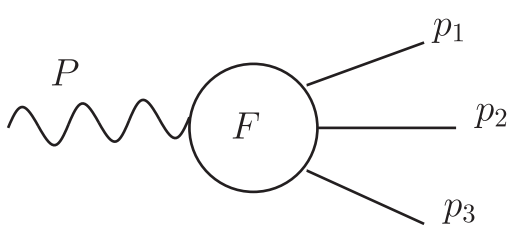
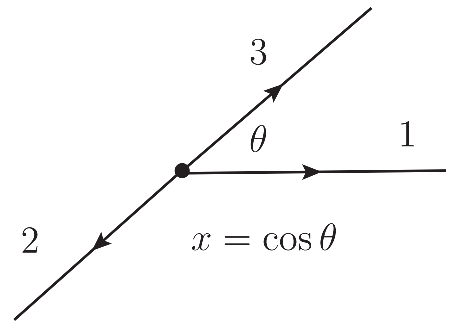
2. The isobar model: troubles and corrections

2.1 Kinematics and the Isobar Model





For simplicity, $J^P = 0^+$;final state particles spinless unit mass

invariant variables

$s = (p_2 + p_3)^2, t = (p_1 + p_3)^2, u = (p_1 + p_2)^2$

$P^2 = (p_1 + p_2 + p_3)^2 = m^2;\ s + t + u = 3 + m^2$

$$\boxed{t(s, x, m^2) = \frac{3+m^2-s}{2} - 2p(s, m^2)q(s)x}$$

$p(s, m^2) = \{[s-(m-1)^2][s-(m+1)^2]\}^{1/2}/2\sqrt{s}$

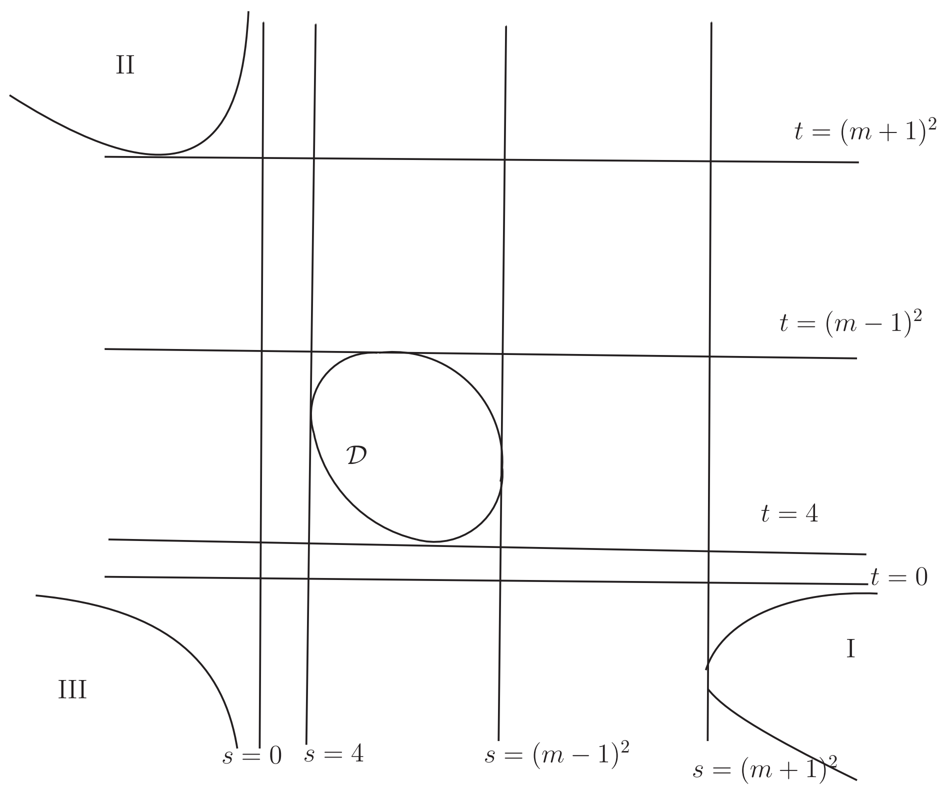
$q(s) = (s-4)^{1/2}/2$

Physical region for decay:

