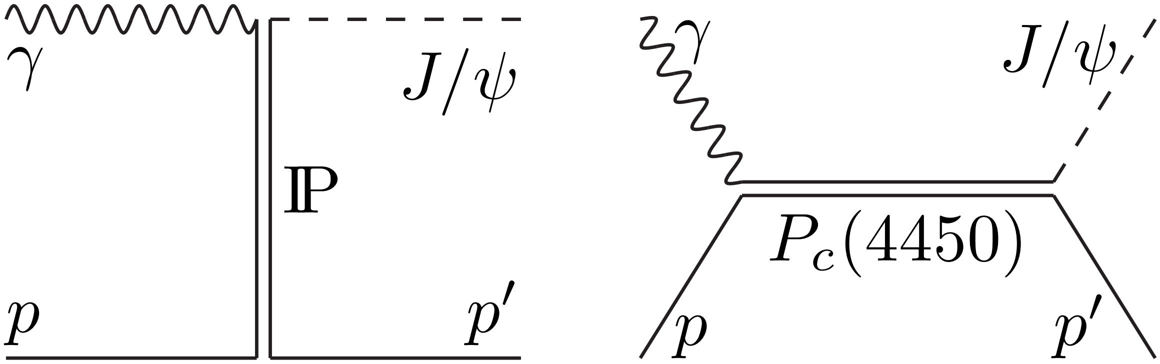

(a) Pomeron exchange

(b) Resonant contribution

FIG. 1. Dominant contributions to the J/ψ photoproduction. The nonresonant background is modeled by an effective Pomeron exchange (a) while the resonant contribution of the $P_c(4450)$ in the direct channel (b) is modeled by a Breit-Wigner amplitude.

I. FORMALISM

This section follows closely our publication in Ref. [1].

The processes contributing to $\gamma\, p \to J/\psi\, p$ are shown in Fig. 1. The nonresonant background is expected to be dominated by the t-channel Pomeron exchange, and we saturate the s-channel by the $P_c(4450)$ resonance. In the following we consider only the most favored $J_r^P = 3/2^-$ and $5/2^+$ spin-parity assignments for the resonance. We adopt the usual normalization conventions [4], and express the differential cross section in terms of the helicity amplitudes $\langle \lambda_\psi \lambda_{p'} |\, T_r \,| \lambda_\gamma \lambda_p \rangle$,

$$\frac{d\sigma}{d\cos\theta} = \frac{4\pi\alpha}{32\pi s}\frac{p_f}{p_i}\frac{1}{4}\sum_{\lambda_\gamma,\lambda_p,\lambda_\psi,\lambda_{p'}} \left| \langle \lambda_\psi \lambda_{p'} |\, T \,| \lambda_\gamma \lambda_p \rangle \right|^2 . \tag{1}$$

Here, p_i and p_f are the incoming and outgoing center-of-mass frame momenta, respectively, θ is the center-of-mass scattering angle, and $W = \sqrt{s}$ is the invariant mass.

Note that the electric charge $\sqrt{4\pi\alpha}$ is explicitly factored out from the matrix element. The contribution of the $P_c(4450)$ resonance is parametrized using the Breit-Wigner ansatz [3],

$$\langle \lambda_\psi \lambda_{p'} |\, T_r \,| \lambda_\gamma \lambda_p \rangle = \frac{\langle \lambda_\psi \lambda_{p'} |\, T_{\text{dec}} \,| \lambda_r \rangle \, \langle \lambda_r |\, T^\dagger_{\text{em}} \,| \lambda_\gamma \lambda_p \rangle}{M_r^2 - W^2 - \mathrm{i}\Gamma_r M_r}. \tag{2}$$

The numerator is given by the product of photo-excitation and hadronic decay helicity amplitudes. The measured width is narrow enough to be approximated with a constant, $\Gamma_r = (39 \pm 24)$ MeV. The angular momentum conservation restricts the sum over λ_r, the spin projection along the beam direction in the center of mass frame, to $\lambda_R = \lambda_\gamma - \lambda_p$. The hadronic helicity amplitude T_{dec}, which represents the decay of the resonance of spin J to the $J/\psi\, p$ state, is given by

$$\langle \lambda_\psi \lambda_{p'} |\, T_{\text{dec}} \,| \lambda_r \rangle = g_{\lambda_\psi \lambda_{p'}}\, d^J_{\lambda_r, \lambda_\psi - \lambda_{p'}}(\cos\theta), \tag{3}$$

where $g_{\lambda_\psi \lambda_{p'}}$ are the helicity couplings between the resonance and the final state. There are three independent couplings with $\lambda_{p'} = \frac{1}{2}$, $\lambda_\psi = \pm 1, 0$, being the other three related by parity. For simplicity, we assume all these couplings to be equal, *i.e.* $g_{\lambda_\psi \lambda'_p} \equiv g$. The helicity amplitudes and the partial decay width $\Gamma_{\psi p}$ are related by

$$\Gamma_{\psi p} = \mathcal{B}_{\psi p}\, \Gamma_r = \frac{\bar{p}_f}{32\pi^2 M_r^2}\frac{1}{2J_r+1}\sum_{\lambda_R}\int d\Omega \left| \langle \lambda_\psi \lambda_{p'} | T_{dec} | \lambda_R \rangle \right|^2 = \frac{\bar{p}_f}{8\pi M_r^2}\frac{6g^2}{2J_r+1}, \tag{4}$$

with $\mathcal{B}_{\psi p}$ being the branching ratio of $P_c \to J/\psi\, p$ and $\bar{p}_f$ the momentum p_f evaluated at the resonance peak. We assume that the $P_c(4450)$ decay is dominated by the lowest partial wave, with angular momentum $\ell = 0$ for

$J_r^P = 3/2^-$ and $\ell = 1$ for $J_r^P = 5/2^+$. We recall that the following near-threshold behavior of the helicity amplitudes holds: $g \propto p_f^\ell$.

