Total Cross Section Code for Coupled-Channel $\bar{K}N$ Scattering Release Notes

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(for the Joint Physics Analysis Center)

These notes constitute a reprint of what can be found at the JPAC Webpage and cover the usage of the Fortran code for the calculation of the $\bar{K}N$ scattering total cross section. You are free to redistribute the code but we encourage to enclose both the `README.tex` and `README.pdf` files with these notes. If you use this code, please remember to cite [1] in any associated publication. We also encourage you to contact the author with questions and comments.

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I. INTRODUCTION

Code for the calculation of the total cross section for the following reactions:

$$K^-p \to K^-p\,, K^-p \to \bar{K}^0 n\,, K^-p \to \pi^0\Lambda\,, K^-p \to \pi^-\Sigma^+\,, K^-p \to \pi^+\Sigma^-\,, K^-p \to \pi^0\Sigma^0\,.$$

II. SUMMARY OF THE FORMALISM

The full model, fitting procedure, and results are detailed in [1]. We report here only the main features of the model.

A. Observables

The differential cross section and polarization observable for the processes $\bar{K}N, \pi\Sigma, \ldots \to \bar{K}N, \pi\Sigma, \ldots$ are given by

$$\frac{d\sigma}{d\Omega}(s,\theta) = \frac{1}{q^2}\left[|f(s,\theta)|^2 + |g(s,\theta)|^2\right], \tag{1}$$

$$P(s,\theta) = \frac{2\,\mathrm{Im}\left[f(s,\theta)\,g^*(s,\theta)\right]}{|f(s,\theta)|^2 + |g(s,\theta)|^2}, \tag{2}$$

where q is the center of mass momentum of the incoming kaon, θ is the scattering angle in the center of mass frame. The amplitudes $f(s,\theta)$ and $g(s,\theta)$ give the contribution from no spin-flip and spin-flip, respectively.

Specifically, in this work we consider the following cases which have been measured (dropping the s and θ dependence)

$$f^{K^-p\to K^-p} = \frac{1}{2} f^1_{\bar{K}N\to\bar{K}N} + \frac{1}{2} f^0_{\bar{K}N\to\bar{K}N}, \tag{3}$$

$$f^{K^-p\to \bar{K}^0 n} = \frac{1}{2} f^1_{\bar{K}N\to\bar{K}N} - \frac{1}{2} f^0_{\bar{K}N\to\bar{K}N}, \tag{4}$$

$$f^{K^-p\to \pi^-\Sigma^+} = -\frac{1}{2} f^1_{\bar{K}N\to\pi\Sigma} - \frac{1}{\sqrt{6}} f^0_{\bar{K}N\to\pi\Sigma}, \tag{5}$$

$$f^{K^-p\to \pi^+\Sigma^-} = \frac{1}{2} f^1_{\bar{K}N\to\pi\Sigma} - \frac{1}{\sqrt{6}} f^0_{\bar{K}N\to\pi\Sigma}, \tag{6}$$

$$f^{K^-p\to \pi^0\Sigma^0} = \frac{1}{\sqrt{6}} f^0_{\bar{K}N\to\pi\Sigma}, \tag{7}$$

$$f^{K^-p\to \pi^0\Lambda} = \frac{1}{\sqrt{2}} f^1_{\bar{K}N\to\pi\Lambda}, \tag{8}$$

and similarly for $g(s,\theta)$.

These amplitudes are related to the s-channel isospin $I=0$ and $I=1$ amplitudes through a general relation

$$f(s,\theta) = \alpha^0\, f^0_{kj}(s,\theta) + \alpha^1\, f^1_{kj}(s,\theta), \tag{9}$$

$$g(s,\theta) = \alpha^0\, g^0_{kj}(s,\theta) + \alpha^1\, g^1_{kj}(s,\theta), \tag{10}$$

where $f^I_{kj}(s,\theta)$ and $g^I_{kj}(s,\theta)$ are the isospin amplitudes. Here α^0 and α^1 are the corresponding Clebsch-Gordan coefficients for isospin zero and one, respectively, and kj label the initial (k) and final (j) state, respectively.