$4 \leq s \leq (m-1)^2; |x| \leq 1$

Second condition is $stu - (m^2-1)^2 \geq 1$

or $\Gamma(s,t,m^2) \equiv st(3+m^2-s-t)-(m^2-1)^2 \geq 0$



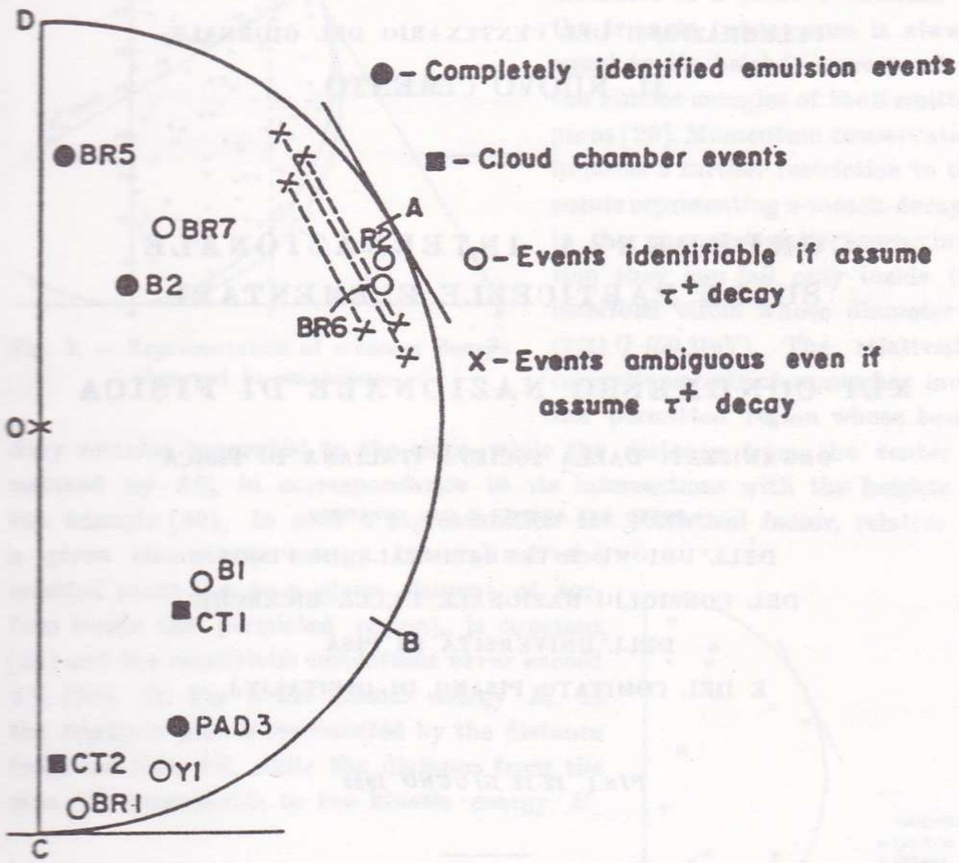
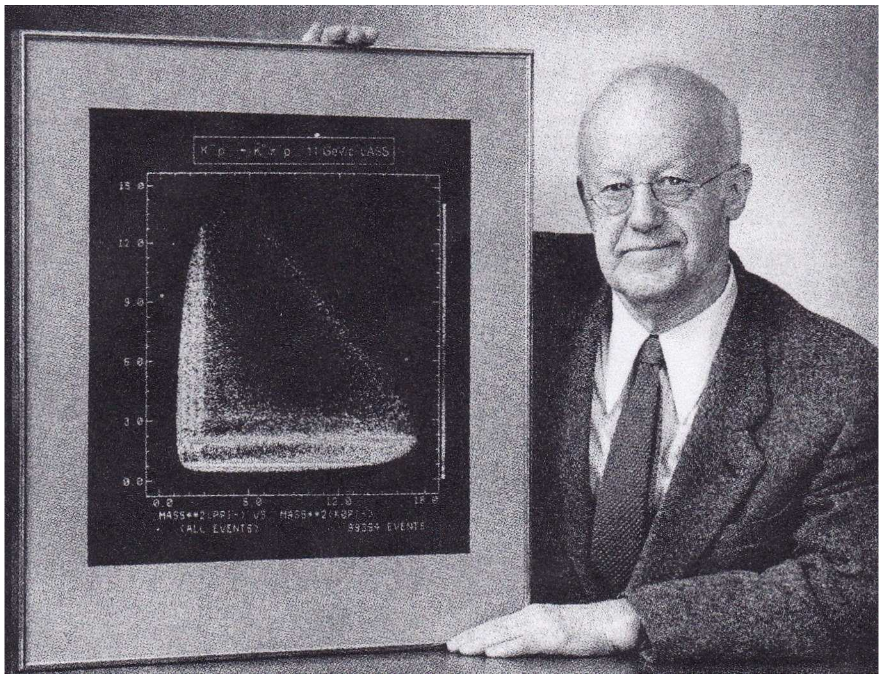


FIG. 3. The data on τ-meson decay events in which the signs of π-meson charges are established.

