^{n+1} \frac{1}{k} - \frac{2\delta_{h_1 \bar{h}_2}}{(n+1)(n+2)} \right) \geq 0 .$$

Fixed $\tau = t + z/c$



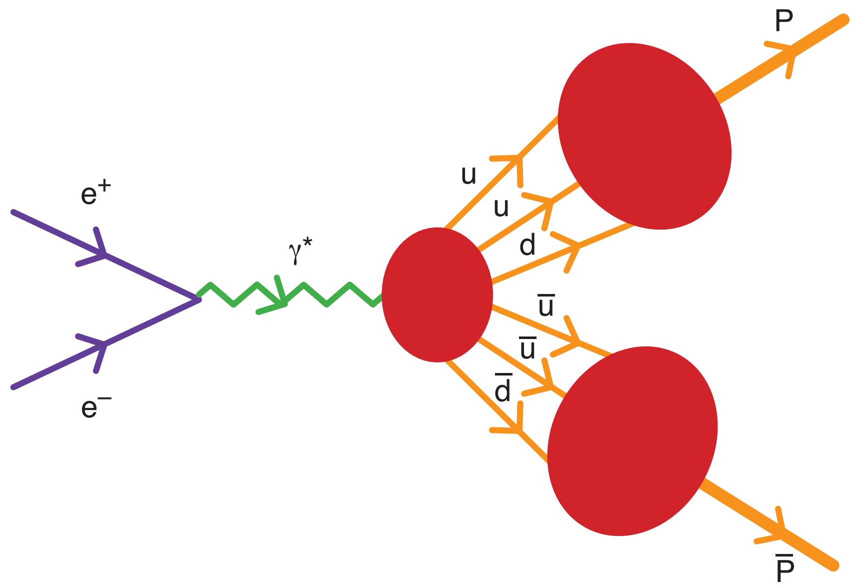


Evolves from $\sqrt{x(1-x)}$ to $x(1-x)$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

AdS/QCD

Timelike proton form factor in PQCD



$$G_M(Q^2) \quad \to \quad \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} b_{nm} \left(\log \frac{Q^2}{\Lambda^2} \right)^{\gamma_n^B + \gamma_n^B}$$

$$\times \quad \left[1 + \mathcal{O}\left(\alpha_s(Q^2), \frac{m^2}{Q^2} \right) \right]$$

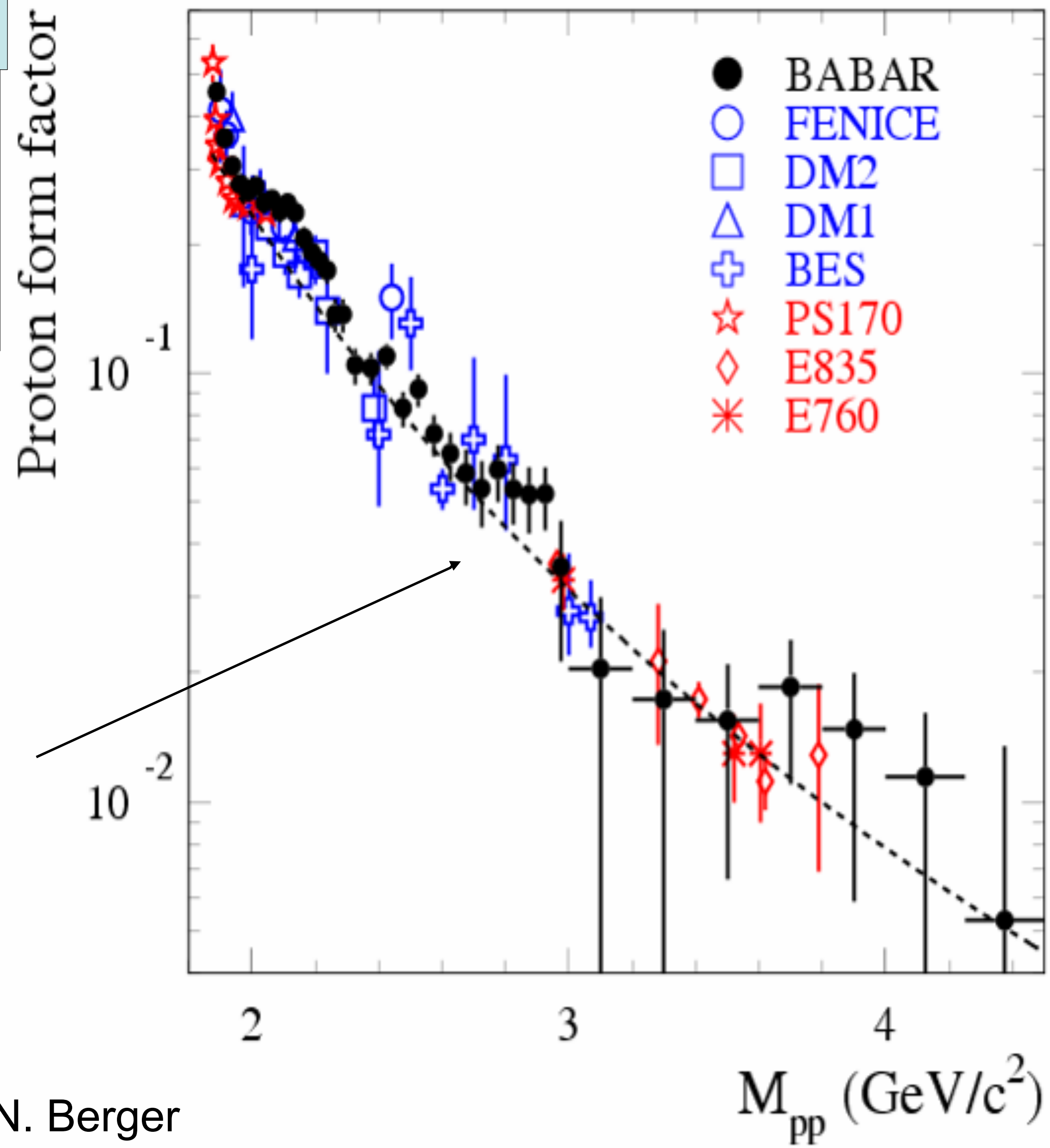
Lepage and Sjb

- Define "Effective" form factor by

$$\sigma = \frac{4\pi\alpha^2\beta C}{3m_{p\bar{p}}^2}|F|^2,\ |F| = \sqrt{|G_M|^2 + \frac{2m_p^2}{m_{p\bar{p}}^2}|G_E|^2}.$$

- Peak at threshold, sharp dips at 2.25 GeV, 3.0 GeV.
- Good fit to pQCD prediction for high m_{pp}.

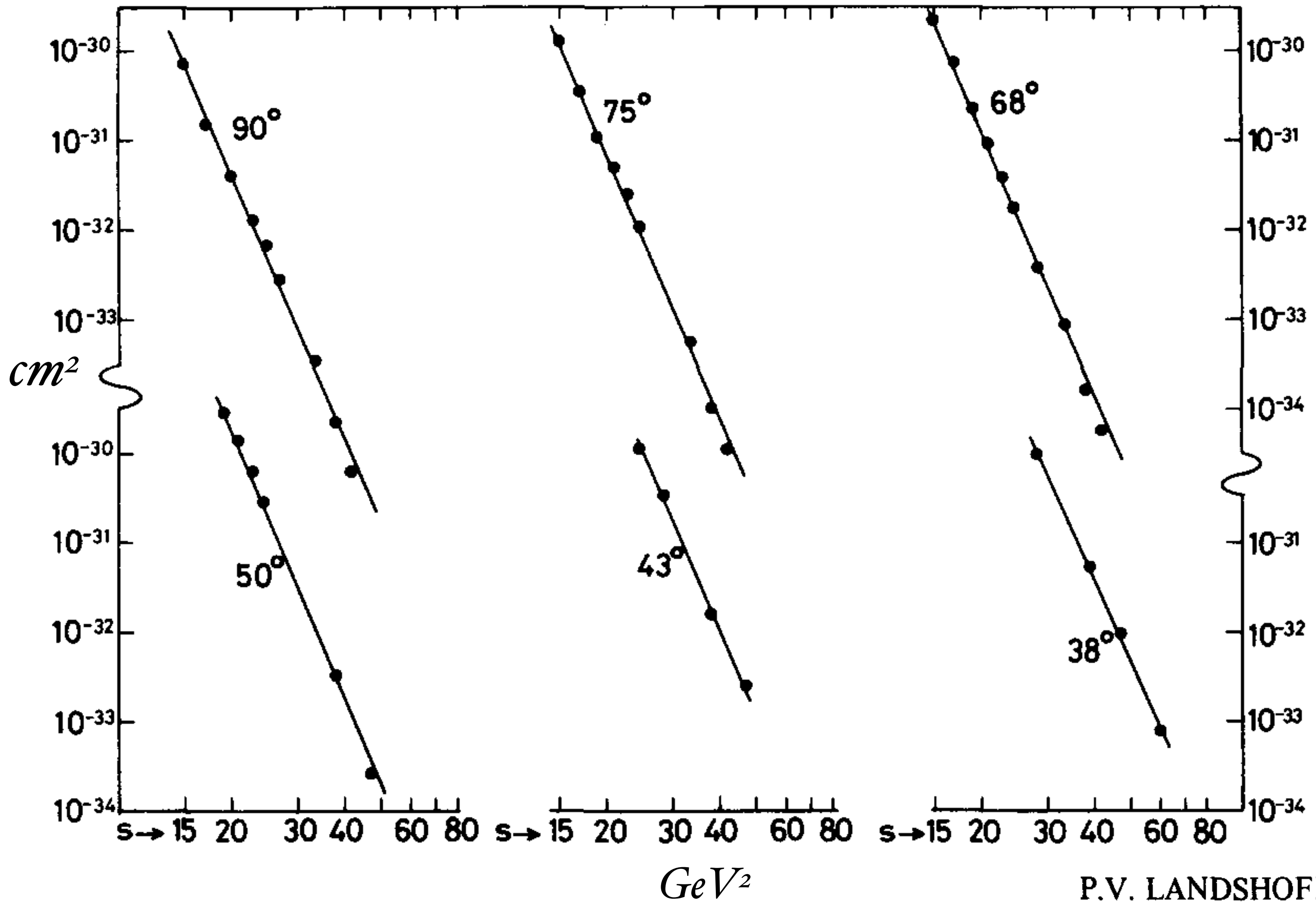
$$F(s) \propto \frac{\log^{-2}\frac{s}{\Lambda^2}}{s^2}$$



Hard Exclusive Processes

- **PQCD Factorization**
- **Convolution of Hadron Distribution Amplitudes with Hard QCD**
- **Leading Twist: Counting Rules**
- **Hadron Helicity Conservation**
- **Color Transparency**
- **BBG Quark Interchange**
- **Absence of Landshoff Amplitudes**
- **Puzzle: Huge Krisch R_{NN}**

Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $n = 4 \times 3 - 2 = 10$



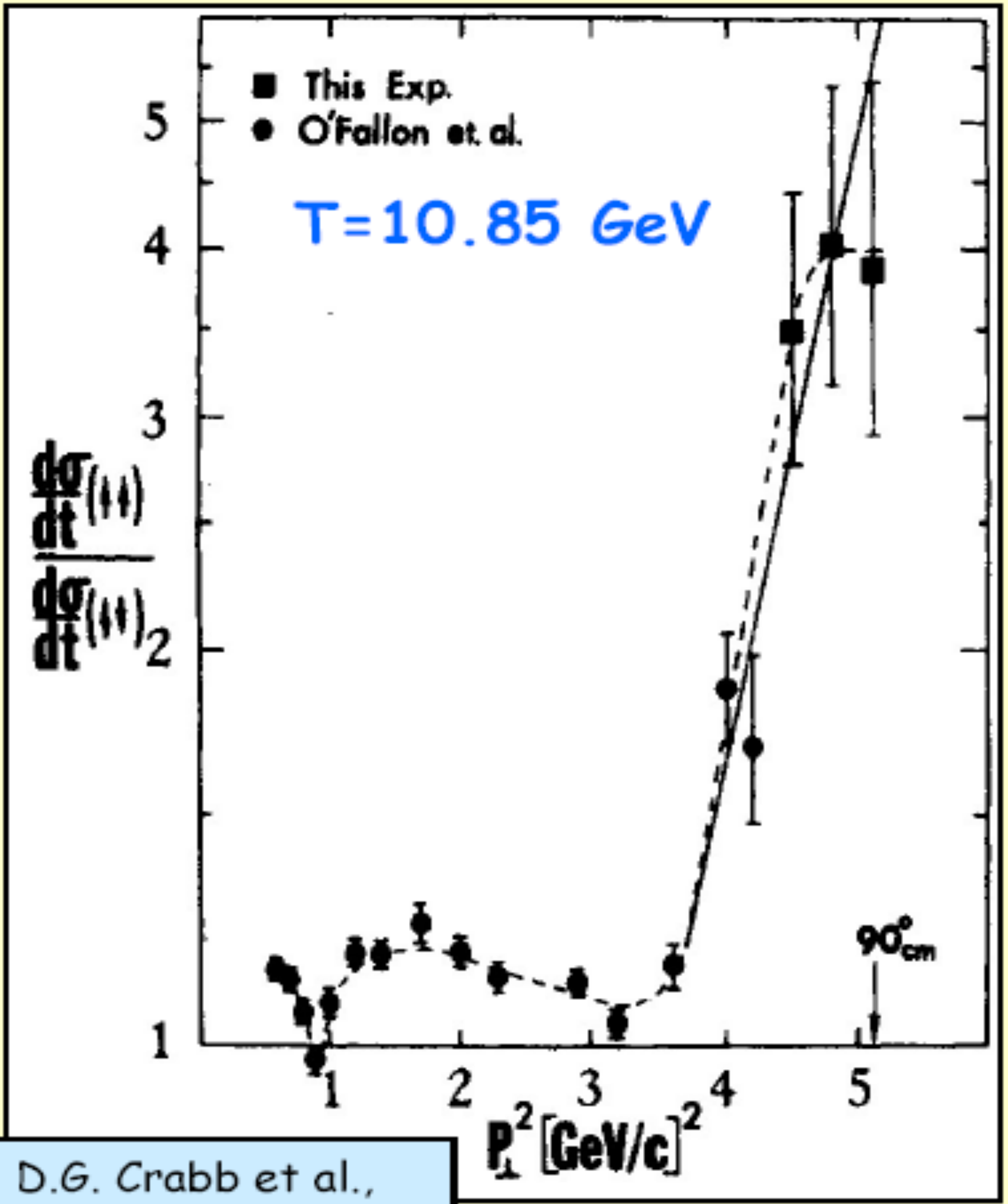
P.V. LANDSHOFF and J.C. POLKINGHORNE

Best Fit

$n = 9.7 \pm 0.5$

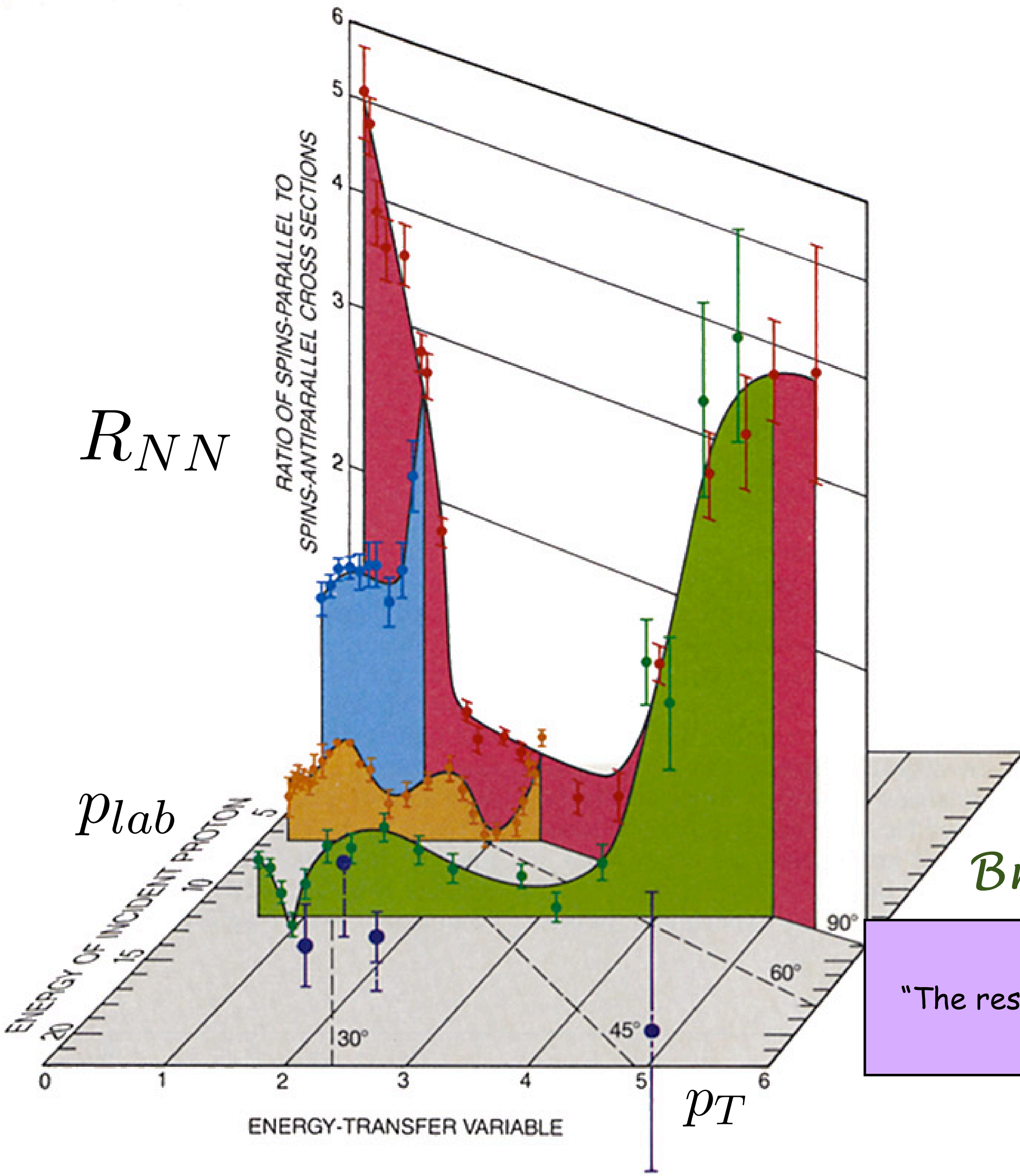
Reflects underlying conformal scale-free interactions

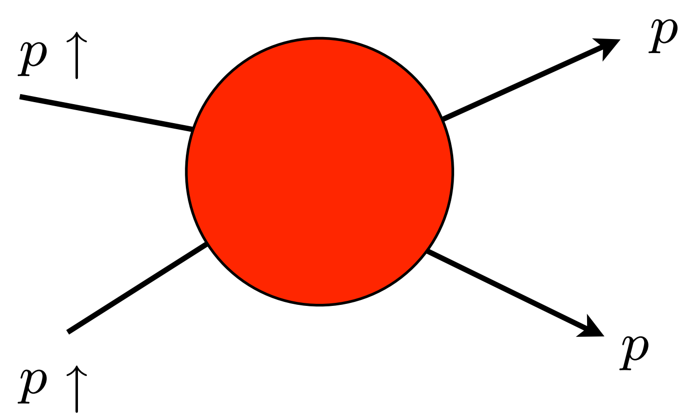
Unexpected spin effects in pp elastic scattering



D.G. Crabb et al., PRL 41, 1257 (1978)

Spin Correlations in Elastic $p-p$ Scattering





polarization normal to scattering plane

Ratio reaches 4:1 !

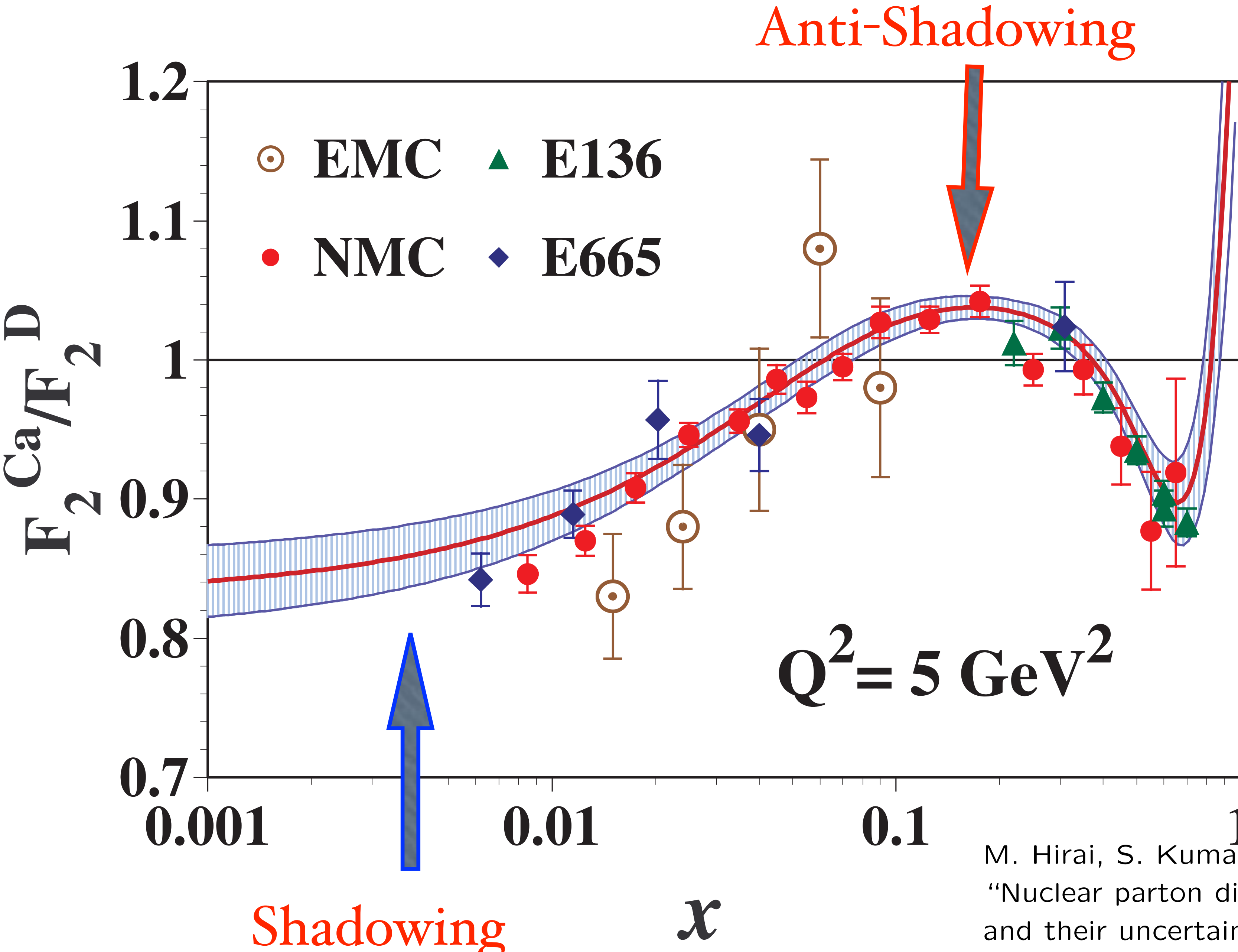
Heppelmann et al.

Breakdown of Color Transparency

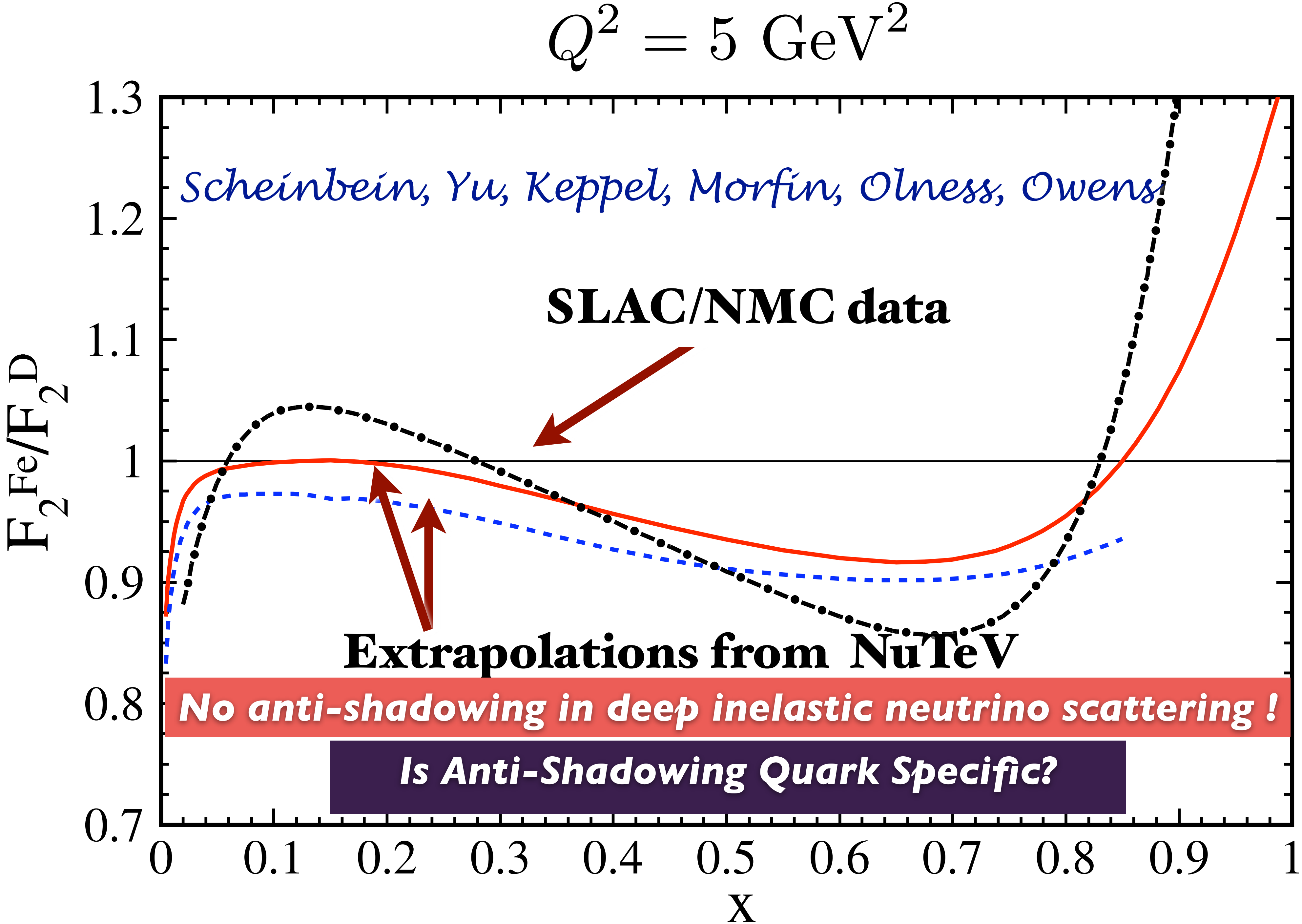
A. Krisch, Sci. Am. 257 (1987)
"The results challenge the prevailing theory that describes the proton's structure and forces"

de Teramond & sjb: B=2 Resonance near Charm Threshold

$$|uuduudc\bar{c}>$$



M. Hirai, S. Kumano and T. H. Nagai, "Nuclear parton distribution functions and their uncertainties," Phys. Rev. C **70**, 044905 (2004) [arXiv:hep-ph/0404093].

