Production Dynamics of Axial Vector Mesons

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- Thanks to Adam Szczepaniak, Geoffrey Fox, …., Moya Wright for the opportunity to be here

Axial vector (pseudo-vector) mesons

- Spin J and parity P $J^P = 1^+$
- Quark model $q\bar{q}$ states with orbital angular momentum $\ell = 1$
- Multiplet like the pseudo-scalar $J^P = 0^-$ and vector $J^P = 1^-$ states (whose $\ell = 0$)
- Particle Data Book names:
  — a_1 (1260) Isospin I = 1 $J^{PC} = 1^{++}$
  — b_1 (1235) I = 1 $J^{PC} = 1^{+-}$
- — K_1 (1270), K_1 (1400) I = 1/2, strangeness S = +/- 1
- Focus in these lectures primarily on the I = 1 a_1 state; with some discussion of the K_1 states
- a_1 forbidden to decay in two pions; 3 pions is the simplest

Outline - 1

1. Axial-vector mesons — names, symbols
2. Features of the data
3. Production dynamics
   - Deck model (non-resonant)
   - Final state interactions
   - Unitarity and analyticity
4. Phenomenology of the a_1
   - One pole, one channel case ($\rho\pi$)
   - One pole, two channel case ($\rho\pi$ and K^*K)
   - Heavy lepton decay $\tau \to a_1\nu$
5. Photoproduction of $\pi\pi$, and final state interactions

Outline - 2

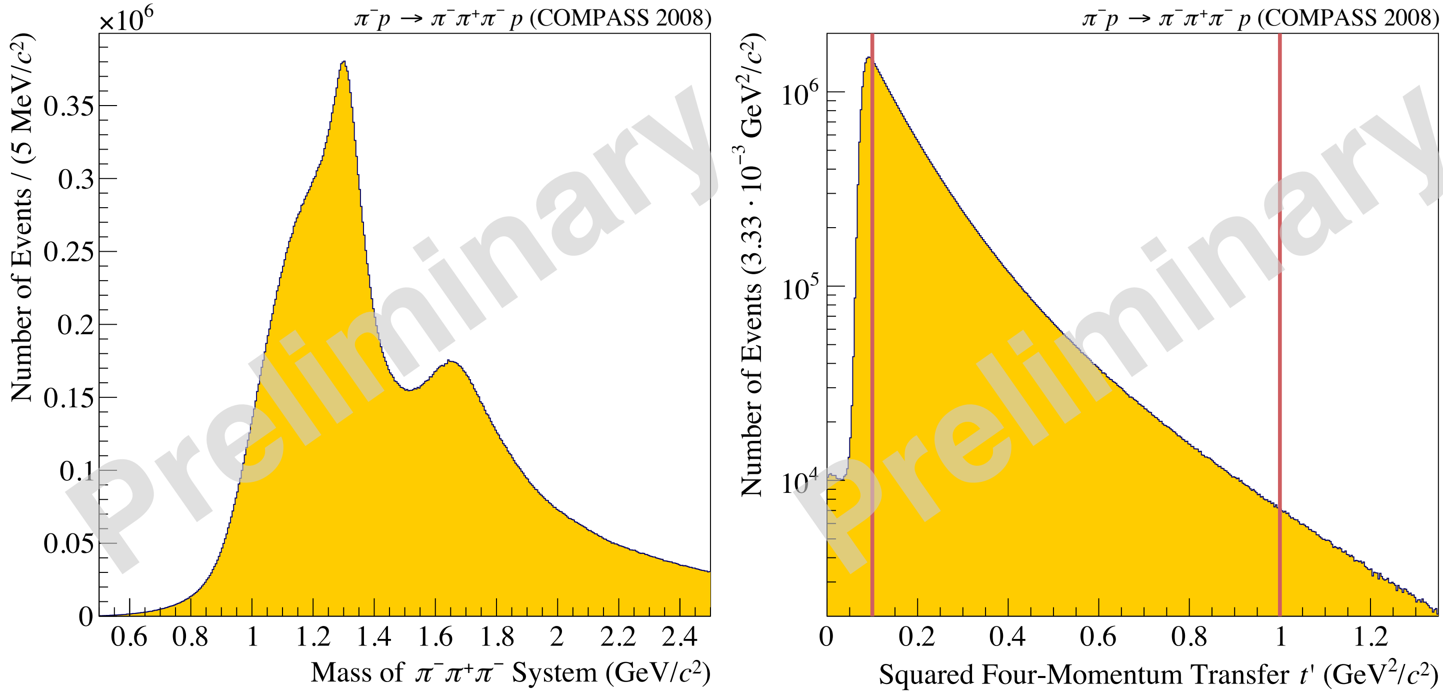
1. Axial-vector mesons in $K\pi\pi$
   - Two resonances (poles), two peaks
   - Mixing
2. Fast forward to 2014 - 2015, the a_1 again
   - One or two axial vector $\pi\pi\pi$ states?
   - One resonance pole, two peaks
   - Extraction of the axial vector mass, width, and branching fractions

How is the a_1 produced?

- Quantum numbers forbid a_1 decay into $\pi\pi$; $\pi\pi\pi$ is simplest
- Well studied reaction is $\pi p \to \pi\pi\pi p$. At high energy and small momentum transfer to the target, “beam excitation” meson systems can be separated from “target excitation” N*’s
- Another environment is a weak process such as heavy lepton decay $\tau \to \pi\pi\pi\nu$
- Selected hadronic experiments:
  - CERN-IHEP group, 25 and 40 GeV/c, 70K events, Antipov, Ascoli et al, Nucl Phys B63, 153 (1973); Phys Rev D7, 669 (1973)
  - ACCMOR, 63 and 94 GeV/c, ~600 K events, Daum et al Nucl Phys B182, 269 (1981)
  - E-852, 18 GeV/c, ~5 M events, Dzierba et al PRD 73 (2006)
  - COMPASS 190 GeV/c, 50 M events, Adolph et al arXiv: 1501.05732 [hep-ex] (2015)

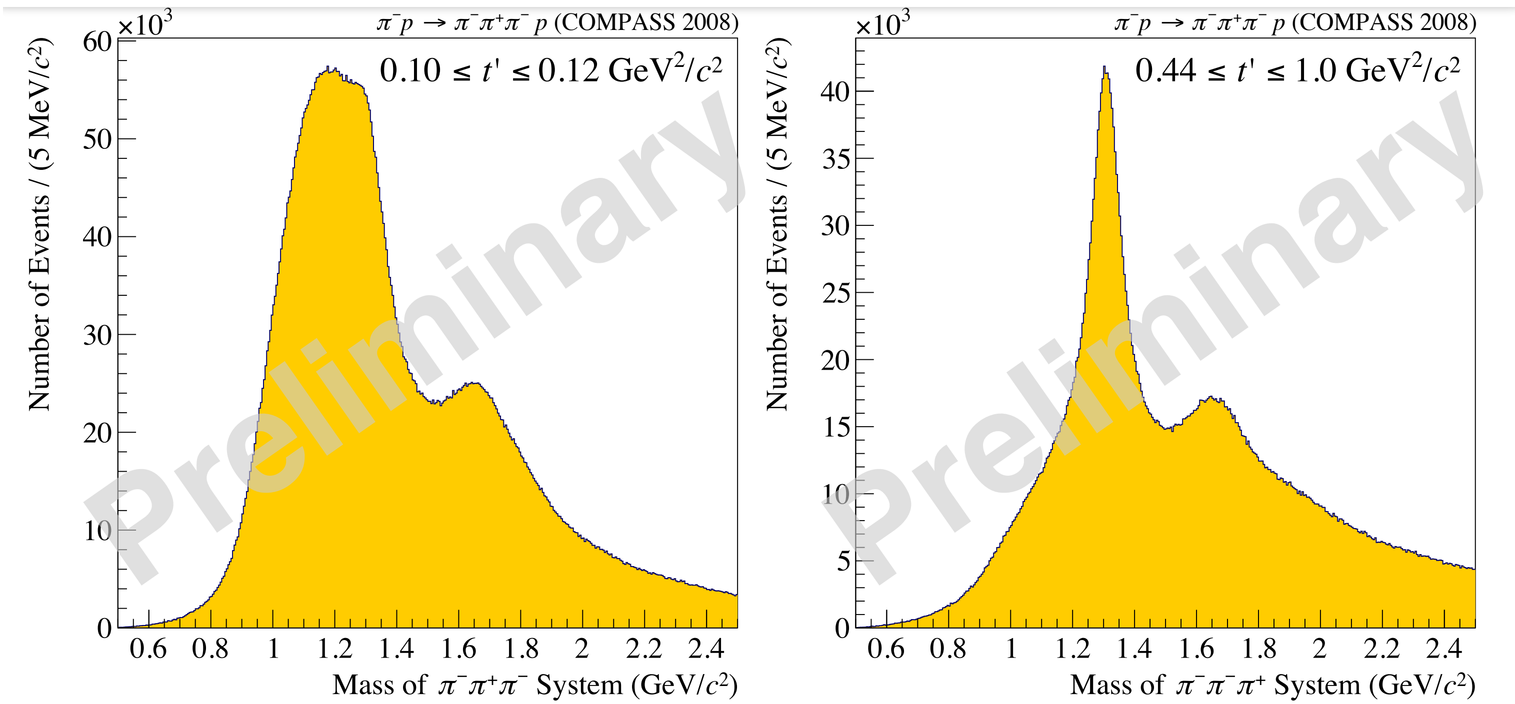
Characteristics of the hadronic data - 1

- Broad invariant mass distribution (COMPASS data as example) and sharply falling momentum transfer distribution



Characteristics of the hadronic data - 2

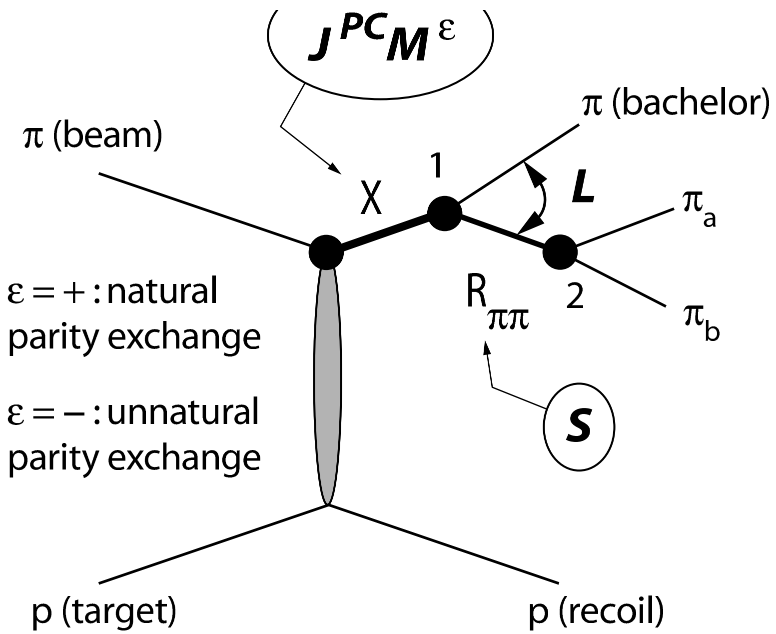
- Mass distribution changes with selections on the range of momentum transfer



- This correlation of the slope with mass is reasonably well understood theoretically (feature of the doubly-peripheral Deck mechanism)

Spin & parity content of the distribution

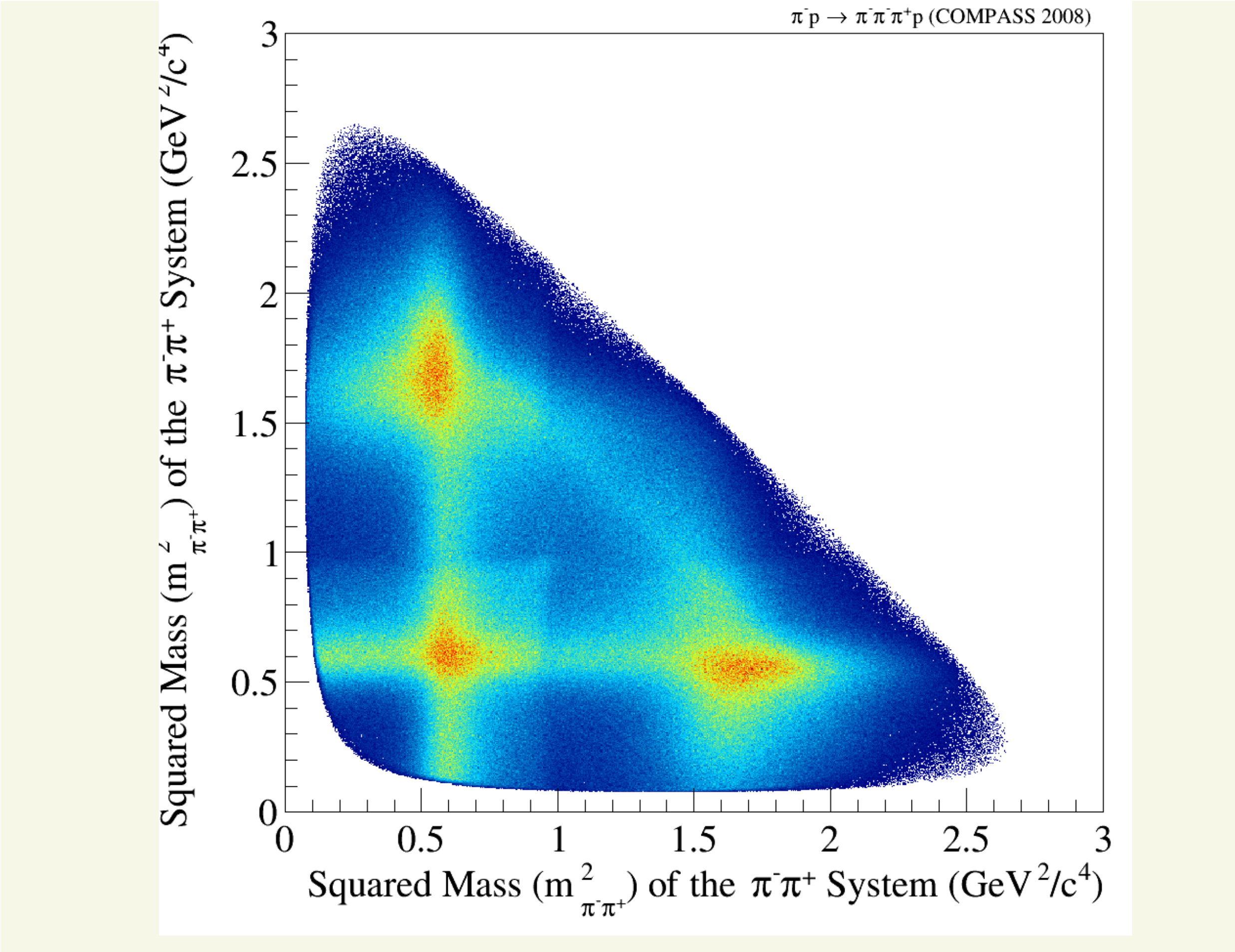
- Extracting the spin and parity of 3 body systems is more difficult for 3 body systems than for 2 body systems
- Special methods developed based on an isobar approach
- Think of the 3pi system X as a superposition of quasi two body systems $X = \rho\pi,\ f_0(980)\pi, ...$



- Are there 3-body contributions not well represented this way?