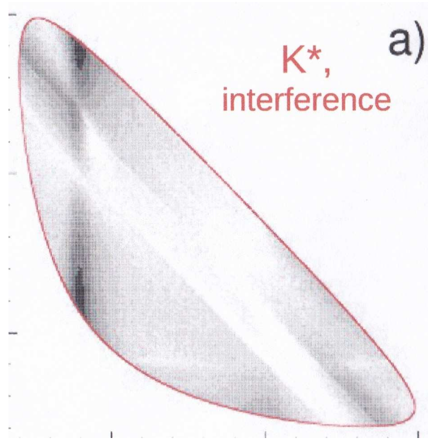
R. H. Dalitz Phys. Rev. 94, 1046 (1954)



$K^- p \rightarrow \bar{K}^0 \pi^- p$ 11 GeV LASS

$p\pi^-$ versus $\bar{K}^0 \pi^-$

Dalitz Conference, Oxford, 1990



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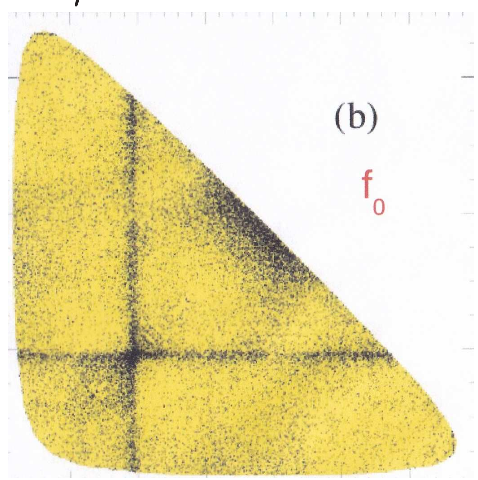
540,000 $K\pi\pi$ events



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131,719 $\pi\pi\pi$ events

Three particle phase space $\propto \frac{\mathrm{d}s\mathrm{d}t}{m^2}$

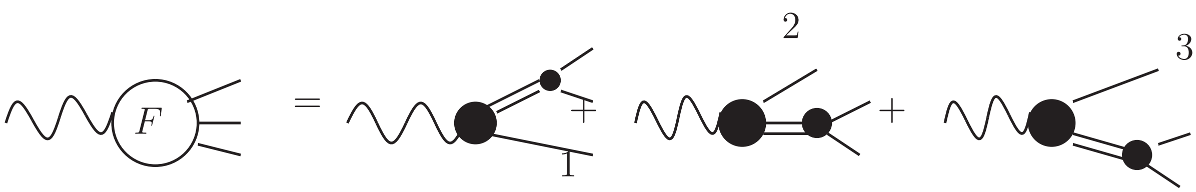
Constant matrix element $\rightarrow$ uniform population of D plot