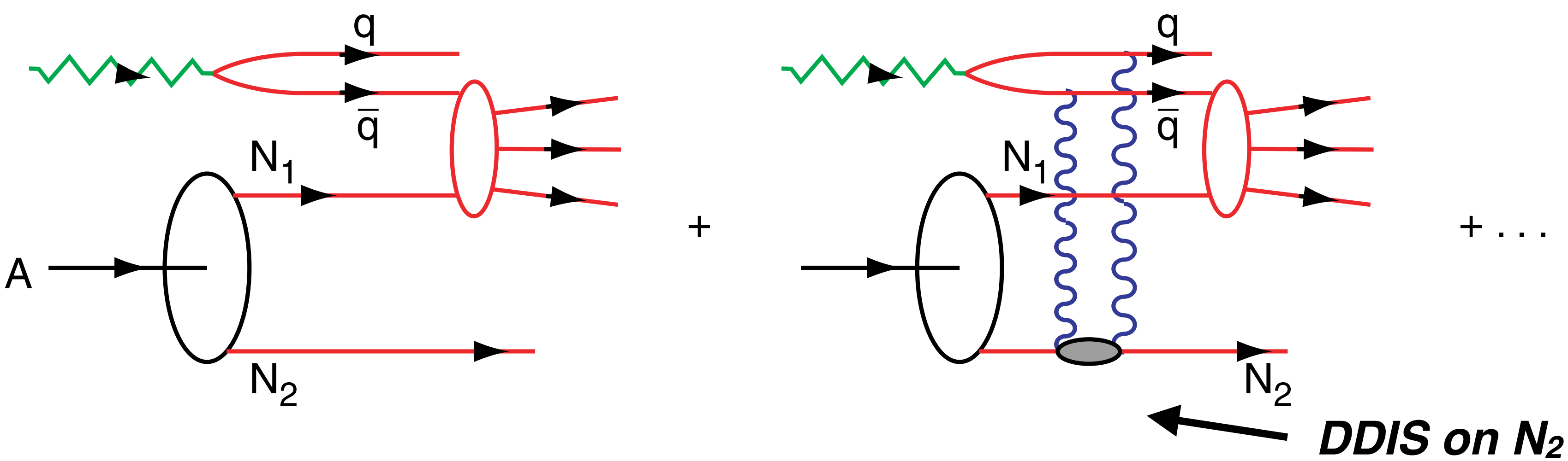


$Q^2 = 5\ \mathrm{GeV}^2$
Scheinbein, Yu, Keppel, Morfin, Olness, Owens
SLAC/NMC data
Extrapolations from NuTeV
No anti-shadowing in deep inelastic neutrino scattering !
Is Anti-Shadowing Quark Specific?
F_2^{Fe}/F_2^D
x
1.3
1.2
1.1
1
0.9
0.8
0.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Is Antishadowing in DIS Non-Universal, Flavor-Dependent?

Stodolsky
Pumplin, sjb
Gribov

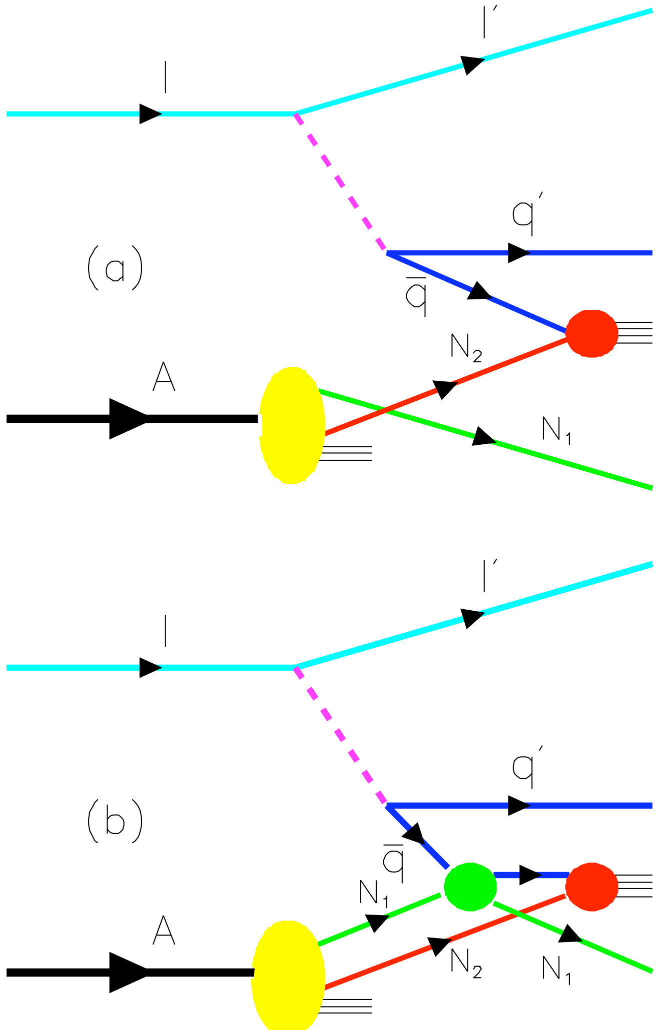
Nuclear Shadowing in QCD



Shadowing requires leading-twist diffractive DIS

Nuclear Shadowing not included in nuclear LFWF !

Dynamical effect due to virtual photon interacting in nucleus



The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken x_B :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.

If the scattering on nucleon N_1 is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\overline{q}$ flux reaching N_2.

Interior nucleons shadowed

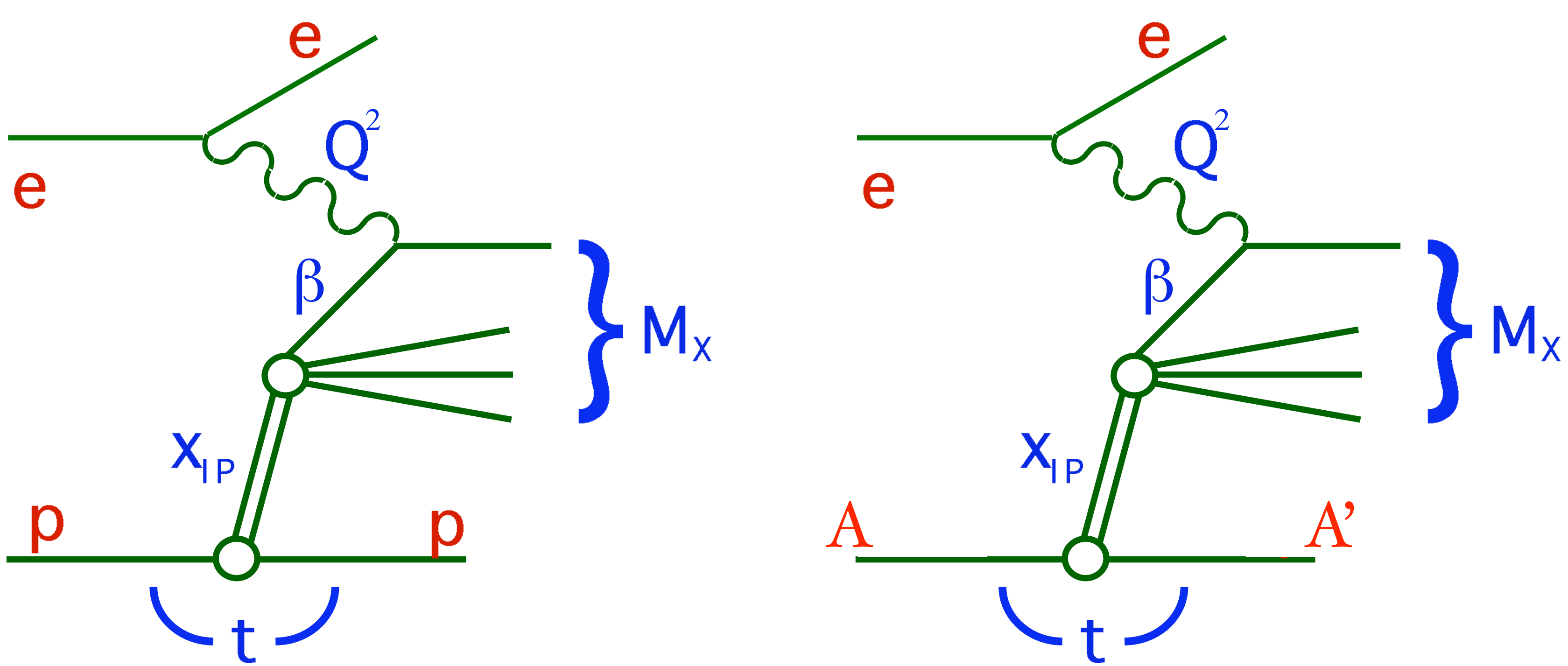
→ Shadowing of the DIS nuclear structure functions.

Observed HERA DDIS produces nuclear shadowing

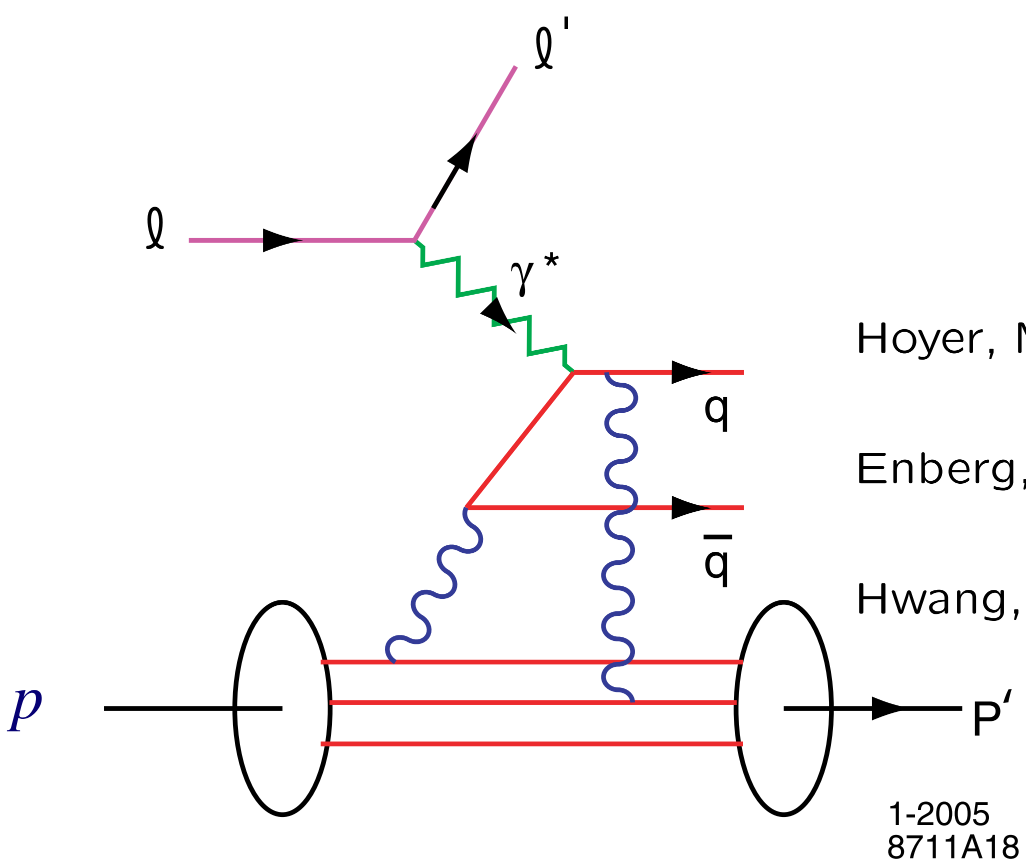
Diffractive Deep Inelastic Scattering

Diffractive DIS $ep \to epX$ where there is a large rapidity gap and the target nucleon remains intact probes the final state interaction of the scattered quark with the spectator system via gluon exchange.

Diffractive DIS on nuclei $eA \to e'AX$ and hard diffractive reactions such as $\gamma^* A \to VA$ can occur coherently leaving the nucleus intact.