Example of a Dalitz plot

- Take slices of 3 pi mass and look at the Dalitz plot — evidence of isobars is evident
- $\pi_2(1670)$ region exhibits $\rho\pi,\, f_2(1270)\pi,\, f_0(980)\pi$

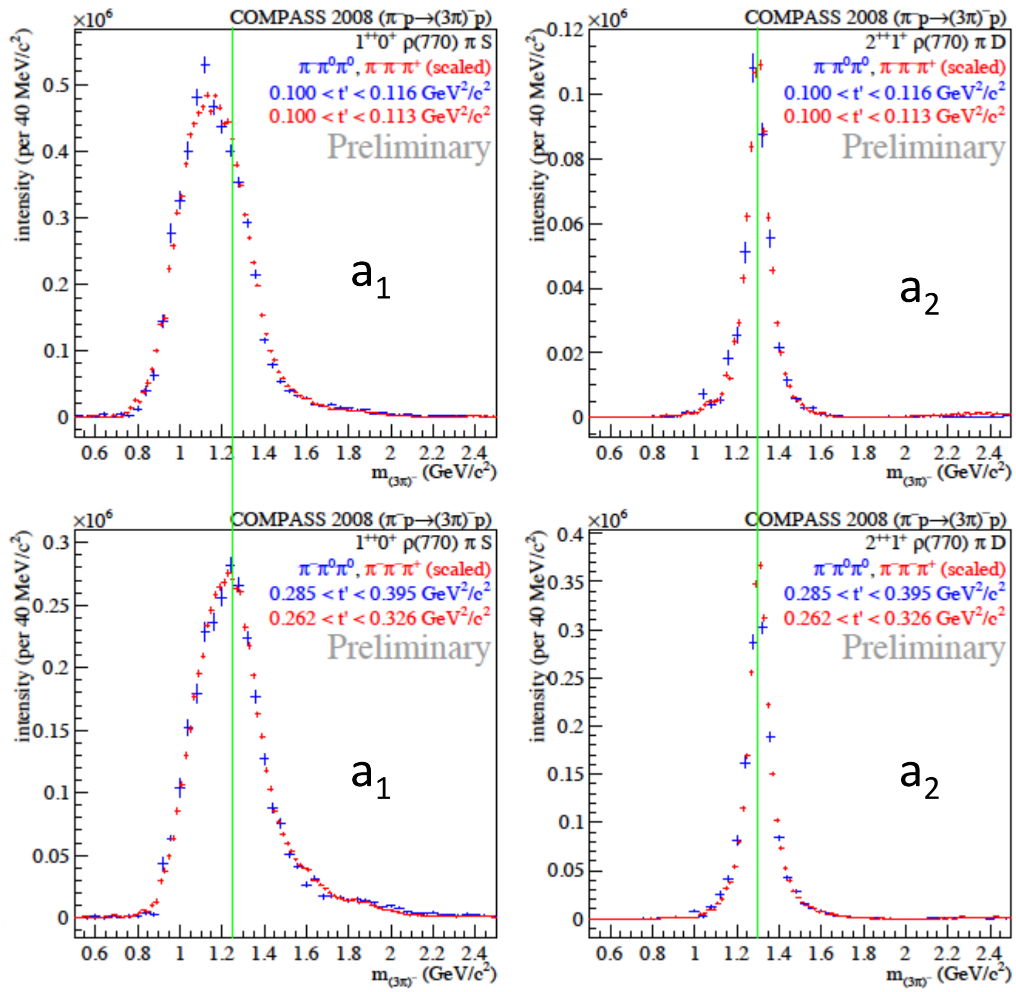


Spin & parity content of the distribution

- The intensities in the different spin-parity J^{PC} waves, and the relative phases among them are extracted from the isobar model fits to data

Two prominent waves at low mass

- Intensities of the $J^P = 1^+ \rho\pi$ S and $2^+ \rho\pi$ D waves



low t

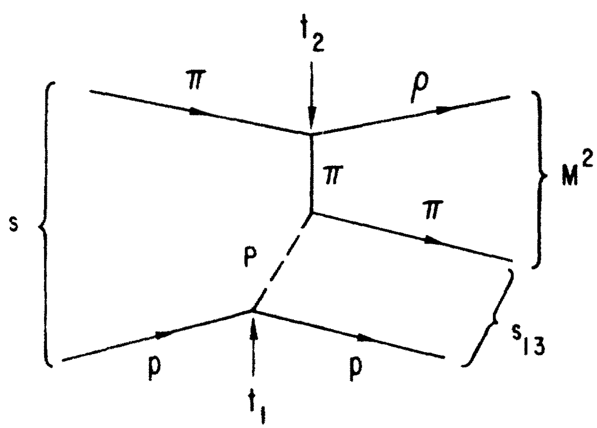
high t

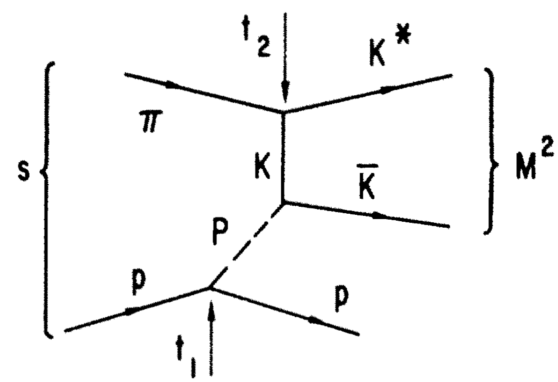
Interpretation of the J^P = 1+ wave

- There is no longer the uncertainty there was in ~1975 that the enhancement is resonant
- Nevertheless, determination of the mass and width of the a_1 resonance requires an understanding of what else in going on in that partial wave
- One question: why does the phase of the 1+ wave change slowly with respect to other partial waves, unlike the rapid behavior expected of a Breit-Wigner amplitude
- In mid 1970’s we treated $\pi p \to \pi\pi\pi p \text{ and } \pi p \to K\bar{K}\pi p$ as a coupled channel system
- Theoretical reasons: extra channel provides inelasticity and affects phase motion; SU(3) …..

Deck Production Mechanism

- Examine $\pi p \to \pi\pi\pi p$ at large incident π momentum and small momentum transfer to the target.
- Think of the 3 pion system as a superposition of quasi-two-body systems $(\pi\pi\pi) \to \pi\rho, \pi f_0, \ldots.$ likewise for $K\bar{K}\pi$
- One pion (one Kaon) exchange production, followed by diffractive scattering of the virtual pion from the target:





- Also graphs in which the rho or the K* are exchanged

Deck Amplitude - 1

- Deck amplitude for $\pi p \to \rho\pi p$

$$T_D^\rho = g_{\rho\pi\pi} K_\rho(t_2) \frac{1}{m_\pi^2 - t_2} i s_{13} e^{bt_1} \sigma_{\pi p}$$

- Similar expression for $\pi p \to K^* \bar{K} p$
- In the $\rho\pi$ rest frame, the Deck amplitude contributes to several partial waves.
- For $J^{PC} = 1^{++}$ we must project out the S wave component
- Re-express the invariants s_{13} and t_2 in terms of t-channel angles

$$t_2 = g_1(M, t_1) + g_2(M, t_1) \cos\theta_t$$

$$s_{13} = g_3(s, M) + g_4(s, M, t_1) \cos\theta_t + g_5(s, M, t_1) \sin\theta_t \cos\phi_t$$

- Deck amplitude is a rational function so we can project analytically all partial waves S, P, D, … (all m), for any value of t_1.

Deck Amplitude - 2

- Perform an expansion for small t_1 (where the data are concentrated) of the partial wave projections of the Deck production amplitude. Define a dimensionless expansion parameter

$$\Theta_1 = \frac{t_1}{(M^2 - m_\pi^2)}.$$

- The S-wave projection is

$$T_S^{Deck} = -\frac{s}{(M^2 - m_\pi^2)} \times \left(1 - \frac{1}{2}\Theta_1 \left(\frac{(3M^2 + m_\pi^2)}{(M^2 - m_\pi^2)} - \frac{E_\rho}{E_\pi}\right)\left(\frac{1}{y}\ln\frac{1+y}{1-y}\right)\right)$$

- Note that the Deck amplitude is a pure S-wave at $t_1 = 0$. Angular dependence cancels in the numerator and denominator.

- Aside: Would not be right to think of pion exchange here as feeding only high partial waves. Also question of how seriously to take details.

Deck Amplitude - 3

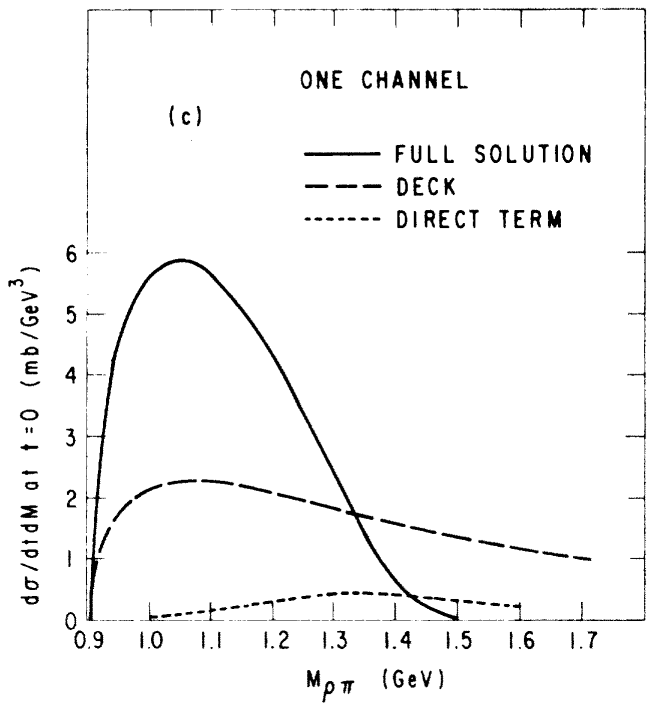
- Deck amplitudes with isospin 1 and t-channel helicity 0, written as a two-component vector:

$$T_D^{J^P=1^+}(M^2, s, 0) \equiv \begin{bmatrix} T_{\text{Deck}}(\rho\pi) \\ T_{\text{Deck}}(K^*\bar{K}) \end{bmatrix}$$

$$= \frac{2i\sqrt{2}\; s}{(M^2 - m_\pi^{\;2})} \begin{bmatrix} g_{\rho^0\pi^+\pi^-} K_\rho \sigma_{\pi p} N_\pi \\ g_{K^{*0}K^+\pi^-} K_{K^*} \sigma_{Kp} N_K \end{bmatrix}.$$

- Upper component refers to $\rho\pi$ and the lower to $K^*\bar{K}$

Low mass enhancement



- The differential cross section $d\sigma/dMdt_1$ at $t_1 = 0$ for $\pi p \to \rho\pi p$
- Focus now only on the LONG DASHED curve: The non-resonant Deck amplitude provides a broad enhancement just above threshold. Discuss the solid and short dashed curves later

Unitarization - 1

- Deck amplitude T^{Deck} produces $J^{PC} = 1^{++}$ non resonant enhancements near threshold in $\rho\pi$ and $K^*\bar{K}$.
- The $\rho\pi$ and $K^*\bar{K}$ are strongly interacting systems. They interact in the final state, even in the one channel $\rho\pi$ case. These final state interactions must be incorporated in the full amplitude. They are inevitable and non- negligible if there is a resonance, such as the $J^{PC} = 1^{++}$ $q\bar{q}$ state of the quark model.
- Construct a full amplitude T^u_{Deck} that includes final state interactions, the $q\bar{q}$ state, and respects unitarity (no double counting)
- For essential details see Basdevant and Berger, Phys Rev D16, 657 (1977)

Unitarization - 2

- Impose unitarity by requiring that the full amplitude $T_D^u(M)$ satisfies proper discontinuity relations in the variable M.
- In the unitarity relation, we retain 2-body intermediate states ($\rho\pi, K^*\bar{K}, ...$) treating the vector mesons as stable and restricting to S-wave orbital angular momentum states
- $T_D^u(M)$ has a right-hand unitarity discontinuity starting at the lowest threshold, $M = m_\rho + m_\pi$
- T^+ is its value above the cut; T^- is the value below the cut.
- Unitarity relationship $T^+ = ST^-$; S is the strong interaction unitary S matrix that describes

$$\rho\pi \to \rho\pi, K^*\bar{K} \to K^*\bar{K}, \rho\pi \to K^*\bar{K}$$

- Aside: We do not know S. We will parametrize it in terms of a K matrix and determine the parameters by comparing with data.

Analyticity and Unitarity - 1

- Theory task: Construct an analytic and unitary T_D^u from knowledge of its singularities: (a) right hand unitarity discontinuities; and (b) “left-hand” pole singularity supplied by the Deck production amplitude, $T_D^{-1} \sim (M^2 - m_\pi^2)$.
- Solution in terms of an analytic 2 X 2 $D(M^2)$ matrix that has only a right hand unitarity discontinuity: $D^+(M) = SD^-(M)$; also invertible — determinant of D should not vanish anywhere on the first sheet.
- By construction, $D^{-1}[T_D^u - T_D]$ has only a right-hand discontinuity,
- Write dispersion integral for $D^{-1}[T_D^u - T_D]$
- Dispersion integral leads to

$$T_D^u(M^2) = T_D(M^2) - \frac{1}{\pi} D(M^2) \times \int_{(m_\rho + m_\pi)^2}^{\infty} ds' \, \frac{ImD(s')T_D(s')}{(s' - M^2)} \quad . \qquad (1)$$

Analyticity and Unitarity - 2

- Dispersion integral leads to

$$T_D^u(M^2) = T_D(M^2) - \frac{1}{\pi} D(M^2) \times \int_{(m_\rho+m_\pi)^2}^{\infty} ds' \, \frac{ImD(s')T_D(s')}{(s'-M^2)} \quad . \tag{1}$$

- This expression is our production amplitude (modified Deck amplitude) with resonant final state interactions included.
- Properties: (a) same left-hand production singularity as T_{Deck} ; (b) satisfies unitarity; (c) reduces to T_{Deck} if no rescattering.

Unitarization - “Practical” details

- Parametrize the coupled channel S matrix in terms of a K matrix:

$$K(M^2) = \begin{pmatrix} \frac{g_1^2}{s_1 - M^2} & \frac{g_1 g_2}{s_1 - M^2} \\ \frac{g_1 g_2}{s_1 - M^2} & \frac{g_2^2}{s_1 - M^2} \end{pmatrix} .$$

- Simple pole parametrization yields analytic expression for D matrix. g_1, g_2 are coupling strengths to the two channels.

$$D(M^2) = \frac{1}{\mathcal{D}_0(M^2)} \begin{pmatrix} g_1 & -g_2(s_1 - M^2 - \alpha^2 C_2) \\ g_2 & g_1(s_1 - M^2 - \alpha^2 C_1) \end{pmatrix}$$

- The denominator $\mathcal{D}_0(M^2) = (s_1 - M^2 - g_1^2 C_1(M^2) - g_2^2 C_2(M^2))$. has the appearance of a resonance factor. In the one channel case $\mathcal{D}_0^{-1}(M^2) \sim e^{i\delta} \sin\delta$

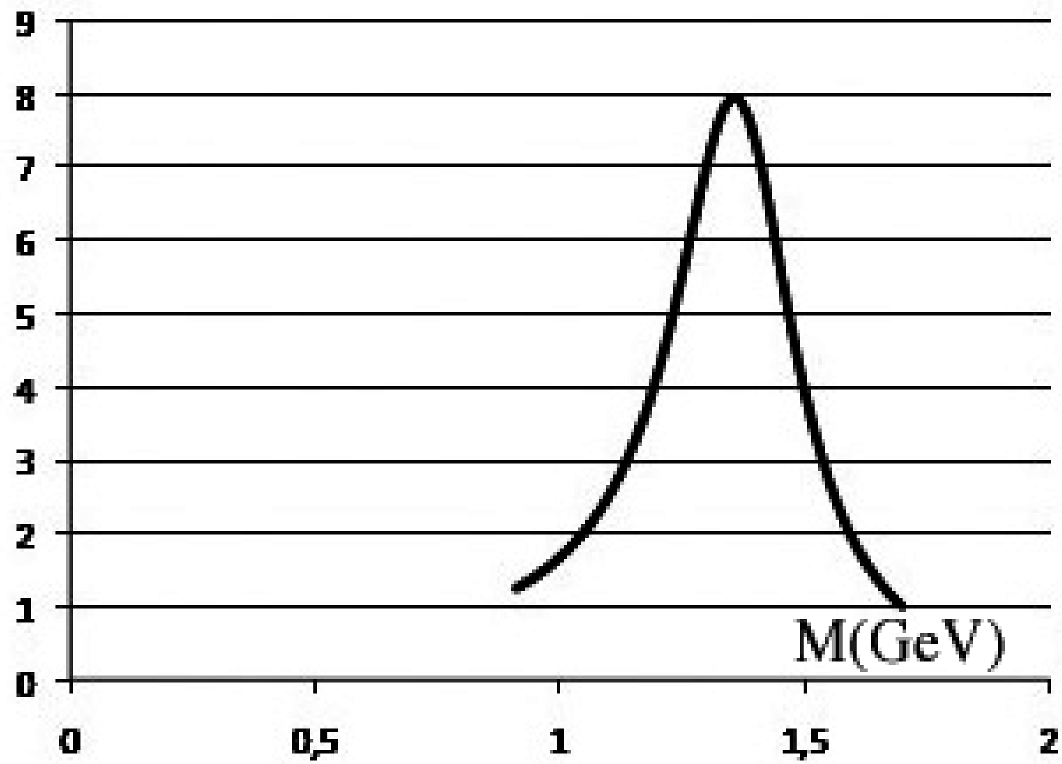
- $\alpha^2 = g_1^2 + g_2^2;\ C_1$ and C_2 are Chew-Mandlestam functions