But in fact dominated by strong two-body fsi

Simple model: THE ISOBAR MODEL

Toy version: $F(s,t,u,m^2) =$

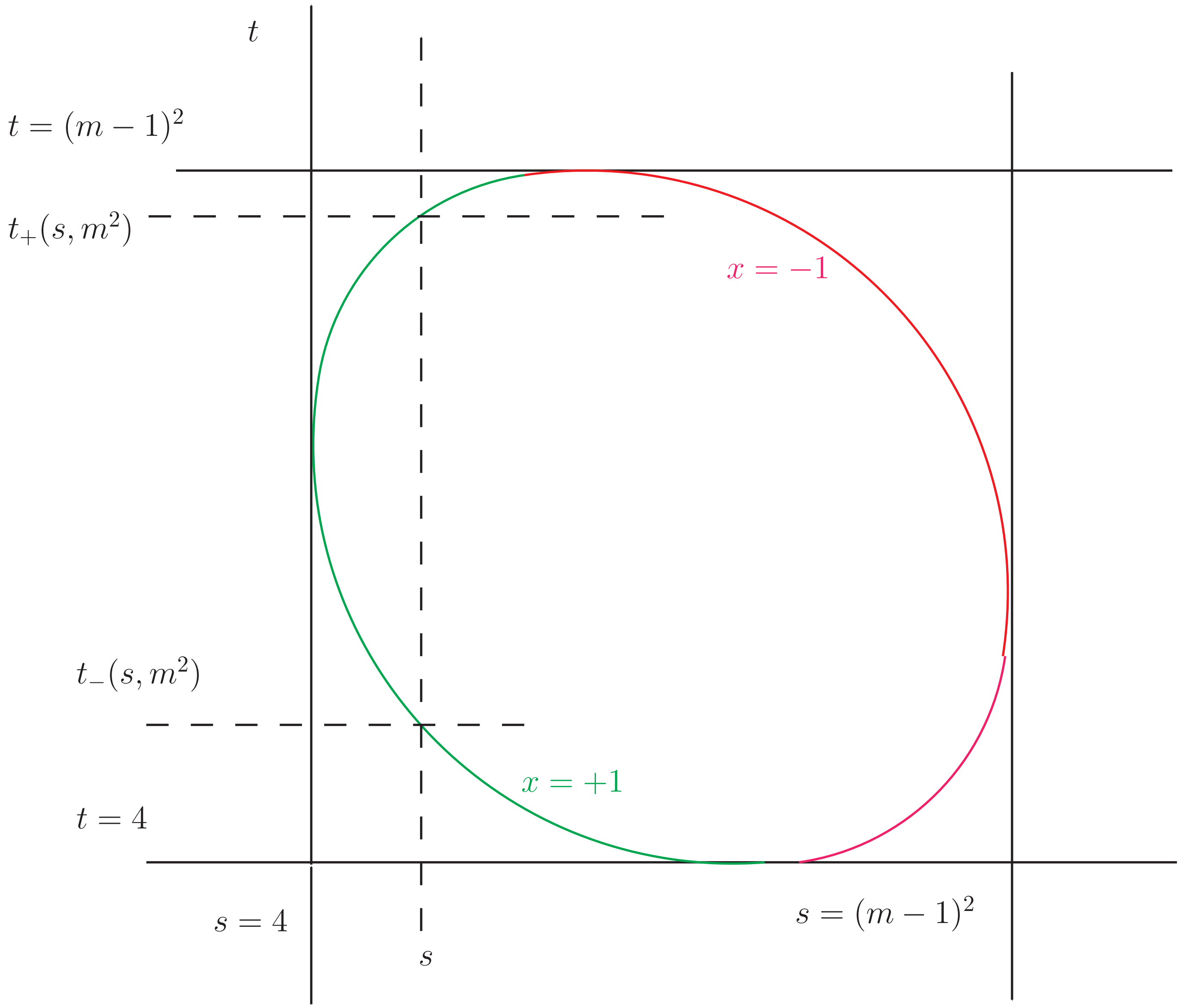
$$C(m^2)M(s) + C(m^2)M(t) + C(m^2)M(u)$$



$C(m^2)$ is "production vertex" $M(s)$ is elastic $2 \rightarrow 2$ amplitude in the 2+3 channel

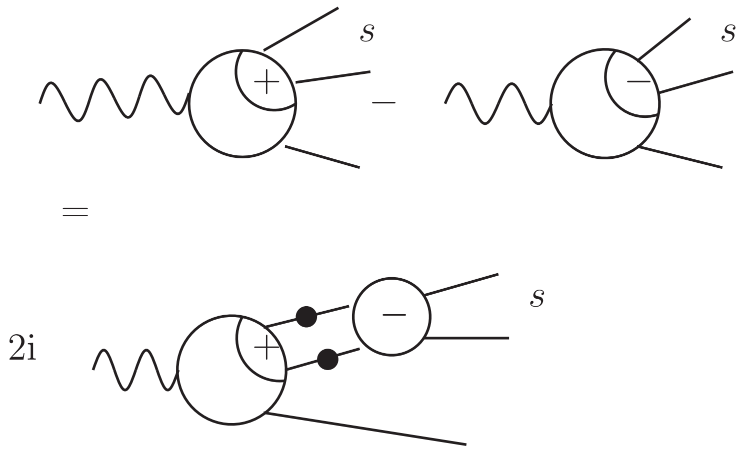
Factorization of each term into product of function of m^2 times function of only one subenergy variable is essential to isobar model

But it is inconsistent with unitarity!