Partial wave expansion of isospin amplitudes is given by

$$f^I_{kj}(s,\theta) = \sum_{\ell=0}^{\infty}\left[(\ell+1)R^{I,kj}_{\ell+}(s) + \ell R^{I,kj}_{\ell-}(s)\right] P_\ell\,(\theta)\,, \tag{11}$$

$$g^I_{kj}(s,\theta) = \sum_{\ell=1}^{\infty}\left[R^{I,kj}_{\ell+}(s) - R^{I,kj}_{\ell-}(s)\right] P^1_\ell\,(\theta)\,, \tag{12}$$

where $P_\ell\,(\theta)$ is the Legendre polynomial with $P^1_\ell\,(\theta) = \sin\theta dP_\ell\,(\theta)\,/d\cos\theta$, $R^{I,kj}_{\ell\tau}(s)$ $(\tau=\pm)$ are the partial waves which are to be considered as kj elements of the channel-space matrix $R_{\ell\tau}(s)$ as defined below, ℓ is the orbital angular momentum of the partial wave and $J = \ell + \tau/2$ is the total angular momentum for $R^{I,kj}_{\ell\tau}(s)$. The orbital angular momentum ℓ coincides with the orbital angular momentum of the initial $\bar{K}N$ state in $R^{I,kj}_{\ell\tau}(s)$ but it is not necessarily the orbital angular momentum of other possible initial states. For example, for the $I = 1, \ell = 0$ partial wave it is possible to have $\bar{K}\Delta(1232)$ in a D wave state ($L = 2$) as initial (final) state.

Finally, the total cross section can be expressed in terms of the partial waves

$$\sigma(s) = \frac{4\pi}{q^2} \sum_{\ell=0}^{\infty} \left[(\ell+1) |R_{\ell+}(s)|^2 + \ell \, |R_{\ell-}(s)|^2 \right], \tag{13}$$

where $R_{\ell\tau}(s) = \alpha^0 R^{0,kj}_{\ell\tau}(s) + \alpha^1 R^{1,kj}_{\ell\tau}(s)$.

B. Partial wave scattering matrix

For a given partial wave we write the scattering amplitude as a matrix in the channel-space

$$S_\ell = \mathbb{I} + 2i R_\ell(s) = \mathbb{I} + 2i \left[C_\ell(s) \right]^{1/2} T_\ell(s) \left[C_\ell(s) \right]^{1/2}, \tag{14}$$

where $\mathbb{I}$ is the identity matrix, $C_\ell(s)$ is a diagonal matrix which accounts for the phase space and $T_\ell(s)$ is the analytical partial wave amplitude matrix. We write $T_\ell(s)$ in terms of a K matrix to ensure unitarity

$$T_\ell(s) = \left[K(s)^{-1} - i\rho(s,\ell) \right]^{-1}. \tag{15}$$

For real s, $K(s)$ is a real symmetric matrix and $\rho(s,\ell)$ is a diagonal matrix. To ensure that $\rho(s,\ell)$ is free from kinematical cuts and has only the square-root branch point demanded by unitarity, we write it as a dispersive integral over the phase space matrix $C_\ell(s)$, ¡i¿a.k.a.¡/i¿ as the Chew-Mandelstam representation,

$$i\rho(s,\ell) = \frac{s - s_k}{\pi} \int_{s_k}^{\infty} \frac{C_\ell(s')}{s' - s} \frac{ds'}{s' - s_k}. \tag{16}$$

Here s_k is the threshold center of mass energy squared of the corresponding channel k and we define

$$C_\ell(s) = \frac{q_k(s)}{q_0} \left[\frac{r^2 q_k^2(s)}{1 + r^2 q_k^2(s)} \right]^\ell. \tag{17}$$

The first factor on the r.h.s is related to the breakup momentum near threshold. For a meson-baryon pair with masses m_1 and m_2, respectively, $s_k = (m_1 + m_2)^2$, and

$$q_k(s) = \frac{\sqrt{(s - (m_1+m_2)^2)(s - (m_1-m_2)^2)}}{2\sqrt{s}} \simeq \frac{\sqrt{m_1 m_2}}{(m_1+m_2)} \sqrt{s - s_k}. \tag{18}$$

The remaining factor ensures the threshold behavior and introduces the effective interaction radius, $r = 1$ fm. Finally, $q_0 = 2\,\text{GeV}$ is a normalization factor for the momentum in the resonance region. Evaluation of the dispersive integral can be found in [1].