Chew-Mandelstam function

- Chew-Mandelstam function: analytic function of the invariant mass squared s of two particles, with a right hand cut where the imaginary part is equal to the phase space factor $2p/\sqrt{s}$; p is the c.m. momentum:

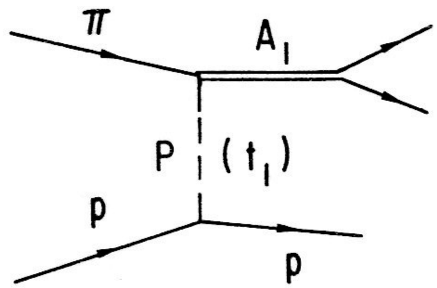
$$
\begin{aligned}
& C_{m,\mu}(s) \equiv C(s;m,\mu) = \\
& -\frac{2}{\pi}\{-\frac{1}{s}[(m+\mu)^2-s]^{1/2}[(m-\mu)^2-s]^{1/2} \\
& \times \ln \frac{[(m+\mu)^2-s]^{1/2}+[(m-\mu)^2-s]^{1/2}}{2(m\mu)^{1/2}} \\
& +\frac{m^2-\mu^2}{2s}\ln\frac{m}{\mu}-\frac{m^2+\mu^2}{2(m^2-\mu^2)}\ln\frac{m}{\mu}-\frac{1}{2}\}.
\end{aligned}
$$

Behavior of D(M)



Direct Production Term

- In addition to its FSI affects manifest in the unitarized Deck amplitude, the resonance may be produced directly via a diffractive coupling, $\pi p \rightarrow a_1 p$



$$T_{dir}(s, M^2) = \frac{is\sigma_{\pi p} G}{\mathcal{D}_0(m^2)} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix},$$

- Those acquainted with the study of $\pi\pi$ scattering in photo-production, $\gamma p \rightarrow \pi\pi N$, will recognize this term as the analog of the “vector- dominance” term in ρ production; the Deck term in the photo production case plays a role in modifying the ρ line shape (e.g., Paul Soding, 1966).

Discussion of the one-channel case - 1

$$K = g^2/(s_1 - M^2)$$

$$D^{-1}(M^2) = s_1 - M^2 - g^2 C_1(M^2)$$

- Very simple parameterization of resonant amplitude
- For a narrow resonance of $\text{mass } m \text{ and width } \Gamma$, the parameters are fixed by

$$s_1 \simeq m^2 + g^2 ReC_1(M^2) \simeq m^2$$

$$g^2 \text{Im} C_1(m^2) \simeq m\Gamma$$

- Introduce bare Deck amplitude $T_D(M^2) = \frac{\alpha}{M^2 - s_o}$
- Yields unitarized Deck amplitude

$$T_D^u(M^2) = \frac{\alpha}{M^2 - s_o} \frac{s_1 - M^2 - g^2 C_1(s_o)}{s_1 - M^2 - g^2 C_1(M^2)}$$

- This amplitude has a real zero near $M^2 = s_1$

Discussion of the one-channel case - 2

- Unitarized Deck amplitude

$$T_D^u(M^2) = \frac{\alpha}{M^2 - s_o} \frac{s_1 - M^2 - g^2 C_1(s_o)}{s_1 - M^2 - g^2 C_1(M^2)}$$

- This amplitude has a real zero near $M^2 = s_1$
- For a narrow resonance, the zero occurs near the resonance
- Thus, the unitarized production amplitude changes sign near the resonance position and its phase jumps by π
- Because $\cos\delta$ vanishes near $M^2 = s_1$, for a narrow resonance, we can write the M dependence as

$$T_D^u(M^2) \sim \exp i\delta \cos\delta$$

- Even for a broad resonance, the net effect of the zero is significant and causes a sharp structure in the mass distribution near the resonance position, seen near M = 1.3 GeV in the a_1 case

Discussion of the one-channel case - 3

- Unitarized Deck amplitude

$$T_D^u(M^2) = \frac{\alpha}{M^2 - s_o} \frac{s_1 - M^2 - g^2 C_1(s_o)}{s_1 - M^2 - g^2 C_1(M^2)}$$

- This amplitude has a real zero near $M^2 = s_1$
- Now add a direct production term

$$T^{dir}(M^2) = \frac{\beta}{s_1 - M^2 - g^2 C_1(M^2)}$$

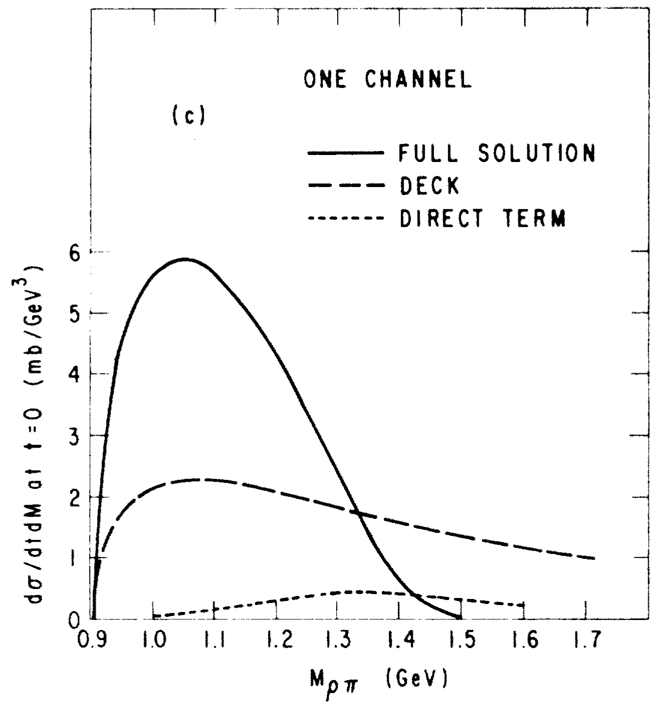
- Resulting full amplitude has its zero shifted to

$$M^2 = \frac{\alpha[s_1 - g^2 C_1(s_o)] - \beta s_o}{\alpha - \beta}$$

- Shift is accompanied by an enhancement compared to the Deck amplitude.
- Example 1: if $\alpha = \beta$, amplitude enhanced by $\sim m/\Gamma$
- Example 2: if $\alpha >> \beta$, resonance produces a dip in $d\sigma/dM$

One channel solution

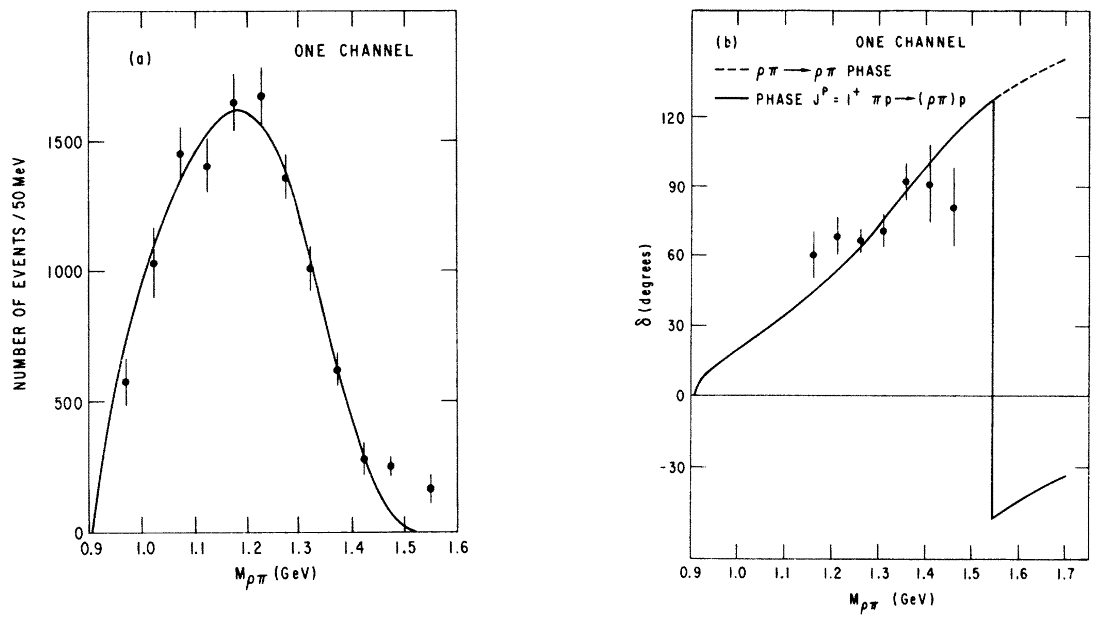
- One channel case corresponds to purely elastic rho pi scattering, parametrized with a simple one-pole K matrix. No inelasticity.



- Solid curve is the full result. Sharp decrease from 1.2 to 1.4 GeV arises from FSI (e.g., zero near s_1). More pronounced if direct production (short dashed curve) is omitted. Large peak in range 1.1 to 1.2 GeV is the FSI enhanced Deck, NOT the resonance

One channel case and the data

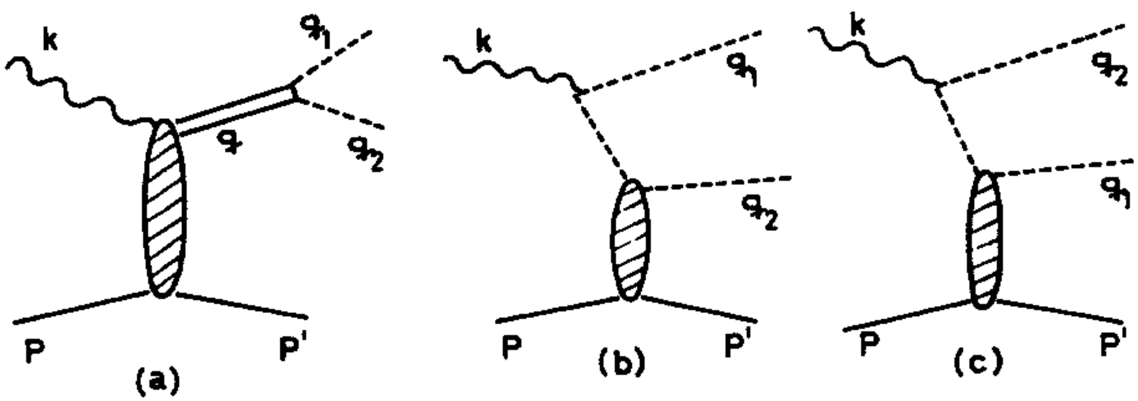
- Mass spectrum in the one-channel case agrees well with data but the phase does not. The unitary amplitude changes sign owing to the zero near s_1, and the phase changes by 180. (Data Antipov, Ascoli, et al)



- Resonance in this one-channel case is 1.36 GeV, close to K*Kbar threshold — cannot avoid including inelasticity if want to deal with the region above 1.4 GeV.

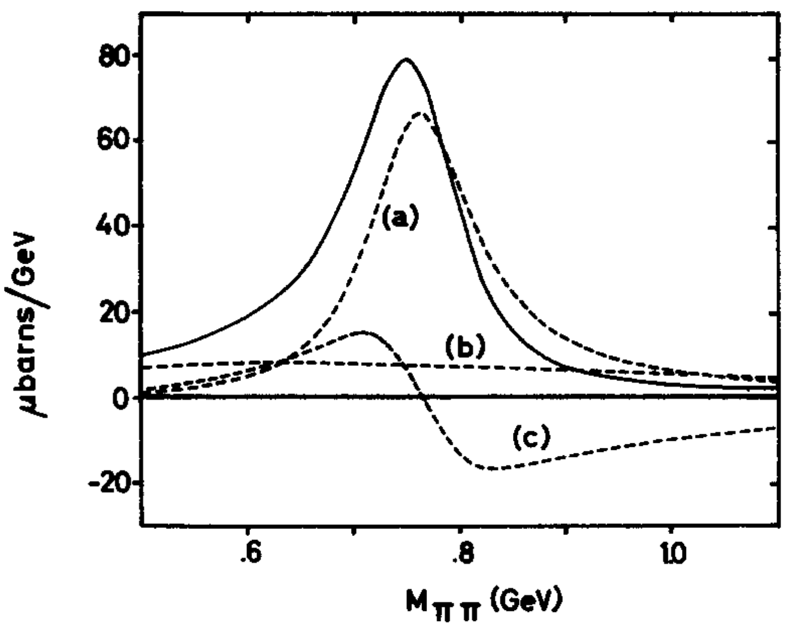
Shift of the rho mass in photoduction

- Simpler case: $\gamma p \to \pi\pi p$
- Diagrams show (a) the direct coupling (vector dominance) coupling to the ρ, $\gamma p \to \rho p$, and (b,c) the Deck graphs for $\gamma p \to \pi\pi p$



Shift of the rho mass in photoduction

- $\gamma p \to \pi\pi p$
- P. Soding, Phys. Lett. 19, 702 (1966)



- Notice the unitary preserving zero in the unitarized Deck curve (c) and shift plus enhancement of the peak on the low mass side in the final result (solid curve)

Two channel, one resonance case - 1

- Since the mass of the a_1 can extend into the range above 1350 MeV, it is interesting, if not necessary, to include the K*Kbar channel
- K matrix in the two-channel case has a single factorized pole, and ratio of couplings $g_1/g_2 = \sqrt{2}$ expected from SU(3)

Two channel, one resonance case - 2

- The final $J^P = 1^+$ partial wave amplitude becomes

$$T_D^u(M^2) = \frac{1}{s_1 - M^2 - g_1{}^2 C_1(M^2) - g_2{}^2 C_2(M^2)} \begin{bmatrix} T_D(\rho\pi)[s_1 - M^2 - g_2{}^2 C_2(M^2) - g_1{}^2 C_1(m_\pi{}^2)] \\ + g_1 g_2 T_D(K^*\bar{K})[C_2(M^2) - C_2(m_\pi{}^2)] \\ T_D(K^*\bar{K})[s_1 - M^2 - g_1{}^2 C_1(M^2) - g_2{}^2 C_2(m_\pi{}^2)] \\ + g_1 g_2 T_D(\rho\pi)[C_1(M^2) - C_1(m_\pi{}^2)] \end{bmatrix}$$