Quark Rescattering



Hoyer, Marchal, Peigne, Sannino, SJB (BHMPS)

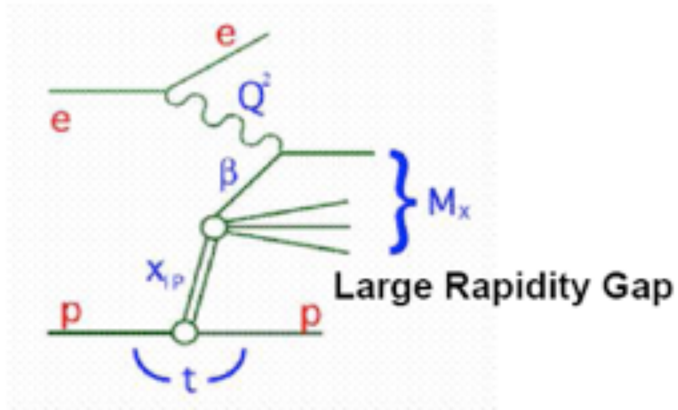
Enberg, Hoyer, Ingelman, SJB

Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron

de Roeck

Diffractive Structure Function F_2^D



Diffractive inclusive cross section

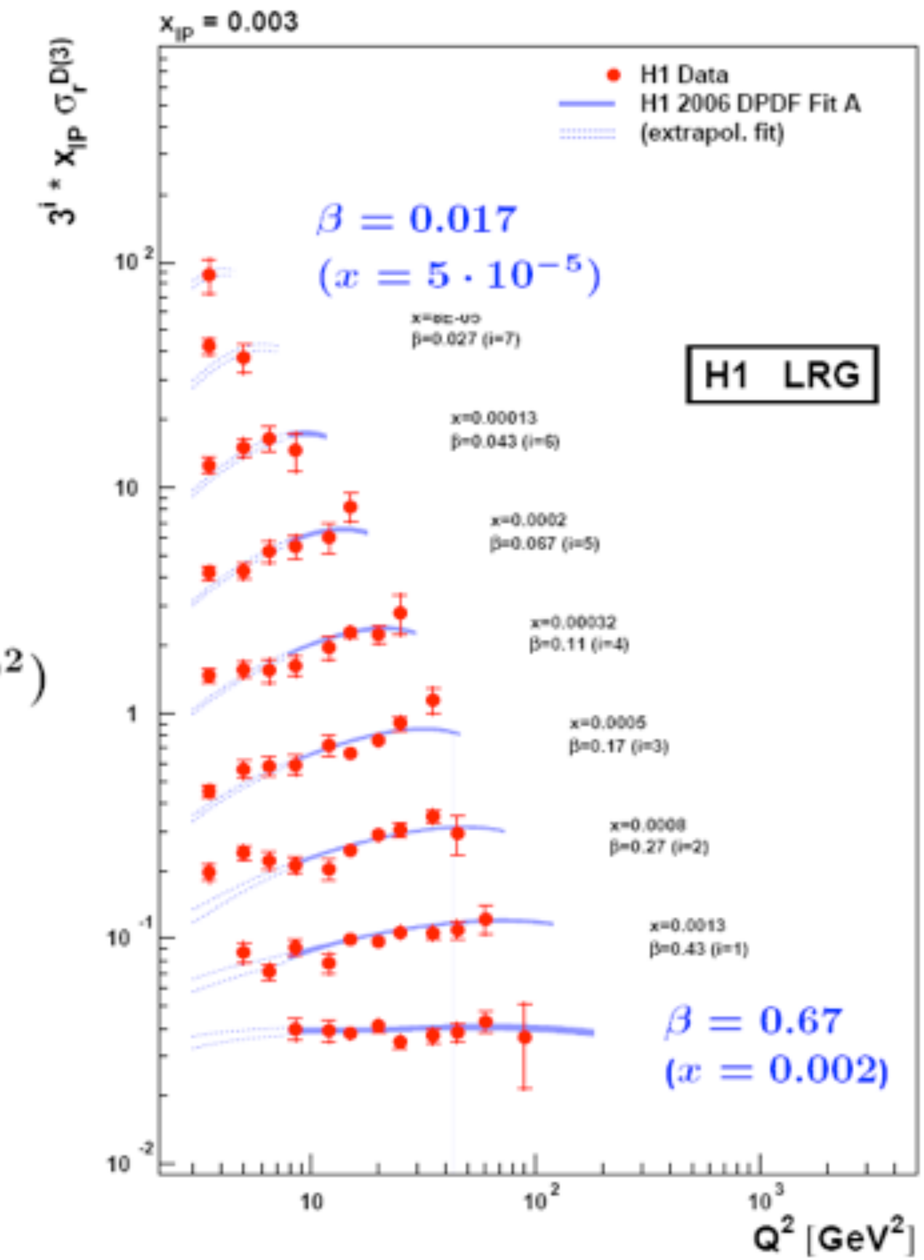
$$\frac{d^3\sigma_{NC}^{diff}}{dx_{I\!P}\, d\beta\, dQ^2} \propto \frac{2\pi\alpha^2}{xQ^4} F_2^{D(3)}(x_{I\!P}, \beta, Q^2)$$

$$F_2^D(x_{I\!P}, \beta, Q^2) = f(x_{I\!P}) \cdot F_2^{I\!P}(\beta, Q^2)$$

extract DPDF and $xg(x)$ from scaling violation

Large kinematic domain $3 < Q^2 < 1600\ \mathrm{GeV}^2$

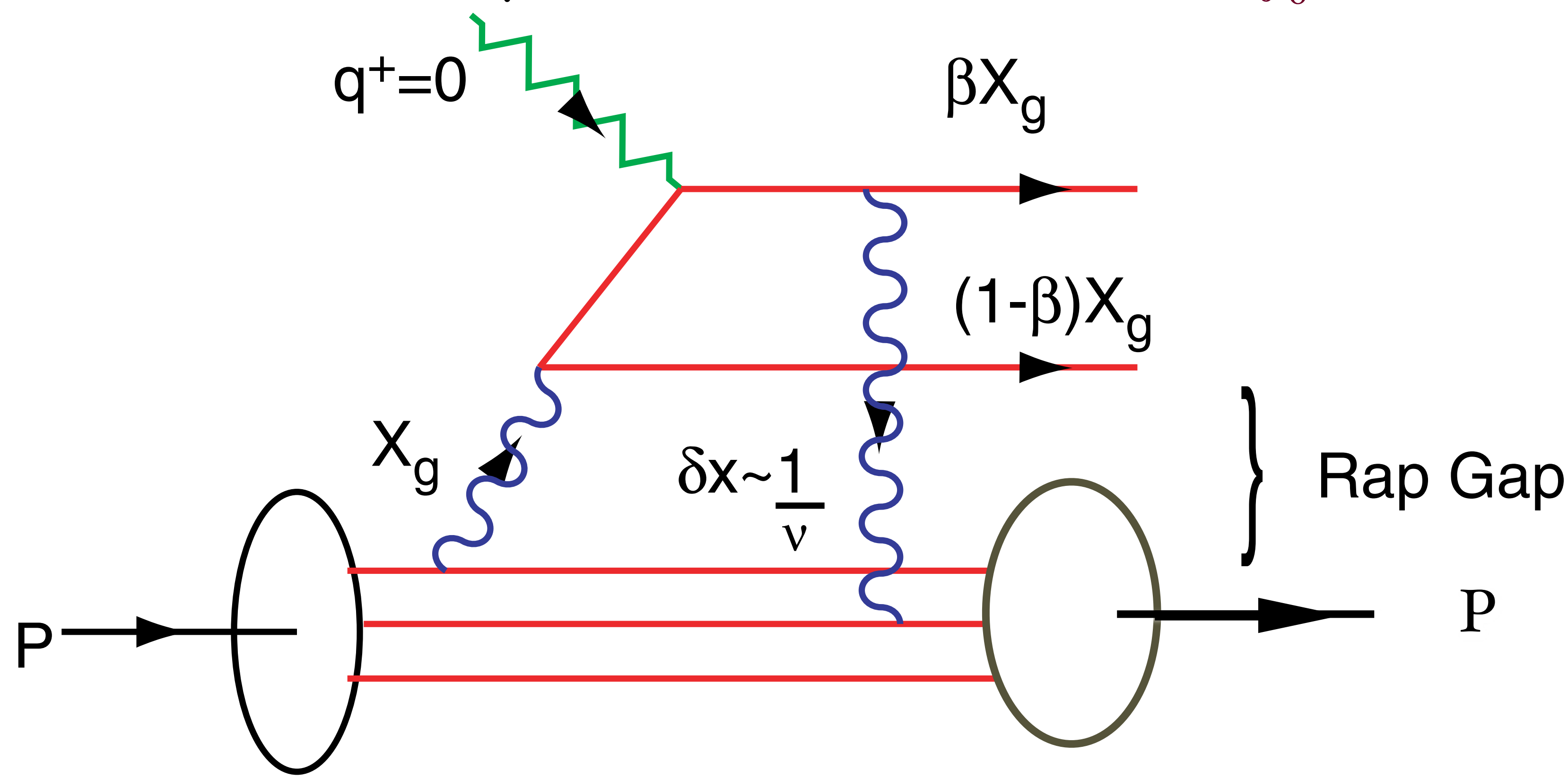
Precise measurements *sys 5%, stat 5–20 %*



Hoyer, Marchal, Peigne, Sannino, sjb

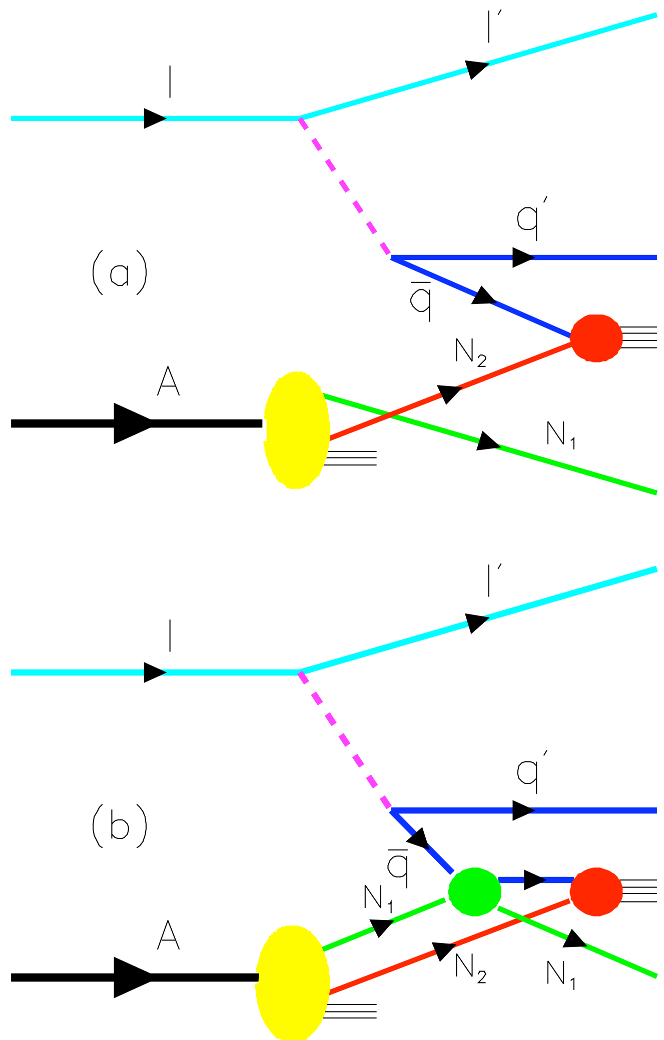
QCD Mechanism for Rapidity Gaps

Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$



Reproduces lab-frame color dipole approach

Schmidt, Lu, Yang, sjb



The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken x_B :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.

If the scattering on nucleon N_1 is via ~~pomeron~~ Regge exchange, the one-step and two-step amplitudes are ~~opposite in phase, thus diminishing the $\bar{q}$ flux reaching N_2.~~

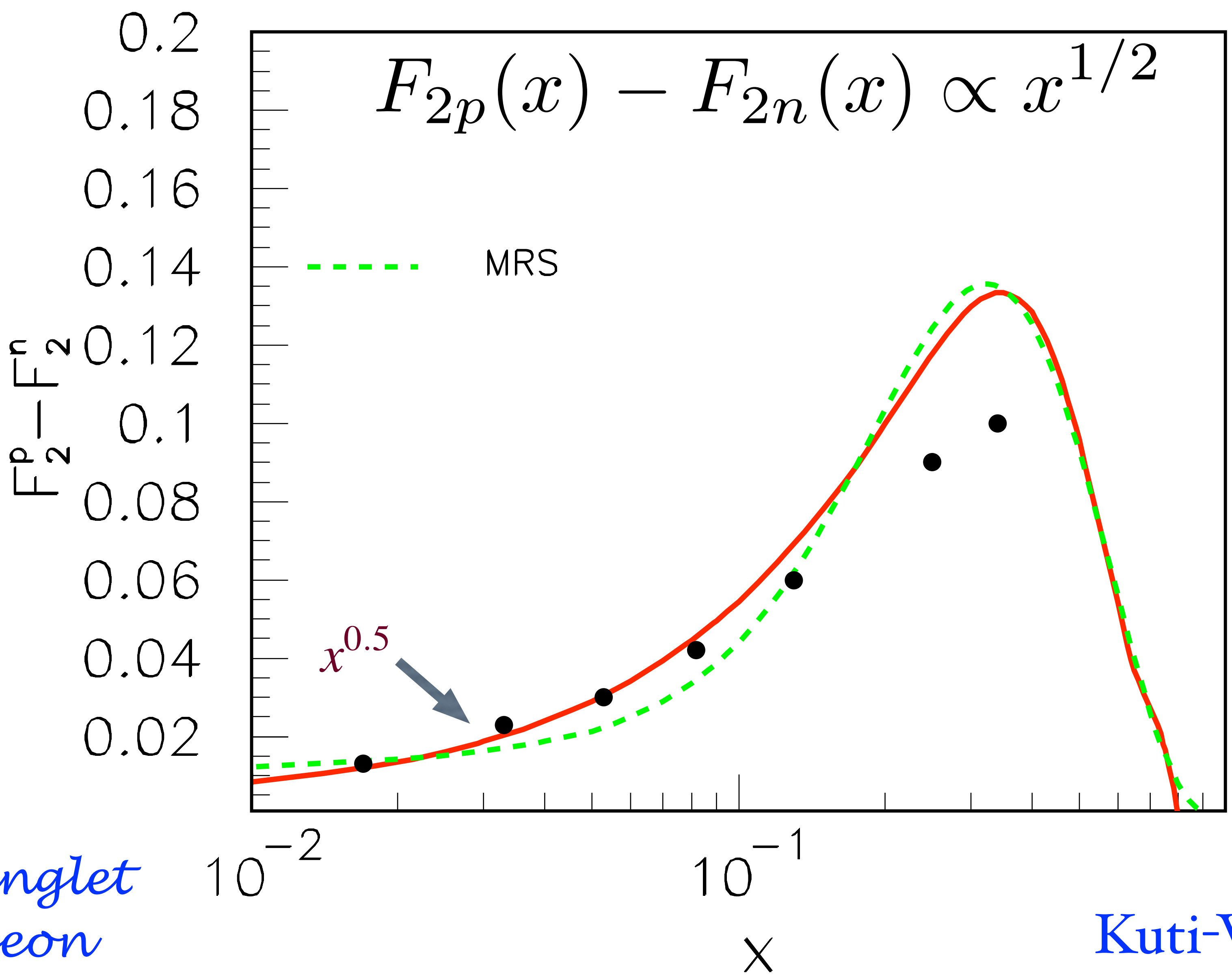
constructive in phase

thus ***increasing*** the flux reaching N_2

Interior nucleons anti-shadowed

Regge Exchange in DDIS produces nuclear anti-shadowing

Regge contribution: $\sigma_{\bar{q}N} \sim \hat{s}^{\alpha_R - 1}$ $\alpha_R \simeq 1/2$



Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior

Landshoff, Polkinghorne, Short ***Close, Gunion, SJB***

Reggeon Exchange

Regge contribution: $\sigma_{\bar{q}N} \sim \hat{s}^{\alpha_R - 1}$ $\alpha_R \simeq 1/2$

Phase of two-step amplitude relative to one step:

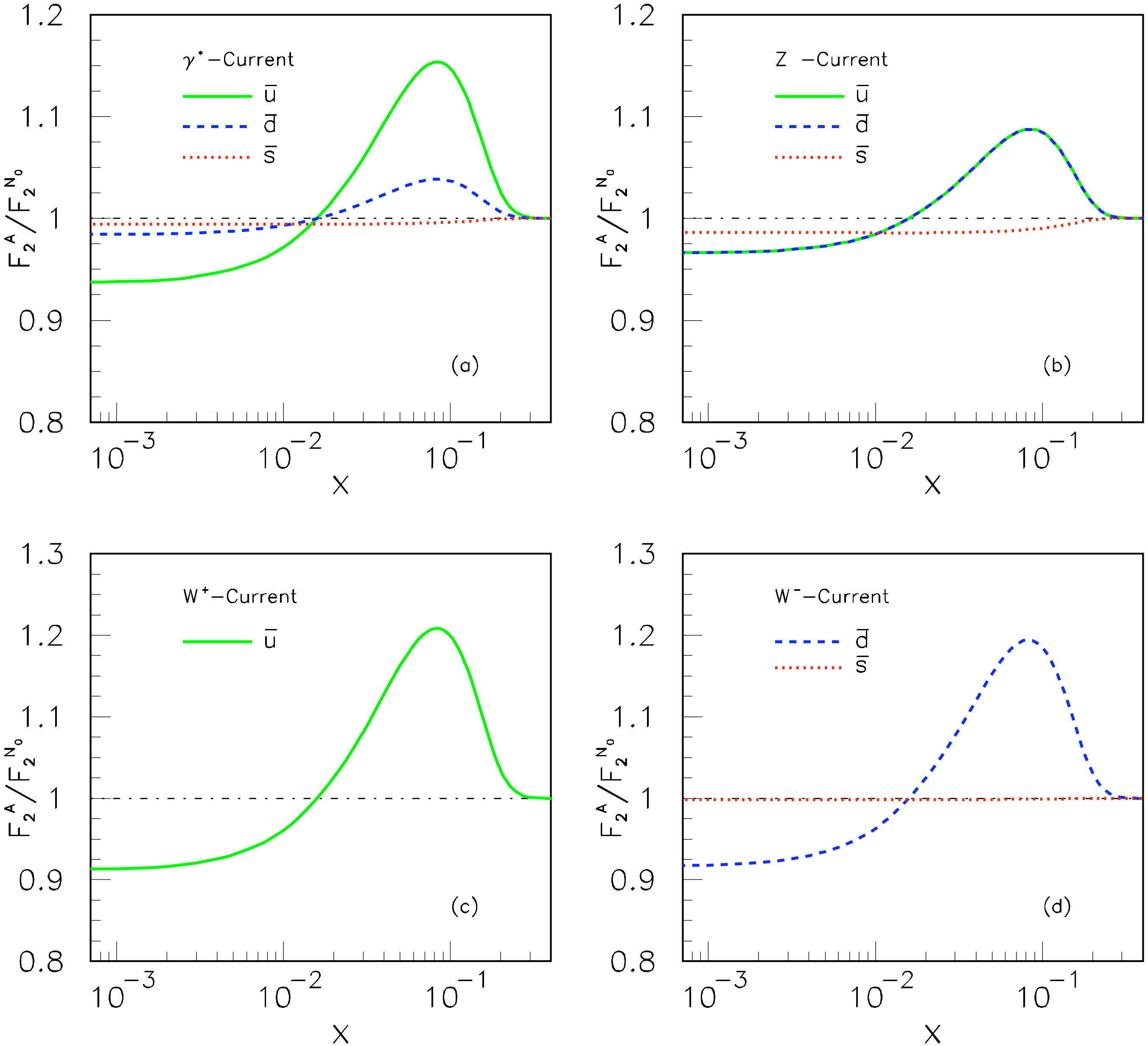
$$\frac{1}{\sqrt{2}}(1-i) \times i = \frac{1}{\sqrt{2}}(i+1)$$

Constructive Interference

Depends on quark flavor!

Thus antishadowing is not universal

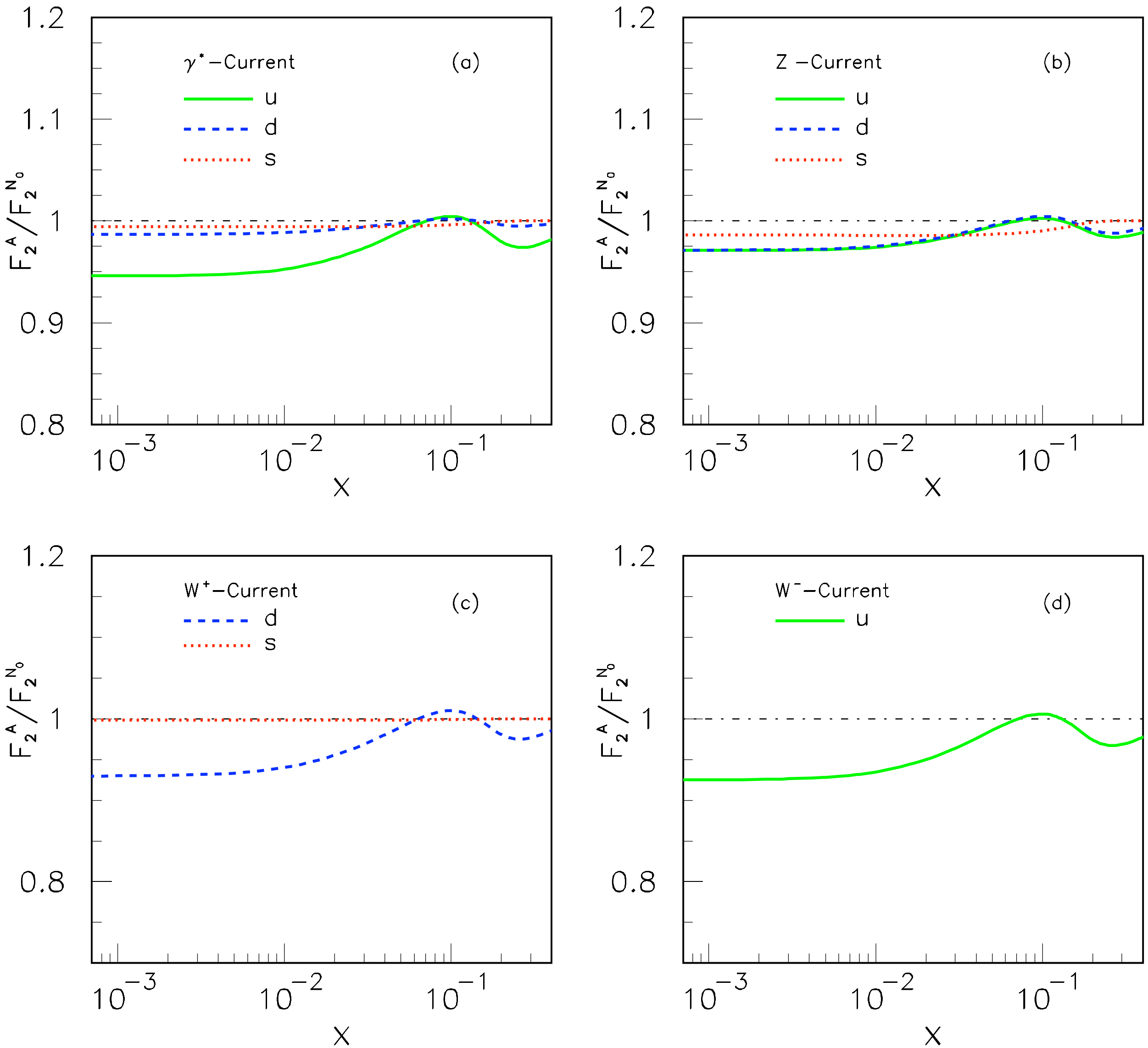
Different for couplings of $\gamma^*, Z^0, W^\pm$



Schmidt, Lu, Yang, sjb

Nuclear Antishadowing not universal !

Shadowing and Antishadowing of DIS Structure Functions



S. J. Brodsky, I. Schmidt and J. J. Yang, "Nuclear Antishadowing in Neutrino Deep Inelastic Scattering," Phys. Rev. D 70, 116003 (2004) [arXiv:hep-ph/0409279].

Modifies NuTeV extraction of $\sin^2\theta_W$

Test in flavor-tagged lepton-nucleus collisions

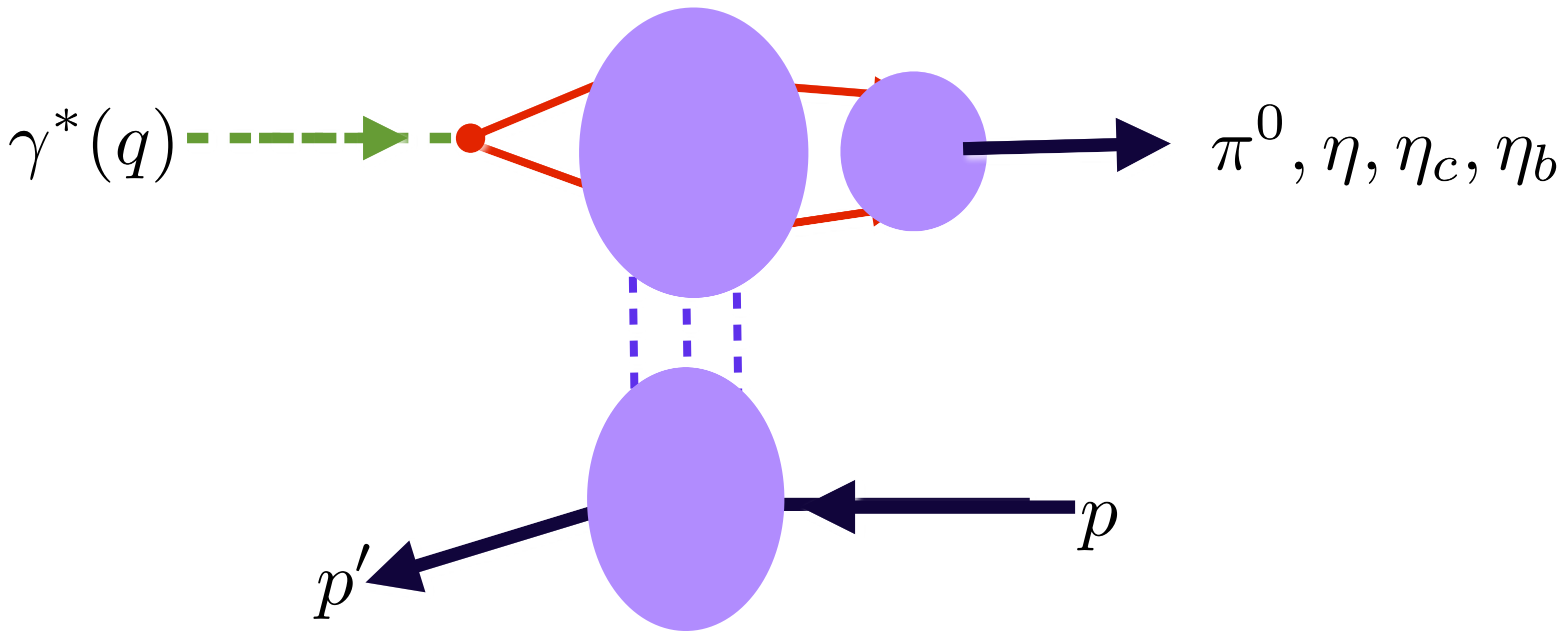
Crucial JLab Experiments

- *Measure Diffractive DIS: Agree with Shadowing of Nuclear Structure Functions?*