t
$t = (m-1)^2$
$t_+(s, m^2)$
$x = -1$
$t_-(s, m^2)$
$x = +1$
$t = 4$
$s = 4$
s
$s = (m-1)^2$

2.2 Isobar model violates unitarity



$$s \geq 4:$$

$$F(s_+, t, u, m^2) - F(s_-, t, u, m^2) =$$

$$\mathrm{i}\rho(s) \int_{-1}^{1} \mathrm{d}x F(s_+, t(s_+, x), u(s_+, x), m^2) M(s_-).$$

LHS:

$$C(m^2)(M_+ - M_-) = C(m^2) 2\mathrm{i}\rho M_+ M_-$$

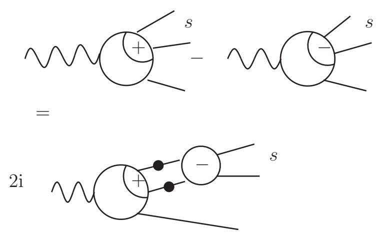
Only = RHS if $M(t)$ and $M(u)$ terms (i.e. parallel fsi channels) are absent

Therefore must modify isobar expansion in order to satisfy subenergy unitarity

2.3 Isobar correction factors

Simple fix: $M(s) \to M(s)\phi(s, m^2)$

$$F(s,t,u,m^2) = C(m^2)[M(s)\phi(s,m^2) + M(t)\phi(t,m^2) + M(u)\phi(u,m^2)]$$



subenergy unitarity

LHS: $C(M_+\phi_+ - M_-\phi_-) =$

$C[(M_+ - M_-)\phi_+ + M_-(\phi_+ - \phi_-)]$

$= C[2\mathrm{i}\rho M_+ M_- \phi_+ + M_-(\phi_+ - \phi_-)]$

RHS: $2\mathrm{i}\rho C M_+ M_- \phi_+ +$

$2\mathrm{i}\rho C M_- \int_{-1}^{1} M(t)\phi(t,m^2)\mathrm{d}x$

And so

$$\boxed{\phi_+ - \phi_- = 2\mathrm{i}\rho \int_{-1}^{1} M(t)\phi(t,m^2)\mathrm{d}x}$$

ϕ has disc for $s \geq 4 \Rightarrow$ has Im part
$\Rightarrow$ changes phases

2.4 How implement?

A. "K-matrix" way

$$\phi(s, m^2) \stackrel{?}{=} 1 + \mathrm{i}\rho \int_{-1}^{1} M(t)\phi(t, m^2)\mathrm{d}x$$

integral equation for ϕ this time

First order correction:

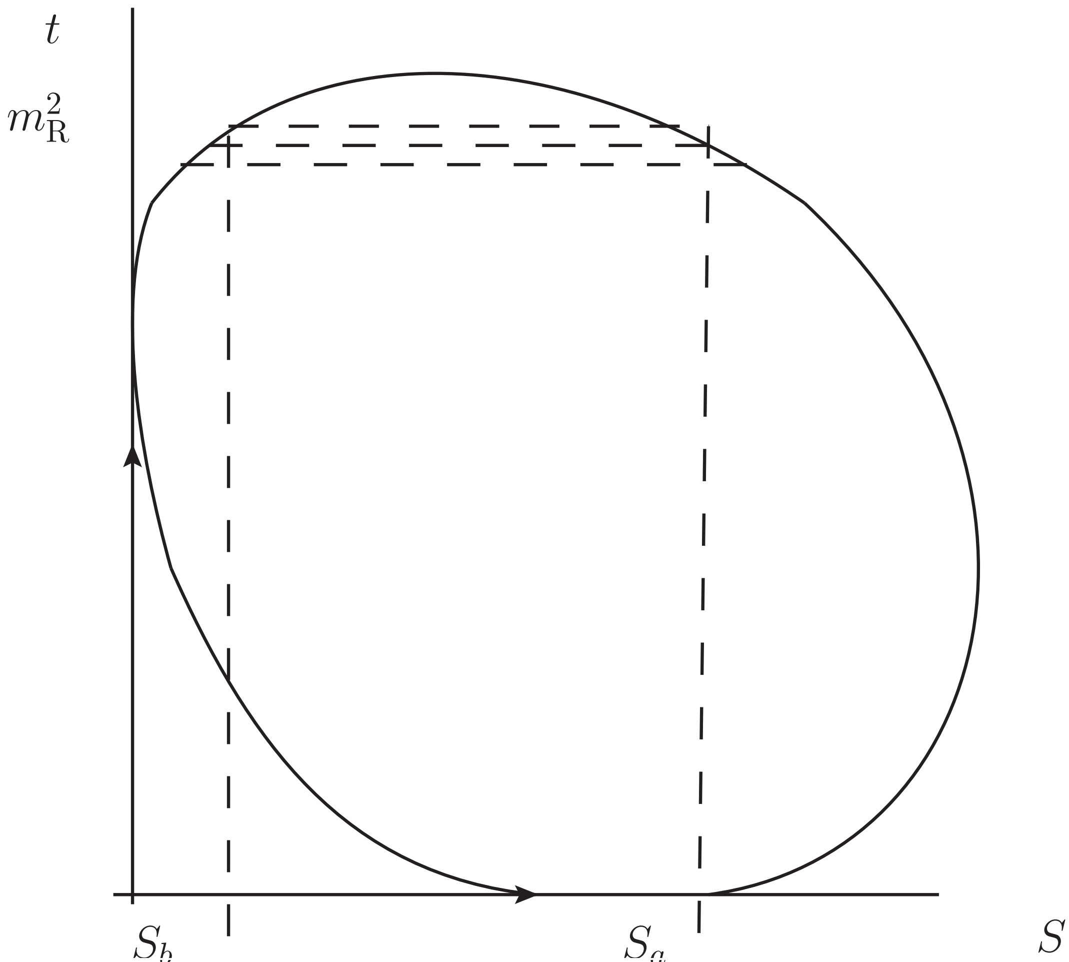
$$\mathrm{i}\rho \int_{-1}^{1} M(t(s,x))\mathrm{d}x$$