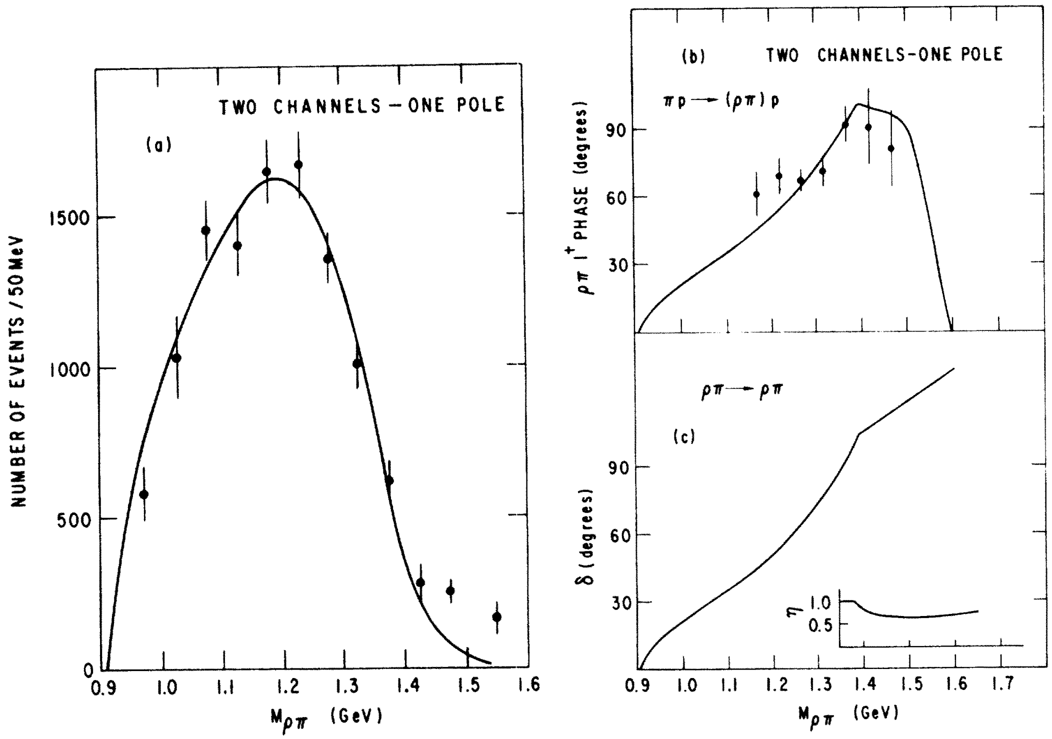
- s_1 is related to the squared mass of the a_1
- For each of the two channels, this expression has the appearance of a resonance factor

$$D_0{}^{-1}(M^2) \equiv [s_1 - M^2 - g_1{}^2 C_1(M^2) - g_2{}^2 C_2(M^2)]^{-1}$$

$$\approx e^{i\delta} \sin\delta$$

- Multiplied by a complex zero near $M^2 = s_1$. The zero is shifted into the complex plane, leading to slow phase variation

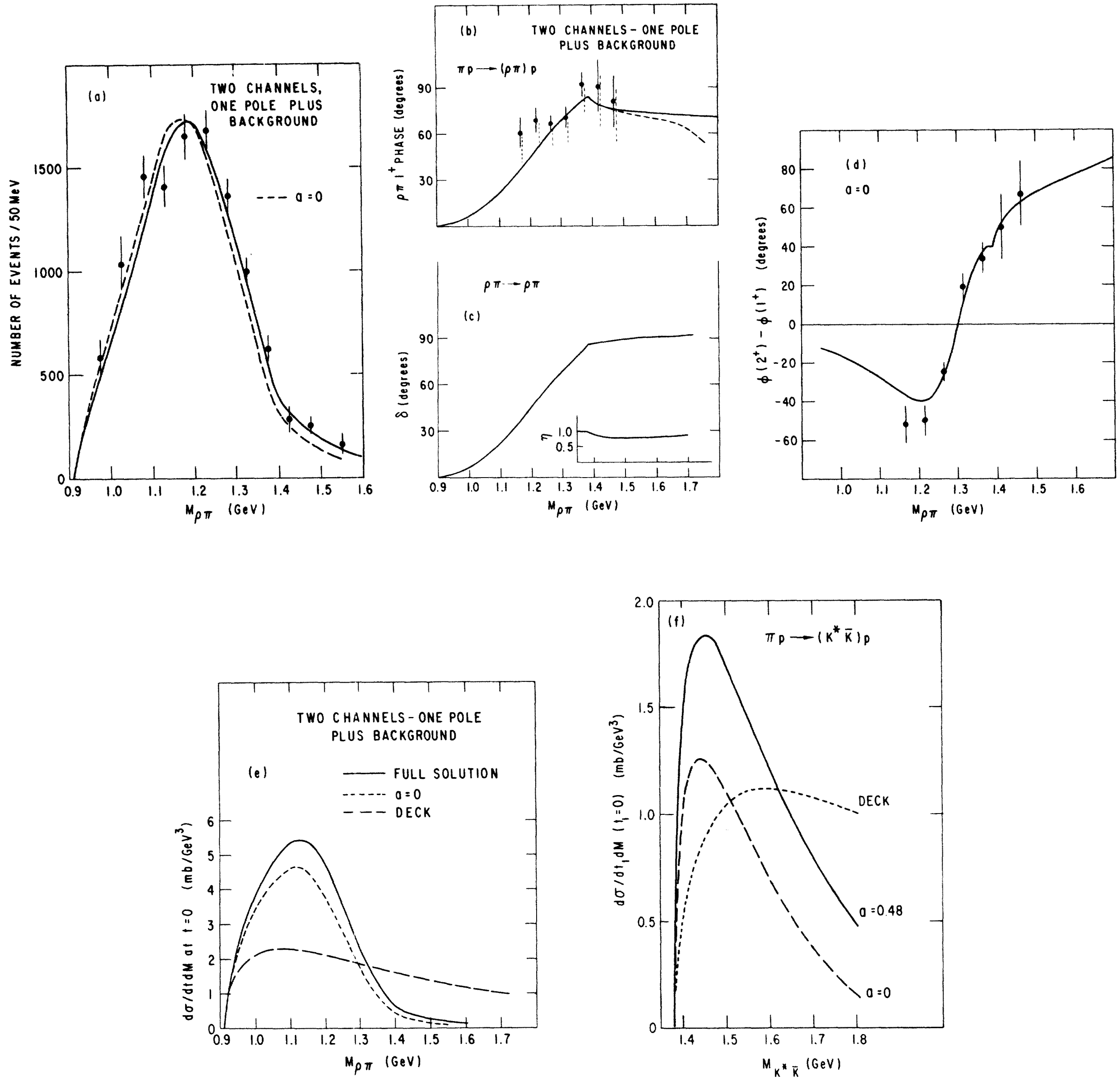
Two-channel, one resonance



- Resonance position and width are very close to one-channel solution, but the phase is more acceptable, up to ~ 1.5 GeV. Passes through 90^o near 1.36 GeV; cusp at $K^*\bar{K}$ threshold

- Phase of the rho pi to rho pi shown in (c) follows unitarity circle until 1.39 GeV, where it enters sharply and has elasticity $\eta = 0.7$

Solution with all the bells and whistles

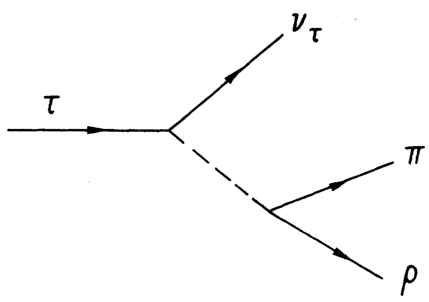


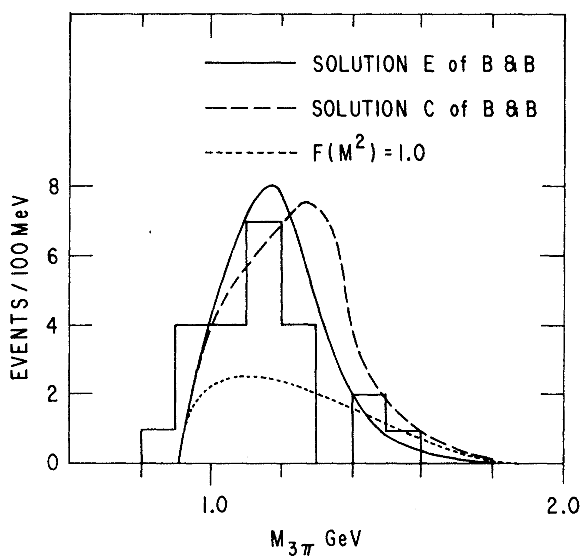
Extraction of mass and width

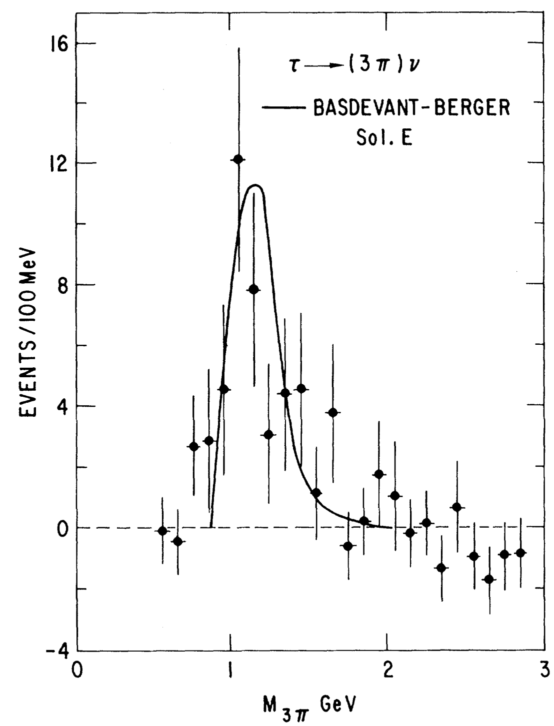
- Our mass and width determinations in 1977 of the a_1 from the pole positions on the second sheet, averaged over various solutions
  - $M = 1.3 \pm 0.15$ GeV
  - $\Gamma = 400 \pm 100$ MeV
- Uncertainties: how much do reasonable variations of the bare Deck amplitude affect these values of the mass and width?; additional channels in the analysis?; amplitude is analytic and unitary, but not crossing symmetric; …

Heavy lepton decay to a_1

- The three pion tau decay spectrum based on two of the solutions found in the hadronic study (Basdevant and Berger, Phys Rev Lett 40, 994 (1978))



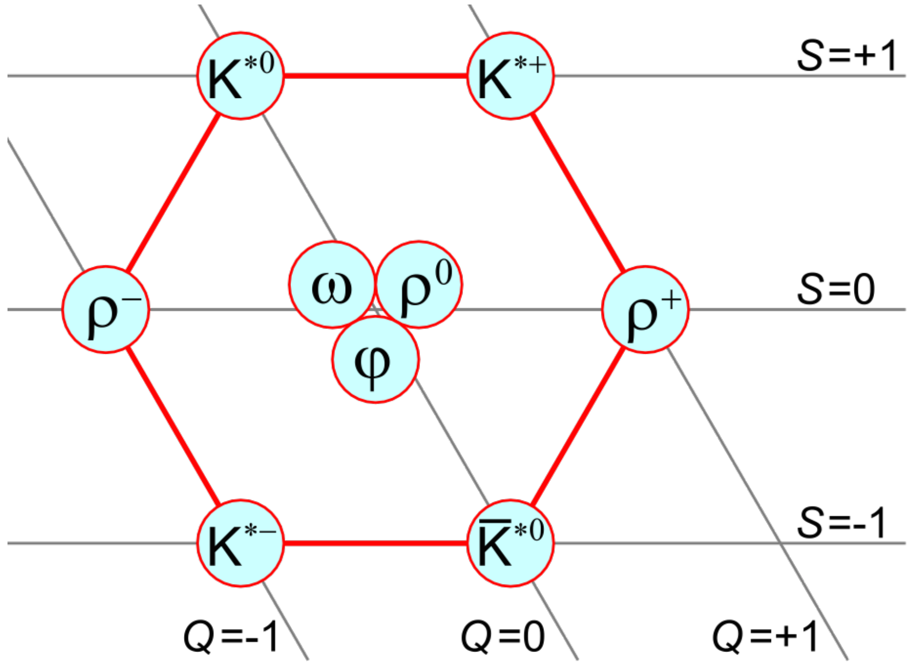




- DORIS data on the left and SPEAR data on the right.

Strangeness +/- 1 axial vector mesons

- Illustration of the vector multiple of the quark model



- Turn attention to S = +1 and S= -1 members of the axial vector multiplets. Notation is K_1. Formerly Q.
- Studied in $Kp \to K\pi\pi p$
- Much less data than in the case of the a_1

$K^{\pm}\pi^{+}\pi^{-}$ data

- Brandenberg et al publications based on 13 GeV/c SLAC data from $K^{\pm}p \to K^{\pm}\pi^{+}\pi^{-}p$ Phys Rev Lett 36, 703 and 706 (1976)
- $72000\ K^{+}p^{+}\pi^{-}$ events and $56,000\ \mathrm{K^{-}p^{+}}\pi^{-}$ events in the mass interval $1.0 < M(K\pi\pi) < 1.6$ GeV
- Partial wave analysis shows that two $J^{P} = 1^{+}$ states are produced: "Q_1" and "$\mathrm{Q_2}$"

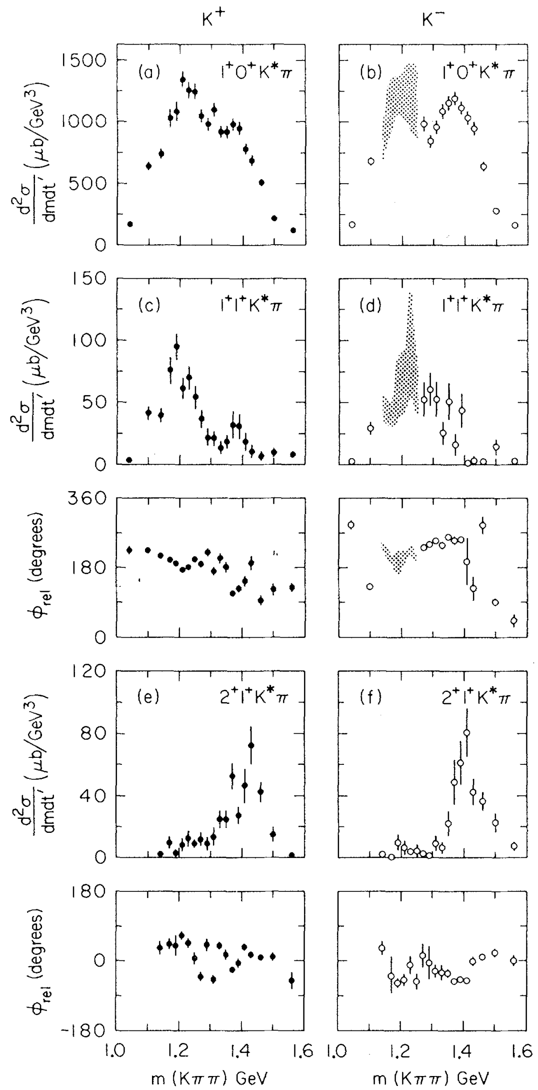
  "Q_1"$(1300),\ \Gamma \sim 200$ MeV couples principally to ρK

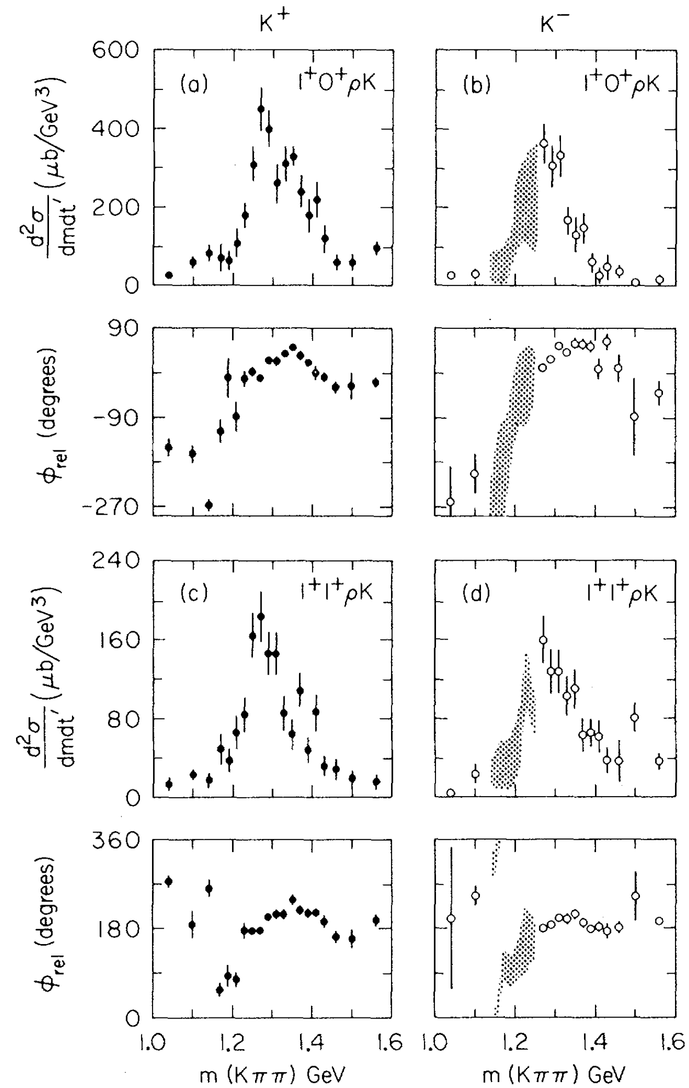
  "Q_2"$(1400),\ \Gamma \sim 160$ MeV couples principally to $K^{*}\pi$