- *Isospin Dependence of Diffractive DIS — Reggeon Exchange*

- *Flavor Dependence of Antishadowing: Tagged Quark Distributions?*

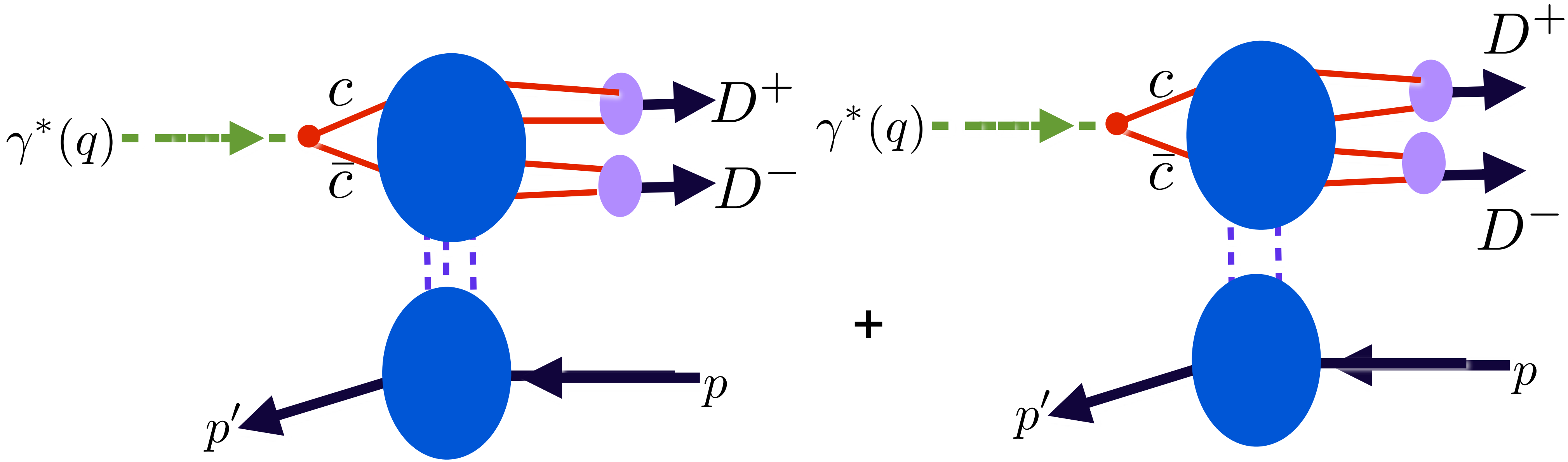
- *Test for Odderon Exchange in DDIS*



Odderon has never been observed!

Look for Charge Asymmetries from Odderon-Pomeron Interference

Merino, Rathsman, sjb



Odderon-Pomeron Interference leads to $K^+ K^-$, $D^+ D^-$ and $B^+ B^-$ charge and angular asymmetries

Odderon at amplitude level

Merino, Rathsman, sjb

Strong enhancement at heavy-quark pair threshold from QCD Sakharov-Schwinger-Sommerfeld effect

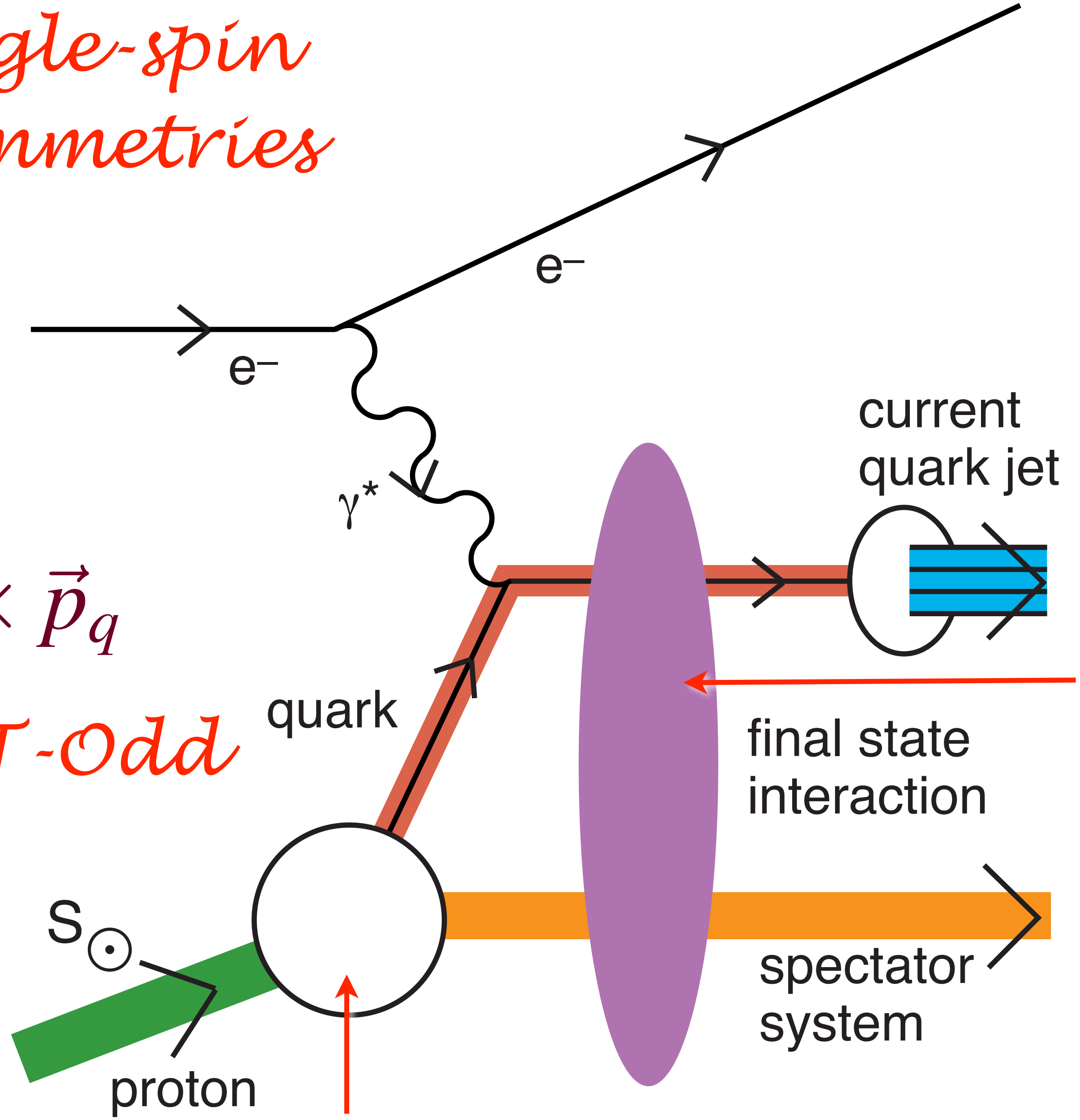
$$\frac{\pi \alpha_s(\beta^2 s)}{\beta}$$

Hoang, Kuhn, sjb

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb



Collins, Burkardt, Ji, Yuan. Xiao, Pasquini, ...

$$i \ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

QCD S- and P- Coulomb Phases --Wilson Line

Pseudo-T-Odd

"Lensing Effect"

QED: Lensing involves soft scales

Light-Front Wavefunction S and P- Waves!

Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!

Single-spin asymmetries in exclusive channels

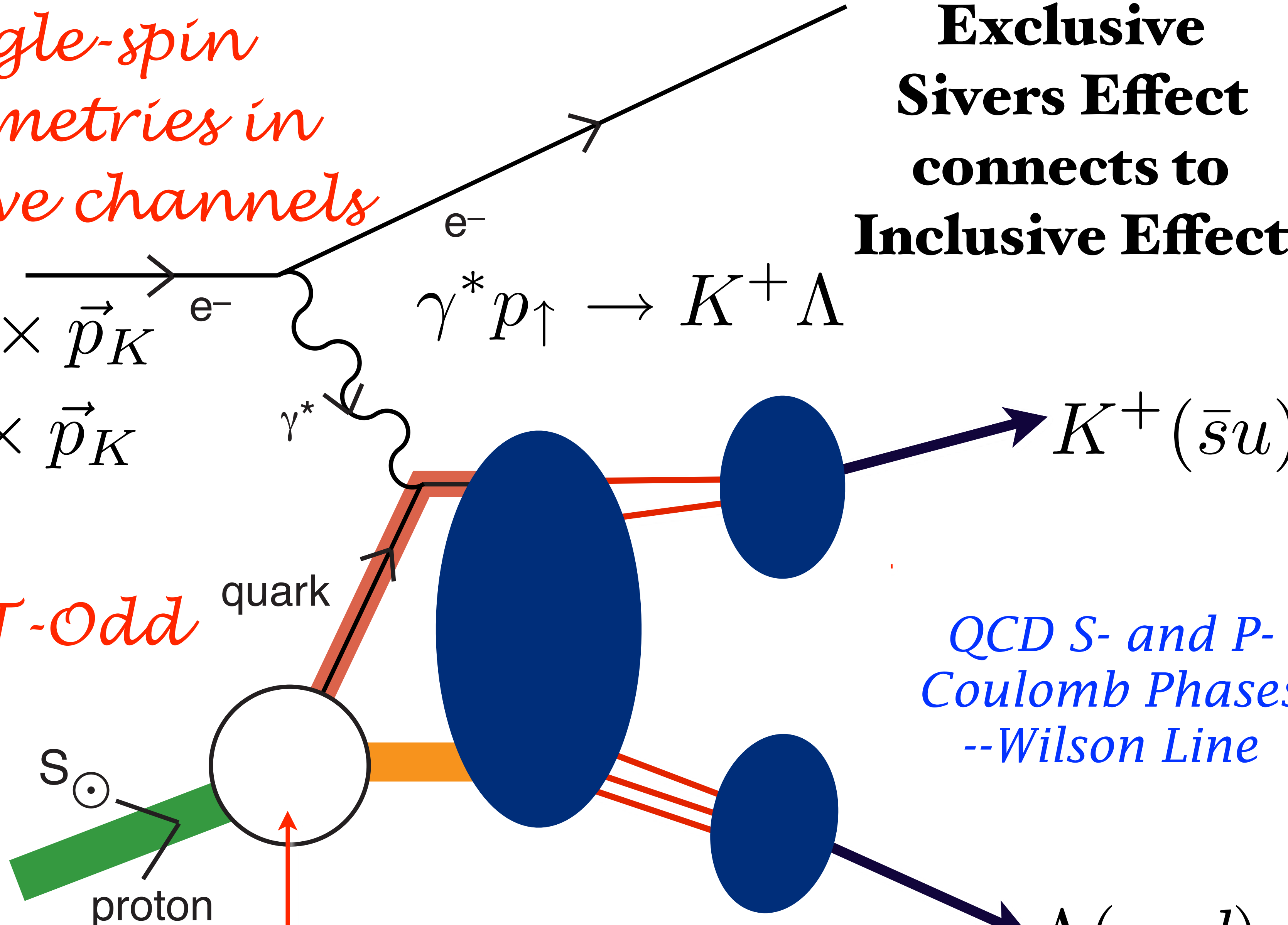
Exclusive Sivers Effect connects to Inclusive Effect

$$i\vec{S}_\Lambda \cdot \vec{q} \times \vec{p}_K$$

$$i\vec{S}_p \cdot \vec{q} \times \vec{p}_K$$

$$\gamma^* p_\uparrow \to K^+ \Lambda$$

Pseudo-T-Odd



QCD S- and P-Coulomb Phases --Wilson Line

Light-Front Wavefunction S and P- Waves

What is measured!