C. Construction of the $K(s)$ matrix

We define the $K(s)$ matrix as the addition of $K_a(s)$ matrices

$$[K(s)]_{kj} = \sum_a x_k^a \, K_a(s) \, x_j^a, \tag{19}$$

where $K_a(s)$ can be of two kinds, pole and background:

$$[K_P(s)]_{kj} = x_k^P \, \frac{M_P}{M_P^2 - s} \, x_j^P, \tag{20}$$

$$[K_B(s)]_{kj} = x_k^B \, \frac{M_B}{M_B^2 + s} \, x_j^B, \tag{21}$$

Each partial wave employs a different amount of pole and background K matrices as well as a different amount of n_C channels. This information is summarized in Table I of Ref. [1].

The $K(s)$ and $T(s)$ matrices are connected through

$$[T(s)]_{kj} = \frac{1}{\mathcal{D}(s)} \sum_{a,b} x_k^a \, c_{ab}(s) \, x_j^b \, , \tag{22}$$

where $\mathcal{D}(s)$ and $c_{ab}(s)$ for the combination of up to six K matrices can be found in the Appendix in Ref. [1].

III. FORTRAN CODE

- Contact person: Cesar Fernández-Ramírez
- Last update: September 2015

A. Zip File Content

- README file: `README.tex` and `README.pdf`
- Fortran Source File: `xsecef.f`
- Input File: `file.inp`
- Parameter files (contain the parameters for each partial wave):
  - `parameters.s01.inp`
  - `parameters.p01.inp`
  - `parameters.p03.inp`
  - `parameters.d03.inp`
  - `parameters.d05.inp`
  - `parameters.f05.inp`
  - `parameters.f07.inp`
  - `parameters.g07.inp`
  - `parameters.s11.inp`
  - `parameters.p11.inp`
  - `parameters.p13.inp`
  - `parameters.d13.inp`
  - `parameters.d15.inp`
  - `parameters.f15.inp`
  - `parameters.f17.inp`
  - `parameters.g17.inp`

B. Input File

Example of input file (`file.inp`):

```
prk-toprok-
1
2.5
4.5
100
```

- The first line indicates the process, the options are:
  - $K^- p \to K^- p$: `prk-toprok-`
  - $K^- p \to \bar{K}^0 n$: `prk-toneuk0`
  - $K^- p \to \pi^0 \Lambda$: `prk-tolapi0`
  - $K^- p \to \pi^- \Sigma^+$: `prk-tos+pi-`
  - $K^- p \to \pi^+ \Sigma^-$: `prk-tos-pi+`
  - $K^- p \to \pi^0 \Sigma^0$: `prk-tos0pi0`
- The second line indicates the fixed kinematical variable, the options are:
  - s (GeV^2): `1`
  - p_{lab} (GeV): `2`
  - E_{lab} (GeV): `3`

  where s is energy squared in the center of mass frame, and p_{lab} and E_{lab} are, respectively, the momentum and the energy of the incoming K^- in the laboratory frame.
- The third line indicates the initial value of the kinematical variable.
- The fourth line indicates the final value of the kinematical variable.
- The fifth line indicates the the amount of points to calculate.There is a limit of 1000 points. It can be changed modifying variable `max_data_points=1000` in module `resonancesizes`.

IV. OUTPUT

The online and the downloadable versions produce an output file (`output.txt`) which contains five columns:

1. s (GeV^2),
2. E_{lab} (GeV),
3. p_{lab} (GeV),
4. the center of mass incoming momentum squared q^2 (GeV^2), and
5. total cross section in milibarn.

V. JPAC WEBPAGE

Further information and latest version of the code can be found at: JPAC Webpage. An online version of the code can also be run at the same webpage.

VI. DISCLAIMERS

- This code follows the *garbage in, garbage out* philosophy. If your parameters do not make sense, the output will not make sense either.
- You can use, share and modify this code under your own responsability.
- This code is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
- No PhD students or postdocs were severely damaged during the development of this project.

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[1] C. Fernández-Ramírez, I. V. Danilkin, D. M. Manley, V. Mathieu, M. R. Pennington, and A. P. Szczepaniak, *Coupled-Channel Model for $\bar{K}N$ Scattering in the Resonant Region*, arXiv:1510.07065 [hep-ph].