- Invitation to consider a coupled channel study with two poles in the K matrix. Two $J^{P} = 1^{+}$ S-wave channels $K^{*}\pi, \rho\pi$
- Basdevant and Berger, Phys Rev D19, 246 (1979)

Intensities and phases

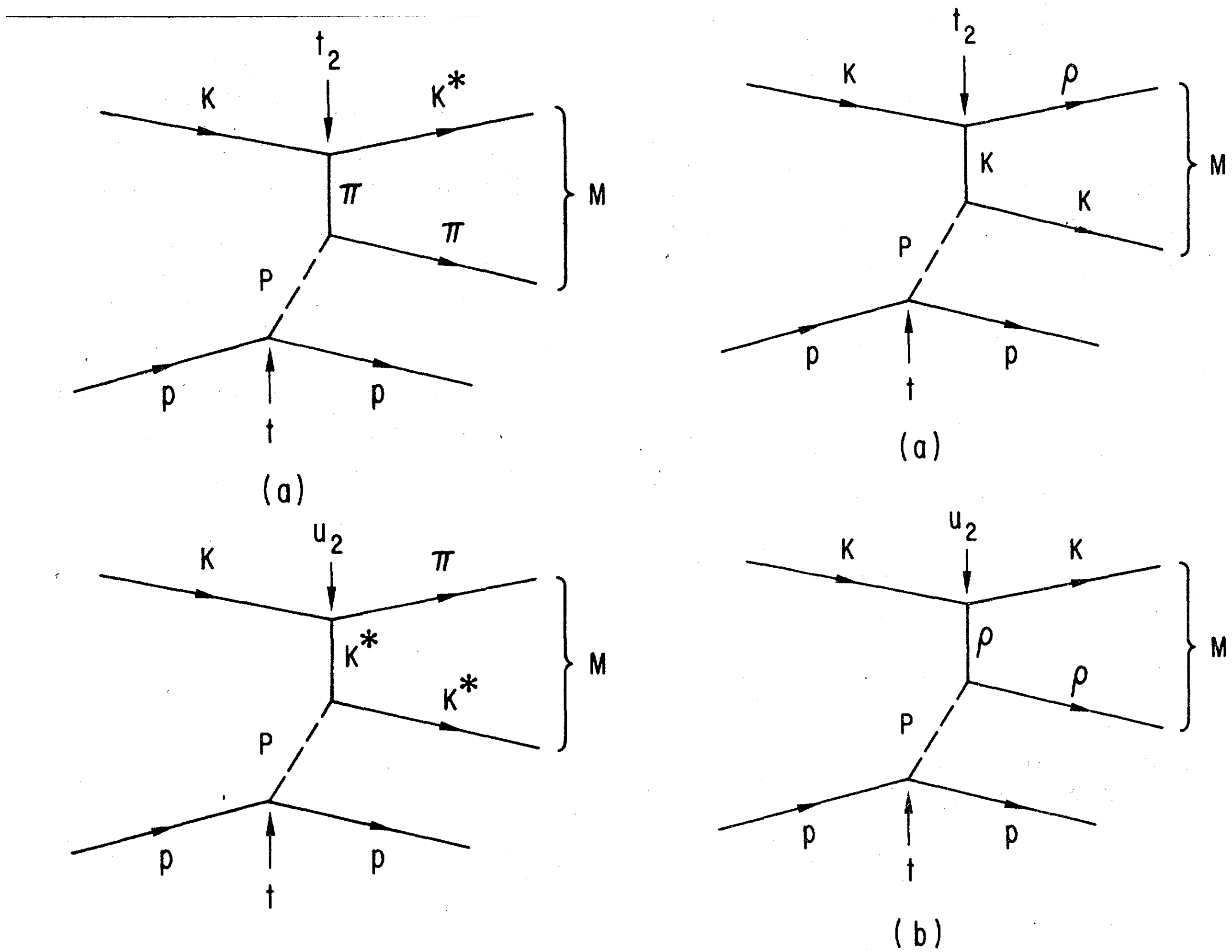
- Mass dependences of the intensities of various waves, and their phases relative to $J^P = 1^+ \lambda_t = 0\ K^*\pi\ (K^*\pi \text{ left; } \rho K \text{ right})$





Basic Deck diagrams

- Basic non-resonant Deck production diagrams: $K^{*}\pi \text{ and } \rho K$



Unitarization - 1

- Deck amplitude T^{Deck} produces $J^{PC} = 1^{++}$ non resonant enhancements near threshold in $K^*\pi$ and ρK.
- The $K^*\pi$ and ρK are strongly interacting systems. They interact in the final state. These final state interactions must be incorporated in the full amplitude. They are inevitable and non-negligible if there is a resonance, such as the $q\bar{q}$ $J^P = 1^+$ state of the quark model.
- Construct a full amplitude T^u_{Deck} that includes final state interactions, the $q\bar{q}$ state, and respects unitarity (no double counting)
- For the full treatment of $K^*\pi$ and ρK see Basdevant and Berger, Phys Rev D19, 246 (1979) and Phys Rev D 19, 239 (1979)

Unitarization - 2

- Impose unitarity by requiring that the full amplitude $T_D^u(M)$ satisfies proper discontinuity relations in the variable M.
- In the unitarity relation, we retain 2-body intermediate states $\rho K \text{ and } \mathrm{K}^*\pi$, this time treating the vector mesons as unstable, but still restricting to S-wave orbital angular momentum states in $\rho K \text{ and } \mathrm{K}^*\pi$
- $T_D^u(M)$ has a right-hand unitarity discontinuity starting at the lowest threshold, $M = m_K + 2m_\pi$
- T^+ is its value above the cut; T^- is the value below the cut.
- Unitarity relationship $T^+ = ST^-$; S is the strong interaction unitary S matrix that describes

$$\rho K \to \rho K, K^*\pi \to K^*\pi, K^*\pi \to \rho K$$

- We do not know S. We will parametrize it in terms of a K matrix and determine the parameters by comparing with data.

Analyticity and Unitarity - 1

- Theory task: Construct an analytic and unitary T_D^u from knowledge of its singularities: (a) right hand unitarity discontinuities; and (b) "left-hand" pole singularity supplied by the Deck production amplitude
- Solution in terms of an analytic 2 X 2 $D(M^2)$ matrix that has only a right hand unitarity discontinuity: $D^+(M) = SD^-(M)$; also invertible — determinant of D should not vanish anywhere on the first sheet.
- By construction, $D^{-1}[T_D^u - T_D]$ has only a right-hand discontinuity,
- Write dispersion integral for $D^{-1}[T_D^u - T_D]$
- Dispersion integral leads to

$$T_D^u(M^2) = T_D(M^2) - \frac{1}{\pi} D(M^2) \int_{(m_K+2m_\pi)^2}^{\infty} \mathrm{Im} D^{-1}(s') T_D(s') \frac{ds'}{s' - M^2} .$$

Two channel, two pole K matrix

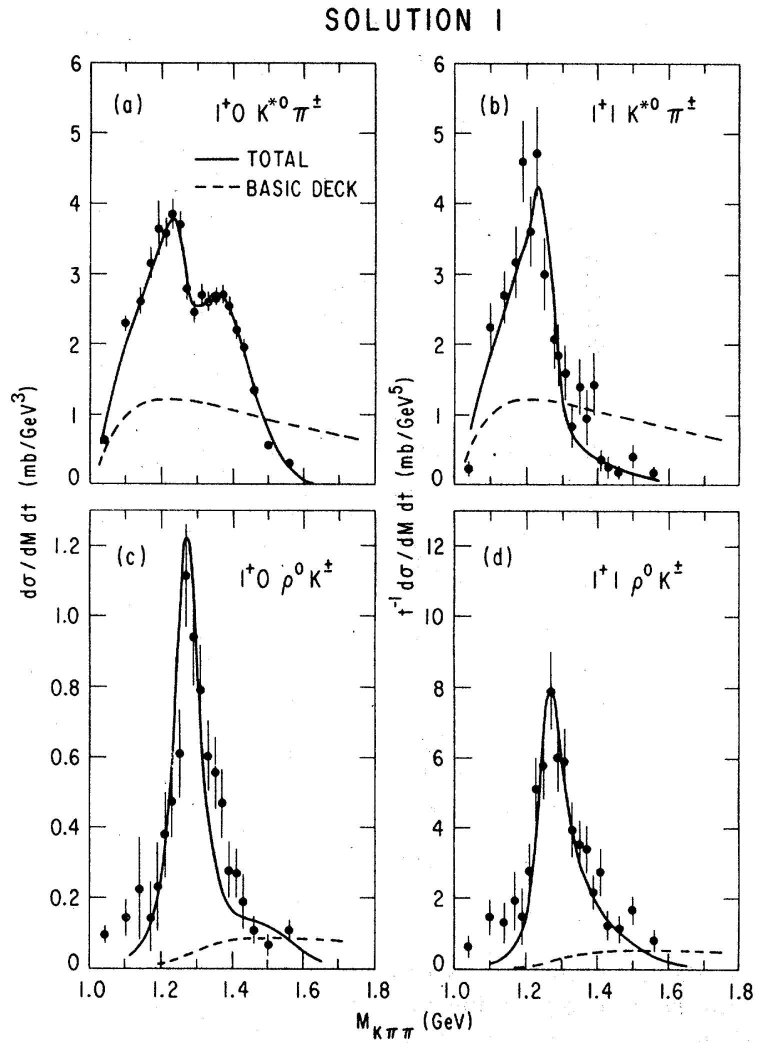
- Include the possibility of a non-resonant part a_{ij}

$$K = \begin{pmatrix} \frac{g_A{}^2}{s_A - M^2} + \frac{g_B{}^2}{s_B - M^2} + a_{11} & \frac{g_A f_A}{s_A - M^2} + \frac{g_B f_B}{s_B - M^2} + a_{12} \\ \frac{g_A f_A}{s_A - M^2} + \frac{g_B f_B}{s_B - M^2} + a_{12} & \frac{f_A{}^2}{s_A - M^2} + \frac{f_B{}^2}{s_B - M^2} + a_{22} \end{pmatrix}$$

- Full unitary solution is $T(M^2) = T_D^u(M^2) + T^{\mathrm{dir}}(M^2),$
- Helicity indices are not shown, but $T(M^2)$ stands for two equations, one for t-channel helicity $\lambda_t = 0, 1$.
- Each $T(M^2)$ is also a two-component vector, upper/lower for $K^*\pi, \rho K$
- Range of good solutions found in fits to data, as in the a_1 case

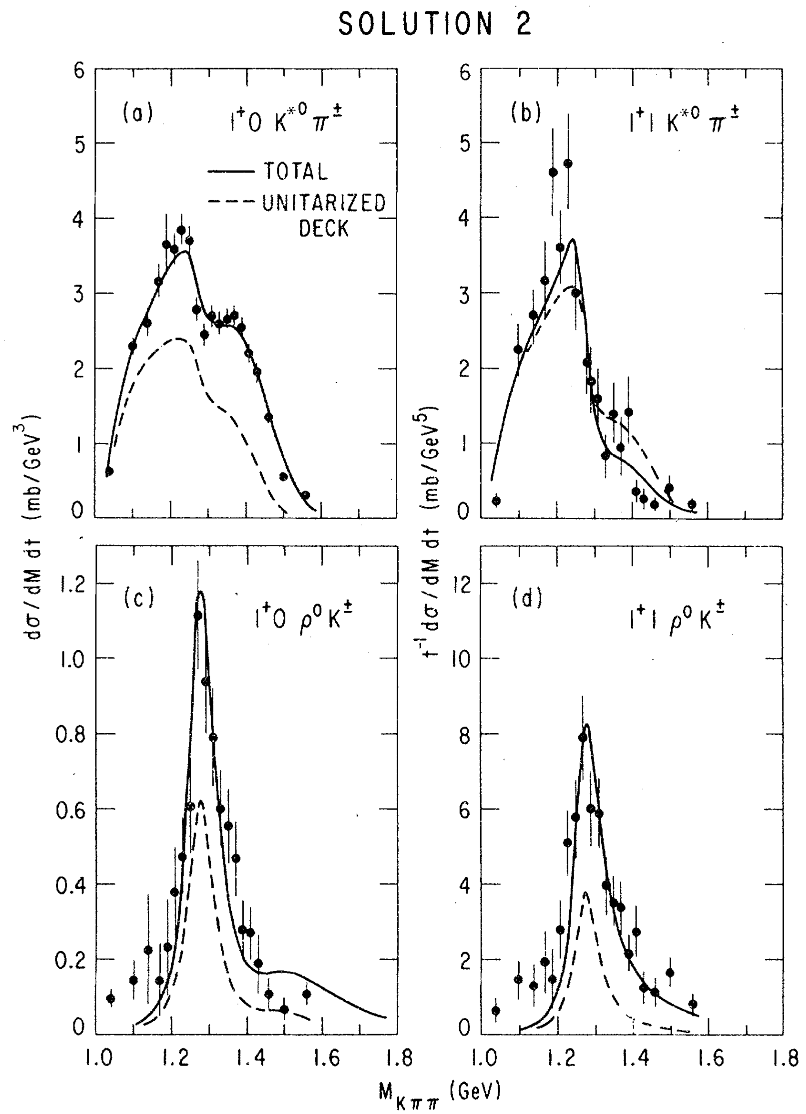
Intensities in solution 1

- Solution 1 shows the significant enhancement and change of shape between the final unitary solution and the basic Deck cross sections (left, helicity 0; right, helicity 1)



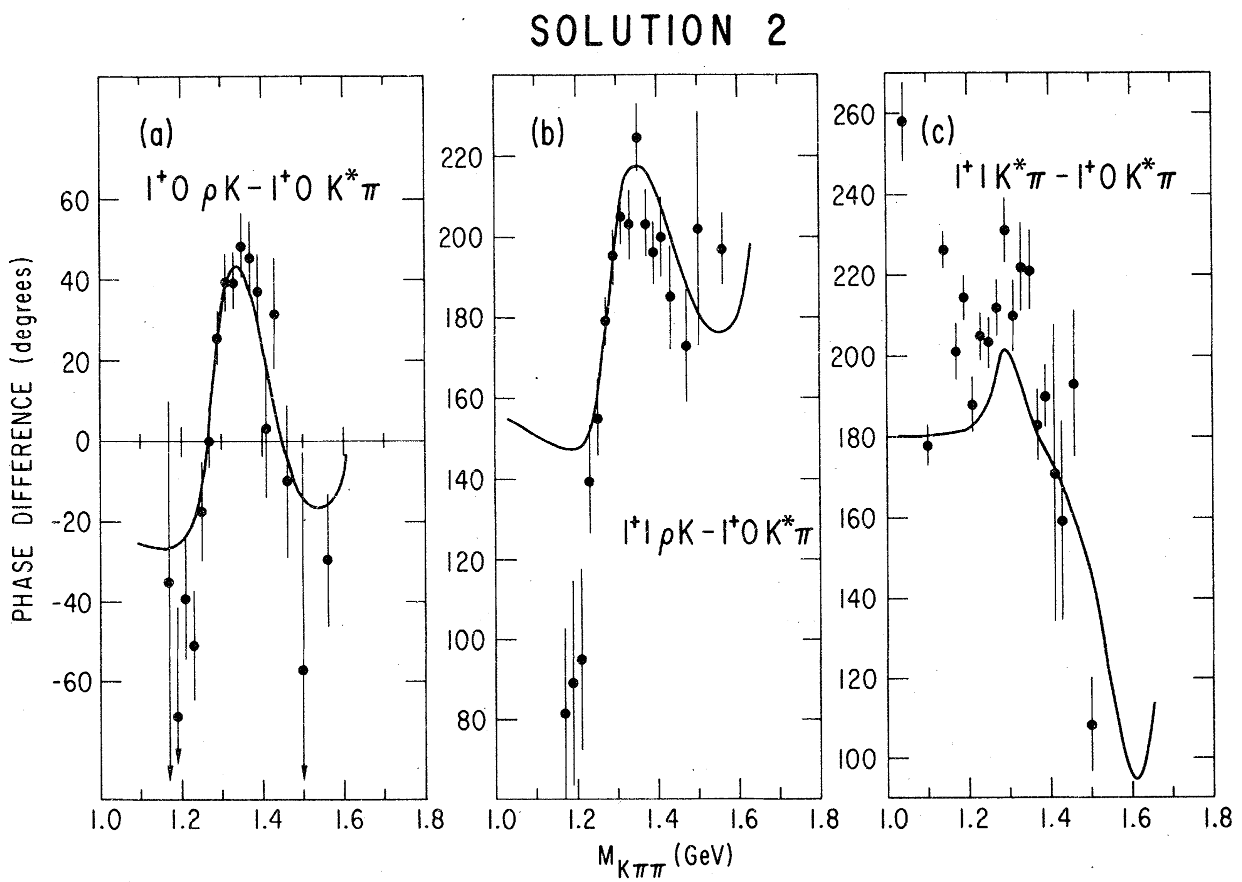
Intensities in solution 2

- Intensities of the various waves in solution - 2 (no constant terms in the K matrix); note the roles of direct production