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven !
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

Hwang, Schmidt, sjb,

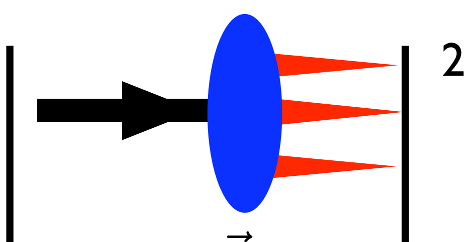
Mulders, Boer

Qiu, Sterman

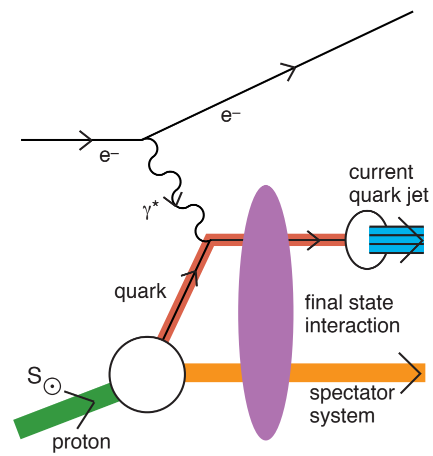
Collins, Qiu

Pasquini, Xiao, Yuan, sjb

Liuti, sjb



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Need a First Approximation to QCD

Comparable in simplicity to
Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Atomic Physics from First Principles

$\mathcal{L}_{QED} \rightarrow$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) \; |\Psi> = E \; |\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})] \; \psi(\vec{r}) = E \; \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}} \frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}} \frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)] \; \psi(r) = E \; \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

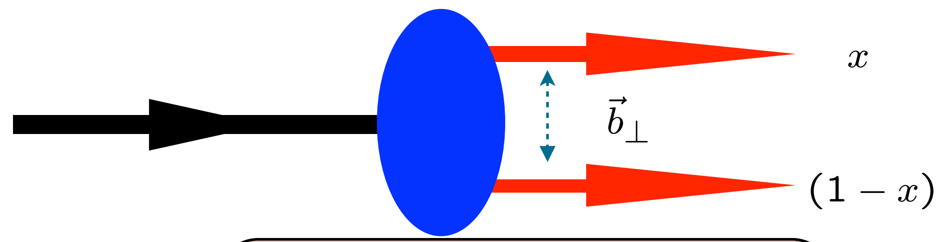
Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

Light-Front QCD

Fixed $\tau = t + z/c$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$



$$\zeta^2 = x(1-x)b^2_\perp$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\text{eff}}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \; \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis

ζ, ϕ

$m_q = 0$

AdS/QCD:

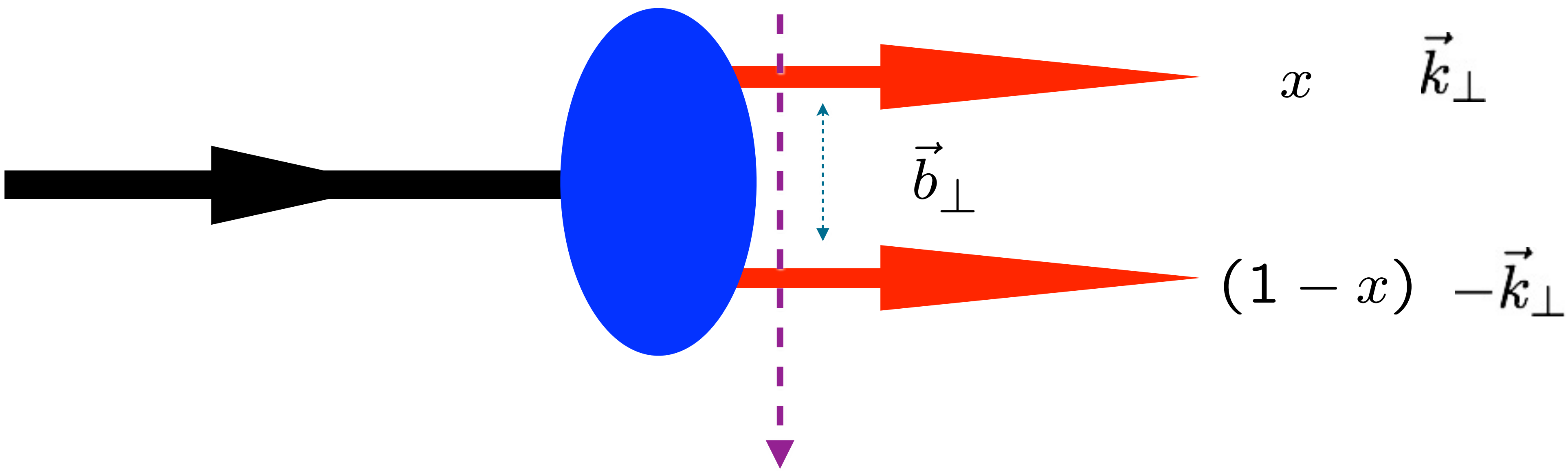
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

Fixed $\tau = t + z/c$



$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

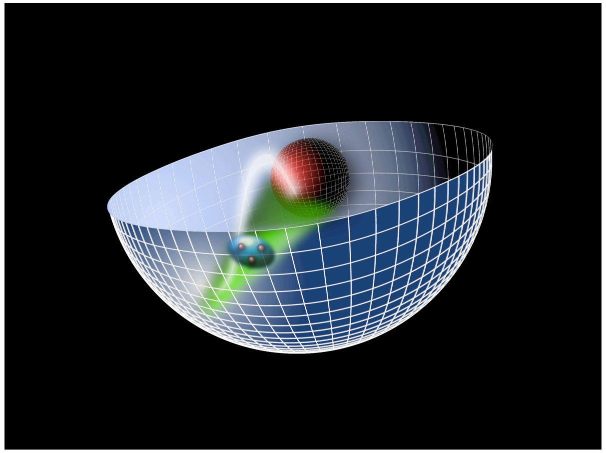
Invariant transverse separation

$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}^2_{q+\bar{q}}$$

$$\int dk^- \Psi_{BS}(P,k) \rightarrow \psi_{LF}(x, \vec{k}_\perp)$$

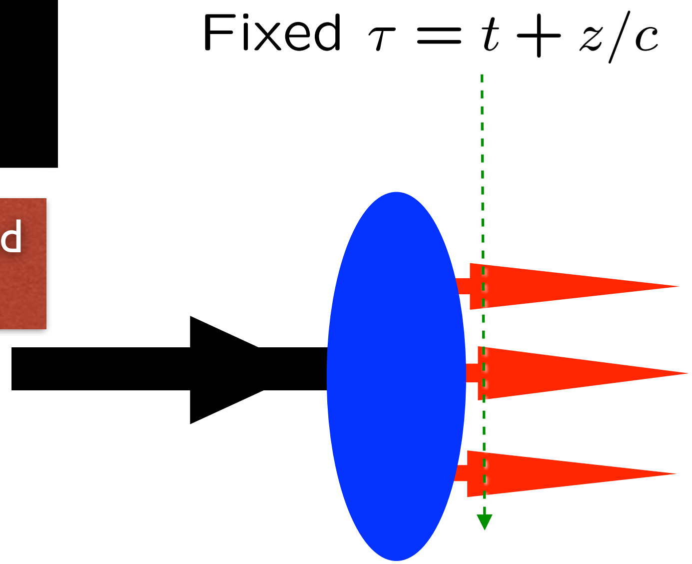
$\phi(z)$

AdS5: Conformal Template for QCD



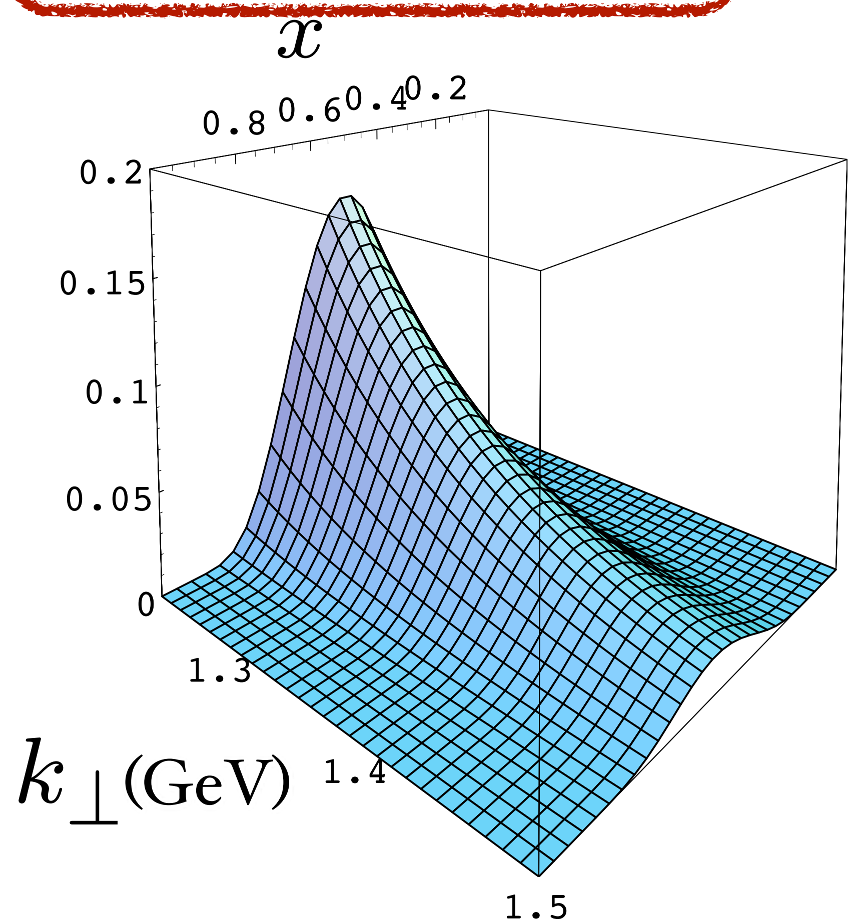
with Guy de Teramond and Hans Guenter Dosch

- *Light-Front Holography*



Duality of AdS$_5$ with LF Hamiltonian Theory

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation
Spectroscopy and Dynamics

Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons,***
- ***Chiral features for m_q=0: m_π=0, chiral-invariant proton***
- ***Hadronic LFWFs***
- ***Counting Rules***
- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

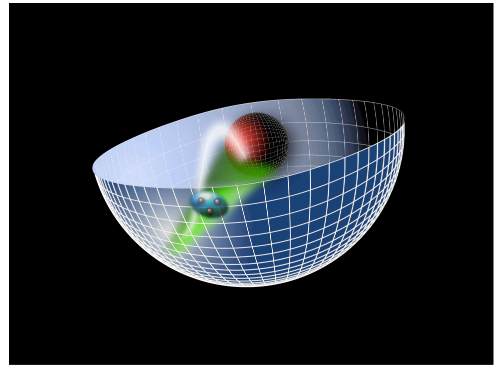
Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6 \ GeV$$

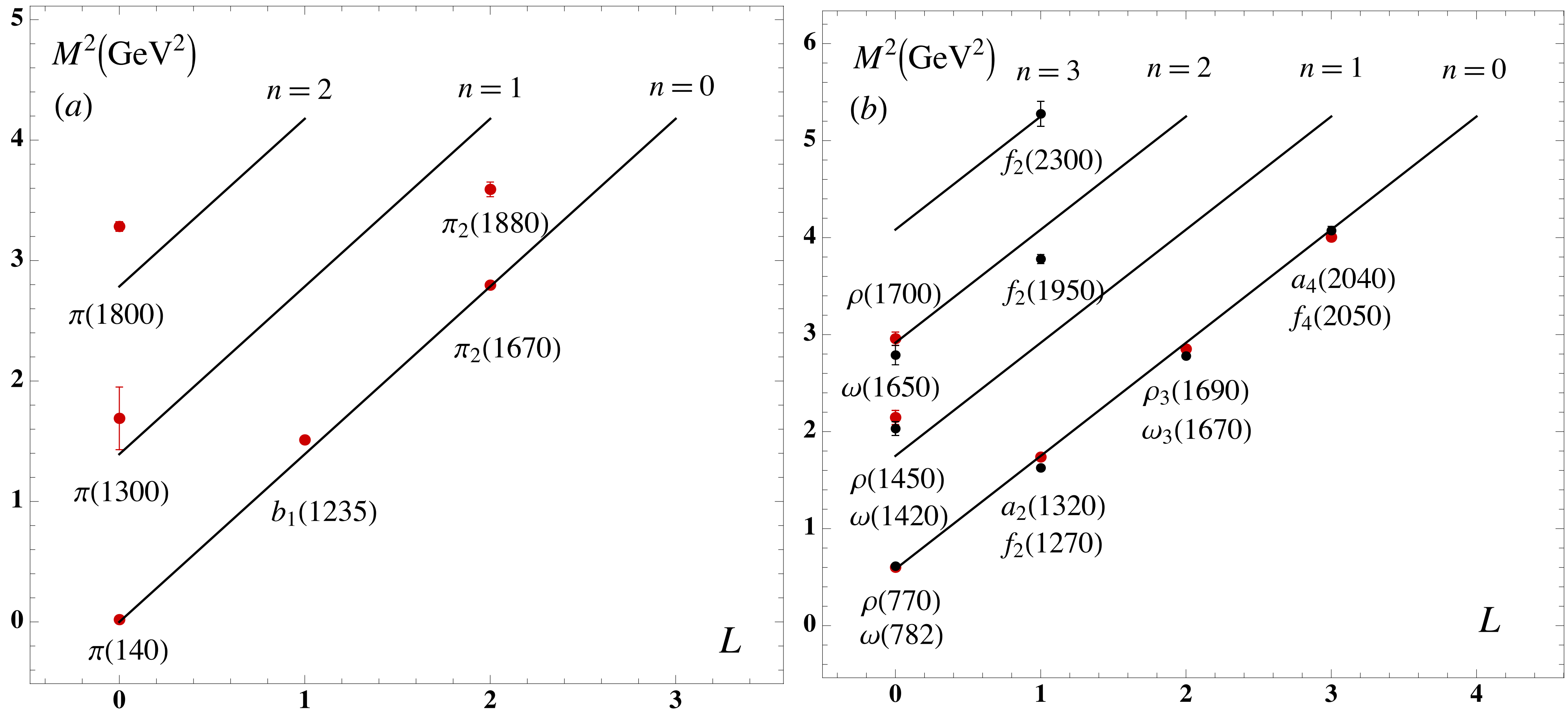
$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