e.g. $M(t) = A/[m_{\mathrm{R}}^2 - t(s,x) - \mathrm{i}\Gamma)]$

Easily integrated $\rightarrow$ logarithmic

singularities at s_a, s_b when

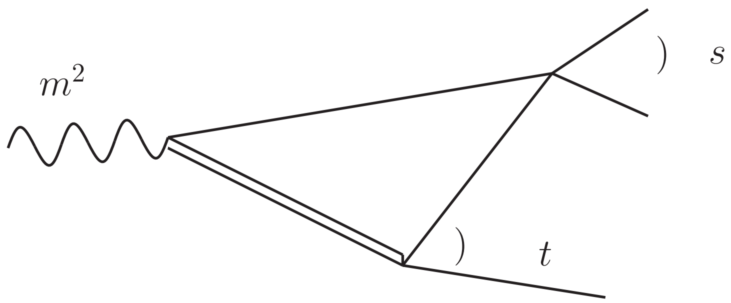
$$M_{\mathrm{R}}^2 - \mathrm{i}\Gamma = t(s, x = \pm 1)$$



spurious!

need analyticity

s_a and s_b are Landau singularities of triangle graph



well studied

which includes analyticity

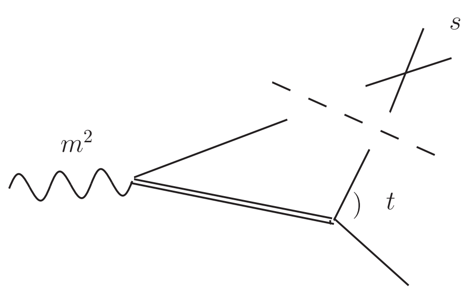
B. Dispersion relation

Put disc ϕ into dispersion relation as usual

$$\phi(s,m^2) =$$
$$1 + \frac{2}{\pi}\int_4^\infty \frac{\rho'}{s'-s}\{\frac{1}{2}\int_{-1}^{1} \mathrm{d}x M(t')\phi(t',m^2)\}\mathrm{d}s'$$
$$t' \equiv t(s',x)$$

Again, integral equation for ϕ, this time with two integrations on RHS

First order correction is just triangle diagram



Define $\Phi(s,m^2) = M(s)\phi(s,m^2)$; then

$$\Phi(s,m^2) = M(s) + 2M(s)\frac{1}{\pi}\int_4^\infty \frac{\rho'}{s'-s}\{\frac{1}{2}\int_{-1}^{1}\Phi(t(s',x))\mathrm{d}x\}$$

isobar model + corrections

Can be derived from Khuri-Treiman

Can proceed on this basis......

But we shall prefer to transform to a version where only one integration on RHS (SVR)

See three-body aspects more clearly