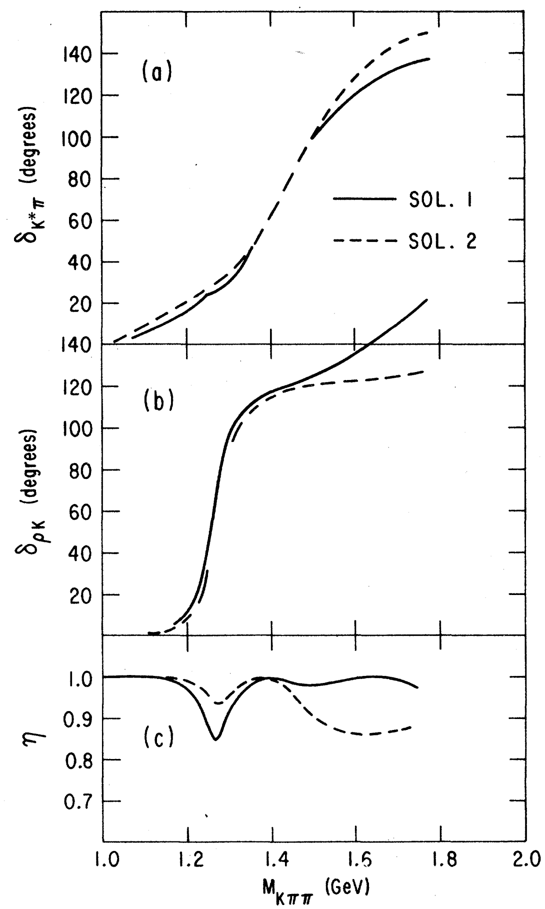
Relative phases in solution 2

- Relative phases of different waves



S-wave phase shifts and inelasticities

- S-wave phase shifts and inelasticities from our solutions



- Behavior of $\delta_{\rho K}$ characteristic of narrow resonance near 1.25 GeV; $\delta_{K^*\pi}$ suggests second broader resonance above 1.4GeV

K_1 Resonance Parameters

- Intensities and phases are consistent with the presence of two resonances, but the interplay of these resonances with the Deck mechanism has its subtleties.

- Parameters of one of the resonances are well determined:

$$M_{Q_1} = 1.28 \pm 0.02 \text{ GeV},$$

$$70 < \Gamma_1^{tot} < 140 \text{ MeV},$$

$$2\% < \frac{\Gamma_1^{K^*\pi}}{\Gamma_1^{tot}} < 10\%.$$

- The second is determined less precisely:

$$M_{Q_2} = 1.42 \pm 0.06 \text{ GeV},$$

$$\Gamma_2^{tot} = 230 \pm 50 \text{ MeV},$$

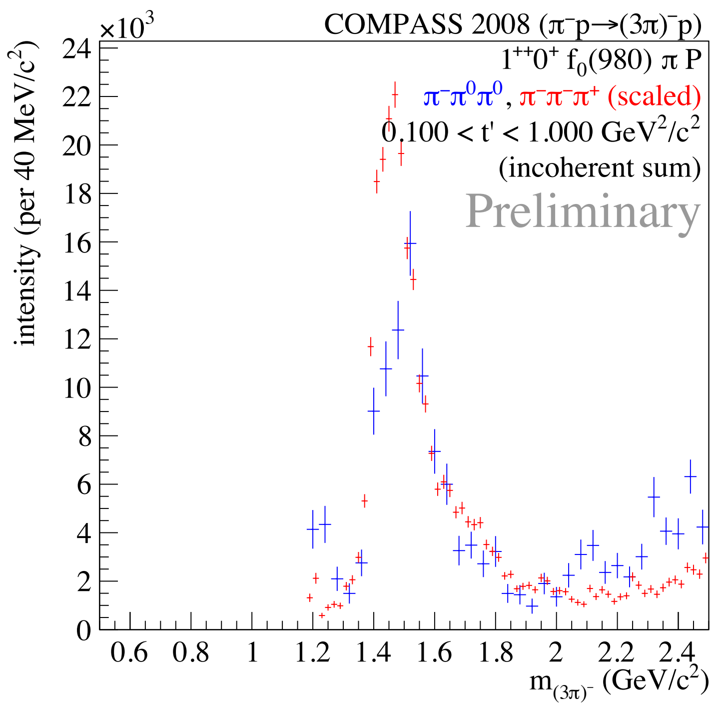
$$1\% < \frac{\Gamma_2^{\rho K}}{\Gamma_2^{tot}} < 20\%.$$

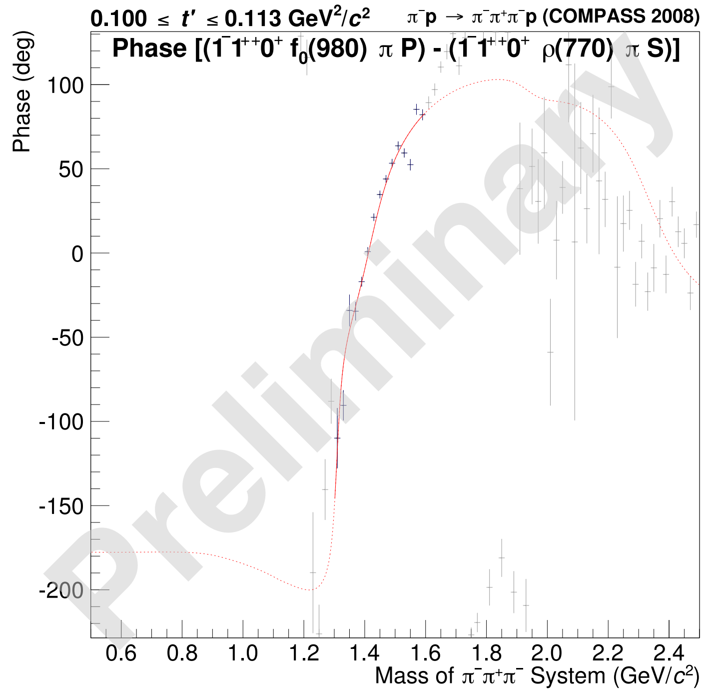
Fast Forward to 2014

- COMPASS: evidence for an axial vector $J^{PC} = 1^{++}$ peak in the P wave $\pi f_0(980)$ channel at about 1420 MeV.
- The usual a_1 is in the S-wave $\pi\rho$ channel at about 1260 MeV.
- Two a_1 so close in mass or is the P wave πf_0 another decay mode of the usual a_1 ? If so, why at a different mass?
  - Counterintuitive to have two states with identical quantum numbers so close in mass ($K\pi\pi$ was a different story)
  - Revive unitary coupled-channel research done in 1975 - 1979 on $\pi\pi\pi,\ \pi K\bar{K}$, this time with both S and P wave channels
- Describe here the new study we published recently.
- Conclusion: One a_1 suffices to explain the two peaks.

New Resonance?

- COMPASS data — intensity and phase





- Observed in the the P wave channel at about 1420 MeV
- Only 0.25% of the intensity in the $\pi^-\pi^+\pi^-$ channel
- Never seen before (statistics)

Repeat previous study: 2 channels, one resonance

- Deck mechanism for production of non-resonant $\rho\pi$ and $f_0\pi$ in the mass range M = 1 to 2 GeV

- Incorporation of resonant behavior $a_1 \rightarrow f_0\pi, \; a_1 \rightarrow \rho\pi$ One resonance. New: one S wave and one P wave channel

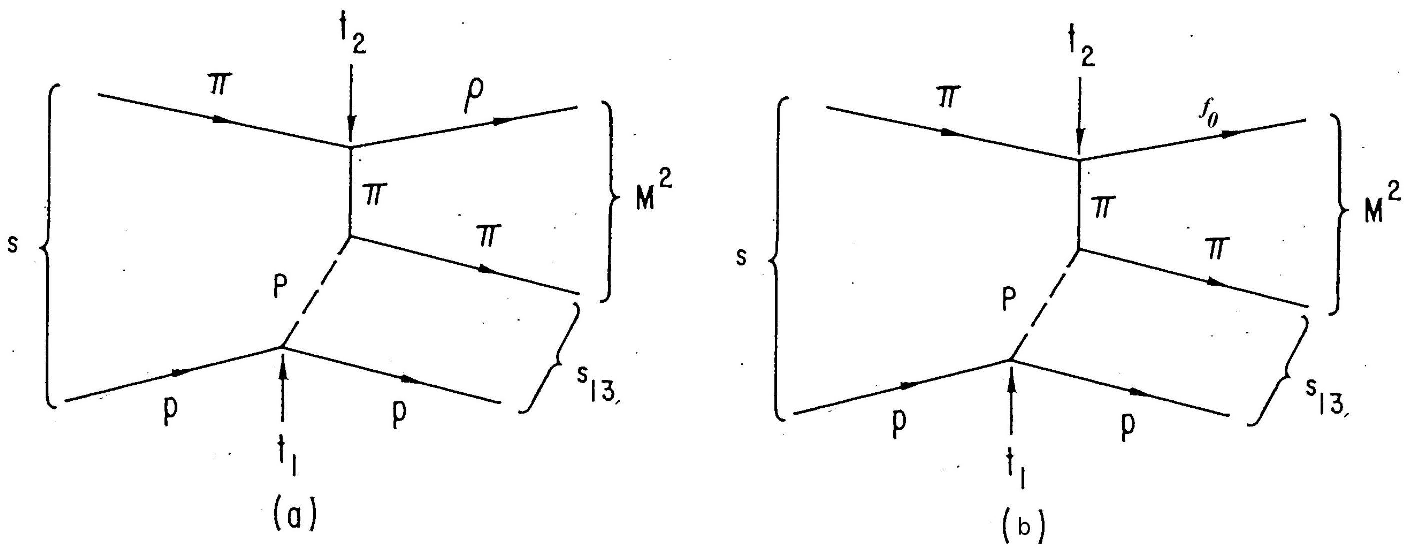
- Final state unitarization: — two channel, strong interaction S matrix, …… reaction amplitude that includes both Deck "background" and resonance

- Results: (a) separate mass peaks in $J^{PC} = 1^{++}$ S wave $\rho\pi$ and P wave $f_0\pi$ channels, and (b) relative phase between the two amplitudes — consistent with data

"Peak locations and relative phase of different decay modes ofthe a_1 axial vector resonance in diffractive production" [arXiv:1504.05955], Phys Rev Lett. 114, 192001 (2015); J.-L. Basdevant and E. L. Berger

Deck Production Mechanism

- Consider $\pi p \to \pi\pi\pi p$ at large incident π momentum and small momentum transfer to the target.
- Think of the 3 pion system as a superposition of quasi-two-body systems $(\pi\pi\pi) \to \pi\rho, \pi f_0, ...$
- One pion exchange production, followed by diffractive scattering of the virtual pion from the target:



Deck Amplitude - 1

- Deck amplitude for $\pi p \to \rho\pi p$

$$T_D^\rho = g_{\rho\pi\pi} K_\rho(t_2) \frac{1}{m_\pi^2 - t_2} i s_{13} e^{bt_1} \sigma_{\pi p}$$

- Similar expression for $\pi p \to f_0 \pi p$
- In the $\rho\pi$ or $f_0\pi$ rest frame, the Deck amplitude contributes to several partial waves.
- For $J^{PC} = 1^{++}$ one must project the S wave component for $\rho\pi$ and the P wave component for $f_0\pi$
- Re-express the invariants s_{13} and t_2 in terms of t-channel angles

$$t_2 = g_1(M, t_1) + g_2(M, t_1)\cos\theta_t$$
$$s_{13} = g_3(s, M) + g_4(s, M, t_1)\cos\theta_t + g_5(s, M, t_1)\sin\theta_t\cos\phi_t$$

- Deck amplitude is a rational function so one can project analytically all partial waves S, P, D, … (all m), for any value of t_1

Deck Amplitude - 2

- Consider an expansion for small t_1 (where the data are concentrated) of the partial wave projections of the Deck production amplitude. Define expansion parameter

$$\Theta_1 = \frac{t_1}{(M^2 - m_\pi^2)}.$$

- The S-wave projection is

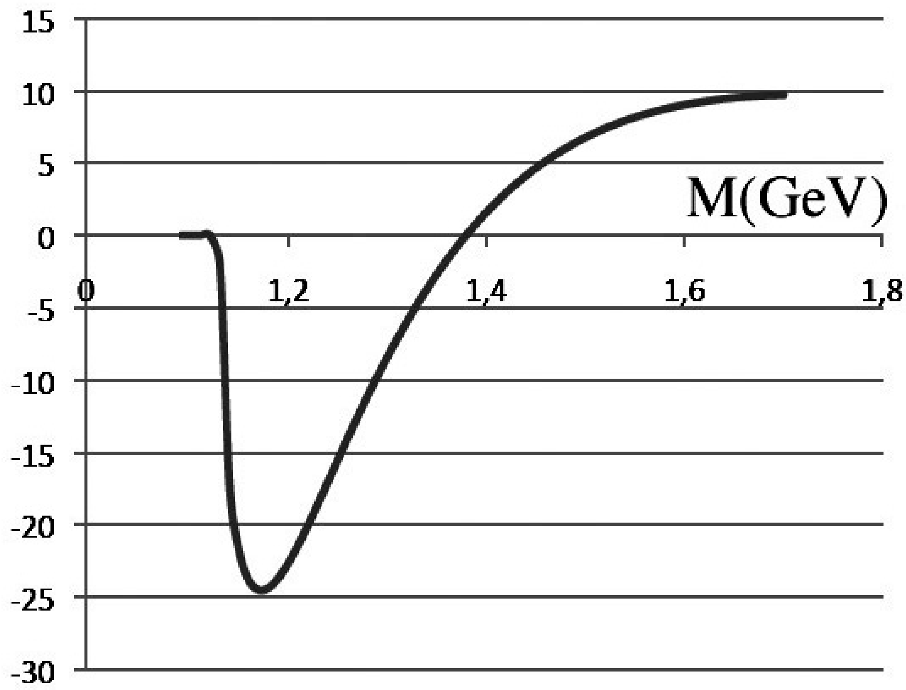
$$T_S^{Deck} = -\frac{s}{(M^2 - m_\pi^2)} \times \left(1 - \frac{1}{2}\Theta_1\left(\frac{(3M^2 + m_\pi^2)}{(M^2 - m_\pi^2)} - \frac{E_\rho}{E_\pi}\right)\left(\frac{1}{y}\ln\frac{1+y}{1-y}\right)\right)$$

- The P-wave projection is

$$T_P^{Deck} = +\frac{3}{2}\frac{s}{(M^2 - m_\pi^2)}\Theta_1 \times \left(\frac{(3M^2 + m_\pi^2)}{(M^2 - m_\pi^2)} - \frac{E_{f_0}}{E_\pi}\right)\left(\frac{-2}{y} + \frac{1}{y^2}\ln\left(\frac{1+y}{1-y}\right)\right)$$

- The P-wave projection vanishes at $t_1 = 0$; more importantly, it passes through 0 for a special value of M — sign change

P wave projection vs M



- P wave amplitude crosses zero near $M \simeq 1.38$ GeV
- This sign change drives the relative phase change between the P wave and other waves
- P wave intensity is also much smaller ($\sim 10^{-3}$) than the S wave

Unitarization - 1

- Deck amplitude T^{Deck} produces $J^{PC} = 1^{++}$ non resonant enhancements near threshold in $\rho\pi \text{ and } f_0\pi$.