The helicity matrix elements of $T_{\rm em}$ are usually parametrized in terms of two independent coupling constants, $A_{1/2}$ and $A_{3/2}$, which are related to the matrix elements with $\lambda_r = 1/2, 3/2$, respectively. The other two helicities $-1/2$ and $-3/2$ are constrained by parity. Using the standard normalization convention, in which the helicity couplings A_{λ_R} have units of $\mathrm{GeV}^{-1/2}$ and are proportional to the unit electromagnetic charge,

$$\langle \lambda_\gamma \lambda_p \left| T_{\rm em} \right| \lambda_R \rangle = \frac{W}{M_r} \sqrt{\frac{8 M_N M_r \bar{p}_i}{4\pi\alpha}} \sqrt{\frac{\bar{p}_i}{p_i}} A_{\lambda_R}, \tag{5}$$

with $\bar{p}_i$ the momentum p_i evaluated at the resonance peak. The electromagnetic decay width Γ_γ is given by

$$\Gamma_\gamma = \frac{\bar{p}_i^2}{\pi} \frac{2M_N}{(2J_r+1)M_r} \left[\left| A_{1/2} \right|^2 + \left| A_{3/2} \right|^2 \right]. \tag{6}$$

The photon helicity amplitudes for a pentaquark are not known. To rely on data as much as possible, we start by following Ref. [2] and assume a VMD relation for the transverse vector-meson helicity amplitudes

$$\langle \lambda_\gamma \lambda_p | \, T_{\rm em} \, | \lambda_r \rangle = \frac{\sqrt{4\pi\alpha} f_\psi}{M_\psi} \, \langle \lambda_\psi = \lambda_\gamma, \lambda_p | \, T_{\rm dec} \, | \lambda_r \rangle \,, \tag{7}$$

with f_ψ being the J/ψ decay constant which is proportional to the electromagnetic current matrix elements, $\langle 0 | \, J^\mu_{\rm em}(0) \, | J/\psi(p,\lambda) \rangle = \sqrt{4\pi\alpha} f_\psi M_\psi \epsilon^\mu(p,\lambda)$. The decay constant is related to the J/ψ wave function via the Van Royen-Weisskopf relation, and can be estimated from the leptonic decay width of the $J/\psi \to l^+ l^-$, yielding $f_\psi = 280$ MeV.

Finally, the VMD leads to

$$\Gamma_\gamma = 4\pi\alpha \, \Gamma_{\psi p} \left(\frac{f_\psi}{M_\psi} \right)^2 \left(\frac{\bar{p}_i}{\bar{p}_f} \right)^{2\ell+1} \times \frac{4}{6}, \tag{8}$$

with the factor 4/6 due to the fact that in Eq. (7) only the transverse polarizations of the J/ψ contribute. Again, we use $\ell = 0$ for $J_r^P = 3/2^-$ and $\ell = 1$ for $J_r^P = 5/2^+$.

With the help of Eqs. (6) and (8), one can constrain the size of the photocouplings.

The background in the resonance region is assumed to be dominated by diffractive production, which we parametrize by an effective, helicity-conserving, Pomeron exchange model [5],

$$\langle \lambda_\psi \lambda_{p'} | T_P | \lambda_\gamma \lambda_p \rangle = iA \, \left(\frac{s - s_t}{s_0} \right)^{\alpha(t)} \, e^{b_0 (t - t_{\rm min})} \delta_{\lambda_p \lambda_{p'}} \delta_{\lambda_\psi \lambda_\gamma}. \tag{9}$$

Here $s_0 = 1$ GeV^2 is fixed. Frequently s_0 is chosen to match the average s of an experiment and that leads to different values for the slope parameter. This is unphysical. The physical value of s_0 is determined by the range of interactions in the s-channel, which should be of the order of the hadronic scale. The Pomeron trajectory is given by $\alpha(t) = \alpha_0 + \alpha' \, t$, where α_0 and α' are parameters to be determined, as well as the normalization A, the effective threshold parameter s_t, and the t-slope parameter b_0.

There seems to be a rapid decrease of the cross section in the threshold region and the shift parameter s_t is introduced to enable a smooth connection between the high energy, $W \sim \mathcal{O}(100$ GeV), and the threshold.

II. INPUT

We invite the users of this website to reproduce our plots with their own set of chosen parameters. In order to do so, the possible choices are for:

- The fitting parameteres of the paper α_0, α', A, s_t, b_0 and $\mathcal{B}_{\psi p}$;
- The physical mass and width of the $P_c(4450)$, M_r and Γ_r, to enable varying them within their errors;
- The spin assignment 3/2 or 5/2;
- The size of the smearing due to the experimental resolution;

- The photocoupling assignment: we fix the value for $A_{1/2}^2 + A_{3/2}^2$ with vector-meson dominance, the user is therefore allowed to choose the ratio $A_{1/2}/\sqrt{A_{1/2}^2 + A_{3/2}^2}$;
- The user is to choose which observable he would like to plot: 1) the differential cross section in the forward direction as a function of near-threshold energies, 2) the total cross section as a function of near-threshold energies, 3) the angular behaviour of the differential cross section at the resonance energy.
- Finally, for observable 1), the user can choose up to which range in energies he wants the plot to be made. The default is up to 11.5 GeV, just around the $P_c(4450)$ peak. Any values above that can be chosen, as long as no smearing is applied to the function, simultaneously. The reason for this is the too high computation time needed online, were one to apply smearing for a wider range of energies on the website.

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