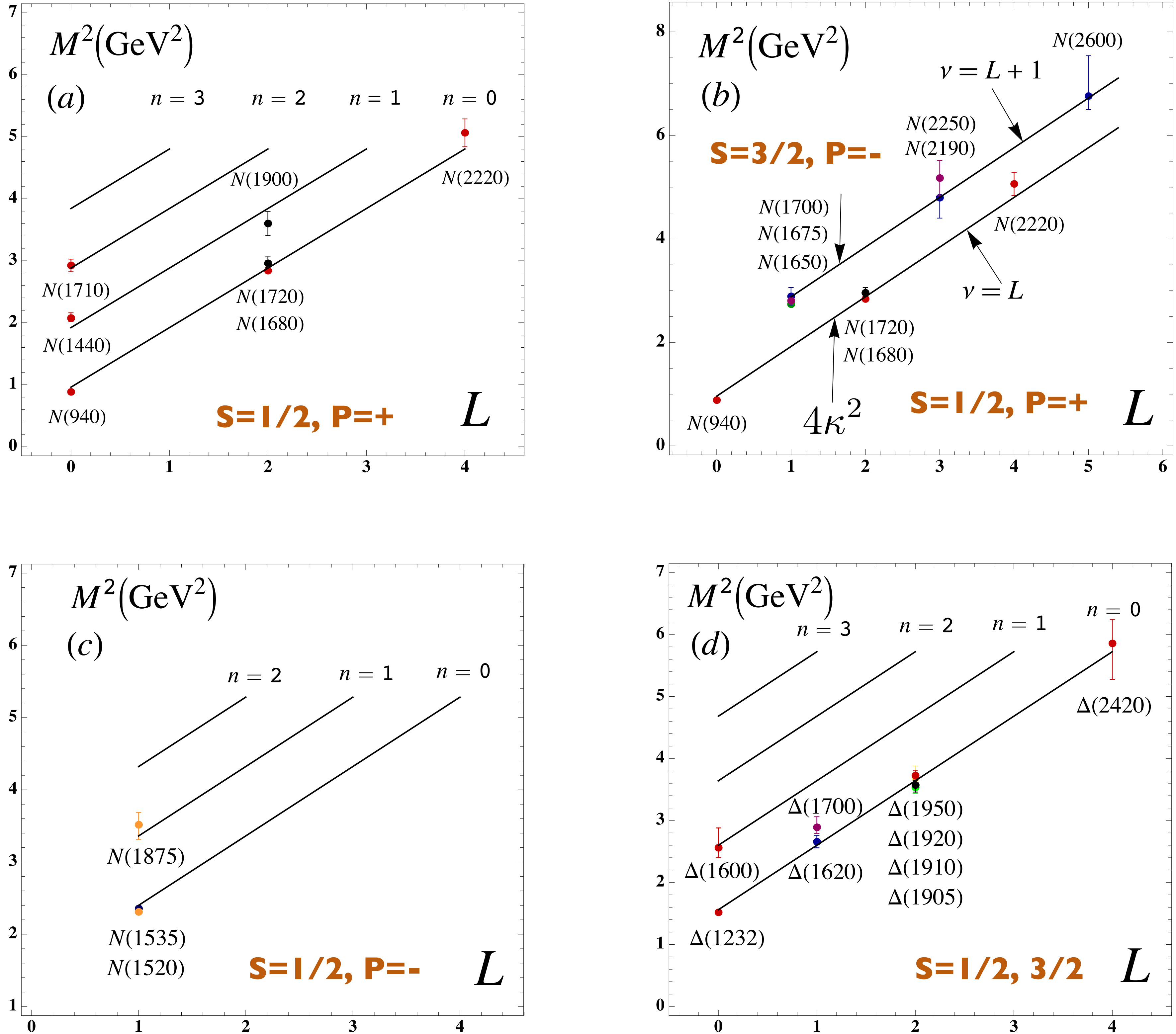
Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

$$m_u = m_d = 0$$

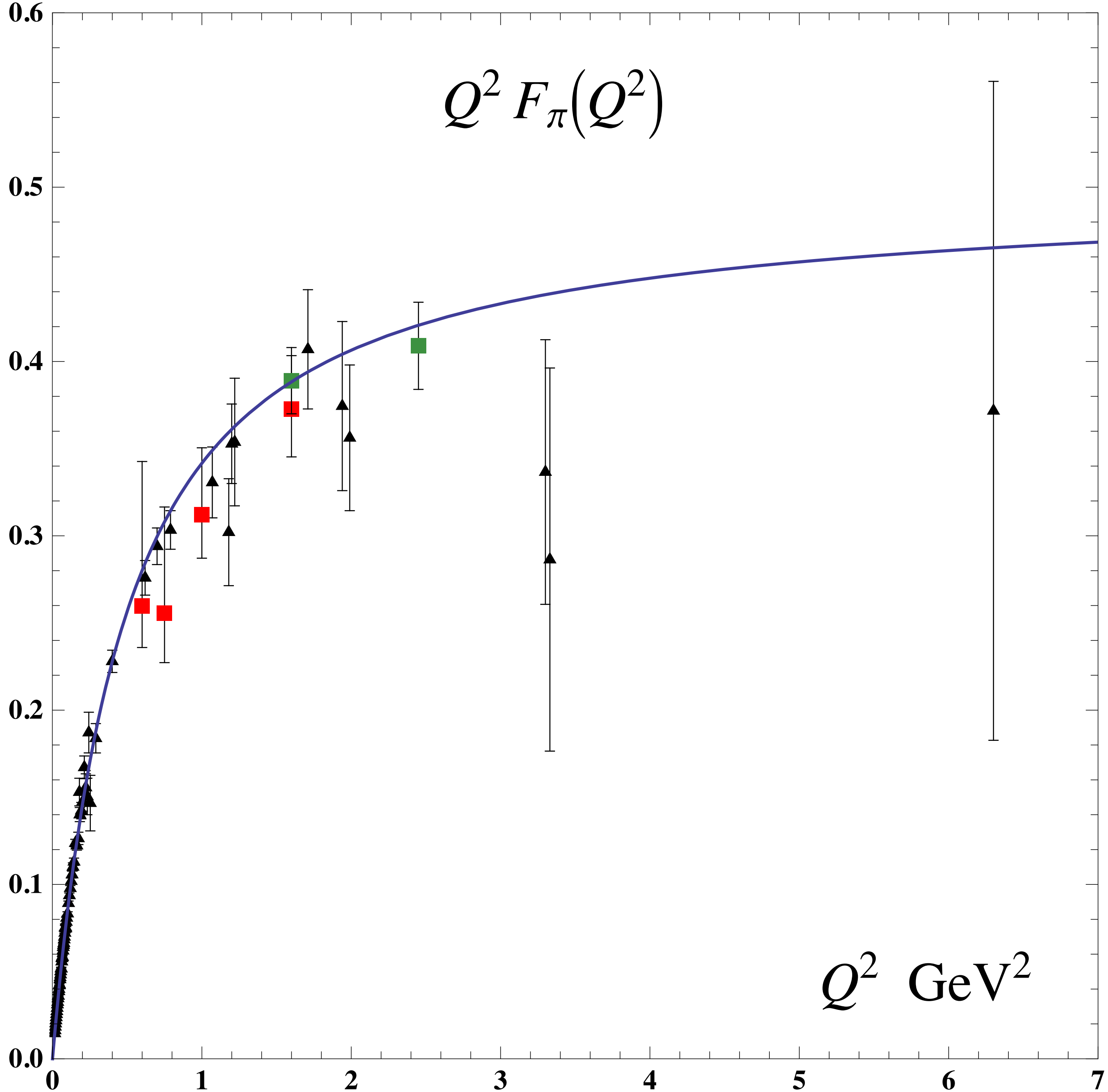
Preview



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$



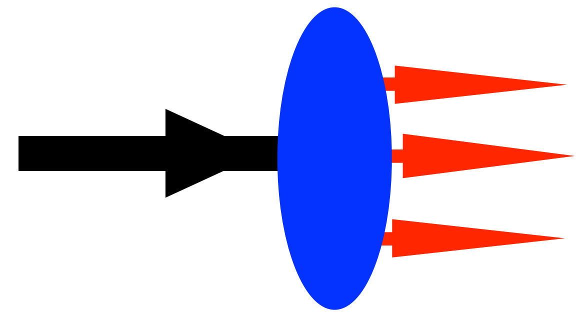
$M^2(\mathrm{GeV}^2)$
(a)
$n = 3$
$n = 2$
$n = 1$
$n = 0$
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
$\nu = L + 1$
N(2600)
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
$n = 2$
$n = 1$
$n = 0$
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
$n = 3$
$n = 2$
$n = 1$
$n = 0$
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

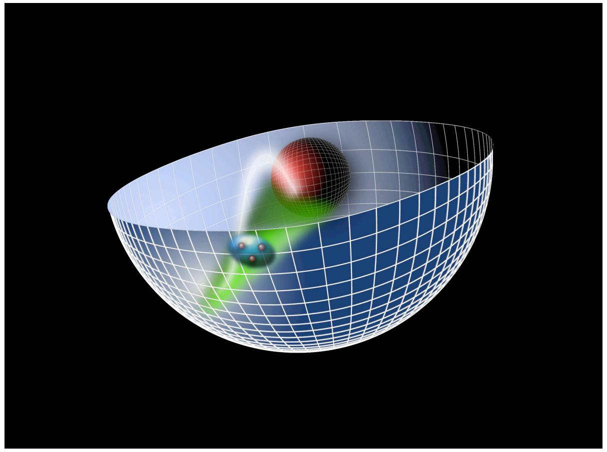
AdS/QCD and Light-Front Holography

- A first, semi-classical approximation to nonpertubative QCD
- Hadron Spectroscopy and LF Dynamics
- Color Confinement Potential
- Running QCD Coupling $\alpha(Q^2)$ at All Scales Q^2
- What sets the QCD Mass Scale?
- Connection of Hadron Masses to $\Lambda_{\overline{MS}}$

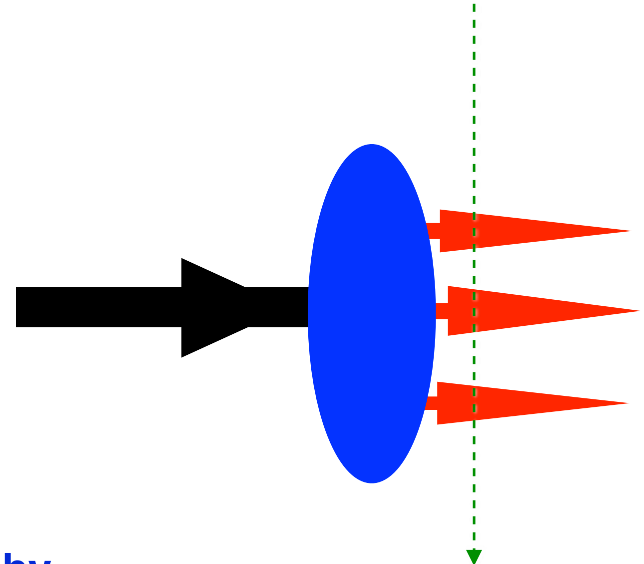


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Scattering Theory and Light-Front QCD

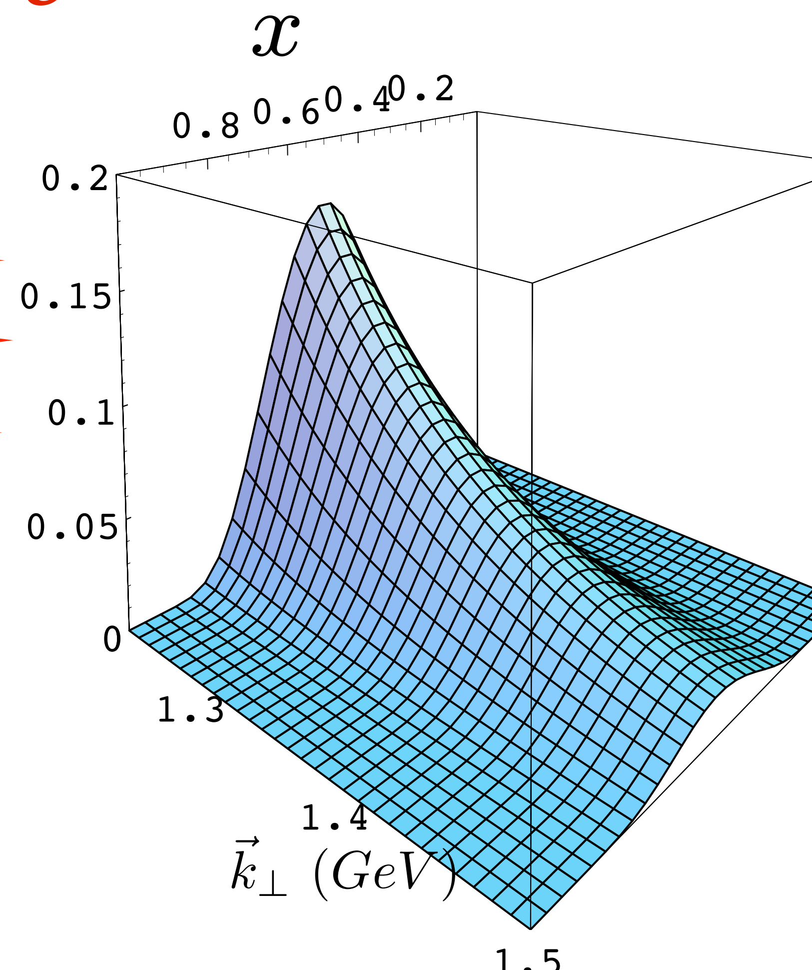


AdS/QCD : Light-Front Holography



Fixed $\tau = t + z/c$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



2015 International Summer Workshop on Reaction Theory

June 11, 2015

INDIANA UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY
DIE LUFT DER FREIHEIT WEHT
1891