- The $\rho\pi \text{ and } f_0\pi$ are strongly interacting systems. They interact in the final state, even in the one channel $\rho\pi$ case. Final state interactions must be included. They are inevitable and non-negligible if there is a resonance, such as the $J^{PC} = 1^{++}\, q\bar{q}$ state of the quark model.

- Construct a full amplitude T^u_{Deck} that includes final state interactions, the $q\bar{q}$ state, and respects unitarity (no double counting).

Unitarization - 2

- Impose unitarity by requiring that the amplitude $T_D^u(M)$ satisfies proper discontinuity relations.
- $T_D^u(M)$ has a right-hand unitarity discontinuity starting at the lowest threshold, $M = m_\rho + m_\pi$
- T^+ is its value above the cut; T^- is the value below the cut.
- Unitarity relationship $T^+ = ST^-$; S is the strong interaction unitary S matrix that describes

$$\rho\pi \to \rho\pi, f_0\pi \to f_0\pi, \rho\pi \to f_0\pi$$

- Aside: we do not know S. Parametrize it in terms of a K matrix and determine the parameters by comparing with data.

Unitarization - 3

- Theory task: Construct an analytic and unitary T_D^u from knowledge of its singularities: (a) right hand unitarity discontinuities; and (b) “left-hand” pole singularity supplied by the Deck production amplitude, $T_D^{-1} \sim (M^2 - m_\pi^2)$.

- Solution in terms of an analytic 2 X 2 $D(M^2)$ matrix that has only a right hand unitarity discontinuity: $D^+(M) = SD^-(M)$.

- Dispersion integral leads to

$$T_D^u(M^2) = T_D(M^2) - \frac{1}{\pi} D(M^2) \times \int_{(m_\rho+m_\pi)^2}^{\infty} ds' \, \frac{ImD(s')T_D(s')}{(s' - M^2)} \quad . \qquad (1)$$

- Expression is our Deck amplitude with resonant final state interactions taken into account.

- Properties: (a) same left-hand production singularity as T_{Deck} ; (b) satisfies unitarity; (c) reduces to T_{Deck} if no rescattering.

K matrix

- Parametrize the coupled channel S matrix in terms of a K matrix:

$$K(M^2) = \begin{pmatrix} \frac{g_1^2}{s_1 - M^2} & \frac{g_1 g_2}{s_1 - M^2} \\ \frac{g_1 g_2}{s_1 - M^2} & \frac{g_2^2}{s_1 - M^2} \end{pmatrix} \quad .$$

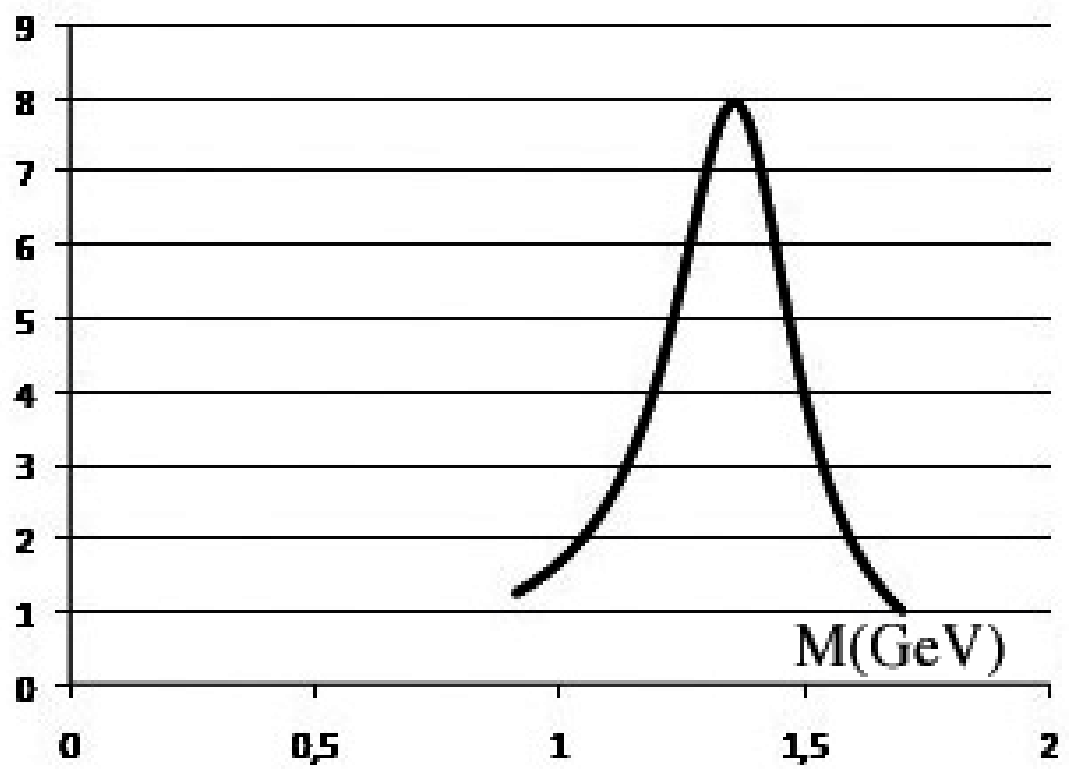
- Simple pole parametrization yields analytic expression for D matrix. g_1, g_2 are coupling strengths to the two channels.

$$D(M^2) = \frac{1}{\mathcal{D}_0(M^2)} \begin{pmatrix} g_1 & -g_2(s_1 - M^2 - \alpha^2 C_2) \\ g_2 & g_1(s_1 - M^2 - \alpha^2 C_1) \end{pmatrix}$$

- The denominator $\mathcal{D}_0(M^2) = (s_1 - M^2 - g_1^2 C_1(M^2) - g_2^2 C_2(M^2))$. has the appearance of a resonance factor. In the one channel case $\mathcal{D}_0^{-1}(M^2) \sim e^{i\delta} \sin\delta$

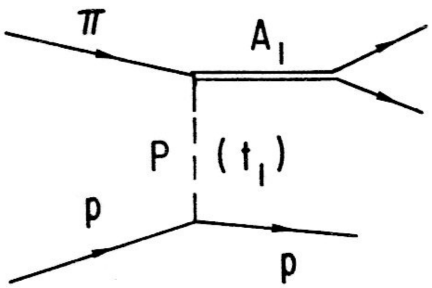
- $\alpha^2 = g_1^2 + g_2^2$; C_1 and C_2 are Chew-Mandlestam functions

Behavior of D(M)



Direct Production Term

- In addition to its affects manifest in the unitarized Deck amplitude, the resonance may be produced directly via a diffractive coupling, $\pi p \to a_1 p$



$$T_{dir}(s, M^2) = \frac{is\sigma_{\pi p} G}{\mathcal{D}_0(m^2)} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix},$$

- Those acquainted with the study of $\pi\pi$ scattering in photo-production, $\gamma p \to \pi\pi N$, will recognize this term as the analog of the “vector- dominance” term in ρ production; the Deck term in the photo production case plays a role in modifying the ρ line shape (e.g., Paul Soding, 1966).

Final Amplitude and Parameters

$$T(M^2) = T_D^u(M^2) + T_{dir}(M^2) \tag{1}$$

- Recall that $T(M^2)$ is a two-dimensional vector; upper and lower components for $\rho\pi$ and $f_0\pi$, respectively
- Parameters are the K matrix pole position s_1 and the pole coupling strengths g_1 and g_2
- Plus the two coupling strengths in the "direct" term, Gf_1 and Gf_2

Comparison with COMPASS data

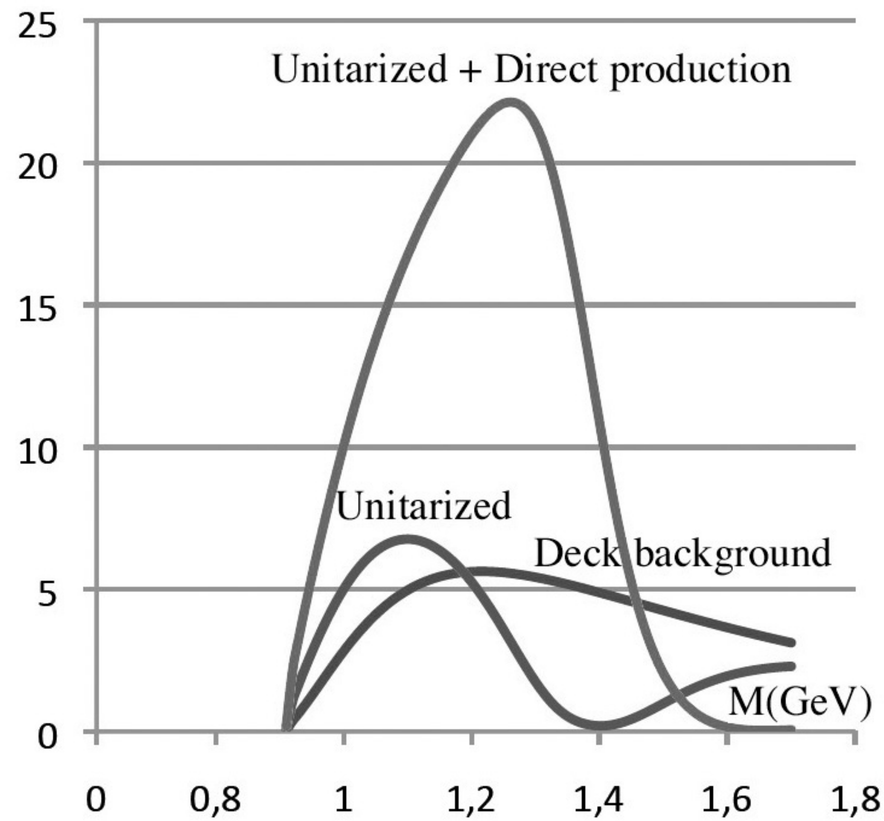
- We have not made a χ^2 fit.
- Focus on the momentum transfer t interval [0.10 to 0.13] GeV^2
- Trial and error: find appropriate values of the a_1 mass and width (defined by the position of the pole on the second sheet) that give the observed mass peaks. Obtain:

$$M(a_1) \simeq 1.40 \pm 0.02\ \mathrm{GeV},$$
$$\Gamma(a_1) \simeq 0.30 \pm 0.05\ \mathrm{GeV}.$$

- These values fix $s_1 \sim 2.002\ \mathrm{GeV}^2;\ g_1 \sim 0.732\ \mathrm{GeV}.$
- The ratio $\gamma = g_2/g_1$ was varied to give the observed relative intensity of the two peaks: central value $\gamma = g_2/g_1 = -0.08.$
- Determine the amount of "direct" production by placing the two peaks at the desired locations:

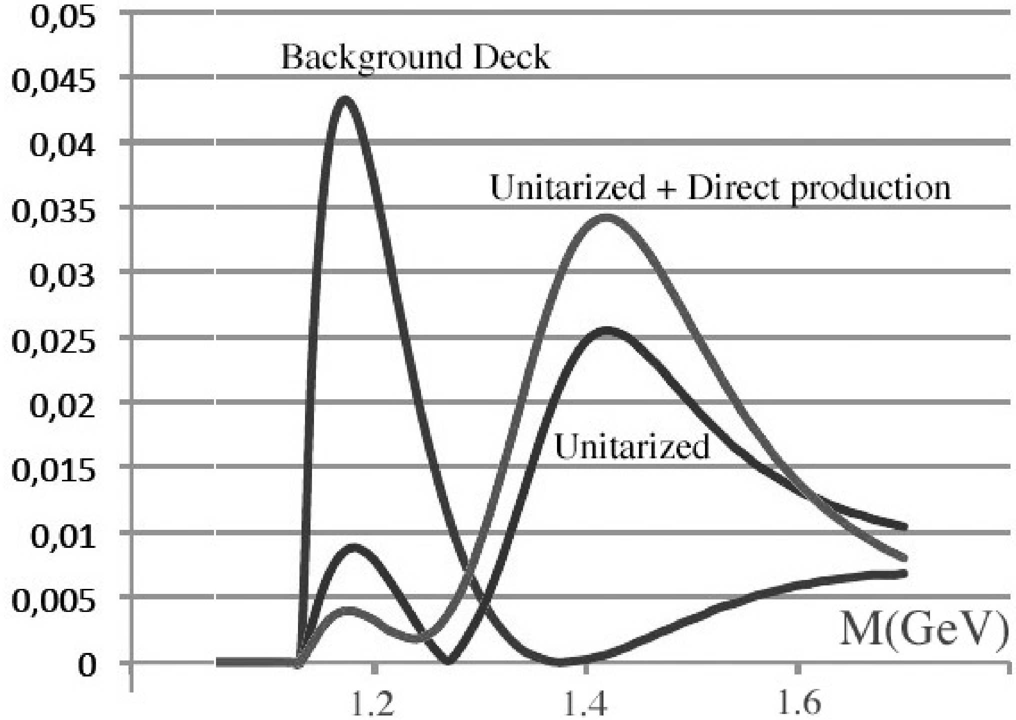
$$G\sigma_{\pi p} f_1 = 120;\ G\sigma_{\pi p} f_2 = 5.5$$

$J^{PC} = 1^{++}\rho\pi$ mass distribution



- Note that unitarization sharpens the Deck amplitude
- Overall peak location at about 1260 MeV, width about 280 MeV
- The peak does not have a symmetric Breit-Wigner form

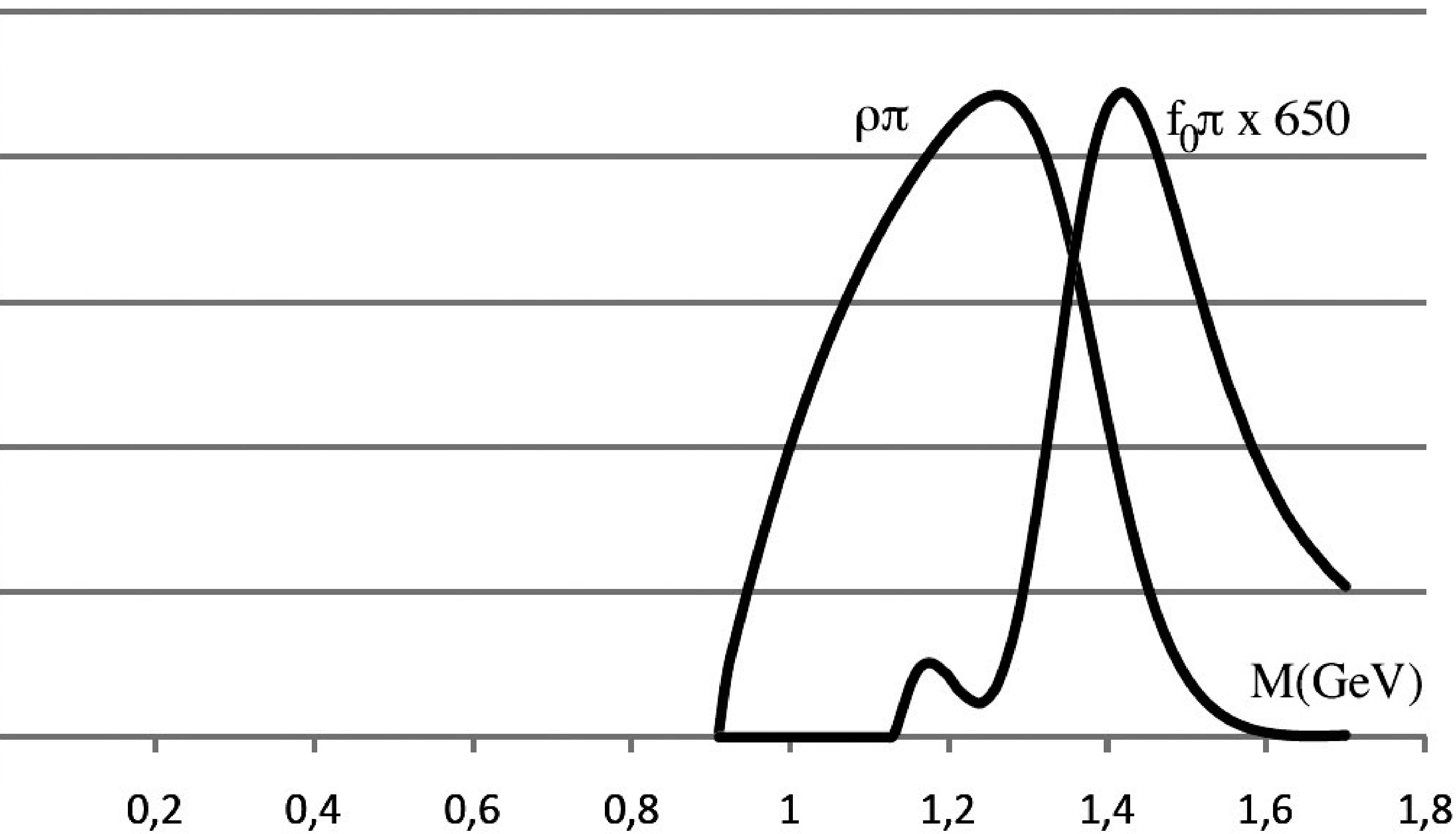
$J^{PC} = 1^{++} f_0\pi$ mass distribution



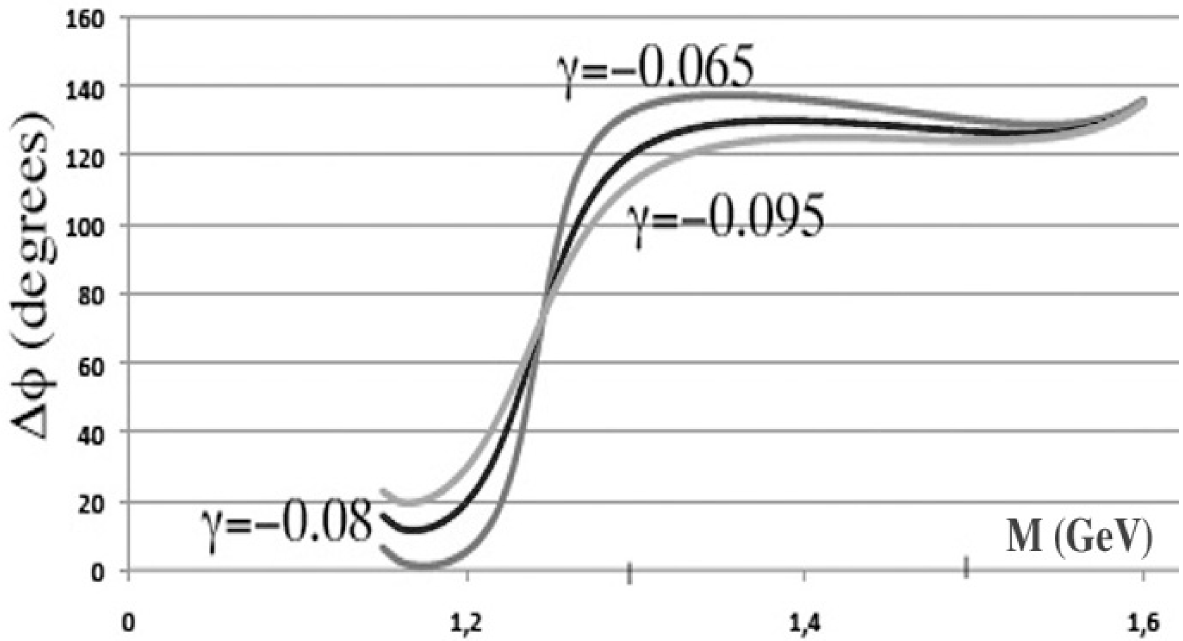
- Deck in $f_0\pi$ is narrow and very near threshold
- The final peak is pushed higher in mass, close to 1420 MeV; width about 140 MeV
- Note the second peak in $f_0\pi$ predicted just below 1200 MeV

Both on the same figure

- Scale up the $f_0\pi$ distribution by X 650



Relative phase



- Curves showing the relative phase as a function of M for three choices of the ratio of coupling strengths.
- Sharp rise of the relative phase related to the zero in the P wave production amplitude.

Dependence on momentum transfer

- We have results for arbitrary values of the momentum transfer to the target, t_1; the changes in mass spectra and phases are modest. Paper in preparation.

- What about the differential cross section as a function of t_1?

- Recall: the final amplitude is a sum of two terms:

$$T(M^2, t_1) = T_D^u(M^2, t_1) + T_{dir}(M^2, t_1) \quad .$$

- Each term has its own t_1 dependence properties; the direct term has the same t_1 dependence for both channels.

- However, there is a (well known) strong mass dependence of the t_1 distribution for the Deck term, both in theory and experiment. The slope is considerably steeper at low M than at higher M. Moreover, there is the kinematic suppression at small t_1 for the P-wave channel.

Outlook and perspectives

- Main features of the COMPASS data, two mass peaks separated by about 160 GeV, with pronounced relative phase motion, are compatible with a single a_1

- New determination of the mass and width of the a_1 along with its branching fraction into $f_0\pi$ possible

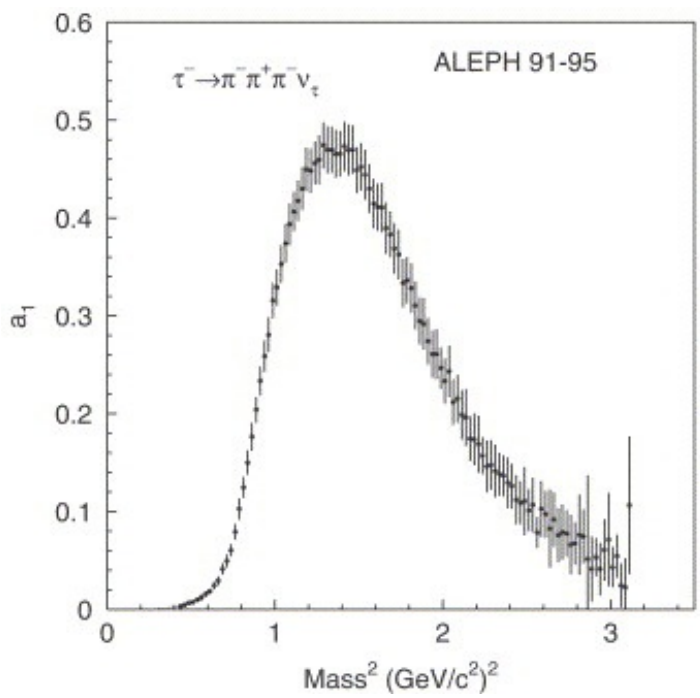
- Rediscovered in this example that, although a peak is often associated with a resonance, its precise mass and width depend also on the dynamics of the mechanism by which it is produced.

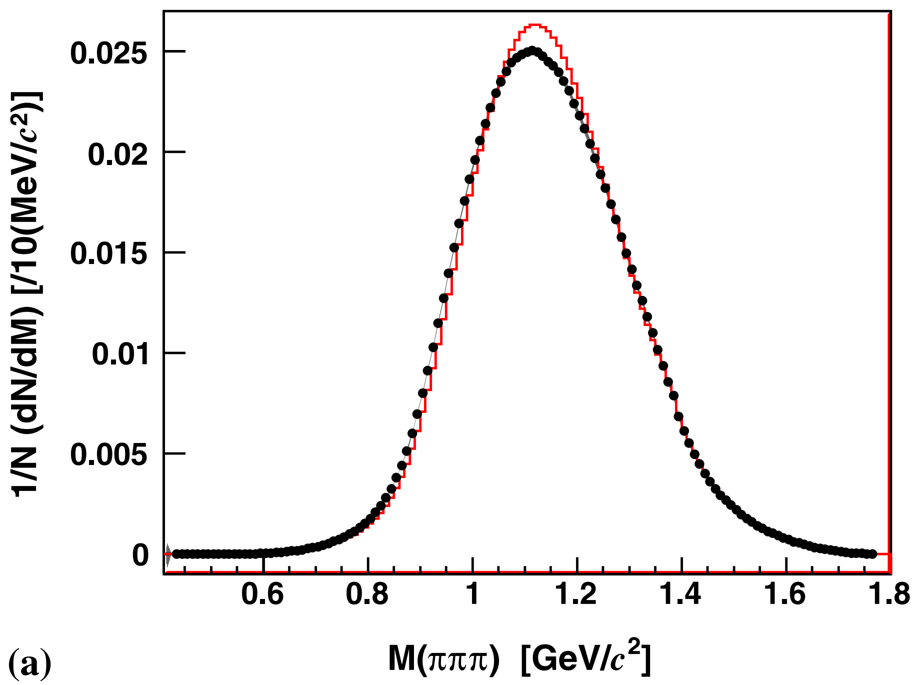
- Here, the same Deck production mechanism has very different character in the S-wave and P-wave channels, leading to a shift by about 160 MeV in the observed positions of the $J^{PC} = 1^{++}$ state.

- If one could do low-energy $\rho\pi$ and $f_0\pi$ elastic scattering, one would observe a single resonance peak with mass and width
$$M \sim 1.36 \text{ GeV and } \Gamma \sim 0.31 \text{ GeV}$$

Heavy lepton decay (under construction)

- Data much improved: ALEPH, Physics Reports 421 (2005) 191–284; CLEO Phys Rev Letters, 90 181802 (2003); BELLE Phys Rev D81, 113007 (2010)

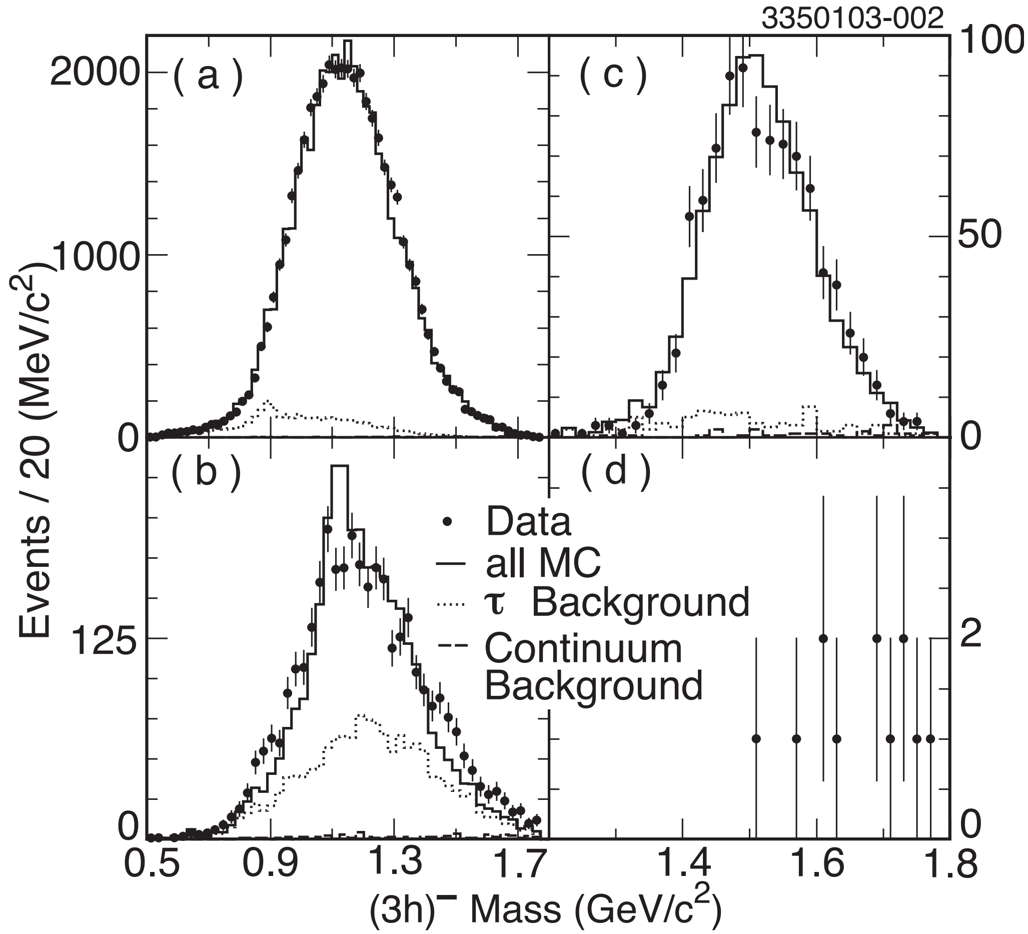




(a)

Heavy lepton decay data

- CLEO data



Summary - 1

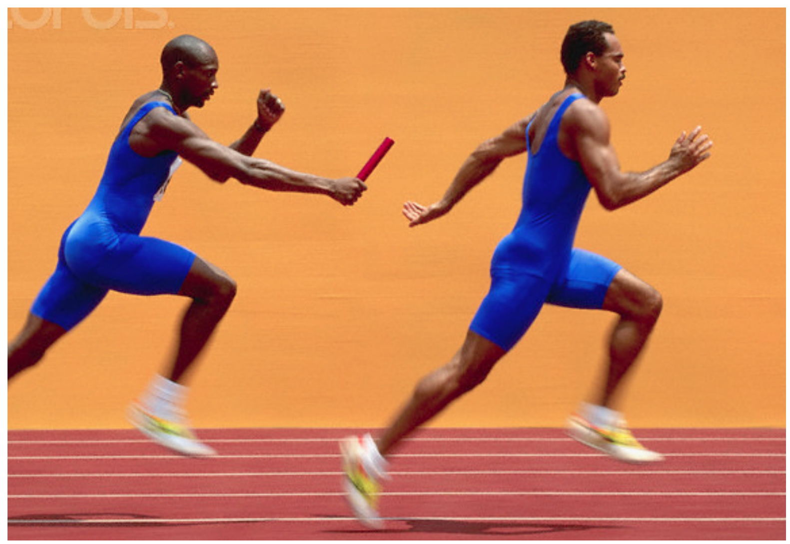
1. Axial-vector mesons — names, symbols
2. Features of the data
3. Production dynamics
   - Deck model (non-resonant)
   - Final state interactions
   - Unitarity and analyticity
4. Phenomenology of the a_1
   - One pole, one channel case ($\rho\pi$)
   - One pole, two channel case ($\rho\pi$ and K^*K)
   - Heavy lepton decay $\tau \to a_1\nu$
5. Photoproduction of $\pi\pi$, and final state interactions

Summary - 2

1. Axial-vector mesons in $K\pi\pi$
   - Two resonances (poles), two peaks
   - Mixing
2. Fast forward to 2014 - 2015, the a_1 again
   - One or two axial vector $\pi\pi\pi$ states?
   - One resonance pole, two peaks
   - Extraction of the axial vector mass, width, and branching fractions

Future

- Other channels, e.g., $\pi_2(1670)$ and π_2'
- Detailed fits to COMPASS data to extract mass, width, branching fractions of the a_1.
- Challenge of data handling — cannot be done by theorists alone
- Heavy lepton decay data; solutions consistent with hadron production
- X, Y, Z
- Pass the baton



Unitarization - "Practical" details

- Parametrize the coupled channel S matrix in terms of a K matrix:

$$K(M^2) = \begin{pmatrix} \frac{g_1^2}{s_1 - M^2} & \frac{g_1 g_2}{s_1 - M^2} \\ \frac{g_1 g_2}{s_1 - M^2} & \frac{g_2^2}{s_1 - M^2} \end{pmatrix} \quad .$$

- Simple pole parametrization yields analytic expression for D matrix. g_1, g_2 are coupling strengths to the two channels.

$$D(M^2) = \frac{1}{\mathcal{D}_0(M^2)} \begin{pmatrix} g_1 & -g_2(s_1 - M^2 - \alpha^2 C_2) \\ g_2 & g_1(s_1 - M^2 - \alpha^2 C_1) \end{pmatrix}$$

- The denominator $\mathcal{D}_0(M^2) = (s_1 - M^2 - g_1^2 C_1(M^2) - g_2^2 C_2(M^2))$. has the appearance of a resonance factor. In the one channel case $\mathcal{D}_0^{-1}(M^2) \sim e^{i\delta} \sin\delta$

- $\alpha^2 = g_1^2 + g_2^2$; C_1 and C_2 are Chew-Mandlestam functions